On the way of finding the non-Abelian Born-Infeld theory

W.Chemissany
R$u$$G$
Supervisor: Dr. Mees De Roo

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Abstract

Various methods have been discussed to find the terms of tree level low energy abelian open superstring effective action with derivative corrections. In this thesis, we will focus on the open superstring tree level sacttering amplitude calculation one. We would try to understand this method up to the six-point function in the hope of reaching a crucial machinery which could help to evaluate the scattering amplitude for any value of N. The main aim of this thesis is to maximize the usefulness of this method to construct generally the nonabelian generalization of this action and particularly the nonabelian Born-Infeld theory up to possible order in field strength $F_{\mu\nu}$.

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Chapter 1

Introduction

In the 30's, M.Born and L.Infeld tried to formulate a new model for nonlinear electrodynamics. By and by, this model is named by Born-Infeld theory. Born and Infeld in their famous artilce [1], discussed some arguments about the necessity for nonlinear generalization of electrodynamics and the possibilities of relating the such theory to the quantum mechanics. furthermore, the nonlinearity existing in this theory creates a very large number of interactions of various type between particles when we use the perturbation approach. It turns out that such nonlinear models gives rise to a crucial description of real interactions.
The Born-Infeld theory has some nice properties, in particular, the nonlinear model put an upper limit on the electromagnetic field strength and offers to the electron a finite energy, in contrast to the case of usual electrodynamics. In spite of the fascination of the Born-Infeld theory, this theory was put on the shelf for a long time, and the breakthrough of the renormalisation in QCD take care of some of these problems such as a regular electric field at radius zero, and the finite total energy of electron. However, recently there has been renewed interest in this theory in connection with moderns theory of strings and p-branes. It turns out that determinantal structures very much like the Born-Infeld Lagrangian frequently appear in this theories. So maybe, the last word about the Born-Infeld theroy has not yet been said. so ,we hope that this thesis will be a useful step on the way of solving this problem.

After almost falling into oblivion for a long time, the Born-Infeld action re-enters into completely different field of physics .It appears in string theory as an exact tree-level effective action of an abelian vector field in the particular case when the field strength is constant and small [1].Now the derivative-independent part of the action involves a determinant $\sqrt{\det\left(\delta_{\mu\nu}+2\pi\alpha' F_{\mu\nu}\right)}$ and is defined in 26 space-time dimensions (in the bosonic string theory). It is only recently,with the discovery of Dirichlet-branes,that the complete significance of the Born-Infeld action has been fully appreciated.It is an essential part of the low energy effective action of a non-perturbative extended object:a Dirichlet-brane,D-brane for short.In the discussion below ,we focus on the massless fields of the theory. the following $(p+1)$-dimensional world-volume integral,the *Dirac-Born-Infeld* action, summarizes the low energy dynamics of a single Dp-brane embedded in a space-time with tn metric $G_{\mu\nu}$:

$$S_p = -T_p \int d^{p+1}x \exp\left(-\phi\right) \sqrt{det\left(G_{mn} + B_{mn} + 2\pi\alpha' F_{\mu\nu}\right)} \qquad (1.1)$$

where T_p is the tension of the Dp-brane -the mass per unit volume. ϕ, G_{mn} and B_{mn}, $m = 0, \ldots, p$ are the pullbacks of the dilaton, metric and the antisymmetric tensor, respectively, to the D-brane world volume. Neglecting all derivatives acting on the field strengths, this low energy effective is a good approximation when the fields are varying slowly. when $B_{mn} = 0$, the action is of quadratic order in the field strengths equivalent to the $U(1)$ Maxwell action reduced to $(p+1)$ dimensions .Obviously, when $B_{mn} = F_{mn} = 0$, this action is nothing but a generalized version of the Nambu-Goto action for higher dimensional objects.

Eq. (1.1), describing a single Dp-brane, is an *Abelian* action, i.e. all fields commute. When instead considering multiple D-branes, matters become more complicated .For instance, the world volume fields become *non-Abelian* and take their values in the Lie algebra $U(N)$, if the open strings stretching between the branes are oriented. Several attempts to generalize the Born-Infeld action describing on D-brane to *non-Abelian* action describing a stack of them have been made. The proper (perhaps closed) form of it is however not known up to date.

In addition the massless A_m, $m = 0, \ldots, p$ living on the Dp-brane, $(25-p)$ transverse scalars X_s, $s = (p+1), \ldots, 25$, also appear. The D25-brane anywhere in space, D25-brans are equivalent to free strings moving in 26-dimensional space-time. One D25-brane with fileds described by the gauge group $U(N)$. Then there is an equivalent description in terms of open strings with Chan-Paton factors -extra non-dynamical degrees of freedom living at the ends of the strings.

The correspondence between bsonic D25-branes and open bosonic strings with "charges" attached to the endpoints suggests a perturbative course of action to obtain the *non-Abelian* effective action of the coincident D25-branes due to the implied equivalence between the S-matrices. One simply obtains the effective action describing both of these systems from calculating the open string scattering amplitudes of *non-Abelian* massless gauge fields up to give order. The effective action describes a point field theory which contains the stringy behavior up to the order under consideration. This approach will pursued in this thesis. Things become rather messy when going to high order in the field strength, so simplification of this complicated calculation is one of the our aims in this thesis. apparently not only the effective actions of the D25-branes are of interest; there also exist lower-dimensional Dp-branes in the theory with $p < 25$. The effective actions of these lower-dimensional Dp-branes are obtained by dimentional reduction from D25-brane by standard procedure.
(I WILL PUT HERE A BRIEF SUMMARY OF THE CHAPTERS OF THE THESIS)

Chapter 2

Superstring Theory

2.1 Free Bosonic String

In this chapter, we will try small description of string theory, starting with the free theory, till reaching the interacted string theory which will be represented by the sacttering amplitude. we will start this chapter making a brief classical analogy between the free string theory and the free particle one. In the course of discussion, we will try to deal with this theory at quantum level, and try to quantize the string, using different old and new approaches which each of them push the understanding of string theory upwards. To achieve our goal from this chapter which is the calculation of the scattering amplitude in the context of superstring theory, so , we should go further in our discussion and look for the supersymmetric version of this string theory.

2.1.1 The Free Praticle Action

We will get a comprehensive understanding, if we will begin with a point particles discussion. Thus, let's take a point particle with mass m moving freely in a target space described by its metric $g_{\mu\nu}$. The metric tensor in this case has to be the Minkowski metric which has one negative component and $D-1$ positive components, with D the dimension of the target space. The action which describes this massive free particle can be written under the form

$$S = -m \int ds. \tag{2.1}$$

The line element is given by ,

$$ds^2 = -g_{\mu\nu}(x)dx^\mu dx^\nu \tag{2.2}$$

with x^μ is the scalar field which describes the trajectory of the particle on the target space. Then (2.1) could be written in the form

$$S = -m\int d\tau\sqrt{-\dot{x}^2} \tag{2.3}$$

where τ is the coordinate which describes the particle on the worldline and

$$\dot{x}^2 \equiv g_{\mu\nu}(x)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \tag{2.4}$$

Unfortunately, this form of the action(2.3) has many disadvantages, one of them the presence of the square root in the action which is not helpful for any advanced discussion, and this action could be extended to describe the massless particle. So that, by using the invariant of this action under the reparametrization transformation

$$\tau \longrightarrow \tilde{\tau},$$

we can put this action under more elegant form using the backgroud field $e(\tau)$

$$S = \frac{1}{2}\int \left(e^{-1}\dot{x}^2 - em^2\right)d\tau. \tag{2.5}$$

So, the e equation of motion is

$$\dot{x}^2 + e^2m^2 = 0. \tag{2.6}$$

It is not that difficult to go from (2.5) to the form(2.3).

Let's take the infinitesimal transformation of the reparameterization symmetry mentioned above

$$\delta x = \xi\dot{x} \tag{2.7}$$

$$\delta e = \frac{d}{d\tau} (\xi e) \,, \tag{2.8}$$

where $\xi(\tau)$ is the parameter of the symmetry.

We can see the influence of the auxiliary field $e(\tau)$ on the X^μ by fixing the gauge of the reparametrization invariance. It turns out that eq.(2.6) can be considered as a constraint on the motion namely the mass shell-condition. We think that this will be suffice to prepare the ground for a study of strings.

2.1.2 String Action and Equations of Motion

Now we are ready to start discussing one of the generalization of the point particle which is the string . Dealing with the string case is not so far from the particle one. Our interest in this thesis is really in strings and especially in open strings (closed string have similar interpretation,throughout our discussion we will avoid to talk about the closed string,to reach our purpose pretty soon). In flat Minkowski space the string theory generalization of the action (2.5) is

$$S = -\frac{T}{2} \int_M d^2\sigma \sqrt{h} h^{\alpha\beta} (\sigma) \, \partial_\alpha X^\mu \partial_\beta X_\mu, \tag{2.9}$$

where $\sigma^0 = \tau$ and the special coordinates $\sigma^1 = \sigma$ that can be chosen to be $0 \leq \sigma \leq \pi$, X^μ in this action is a massless scalar field in a target space with D-dimensinal, and $h^{\alpha\beta}$ is a backgroud field which describes the worldsheet .

Let us turn now to the symmetries of (2.9). These are the world-sheet reparametrization invariances

$$\delta X^\mu = \xi^\alpha \partial_\alpha X^\mu \tag{2.10}$$

$$\delta h^{\alpha\beta} = \xi^\gamma \partial_\gamma h^{\alpha\beta} - \partial_\gamma \xi^\alpha h^{\gamma\beta} - \partial_\gamma \xi^\beta h^{\alpha\gamma} \tag{2.11}$$

$$\delta(\sqrt{h}) = \partial_\alpha(\xi^\alpha \sqrt{h}) \tag{2.12}$$

and the Weyl scaling

$$\delta h^{\alpha\beta} = \Lambda h^{\alpha\beta} \tag{2.13}$$

In addition there are space-time global symmetries. For flat Minkowski space this is just Poincaré invariance, described by

$$\delta X^{\mu} = a^{\mu}{}_{\nu} X^{\nu} + b^{\mu} \tag{2.14}$$

and

$$\delta h^{\alpha\beta} = 0, \tag{2.15}$$

where $a_{\mu\nu} = \eta_{\mu\nu} a^{\rho}{}_{\nu}$ is antisymmetric tensor. ($\eta_{\mu\nu}$ is the Minkowski metric.)

To get similar mass-shell condition to what we got in the case of particle theory, we prefer to start the discussion with defining the two-dimensional energy momentum-tensor

$$T_{\alpha\beta} = -\frac{2}{T}\frac{1}{\sqrt{h}}\frac{\delta S}{\delta h^{\alpha\beta}}. \tag{2.16}$$

it is easy to get

$$T_{\alpha\beta} = \partial_{\alpha} X^{\mu} \partial_{\beta} X_{\mu} - \frac{1}{2} h_{\alpha\beta} h^{\alpha'\beta'} \partial_{\alpha'} X^{\mu} \partial_{\beta'} X_{\mu}. \tag{2.17}$$

Using the Weyl symmetry, we conclude

$$h^{\alpha\beta} T_{\alpha\beta} = 0,$$

deriving the $h^{\alpha\beta}$ field equation gives

$$\frac{\delta S}{\delta h^{\alpha\beta}} = 0$$

as a consequences we reach the constraints which govern the motion of string

$$T_{\alpha\beta} = 0.$$

In fact, as we mention in particle theory case ,that $h_{\alpha\beta}$ is not a dynamical variable, is kind of auxiliary field, and since we have three paremeters of three local symmetries for two tranformations,two are coming from reparemetrazition invariance and one is coming from Weyl invariance, and since also the metric has three independent components, so we could gauge the $h_{\alpha\beta}$ metric away by this gauge choice

$$h_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},$$

the two-dimensional Minkowski metric.

under this choice, the action is

$$S = -\frac{T}{2}\int d^2\sigma \eta^{\alpha\beta}\partial_\alpha X.\partial_\beta X. \tag{2.18}$$

Varying with respect to X^μ ,We then get the following equations of motion

$$(\partial^2\tau - \partial^2\sigma) = 0. \tag{2.19}$$

with

$$X'(\sigma + 2\pi) = X^\mu(\sigma)$$

which verifies the boundary condition in the case of open strings. It is necessary but not sufficient to ensure t that (2.18) is invariant under general variation

$$X^\mu \rightarrow X^\mu + \delta X^\mu$$

.

The general solution to the massless wave equation can be written as a sum of two arbitrary functions

$$X^\mu(\sigma) = X^\mu_R(\sigma^-) + X^\mu_L(\sigma^+), \tag{2.20}$$

where

$$\sigma^- = \tau - \sigma \tag{2.21}$$
$$\sigma^+ = \tau + \sigma. \tag{2.22}$$

$X^{\mu}_{R,L}$ are arbitrary functions, subject only to boundary conditions. They describe the "right"-and "left"-moving modes of the string respectively.

We still have to impose on the solutions of the equations of motion the constraints resulting from the gauge fixed equations of motion for the metric, we have to require that

$$T_{10} = T_{01} = \dot{X}.X' = 0 \tag{2.23}$$

$$T_{00} = T_{11} = \frac{1}{2}(\dot{X}^2 + X'^2) = 0. \tag{2.24}$$

which can alternatively expressed as

$$\frac{1}{2}(\dot{X} \pm X')^2 = 0$$

In the light cone coordinate system, the constraints become

$$T_{++} = \frac{1}{2}(T_{00} + T_{01}) = \partial_+ X.\partial_+ X, \tag{2.25}$$

$$T_{--} = \frac{1}{2}(T_{00} - T_{01}) = \partial_- X.\partial_- X \tag{2.26}$$

$$T_{+-} = T_{-+} = 0 \tag{2.27}$$

where $T_{++} = \frac{1}{2}(T_{00} + T_{01}), T_{--} = \frac{1}{2}(T_{00} - T_{01})$. The last equations express the tracelessness of the energy-momentum tensor, it turns out that

$$\dot{X}_R^2 = \dot{X}_L^2 = 0. \tag{2.28}$$

As mentioned, the analysis of open-string is what we are interested in.

The general solution of the wave equation with these boundary conditions is given by

$$X^{\mu}(\sigma, \tau) = x^{\mu} + l^2 p^{\mu}\tau + il \sum_{n \neq 0} \frac{1}{n} \alpha^{\mu}_{n} e^{-in\tau} \cos n\sigma \tag{2.29}$$

where

$$l = \sqrt{2\alpha'} = 1/\sqrt{\pi T}$$

The open-string boundary conditions cause the left-and right-moving components to combine into standing waves. In particular,

$$2\partial_{\pm} X^{\mu} = \dot{X}^{\mu} \pm X'^{\mu} = l \sum_{-\infty}^{+\infty} \alpha_n^{\mu} e^{-in(\tau \pm \sigma)}$$

where we have set $\alpha_0{}^{\mu} = lp^{\mu}$.

Let us consider now the mode expansions of the constraints $T_{\alpha\beta}$. In the case of open string, the constraint equations amount to the vanishing of T_{++} for $-\pi \leq \sigma \leq \pi$,or equivalently to the vanishing of its Fourier components

$$\begin{aligned} L_m &= T \int_0^{\pi} (e^{im\sigma} T_{++} + e^{-im\sigma} T_{--}) d\sigma \\ &= \frac{T}{4} \int_{-\pi}^{\pi} e^{im\sigma} (\dot{X} + X')^2 d\sigma \\ &= \frac{1}{2} \sum_{-\infty}^{+\infty} \alpha_{m-n} . \alpha_n . \end{aligned} \tag{2.30}$$

A string in a given state of oscillation has a mass squared $M^2 = -p_{\mu} p^{\mu}$.The constraint equation L_0 translates into a very important equation that determines M^2 in terms of the internal modes of oscillation of the string. This is

$$M^2 = \frac{1}{\alpha} \sum_{n=1}^{\infty} (\alpha_{-n} . \alpha_n) \tag{2.31}$$

for open strings. Equation (2.31) is known as the mass-shell condition for open string.

2.1.3 String Quantization

In fact, we have many ways to quantize the bosonic string. Since quantization itself is not our target, and since all the approaches will

offer the same result, we are trying here to follow the old covariant approach on which we will be able to rely and quantize also the supersymmetric string. .
In this approach the X^μ has not been anymore classical field we will deal with it as quantum operator with momentum conjugate

$$P^\mu_\tau = T\dot{X}^\mu,$$

the canonical commutation relations at equal τ.

$$[P^\mu_\tau(\sigma,\tau), X^\nu(\sigma',\tau)] = -i\delta(\sigma-\sigma')\eta^{\mu\nu} \tag{2.32}$$

$$[X^\mu(\sigma,\tau), X^\nu(\sigma',\tau)] = [P^\mu_\tau(\sigma,\tau), P^\mu_\tau(\sigma',\tau)] = 0. \tag{2.33}$$

Besides,

$$[x^\mu, p^\nu] = i\eta^{\mu\nu} \tag{2.34}$$

and the α^μ_m have commutation relations

$$[\alpha^\mu_m, \alpha^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}. \tag{2.35}$$

The α_m are therefore naturally interpreted as harmonic oscillator raising and lowering operators for negative or positive m, respectively.

We saw that the constraints conditions correspond to the vanishing of T_{++} and T_{--}, whose fourier modes give the Virasoro generators

$$L_m = \frac{1}{2}\sum_{-\infty}^{\infty} \alpha_{m-n}.\alpha_n,$$

In quantum field theory discussion the situation is rather different and we have to take care of ordering ambiguities and α_m has to be quantum operators. In fact this problem is restricted to arise only in L_0. So the normal ordering form of L_0 is

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} \alpha_{-n}.\alpha_n. \tag{2.36}$$

To solve this problem, people postulated to introduce a constant a. Somehow, the state to be physical has to obey first, the following condition

$$(L_0 - a) \mid \phi\rangle = 0, \tag{2.37}$$

in the case of open string,

$$M^2 = -2a + 2\sum_{n=1}^{\infty} \alpha_{-n}.\alpha_n. \tag{2.38}$$

The other condition which the state has to obey to be physical is

$$L_m \mid \phi\rangle = 0 \qquad m = 1, 2, \ldots \tag{2.39}$$

and the L_m generators are related by the Virasoro algbra commutation relation

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{1}{12}D(m^3 - m). \tag{2.40}$$

Let us turn now to interpret the bosonic open string spectrum. First we denote the ground state of momentum k^μ as $\mid 0; k\rangle$. And later we impose the mass-shell condition $L_0 = a$ on the mentioned state. As an example we take the first excited state. We denote this state by

$$\mid 1; k\rangle = \zeta \cdot \alpha_{-1} \mid 0; k\rangle$$

with ζ^μ is the D-dimentional polarization vector before using any gauge constraint. Imposing the mass-shell condition we get

$$k^2 = 2(a-1)$$

.
We still have one more condition which we can impose on the first excited state which is

$$L_1 = 0,$$

it implies that

$$\zeta \cdot k = 0.$$

The D number of degrees of freedom beaks down to $D-1$ polarizations under these conditions. The norm of these states is found to be

$$\langle 1; k \mid 1; k \rangle = \zeta^2.$$

We have the choice to interpret the case in any lorentz frame. For instance , if we choose the vector k to lie in $(0,1)$,it turns out that the $D-2$ states with spacelike polarization normal to that plane has the positive norm.
At some stage , we take a such that:

• The first excited state is tachyon, $k^2 > 0$,this implies that $k_0 = 0$ then we get

$$\zeta^2 < 0.$$

• The second case is $k^2 < 0$, so, in this case it is clear that k have only time component, this implies

$$\zeta^2 > 0.$$

• Finally, if $k^2 = 0$ the polarization vector is proportional to k

$$\zeta^2 = 0.$$

Then we can conclude from this interpretation that this spectrum to be ghost-free has to set

$$a \leq 1.$$

And in the case of $a = 1$ the vector particle is massless and the ground state is a tachyon. And the L_1 condition leaves $D-2$ positive-norm states with transverse polarization, and one longitudinal state $\zeta^\mu = k^\mu$ of zero norm.

The general claim which we would like to emphasize in the end of this section, is that the spectrum is ghost-free provided that $a = 1$ and $D = 26$

2.2 Supersymmetry in String Theory

With bosonic open string, we have not yet reach the ideal picture, and that is because of many reasons, one of this reason is that we still deal with the bosonic string, and we still need to interpret the problem in the presence of fermion. besides, the the appearance of tachyon in the spectrum of bosonic string is not desirable. Solving these problems push people to think of extra symmetry, we could introduce it in the form of the bosonic string action in the hope of getting the supersymmetring string theory. This symmetry is the supersymmetry.

The particular string theory described in this section is based on the introduction of a world-sheet supersymmetry that relates the space time $X^{\mu}(\sigma, \tau)$ to the fermionic partners $\psi^{\mu}(\sigma, \tau)$. The latter are two-component world-sheet spinors.

To generalize the free bosonic string theory to supersymmetric version, it is better to start the discussion by rewrite the bosonic string action,

$$S = -\frac{1}{2\pi} \int d^2\sigma \partial_\alpha X_\mu \partial^\alpha X^\mu.$$

In fact, we have few possibility to pick up a relevant spinor which leads to interesting theory. Introducing $D-$plet of Majorana fermions $\psi^{\mu}_{A}(\sigma, \tau)$ was the most appreciated way. Let's not here that this spinor is transforming in the vector representation of the lorentz group(with A denote world-sheet spinor indices) $SO(D-1, 1)$. So, the generalized langrangian is

$$S = -\frac{1}{2\pi} \int d^2 \{\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi^\mu\}. \tag{2.41}$$

Here the symbol ρ^α represents two-dimensional Dirac matrices. A convenient basis is

$$\rho^{\mathbf{0}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \rho^{\mathbf{1}} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}. \tag{2.42}$$

These matrices satisfy

$$\{\rho^{\alpha}, \rho^{\beta}\} = -2\eta^{\alpha\beta}. \tag{2.43}$$

In this basis the spinor ψ can be written as $\psi_\pm$:

$$\psi = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}. \tag{2.44}$$

We have chosen the ρ^α to be purely imaginary, so the Dirac operator $i\varrho^\alpha\partial_\alpha$ is real. such a two-components real spinor is known as Majorana spinor. The symbol $\bar{\psi}$ indicates $\psi^\dagger\rho^0$ as usual. One of the important properties of these Majorana spinors is, if for example, χ and ψ are anticommuting variables, so in this case

$$\bar{\chi}\psi = \bar{\psi}\chi.$$

To make sense of (2.41) we have to face precisely analogous question for the fermions such as in the bosons (in D=26, we have free-ghost) for (2.41) we can deduce the equal τ commutation relations of the fermions,

$$\{\psi_A^\mu(\sigma), \psi_B^\nu(\sigma')\} = \pi\eta^{\mu\nu}\delta_{AB}\delta(\sigma - \sigma'). \tag{2.45}$$

The Virasoro conditions are enough to eliminate the wrong metric modes created by $X^0(\sigma)$ in the purely bosonic model, but to solve the analogous problem for $\psi_A^0(\sigma)$ we have to find a new symmetry and new constraints. This new symmetry is supersymmetry.

2.2.1 World-Sheet Supersymmetry

All what we need from this analysis is to reach the RNS formulation, In our opinion , studying supersymmetry on the world sheet is the short way to get it. Without going to proof, we can say that the action (2.41) is invariant under the infinitesimal transformations

$$\begin{aligned} \delta X^\mu &= \bar{\epsilon}\psi^\mu \\ \delta\psi^\mu &= -i\rho^\alpha\partial_\alpha X^\mu\epsilon, \end{aligned} \tag{2.46}$$

with ϵ is a constant anticommuting spinor.

It seems pretty clear from these formulas that this supersymmetry transforms fermion to boson and vice versa. From the supersymmetric invariance of the action , and by using the Noether theorom, we can get not only the usual energy-momentum tensor including the fermionic degrees of freedom, but in addition, we get an extra current called supercurrent.

Using (2.41) and the subsequent formula (2.46) for the supersymmetry transformation law, one can derive that, if ϵ is a constant, it leaves the action S invariant. If ϵ is not constant, the (2.46) does not leave the action invariant, but its variation is of the general form

$$\delta S = \frac{2}{\pi} \int d^2\sigma (\partial_\alpha \bar{\epsilon}) J^\alpha$$

J^α is then conserved Noether current. applying Noether theorem for supersymmety symmetry (2.46), gives the formula for the supercurrent

$$J_\alpha = \frac{1}{2} \rho^\beta \rho_\alpha \psi^\mu \partial_\beta X_\mu. \tag{2.47}$$

Applying the same theorem to the translation $\delta\sigma^\alpha =$ constant, gives the formula for the energy-momentum tensor:

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu + \frac{i}{4} \bar{\psi}^\mu \rho_\alpha \partial_\beta \psi_\mu + \frac{i}{4} \bar{\psi}^\mu \rho_\beta \partial_\alpha \psi_\mu - (trace). \tag{2.48}$$

In spite of introducing the fermionic field the energy-momentum tensor still have the same properties acquired in the bosonic case. but what it is new here is the analogous restriction imposing on the supercurrent

$$\rho^\alpha J_\alpha = 0, \tag{2.49}$$

and it is easy to obtain as a consequence $\rho^\alpha \rho^\beta \rho_\alpha = 0$.

The fermion equation of motion derived (2.41)is simply the two-dimensional Dirac equation $\rho^\alpha \partial_\alpha \psi = 0$. In the basis for ρ^α given in (2.42) this decoupled equations for the upper and lower components of ψ^μ

$$\begin{aligned} \left(\frac{\partial}{\partial\sigma} + \frac{\partial}{\partial\tau} \right) \psi^\mu_- &= 0 \\ \left(\frac{\partial}{\partial\sigma} - \frac{\partial}{\partial\tau} \right) \psi^\mu_+ &= 0. \end{aligned} \tag{2.50}$$

Thus ψ_- and ψ_+describe right-and left-moving modes, respectively. By introducing light-cone coordinates on the world sheet $\sigma^\pm = \tau \pm \sigma$ and $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$. We are able to write the fermionic part of the action (2.41)

$$S_F = \frac{1}{\pi}\int d^2\sigma(\psi_-\partial_+\psi_- + \psi_+\partial_-\psi_+) \tag{2.51}$$

we actually have a two dimensional chirality operator

$$\bar{\rho} = \rho^0\rho^1$$

actually has $\psi_\pm$ for its eigenstates (to be precise, $\bar{\rho}\psi_\pm = \mp\psi_\pm$.)

In terms of light-cone components we write

$$\begin{aligned} J_+ &= \psi_+^\mu\partial_+X_\mu \\ J_- &= \psi_-^\mu\partial_-X_\mu. \end{aligned} \tag{2.52}$$

They are obviously conserved

$$\partial_+J_+ = \partial_-J_+ = 0,$$

Using the equal τ (anti)commutators

$$\begin{aligned} \{\psi_+^\mu(\sigma), \psi_+^\nu(\sigma')\} = \{\psi_-^\mu(\sigma), \psi_-^\nu(\sigma')\} = \pi\eta^{\mu\nu}\delta(\sigma-\sigma') \\ [\partial_\pm X^\mu(\sigma), \partial_\pm X^\nu(\sigma')] = \pm i\frac{\pi}{2}\eta^{\mu\nu}\delta'(\sigma-\sigma') \\ \{\psi_+^\mu, \psi_-^\nu\} = [\partial_+X^\mu, \partial_-X^\nu] = 0, \end{aligned} \tag{2.53}$$

one can readily calculate the algebra

$$\begin{aligned} \{J_+(\sigma), J_+(\sigma')\} &= \pi\delta(\sigma-\sigma')T_{++}(\sigma) \\ \{J_-(\sigma), J_-(\sigma')\} &= \pi\delta(\sigma-\sigma')T_{--}(\sigma) \\ \{J_+(\sigma), J_(\sigma')\} &= 0. \end{aligned} \tag{2.54}$$

Here T_{++} and T_{--} are the light-cone components of the energy-momentum tensor

$$T_{++} = \partial_+X^\mu\partial_+X_\mu + \frac{i}{2}\psi_+^\mu\partial_+\psi_{+\mu}$$

$$T_{--} = \partial_- X^\mu \partial_- X_\mu + \frac{i}{2}\psi^\mu_- \partial_- \psi_{-\mu}. \tag{2.55}$$

The constraint equations which can eliminate the timelike components of ψ^+ and X^μ alike are the super-Virasoro constraints,

$$T_{--} = T_{++} = J_- = J_+ = 0. \tag{2.56}$$

The boundary conditions derived in the case of open bosonic string will not be modified here for the X^μ, but we still have to look for boundary condition for the fermionic coordinates , so we follow the same procedure of purely bosonic-string ,it comes out that the surface term which has to be vanished at each end of open string is

$$\psi_+ \delta\psi_+ - \psi_- \delta\psi_-.$$

So we set

$$\psi^\mu_+(0,\tau) = \psi^\mu_-(0,\tau). \tag{2.57}$$

In fact, the discussion here come in two cases .

• Ramond(R) boundary conditions)

$$\psi^\mu_+(\pi,\tau) = \psi^\mu_-(\pi,\tau), \tag{2.58}$$

and the mode expansion of the Dirac equation becomes

$$\psi^\mu_-(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{n\in Z} d^\mu_n e^{-in(\tau-\sigma)} \tag{2.59}$$

$$\psi^\mu_+(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{n\in Z} d^\mu_n e^{-in(\tau+\sigma)} \tag{2.60}$$

Where are run over all integers n.

• Neuveu-schwarz(NS) boundary conditions, one chooses

$$\psi^\mu_+(\pi,\tau) = -\psi^\mu_-(\pi,\tau) \tag{2.61}$$

so that the mode expansions become

$$\psi^\mu_-(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{r\in Z+1/2} b^\mu_r e^{-ir(\tau-\sigma)} \tag{2.62}$$

$$\psi_+^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}} \sum_{r\in Z+1/2} b_r^\mu e^{-ir(\tau+\sigma)} \tag{2.63}$$

where now the sums run over half-integer modes r.

The super-Virasoro operators are given by the modes of $T_{\alpha\beta}$ and J_α. For open strings, there is one independent set of L_m's defined,just as in the previous chapter, by linear combination,

$$L_m = \frac{1}{\pi}\int_0^\pi d\sigma\{e^{im\sigma}T_{++} + e^{-im\sigma}T_{--}\} = \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma}T_{++}. \tag{2.64}$$

For the fermionic generators of the algebra we define

$$F_m = \frac{\sqrt{2}}{\pi}\int_0^\pi d\sigma\{e^{im\sigma}J_+ + e^{-im\sigma}J_-\} = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma}J_+ \tag{2.65}$$

in the case of R boundary conditions or

$$G_r = \frac{\sqrt{2}}{\pi}\int_0^\pi d\sigma\{e^{ir\sigma}J_+ + e^{-ir\sigma}J_-\} = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} e^{ir\sigma}J_+ \tag{2.66}$$

in the case of NS boundary conditions.

2.2.2 Quantization and RNS Formulation

So far, our familiar interpretation has been already finished, we become so close to establish the RNS formulation, to reach this point we start our discussion with describing the quantization of the superstring using the techniques described for the bosonic string. As we have seen above we have two kinds of boundary conditions related to two sectors (fermionic and bosonic), and for simplification we prefer to study them seprately.

The dynamics of the coordinates $X^\mu(\sigma,\tau)$ and $\psi^\mu(\sigma,\tau)$ are given by two-dimensional Klein-Gordon and a free Dirac equation supplemented by certain constraints. In addition to the commutators resulted by the quantization of the X^μ coordinates which we saw in the purely bosonic case, we have the quantization of the fermionic coordinates. The canonical anticommutation relations for the coordinates

$\psi^\mu_A(\sigma,\tau)$ are

$$\{\psi^\mu_A(\sigma,\tau),\psi^\mu_B(\sigma',\tau)\} = \pi\delta(\sigma-\sigma')\delta_{AB}. \tag{2.67}$$

This implies that the modes b^μ_r or d^ν_n introduced before satisfy

$$\{b^\mu_r, b^\nu_s\} = \eta^{\mu\nu}\delta_{r+s} \tag{2.68}$$
$$\{d^\mu_m, d^\nu_n\} = \eta^{\mu\nu}\delta_{m+n}. \tag{2.69}$$

The zero-frequency part of the Virasoro constraint gives the mass-shell condition

$$\alpha' M^2 = N + a.$$

And

$$N = N^\alpha + N^d \tag{2.70}$$

or

$$N = N^\alpha + N^b \tag{2.71}$$

where

$$N^\alpha = \sum_{m=1}^{\infty} \alpha_{-m}.\alpha_m \tag{2.72}$$
$$N^d = \sum_{m=1}^{\infty} m d_{-m}.d_m \tag{2.73}$$
$$N^b = \sum_{r=1/2}^{\infty} r b_{-r}.b_r. \tag{2.74}$$

Let's now examine the states of the theory. In doing so we distinguish between two sectors, The R and the NS sectors corresponding to the boundary conditions discussed above. the oscillator groud state in both sectors is defined by

$$\alpha^\mu_m \mid 0\rangle = d^\mu_m \mid 0\rangle = 0 \qquad m > 0 \tag{2.75}$$

or

$$\alpha^\mu_m \mid 0\rangle = b^\mu_r \mid 0\rangle = 0 \qquad m, r > 0 \tag{2.76}$$

An excitation by a raising operator α^μ_{-m} or d^μ_{-m} increases the eigenvalues of $\alpha' M^2$ by m units. Similarly, b^μ_{-r} increases $\alpha' M^2$ r units.

In the R sector we will have the d_0^μ zero modes. They do not change of a given state, in particular the ground state. It is easy to check that

$$[d_0^\mu, M^2] = 0$$

which means the states $\mid 0\rangle$ and d_0^μ degenerate in mass.

It turns out here that in the NS sector there is a unique ground state which must therefore be spin zero (tachyon) . In the R sector the ground state degenerate .
Suprisingly, since the d_0^μ are generators of a clifford algebra

$$\{d_0^\mu, d_0^\nu\} = \eta^{\mu\nu}$$

we conclude that the R ground state is a spinor of $SO(D-1,1)$. This algebra is just the Dirac algebra, so up to normalization the zero modes d_0^μ are Dirac matrices.

The oscillator expressions of the super-Virasoro generators are again undefined without giving an operator ordering prescription ,as in the purely bosonic case we define them by their normal ordered expressions,

$$L_m = L_m^{(\alpha)} + L_m^{(b)} \qquad (NS) \tag{2.77}$$

$$L_m = L_m^{(\alpha)} + L_m^{(d)}, \qquad (R) \tag{2.78}$$

where

$$L_m^{(\alpha)} = \tfrac{1}{2} \sum_{n=-\infty}^{\infty} : \alpha_{-n}.\alpha_{m+n} : \tag{2.79}$$

as before, and

$$L_m^{(b)} = \tfrac{1}{2} \sum_{n=-\infty}^{\infty} (r + \tfrac{1}{2}m) : b_{-r}.b_{m+r} : \tag{2.80}$$

$$L_m^{(d)} = \tfrac{1}{2} \sum_{r=-\infty}^{\infty} (n + \tfrac{1}{2}m) : d_{-n}.d_{m+n} : . \tag{2.81}$$

in each case the normal ordering is only required for $m = 0$. For the fermionic generators one finds

$$G_r = \sum_{n=-\infty}^{\infty} \alpha_n . b_{r+n} \qquad (NS) \tag{2.82}$$

$$F_m = \sum_{n=-\infty}^{\infty} \alpha_{-n} . d_{m+n} \qquad (R) \tag{2.83}$$

The super-Virasoro algebra in the bosonic (or NS) sector is

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + A(m)\delta_{m+n} \\ [L_m, G_r] &= (\tfrac{1}{2}m - r)G_{m+r} \\ \{G_r, G_s\} &= L_{r+s} + B(r)\delta_{r+s}. \end{aligned} \tag{2.84}$$

Here $A(m)$ and $B(r)$ are c-number anomaly terms, analogous to those that arise in the bosonic theory with values

$$\begin{aligned} A(m) &= \tfrac{1}{8}D(m^3 - m) \\ B(r) &= \tfrac{1}{2}D(r^2 - \tfrac{1}{4}) \end{aligned}$$

The anomaly $A(m)$ receives two-thirds of its contribution from the α oscillators and on-third from b oscillators. The fermionic (R) sector has a very similar algebra

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + A(m)\delta_{m+n} \\ [L_m, F_n] &= (\tfrac{1}{2}m - n)F_{m+n} \\ \{F_m, F_n\} &= 2L_{m+n} + B(m)\delta_{m+n}, \end{aligned} \tag{2.85}$$

where now the anomalies are

$$\begin{aligned} A(m) &= \tfrac{1}{8}Dm^3 \\ B(m) &= \tfrac{1}{2}Dm^2 \end{aligned}$$

In the old covariant approach the constraint equations are incorporated into the quantum theory by requiring that their positive-frequency components annihilate physical states. thus, proceeding in analogy to pure bosonic case, we require that a physical bosonic state $\mid \phi\rangle$ satisfy

$$\begin{aligned} G_r \mid \phi\rangle &= 0 \qquad r > 0 && (2.86) \\ L_n \mid \phi\rangle &= 0 \qquad n > 0 && (2.87) \\ (L_0 - a) \mid \phi\rangle &= 0, && (2.88) \end{aligned}$$

where a a is a constant to be determined. The infinite number of conditions in that formulas all follow from the particular two $G_{1/2} \mid \phi\rangle = G_{3/2} \mid \phi\rangle = 0$ as a consequence of the algebra (2.84).

Now we would like to determine the critical values of a and D in the theory. Finding extra zero-norm states that occur for special values of a and D can help us sketch out the ghost-free region.

After small calculation , we find $a = 1/2$ for NS-sector and $a = 0$ for R-sector are the preferred value of a, analogous to $a = 1$ in the bosonic string.

Let us turn our attention now to the fermionic sector. Once again a physical state $\mid \psi\rangle$ is required to be annihilated by the positive-frequency components of the constraint conditions

$$F_n \mid \psi\rangle = L_n \mid \psi\rangle = 0, \qquad n > 0. \tag{2.89}$$

In addition, the zero-mode condition gives a wave equation

$$F_0 \mid \psi\rangle = 0. \tag{2.90}$$

Since $F_0^2 = L_0$, (2.90) implies that

$$L_0 \mid \psi\rangle = 0. \tag{2.91}$$

In both sectors, and after a few algebra and using the constraints conditions, we will find out that the critical D is equal 10 .

Chapter 3

Tree Amplitudes

So, we are almost so close to the Master formula which we are looking for, and on which we will rely to start discussing the possibility of constructing the low energy effective action of the open string. After discussing the RNS supersymmetric formulation, we still should extend this formulation to scattering amplitude terminology. In fact, taking tree amplitude as a leading contribution to evaluate the scattering amplitude, and basing on the perturbation structure of the field theory, we can say that the correlation function which describes the scattering of M- external open strings can be taken as a feasible start to get a simplified a computation of the scattering amplitude. Before going further, let us define what the tree amplitude is, we prefer here to talk about the tree amplitude from point of view of the purpose of using this tree approximation in this thesis

At quantum level, the tree-level M-point function derived from the quantum effective theory is equal to one particle irreducible M-point function in the original quantum field theory, which means that the classical field theory of the quantum effective theory is equivalent to the original quantum field theory, in a sense that the tree approximation of the correlation functions for the new action gives the exact quantum correlation functions of the original theory. To clarify the exact role of the scattering amplitude in describing the interacting strings, let's start our studying in this section with focusing on the scattering amplitude structure and the effective ingredients to simplify it.

3.1 Tree Amplitudes structure in Open Strings

Thinking about scattering amplitude in the context of string theory is not so far from that in the quantum field theory. We would rather to illustrate some differences between the two cases which will be useful in our effective action construction and gets the picture a tiny bit clear for the reader. There are different field theories describing the point-particle behavior, in contrast to the string which has one theory governs its structure;string theory. It comes out that we still have far fewer string diagrams than Feynman diagrams, precisely, the reason is that the string theory with few diagrams, at low energy, reproduces a field theory with many Feynman diagrams. On the other hand, the derivation itself of string scattering amplitude is more complicated and not easy reachable as in the case of field theory. Nevertheless, the complete understanding of the perturbation structure of field theory based on the point-particle correlation function and using the vertex operator method helps to deduce the general form of theoretic string amplitude of M external open strings

$$A_M = g^{M-2}\langle\phi_1 \mid V_2(k_2)\Delta V_3(k_3\cdots\Delta V_{M-1}(K_{M-1}) \mid \phi_M\rangle \qquad (3.1)$$

The vertex operator for this formula represents the vertex interaction, it is the absorption and emission of an external line by an internal line. The vertex operator for a state Λ with momentum k_μ can be given by

$$V_\Lambda(k,\tau) = e^{i\tau L_0}V_\Lambda(k,0)e^{-i\tau L_0} \qquad (3.2)$$

We know from field theory that the inverse of the operator existing in the field equation is the propagator of the field, so the propagator in (3.1) is coming from the the mass-shell condition $(L_0-1)\mid\phi\rangle = 0$ which represents a similar structure to the Klein Gordon equation $(\Box + m^2)\mid\phi\rangle = 0$, we conclude that the theoretic string propagator is

$$\Delta = (L_0-1)^{-1} = \int_0^1 z^{L_0-2}dz \qquad (3.3)$$

It is convenient to define $z = e^{i\tau}$. The vertex operator of (3.2) is required to have conformal dimension $J = 1$, in a sense that

$$[L_m, V_\Lambda(k,z)] = \left(z^{m+1}\frac{d}{dz} + mz^m\right)V_\Lambda(k,z). \qquad (3.4)$$

Vertex operator are constructed from normal-ordered expression based

$$X^{\mu} = x^{\mu} - ip^{\mu} \ln z + i \sum_{n\neq} \frac{1}{n} \alpha_n^{\mu} z^{-n} \tag{3.5}$$

and its derivatives (we prefer here to set the open string Regge slope $\alpha' = \frac{1}{2}$ in this studying and we will pop it up in the final result). For example, the vertex operator for a massless vector particle of momentum k_{μ} and polarization ζ^{μ} is given by

$$V(\zeta, k, z) = \zeta \cdot \dot{X}(z) e^{ik\cdot X(z)}. \tag{3.6}$$

In this case we must set $k^2 = k \cdot \zeta = 0$. Before moving to next subsec- tion, mention that any physical state can $\mid \Lambda; k\rangle$ be obtained by inserting in the far past a suitable operator-namely its vertex operator $V_{\Lambda}(k)$. Thus,

$$\mid \Lambda; k\rangle = \lim_{\tau\to i\infty} e^{-i\tau} V_{\Lambda}(k,\tau) \mid 0; 0\rangle.$$

Likewise, we have a formula analogous to that one for final states

$$\langle \Lambda; k \mid = \lim_{\tau\to -i\infty} e^{i\tau)} \langle 0; 0 \mid V_M(k,\tau).$$

These formulas will be useful for the upcoming calculation.

3.2 Symmetries and Tree Unitarity

All our forthcoming discussions will take the duality and the symmetries of the scattering amplitude as tools to understand the whole expression of the (3.1). Because of lack of information coming from the world-sheet open strings, people tried to make a subsequent conformal mappings for the world-sheet, to get in the end the $SL(2,c)$ symmetry helps to show up the external strings as vertex operator on a plane or on a boundary of disk, which make the expression of A_M symmetric under the cyclic permutation of external particles. Proving this cyclic symmetry is not our purpose in this thesis [3]. After getting the property of duality we can talk about the tree level unitarity requirements. The main obstacle which prevents us proceeding rapidly in our evaluation of this A_M is the presence of the poles which come from two different sources. The first source is derived explicitly from the propagator existing in (3.1). On the other hand, because of the duality property there are extra poles in channels which are easy seen from the cyclic transformation of the M-external particles. These poles arise as singularities in the infinite sum that are implicit in the operator multiplication (further discussion concerning this point will be illustrated in some examples in the end of this chapter). Unitarity property can be applied to the correspondence between string theory and particle theory at low energy and show up the poles in different crossed channels. Besides, one of the interesting properties of unitarity is that the residue of a pole in any subprocess should factorize as the product of tree amplitude of different subprocesses [3]. The last property of duality is the requirement of making the (3.1) free-ghost. It means that V_i in A_M are all required to satisfy the mass shell condition and the Virasoro conditions, furthermore, the propagators which appear as poles have to follow the same property which is imposed by unitarity.

We actually believe in symmetry as a very powerful tool to make generally physics understandable, and to put particularly a large restriction on our scattering amplitude calculation which seems to be complicated and messy with higher order of M. Going on understanding the structure of (3.1) we remark that is invariant under the following gauge tranformation A_M

$$\zeta^\mu \to \zeta^\mu + \epsilon k^\mu. \tag{3.7}$$

We can see from the the quantization of bosonic string that a large number of zero-norm state and its decoupling corresponds to a large number of gauge symmetries [3]]. So, by giving different values to M we will show in our examples for massless states, in the end of this chapter that one small part of A_M should be calculated and the remainder could be deduced by gauge invariance. Understanding the whole usefulness of these symmetries and unitarity is still not feasible, and in our thesis, we will try to shed the light on this problem and put some comments we hope that they will be useful for the forthcoming researches.

3.3 Tree Amplitude in the RNS Formulation

As we defined before, the physical vertex operators, are operators which describes physical state operators of a spectrum generating by mass- shell condition and Virasoro algebra. Nevertheless, mapping physical states to physical states in superstring RNS context requires two conditions, for example, in the case of boson emission from a bosonic state:

• Conformal dimension of vertex operator V has to be one($J = 1$).

• Vertex operator V must commute with G_r (NS Virasoro generators).

Given in (NS) sector, an operator W which gives a vertex operator $V(0)$ independent of r such that for every $r \in Z + \frac{1}{2}$

$$V(0) = [G_r, W(0)].$$

We have

$$G_r^2 = L_{2r},$$

it gives that

$$G_(r), V(0) = [L_{2r}, W(0)].$$

Back to

$$[L_m, V(\tau)] = e^{im\tau}(-i\frac{d}{d\tau} + mJ)V(\tau),$$

we can conclude that V has $J = 1$ if W has comformal dimension $J = 1/2$.

To employ this discussion in its suitable place, we prefer before giving any examples to discuss the GSO projection generally and apply it particularly on our vertex operator method to get the boson emission vertex operator which are looking for. Let us remind the reader that all what we have studied so far, is the world-sheet supersymmetry. Therefore we need the GSO projection to:

1- Have a space-time supersymmetric spectrum.

2- Get rid of the state describing tachyon.

To understand the GSO projection, we discuss it briefly, and separately in bosonic (NS) and fermionic (R) sector.

In bosonic (NS) string

Let us define a quantum number G which is the eigenvalue of the operator

$$G = (-1)^N$$

where N is the world-sheet fermion number operator. Take $\mid 0; k\rangle$ as a NS vacuum, acting G on it yields

$$(-1)^N \mid 0; k\rangle = - \mid 0; k\rangle \quad i.e. \quad G = -1$$

In NS sector, we have

$$N = \sum_{r=1/2} b_{-r} \cdot b_r - 1.$$

A general state in the NS-sector,

$$\alpha^{i_1}_{-n_1} \cdots \alpha^{i_N}_{-n_N} b^{j_1}_{-r_1} \cdots b_{-rM} \mid 0; k\rangle$$

has

$$G = (-1)^M$$

and all states with M even are projected out.

In fermionic (R) string

In the R-sector the equivalent of G is a generalized chirality operator

$$\Gamma = (-1)^N = d^1_0 \cdots d^8_0 (-1)^{\sum\limits_{n=1}^{\infty} d_{-n} \cdot d_n}$$

where $d^1_0 \cdots d^8_0$ is the chirality operator in the 8 transverse dimensions and $\sum\limits_{n=1}^{\infty} d_{-n} \cdot b_n$ the world-sheet fermion number operator.

Then a general state in the R-sector

$$\alpha^{i_1}_{-n_1}\cdots\alpha^{i_n}_{-n_N}b^{j_1}_{-m1}\cdots b^{j_M}_{-m_M}\mid\psi_0\rangle$$

has

$$\Gamma=(-1)^M(-1)^{\sum\limits_i\delta_{i,0}}$$

and

$$\alpha^{i_1}_{-n_1}\cdots\alpha^{i_n}_{-n_N}b^{j_1}_{-m1}\cdots b^{j_M}_{-m_M}\mid\bar{\psi}_0\rangle$$

has

$$\Gamma=-(-1)^M(-1)^{\sum\limits_i\delta_{i,0}}$$

GSO projection demands that all state have either $\Gamma=1$ or $\Gamma=-1$.

In fact, we can deduce two types of states and vertices (bosonic (NS) string) according whether M is odd or even,

• If W is bosonic operator (M even)$\Rightarrow$ V is a fermionic vertex operator as in the case of tachyons. The string states emitted or absorbed by V are called states of odd G-parity.

• If W is fermionic operator (M is odd)$\Rightarrow$ V is bosonic vertex operator as in the case of massless vector bosons. The string states emitted or absorbed by V are called states of even G-parity.

Using the GSO conditions in the NS-sector case, the odd G-parity states do lead to inconsistensies, and we are forced to truncate the spectrum to the even G-parity.

Example: The first excited state (massless vector of polarization ζ^μ and momentum k^μ), we have

$$W(0)=\zeta\cdot\psi(0)e^{ik\cdot X(0)}.$$

For $k^2 = 0$ and constraint condition $\zeta \cdot k = 0$, the physical boson emission vertex operator

$$V = \{G_r, W\} = (\zeta \cdot \dot{X}(0) - \zeta \cdot \psi(0) k \cdot \psi(0)) e^{ik \cdot X(0)}.$$

Discussing Bosonic emission from fermionic (R) string is not much different than what we did for (NS) string, we just have to replace the operators G_r by ones F_m for the fermionic sector. So now we are stuffed with the appropriate ingredients to analyze the tree level amplitudes in the RNS formulation.

In fact, we don't have many differences between interpreting tree scattering amplitude in the purely bosonic state and its interperetation in (NS) bosonic sector context. Therefore, it turns out that we can define superstring tree amplitude as

$$A_M = g^{M-2} \langle \phi_1 \mid V(2) \Delta V(3) \cdots \Delta V(M-1) \mid \phi_M \rangle. \tag{3.8}$$

Nevertheless, in the NS-sector, the tree unitarity and cyclic symmetry require two extra conditions:

- The physical state vertex operator V must be bosonic with $J = 1$.

- The intermediate-state poles Δ has to satisfy the super-conformal conditions G_r as well as those associated by the L_n Virasoro generators.

To reach a simplified formula for A_M, leading to the master formula describing the amplitude of M-massless external open strings, we rely on the "pictures" approach mentioned in [3] which gives rise to the conformal invariant formula

$$A_M = g^{M-2} \int d\mu_M(y) (\prod y_i)^{-1} \langle 0; 0 \mid V(k_1, y_1) \cdots V(k_M, y_M) \mid 0; 0 \rangle. \tag{3.9}$$

Let's now construct the scattering amplitude of M-massless external open superstring using all the ingredients we have dicussed up to now. we basically define the physical boson emission vertex operator as

$$V(\zeta, k) = (\zeta \cdot \dot{X} - \zeta \cdot \psi k \cdot \psi) e^{ik \cdot X} \tag{3.10}$$

where $k^2 = \zeta.k = 0$. It is convenient to write this formula in the following form

$$\frac{1}{y}V(\zeta,k,y) = \int d\phi d\theta \exp(ik\cdot X + \frac{\theta\phi\zeta\cdot\dot{X}}{y} + \frac{\theta k\cdot\psi}{\sqrt{y}} - \frac{\phi\zeta\cdot\psi}{\sqrt{y}}) \quad (3.11)$$

where θ and ϕ are two Grassmann variables. Let me enumerate now some identities which will be useful in the evaluation of the amplitude

$$\langle 0 \mid X^\mu(y_i)\dot{X}(y_i)/y_i \mid 0\rangle = \frac{i}{y_i - y_j}\eta^{\mu\nu} \quad (3.12)$$

$$\langle 0 \mid \frac{\dot{X}^\mu(y_i)}{y_i}\frac{\dot{X}^\nu(y_j)}{y_j} \mid 0\rangle = \frac{1}{(y_i - y_j)^2}\eta^{\mu\nu}. \quad (3.13)$$

Besides we have

$$\langle 0 \mid \frac{\psi^\mu(y_i)}{\sqrt{y_i}}\frac{\psi^\nu(y_j)}{\sqrt{y_j}} \mid 0\rangle = \frac{1}{y_i - y_j}\eta^{\mu\nu} \quad (3.14)$$

$$\langle 0 \mid X^\mu(y_i)X^\nu(y_j) \mid 0\rangle = -\eta^{\mu\nu}\ln(y_i - y_j)\lambda. \quad (3.15)$$

Here λ is an infrared cutoff that cancels out of all really well defined formulas. (detailed derivations of these relations are available [3]).

Evaluating of (3.9) leads to

$$\langle 0 \mid \frac{V(\zeta_1,k_1,y_1)}{y_1}\cdots\frac{V(\zeta_M,k_M,y_M)}{y_M} \mid 0\rangle = \int (\prod d\theta_i)\prod_{i<j}(y_i - y_j - \theta_i\theta_j)^{k_i\cdot k_j}F_M(\zeta,k,y,\theta), \quad (3.16)$$

where

$$F_M = \int (\prod d\theta_i)\prod_{i<j}\exp[\frac{(\theta_i-\theta_j)(\phi_i\zeta_i\cdot k_j + \phi_j\zeta_j\cdot k_i)}{y_i - y_j} - \frac{\phi_i\phi_j\zeta_i\cdot\zeta_j}{y_i - y_j} - \frac{\theta_i\theta_j\phi_i\phi_j\zeta_i\cdot\zeta_j}{(y_i-y_j)^2}].$$

So we are ready now to evaluate this formula for different values of M. In the course of our discussion, we will study the impasse that we have met with choosing higher values of M. Before leaving to give some examples concerning this formula let me denote that the non abelian symmetries can be introduced by attaching charges at the ends of open-string states.

Attaching charges to open strings is of course only relevant in string theories that do have such open strings. A technique for introducing $U(n)$ gauge symmetry Appendix B.3 in the open string sector of the bosonic string theory was proposed by Chan and Paton.

3.4 Applications

Here, in this part, we prefer to write the obtained formula in more convenient way [4] which makes it easy evaluated and the dealing with it more flexible. Basing on the dimensional analysis in Appendix A.1, the tree level scattering amplitude of M massless vector bosons calculated in Open Superstring theory takes;

- <u>In abelian open superstring theory (without Chan-Paton factors)</u>

the form

$$\begin{aligned}\mathscr{A}^{(M)}_{abl.} &= (2\pi)^{10}(\delta)^{10}(k_1+k_2+\cdots+k_M)\times \\ &\times \sum_{non-cyclic} A(1,2,\cdots,M),\end{aligned} \tag{3.17}$$

and the sum is over all the *non cyclic* permutations of the sets

$$\{\zeta_1,k_1\},\{\zeta_2,k_2\}\cdots,\{\zeta_M,k_M\}.$$

And

- <u>In nonabelian open superstring theory(with Chan-Paton factos)</u>

the following form

$$\begin{aligned}\mathscr{A}^{(M)}_{nonabel.} &= i(2\pi)^{10}(\delta)^{10}(k_1+k_2+\cdots+k_M)\times \\ &\times \sum_{non-cyclic} tr(\lambda^{a_1}\lambda^{a_2}\cdots\lambda^{a_n})A(1,2,\cdots,M),\end{aligned} \tag{3.18}$$

where λ^{a_n} are the $U(n)$ generators satisfying the relations in Appendix

B.1 , and the sum is over all the *non cyclic* permutations of the sets

$$\{\zeta_1, k_1, a_2\}, \{\zeta_2, k_2, a_2\} \cdots, \{\zeta_M, k_M, a_M\}.$$

and in both cases

$A(1, 2 \cdots, M)$ is a factor corresponds to M-particle scattering amplitude in open superstring theory which do not carry color indices,

$$\begin{aligned} A(1,2,\cdots,M) = 2\frac{g^{M-2}}{(2\alpha')^{7M/4+2}}&(x_{M-1}-x_1)(x_M-x_1)\times \\ &\times\int dx_2\cdots dx_{M-2}\int d\theta_1\cdots d\theta_{M-2}\times \\ &\times\prod_{i>j}^{M}|x_i-x_j-\theta_i\theta_j|^{2\alpha' k_i\cdot k_j}\times \\ &\times\int d\phi_1\cdots d\phi_M e^{f_M(\zeta,k,\theta,\phi)}, \end{aligned} \tag{3.19}$$

where

$$f_M(\zeta,k,\theta,\phi) = \sum_{i\neq j}^{M}\frac{(\theta_i-\theta_j)\phi_i(\zeta_i\cdot k_j)(2\alpha')^{11/4}-1/2\phi_i\phi_j(\zeta_i\cdot\zeta_j)(2\alpha')^{9/2}}{x_i-x_j-\theta_i\theta_j} \tag{3.20}$$

the powers of $(2\alpha')$ that appear in (3.19) is due to dimensional analysis Appendix A.1 and any α' expansion of (3.19) contains only integer powers of it. Furthermore, as we mentioned, the amplitude $A(1, 2, \cdots M)$ is invariant under $SL(2, C)$ local transformations of all the x_i and θ_i. By choosing a convenient gauge choice, the variables $x_1, x_{M-1}, x_M, \theta_{M-1}$ and θ_M, all can be considered as free parameters and the final answer has to be independent of them. From the Proof of cyclic symmetry of this formula the the superstrings have to obey $x_1 < x_2 < x_3 \cdots < x_M$. The complete evaluation of (3.19) is achieved up to $M = 5$.

- Three point amplitude

We start first with evaluating the three tree amplitude of open superstrings [3];

$$A(1,2,3) = 2\frac{g}{(2\alpha')^{29/4}}(x_2 - x_1)(x_3 - x_1 \times \\ \times \int d\theta_1 |x_2 - x_1 - \theta_2\theta_1|^{2\alpha' k_2\cdot k_1} |x_3 - x_1 - \theta_3\theta_1|^{2\alpha' k_3\cdot k_1} \times \\ \times |x_3 - x_2 - \theta_3\theta_2|^{2\alpha' k_3\cdot k_2} \int d\phi_1 d\phi_2 d\phi_3 e^{f_3(\zeta,k,\theta,\phi)}, \quad (3.21)$$

Remarks about $A(1,2,3)$

-There is no x integration.

-The ordering of superstrings is $x_1 < x_2 < x_3$ and the we have $\theta_2 = \theta_3 = 0$.

-The conservation of momenta

$$k_1 + k_2 + k_3 = 0 \Rightarrow k_2\cdot k_1 = k_3 \cdot k_1 = k_3 \cdot k_2 = 0$$

-The expression of $A(1,2,3)$ is

$$A(1,2,3) = g[(\zeta_3 \cdot k_1)(\zeta_1\cdot\zeta_2) - (\zeta_2\cdot k_1)(\zeta_1\cdot\zeta_3)] + \\ \text{(cyclic perm)}. \quad (3.22)$$

To evaluate the whole expression of the open superstring tree amplitude, we should also distinguish two cases;

- Abelian case

The complete amplitude can be expressed as

$$\mathscr{A}^{(3)}_{abel.} = i(2\pi)^{10}\delta^{10}(k_1 + k_2 + k_3) \sum_{non-cyclic} A(1,2,3) \quad (3.23)$$

it comes out that $\mathscr{A}^{(3)}_{abel.} = 0$. We actually have another way to check that $\mathscr{A}^{(3)}_{abel.}$ vanishes. We know that the open superstring scattering amplitude is gauge invariant, so making use the gauge freedom

$$\zeta_m^{\mu_m} \to \zeta_m^{\mu_m} + \varepsilon_m k_m^{\mu_m} \qquad (m = 1,2,3)$$

the gauge choice

$$\zeta_m^0 = 0$$

implies that

$$\vec{\zeta}_m \perp \vec{k}_m.$$

On the other hand, the physical conditions lead to

$$\vec{k}_1 \parallel \vec{k}_2 \text{ and } \vec{k}_1 \parallel \vec{k}_3$$

Thus, the three polarizations vectors of the external lines are perpendicular to $k_1, k_2,$ and k_3, and eventually, we come up with $\mathscr{A}^{(3)}_{abel.}$ equals zero. Because of the gauge invariance, it is always zero.

. Nonabelian case

The tree amplitude expression in this case becomes

$$\begin{aligned}\mathscr{A}^{(3)}_{nonabel.} = {} & 2ig(2\pi)\delta^{10}(k_1+k_2+k_3) \times \\ & A(1,2,3)\mathrm{tr}(\lambda^{a_1}[\lambda^{a_2},\lambda^{a_3}]).\end{aligned} \tag{3.24}$$

$\mathscr{A}^{3}_{nonabel.}$ is independent of α', so, it is exactly the cubic interaction of the Yang-Mills theory which in a such way doesn't have any superstring corrections.

- Four point amplitude

The four point function was already evaluated a long time ago [3], the $A(1,2,3,4)$ factor in this case is

$$\begin{aligned}A(1,2,3,4) = {} & 2\frac{g^2}{(2\alpha')^9}(x_3-x_1)(x_4-x_1)\times \\ & \times \int_{x_1}^{x_3} dx_2 \int d\theta_1 d\theta_2 |x_2-x_1-\theta_2\theta_1|^{2\alpha' k_2\cdot k_1} \times \\ & \times |x_3-x_1-\theta_3\theta_1|^{2\alpha' k_3\cdot k_1}|x_4-x_1-\theta_4\theta_1|^{2\alpha' k_4\cdot k_1}\times \\ & \times |x_3-x_2-\theta_3\theta_2|^{2\alpha' k_3\cdot k_2}|x_4-x_2-\theta_4\theta_2|^{2\alpha' k_4\cdot k_2}\times \\ & \times |x_4-x_3-\theta_4\theta_3|^{2\alpha' k_4\cdot k_3}\int d\phi_1 d\phi_2 d\phi_3 d\phi_4 e^{f_4(\zeta,k,\theta,\phi)}.\end{aligned} \tag{3.25}$$

The cyclic ordering of open superstrings , and the gauge choice of $SL(2,C)$, imply that the following variables could be chosen as

$$x_1 = 0,\ x_3 = 1,\ x_4 \to +\infty,\ \text{and } \theta_3 = \theta_4 = 0.$$

Here two cases should also be taken into consideration;

.<u>The abelian case</u>

The whole expression of the four point tree amplitude in the abelian open superstring bahaves

$$\begin{aligned}\mathscr{A}^{(4)}_{abel.} &= 8ig^2(\alpha')^2(2\pi)^{10}\delta^{10}(k_1+k_2+k_3+k_4)\times \\ &\times\left\{\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}+\frac{\Gamma(-\alpha' t)\Gamma(-\alpha' u)}{\Gamma(1-\alpha' t-\alpha' u)}+\right. \\ &+\left.\frac{\Gamma(-\alpha' u)\Gamma(-\alpha' s)}{\Gamma(1-\alpha' u-\alpha' s)}\right\}K(\zeta_1,k_1;\zeta_2,k_2;\zeta_3,k_3;\zeta_4,k_4)\end{aligned} \tag{3.26}$$

where (K) is kinematic factor given by

$$\begin{aligned}K = &\frac{-1}{4}[st(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_4)+us(\zeta_2\cdot\zeta_3)(\zeta_1\cdot\zeta_4)+ut(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)]+ \\ &+\frac{1}{2}s[(\zeta_1\cdot k_4)(\zeta_3\cdot k_2)(\zeta_2\cdot\zeta_4)+(\zeta_2\cdot k_3)(\zeta_4\cdot k_1)(\zeta_1\cdot\zeta_3)+ \\ &+(\zeta_1\cdot k_3)(\zeta_4\cdot k_2)(\zeta_2\cdot\zeta_3)+(\zeta_2\cdot k_4)(\zeta_3\cdot k_1)(\zeta_1\cdot\zeta_4)]+ \\ &+\frac{1}{2}t[(\zeta_2\cdot k_1)(\zeta_4\cdot k_3)(\zeta_3\cdot\zeta_1)+(\zeta_3\cdot k_4)(\zeta_1\cdot k_2)(\zeta_2\cdot\zeta_4)+ \\ &+(\zeta_2\cdot k_4)(\zeta_1\cdot k_3)(\zeta_3\cdot\zeta_4)+(\zeta_3\cdot k_1)(\zeta_4\cdot k_2)(\zeta_2\cdot\zeta_1)]+ \\ &+\frac{1}{2}u[(\zeta_1\cdot k_2)(\zeta_4\cdot k_3)(\zeta_3\cdot\zeta_2)+(\zeta_3\cdot k_4)(\zeta_2\cdot k_1)(\zeta_1\cdot\zeta_4)+ \\ &+(\zeta_1\cdot k_4)(\zeta_2\cdot k_3)(\zeta_3\cdot\zeta_4)+(\zeta_3\cdot k_2)(\zeta_4\cdot k_1)(\zeta_1\cdot\zeta_2)]\end{aligned} \tag{3.27}$$

K can be written in a more convenient way which will be useful for the forthcoming discussions, as

$$K(1,2,3,4) = t^{ijklmnpq}k^1_i\zeta^1_j k^2_k\zeta^2_l k^3_m\zeta^3_n k^4_p\zeta^4_q,$$

when t_8 is an eight rank tensor defined in Appendix A.2.1.

We actually have two crucial properties of (K):

i) It has itself, total symmetry in the four external particles (so K may be written as a common factor in $\mathscr{A}^{(4)}_{abel.}$ expression).

ii) It vanishes whenever any ζ_i is substituded by the corresponding

k_i, after using the physicsl conditions, so K itself, on shell gauge-invariant.

And s, t and u are the Mandelstam variables,

$$s = -2k_1 \cdot k_2, \qquad t = -2k_1 \cdot k_4, \qquad u = -2k_1 \cdot k_3,$$

satisfying the following relation

$$s + t + u = 0 \tag{3.28}$$

Let's denote in general that $\mathscr{A}^{(4)}_{abel.}$ contains an infinite number of higher order in α'. The higher order corrections of four point amplitude are due to the α' expansion of

$$\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} = \frac{1}{\alpha'^2 st} - \frac{\pi^2}{6} - \zeta(3)(s+t)\alpha' + \mathcal{O}\big(\alpha'^2\big)\,.$$

The coefficient of this α' expansion can all be determined in terms of the Riemann Zeta functions, evaluated in integer values.

Substituting all the terms involving Gamma functions and using (3.26) lead to

$$\begin{aligned}\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} + \frac{\Gamma(-\alpha' t)\Gamma(-\alpha' u)}{\Gamma(1-\alpha' t-\alpha' u)} + \frac{\Gamma(-\alpha' u)\Gamma(-\alpha' s)}{\Gamma(1-\alpha' u-\alpha' s)} = \\ \underbrace{(\frac{1}{st} + \frac{1}{tu} + \frac{1}{us})}_{0}\frac{1}{\alpha'^2} - \frac{\pi^2}{2} - 2\zeta(3)\underbrace{(s+t+u)}_{0}\alpha' \\ + \mathcal{O}\big(\alpha'^2\big)\,. \end{aligned} \tag{3.29}$$

It turns out that the leading term of $\mathscr{A}^{(4)}$ in abelian open superstrings theory is

$$\begin{aligned}\mathscr{A}^{(4)}_{abel.\alpha'^2} = 8ig^2(\alpha')^2(2\pi)^{10}\delta^{10}(k_1+k_2+k_3+k_4)(-\frac{\pi^2}{2}) \times \\ \times K(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4)\end{aligned} \tag{3.30}$$

- <u>Nonabelian case</u>

In this case, the color indices must be involved in the expression of the tree amplitude

$$\begin{aligned}\mathscr{A}^{(4)}_{nonabel.} &= 8ig^2(\alpha')^2(2\pi)^{10}\delta^{10}(k_1+k_2+k_3+k_4)\\ &\times\Big\{\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}[\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_3}\lambda^{a_2})]\\ &+\frac{\Gamma(-\alpha' t)\Gamma(-\alpha' u)}{\Gamma(1-\alpha' t-\alpha' u)}[\mathrm{tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_2}\lambda^{a_3})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_2}\lambda^{a_4})]\\ &+\frac{\Gamma(-\alpha' u)\Gamma(-\alpha' s)}{\Gamma(1-\alpha' u-\alpha' s)}[\mathrm{tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_4}\lambda^{a_2})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_4}\lambda^{k)a_3})]\Big\}\\ &\times K(\zeta_1,k_1;\zeta_2,k_2;\zeta_3,k_3;\zeta_4,k_4) \qquad (3.31)\end{aligned}$$

where (K) is the same kinematic factor mentioned in the abelian case. After performing the α' expansion of all the terms involving Gamma functions, we conclude that

i) The leading term of $\mathscr{A}^{(4)}_{nonabel.}$ is of order zero in α', and is nothing else than the Yang-mills four-gluon tree amplitude.

ii) The first superstring corrections to the Yang-Mills 4-gluon tree amplitude, is of order two in α'.

- Five point amplitude

In fact, the evaluation of five point function is rather messy and lengthy but it is doable [4].

Remarks about the five point amplitude

- The free parameters has to be fixed

$$x_1=0,\ x_4=1,\ x_5\to\infty \text{ and } \theta_4=\theta_5=0.$$

- The final result of the Lorentz factor $A(1,2,3,4,5)$ up to order three in α' terms has the following form

$$\begin{aligned}A(1,2,3,4,5) &= A^{(0)}(1,2,3,4,5)+A^{(2)}(1,2,3,4,5)\cdot\alpha'^2+\\ &+A^{(3)}(1,2,3,4,5)\cdot\alpha'^3+\mathcal{O}\big(\alpha'^4\big) \qquad (3.32)\end{aligned}$$

- After ϕ-integration and θ-integration, we conclude that $A(1,2,3,4,5)$ consists of *two* types of terms

- Terms of the type

$$(\zeta_a \cdot \zeta_b)(\zeta_c \cdot \zeta_d)(\zeta_e \cdot k_f) \times \{\text{kinematic factor}\}.$$

- Terms of the type

$$(\zeta_a \cdot \zeta_b)(\zeta_c \cdot k_d)(\zeta_e \cdot k_f)(\zeta_g \cdot k_h) \times \{\text{kinematic factor}\}.$$

These kinematic factors are double integrals depending on α' and the momenta k_i. And as we mentioned in §3.2, the final answer of these double integrals have poles which can be hidden in a multiplication of two Beta functions Appendix B.1. Moreover, there are relations among these kinematic factors, which could make their evaluation easy to achieve.[1].

• Toward six point amplitude

Going up with the values of M makes the computation of the superstring amplitude more complicated and lengthy. The evaluation of the six point superstring scattering amplitude is still under investigation, and in our opinion, to generally simplify the (3.18) , we still have at leat to understand how this six point amplitude works out.

Remarks on the six point amplitude We have not had the full answer yet of the six point function, nevertheless, in the course of our interpretation of the six point amplitude we could say

- The fixed free parameters are

$$x_1 = 0,\ x_5 = 1,\ x_6 \to \infty \text{ and } \theta_5 = \theta_6 = 0.$$

- We have mentioned at the very beginning of this chapter that, at low energy, the superstring diagram can be equivalent to many field theory diagrams corresponding to poles in different subchannels. Hence, constructing the possible six point Feynman diagrams Appendix A.3 leads to

i) The final answer of $A(1, 2, 3, 4, 5, 6)$ of open superstrings with Chan

[1]The detailed evaluation of five point amplitude is in [4]

Paton factors, (color indices), has to be written as

$$\begin{aligned} A(1,2,3,4,5,6) = {} & A^{(0)}(1,2,3,4,5,6) + A^{(2)}(1,2,3,4,5,6)\cdot\alpha'^2 + \\ & + A^{(3)}(1,2,3,4,5,6)\cdot\alpha'^3 + A^{(4)}(1,2,3,4,5,6)\cdot\alpha'^4 + \\ & + \mathcal{O}\big(\alpha'^5\big)\,. \end{aligned} \tag{3.33}$$

It comes out that, in this case, the leading term is of order zero in α'.

ii) The final answer of $A(1,2,3,4,5,6)$ of open superstrings (without non-abelian symmetry) takes the following form

$$A(1,2,3,4,5,6) = +A^{(4)}(1,2,3,4,5,6)\cdot\alpha'^4 + \mathcal{O}\big(\alpha'^5\big)\,.$$

And this is due to the vanishing of cubic vertex and the five point vertex in the abelian case (no color indices). As a consequence, the leading term in the abelian case is of order fourth in α'.

- After ϕ-integration and θ-integration, we conclude that $A(1,2,3,4,5)$ consists of *four* types of terms

. terms of the type

$$(\zeta_a\cdot\zeta_b)(\zeta_c\cdot\zeta_d)(\zeta_e\cdot\zeta_f)\times\{\text{kinematic factor}\}.$$

. And we have Terms of the type

$$(\zeta_a\cdot\zeta_b)(\zeta_c\cdot\zeta_d)(\zeta_e\cdot k_f)(\zeta_g\cdot k_h)\times\{\text{kinematic factor}\}.$$

. The third type is

$$(\zeta_a\cdot\zeta_b)(\zeta_c\cdot k_d)(\zeta_e\cdot k_f)(\zeta_g\cdot k_h)(\zeta_k\cdot k_l)\times\{\text{kinematic factor}\}.$$

. The last one

$$(\zeta_a\cdot k_b)(\zeta_c\cdot k_d)(\zeta_e\cdot k_f)(\zeta_g\cdot k_h)(\zeta_k\cdot k_l)(\zeta_m\cdot k_n)\times\{\text{kinematic factor}\}.$$

and this term, due the interchange under the external lines gives zero.

Remarks on these terms

i) In the six point amplitude, The kinematic factors are triple x-integration which depend on α' and the momenta k_i, and contain all the six point amplitude pole[2].

[2]These integrals are still under evaluation.

ii) Since the superstring tree amplitudes are on-shell gauge invariants, there is possibility to relate the four types of terms to each other, thereafter, we still need to evaluate one of them and the remainder will be calculated by gauge invariance[3].

- In the abelian case of the six point amplitude, in contrast to the four point one, we still have poles in the amplitude Appendix A.3.

So, we can now move to the next step, which is how to employ these superstring scattering amplitude to construct generally the low energy tree level open superstring effective action and particularly the ten-dimensional Born Infeld action (abelian and non abelian) which is the leading action of the former.

[3]Generalization of this approach has been part of our research . In this thesis we have some steps toward this aim A.3.

Chapter 4

The Effective Action in Open Superstring Theory

In this chapter, we will try to employ the open superstring scattering tree amplitude formalism in the constructing of tree level open superstring effective action. In fact, there are plenty of methods which discuss this problem for a long time ago. But, in our opinion, the scattering tree amplitude calculation one, regardless of the hardship and the toil that we have met in going to the higher values of M, we still have the belief in getting something from it. We actually have two versions of the open superstring effective action which is due to the presence and the absence of the Chan-Paton factors at the free ends of open superstring. And we would prefer to interpret separately the two cases corresponding to the two versions of open supersting tree amplitude.

4.1 Abelian tree level open superstring effective action

Unfortunately, few sectors of this action are known so far, and these sectors has been reached from several sources following different methods. To make clear difference between what is known and what is unknown, first, it is desirable for the higher order sectors to follow the notation already existing in [6]. Such Terms can be written as

$$\mathcal{L}_{(m,n)} = \alpha'^m \left(\partial^n F^p + \partial^{n+1} F^{p-2} \bar{\chi}\gamma\chi \right) . \tag{4.1}$$

the dimensional analysis of this relation leads to

$$2p - 2m + n - 4 = 0.$$

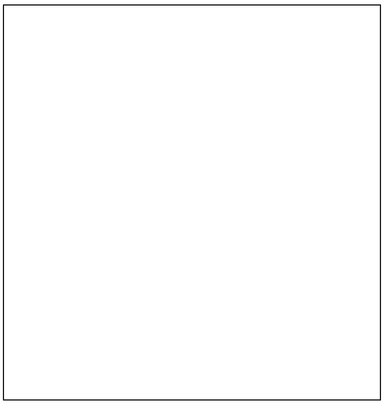

Figure 4.1: Here we represent the sectors on this figure as follow. Black dots indicate the occupied sectors which are explicitly known bosonic and fermionic partner . Empty white dots represent the sectors which are empty up to field redefinitions. The red dots correspond to the sectors which has the bosonic terms known and the the fermionic partner under construction. Finally the yellow dots stand for the sectors which must be empty but they have not been constructed yet.

Second, it is plausible to represent these terms using fig.(4.1)

- More informations about fig.(4.1)

- In [7] it was shown that there are no correction quadratic in derivatives to all orders in α'.
- The four derivative bosonic terms were derived in [5].
- Due to the vanishing of the abelian open superstring theory scattering tree amplitude, there are no corrections with an odd number of fields strength.
- All bosonic terms of the form $\partial^4 F^{2r}$(r is an integer) were evaluated in [8].

In this thesis, we are interested in the construction of the 4-point open superstring effective action from the 4-point photons[1] scattering amplitude. And any advance in the evaluation of superstring scattering tree amplitude for any value of M , we would be able similarly to the 4-point effective action, construct the corresponding effective

[1]throughout this chapter, the photon stands for the abelian massless bosonic field and gluon for non abelian one.

action.

4.1.1 Abelian open string 4-point effective action

Let's start first with the 4-point tree level open string effective action, this sector of the whole action takes the following form

$$S^{abel}_{(2,0)} = \frac{1}{8}(2\pi g\alpha')^2 \int d^{10}x \left(\mathrm{tr}F^4 - \frac{1}{4}(\mathrm{tr}F^2)^2 \right) \tag{4.2}$$

where the strength in the abelian string is $F_{ij} = \partial_i A_j - \partial_j A_i$.

The open string 4-photon tree amplitude §3.4 factorizes in product of two terms

$$\begin{aligned} \mathscr{A}^{(4)} &= -16ig^2\alpha'^2\delta^{10}(k_1+k_2+k_3+k_4) \times \\ &\quad \times \mathfrak{g}(k_1,k_2,k_3,k_4)K(1,2,3,4) \end{aligned} \tag{4.3}$$

. The first term (K), which depends on the polarization vector of the external 4-bosons and their wave functions, predicts the form which the fields should take in the effective action, and has the following significant form

$$K(1,2,3,4) = t^{ijklmnpq} k^1_i \zeta^1_j k^2_k \zeta^2_l k^3_m \zeta^3_n k^4_p \zeta^4_q.$$

. The second term ($\mathfrak{g}$), which is proportional to Veneziano amplitude. This term has some interesting properties:

- ($\mathfrak{g}$) depends on the monenta and contains the α' dependence.

- It replaces in $\mathscr{A}^{(4)}$ the following expression

$$\begin{aligned} \mathfrak{g}(k_1,k_2,k_3,k_4) &= h(s,t)+h(t,u)+h(u,s) \\ &= \frac{\Gamma(-\alpha's)\Gamma(-\alpha't)}{\Gamma(1-\alpha's-\alpha't)} + \frac{\Gamma(-\alpha't)\Gamma(-\alpha'u)}{\Gamma(1-\alpha't-\alpha'u)} + \\ &\quad + \frac{\Gamma(-\alpha'u)\Gamma(-\alpha's)}{\Gamma(1-\alpha'u-\alpha's)}. \end{aligned} \tag{4.4}$$

- Up to momentum conservation and mass shell condition discussed previously, we prefer to write the mandelstam variables in such a way

$$\begin{aligned} s &= -k_1 \cdot k_2 - k_3 \cdot k_4 \\ t &= -k_1 \cdot k_3 - k_2 \cdot k_4 \\ s &= -k_1 \cdot k_4 - k_2 \cdot k_3 \end{aligned} \tag{4.5}$$

that $\mathfrak{g}$ obviously *symmetric* in k_a.

- In the precedent chapter, we mentioned that in the abelian case we don't have any pole in the α' expanded expression of $\mathscr{A}^{(4)}$, and, in the context of $\mathfrak{g}$, this is can be shown by the expanding in α'. It turns out that $\mathfrak{g}$ is *regular* when k_a goes to zero. And the leading order contribution for $\mathscr{A}^{(4)}$ is due to

$$\mathfrak{g}(k_1, k_2, k_3, k_4) = -\frac{\pi^2}{2} + \mathcal{O}\big(\alpha'^2\big) \tag{4.6}$$

We conclude that the leading term of $\mathscr{A}^{(4)}$ is

$$\mathscr{A}^{(4)}_{\alpha'^2} = 8i\pi^2 g^2 \alpha'^2 K(1,2,3,4) \tag{4.7}$$

which can be easily reproduced by $S_{(2,0)}$.

Let's now revive the t_8 tensor in the expression of (4.2), which can be rewritten as

$$S^{abel}_{(0,2)} = \frac{1}{8}(2\pi g\alpha')^2 \int d^{10}x \frac{1}{24} t_{ijklmnpq} F^{ij} F^{kl} F^{mn} F^{pq}. \tag{4.8}$$

We can observe so far the following:

a- The only difference between $\mathscr{A}^{(4)}$ and $\mathscr{A}^{(4)}_{\alpha^2}$ is the extra momentum factors which are due to the existence of $(\mathfrak{g})$ in the former.

b- Every momentum factor k_a in (K) is reproduced by acting of derivative operator on the appropriate field in (4.8). We come up with reproducing of $(\mathfrak{g})$.

c- $(\mathfrak{g})$ expands into an infinite series in α', and determines how the derivatives should act on the fields.

- <u>How can we construct the complete 4-point effective action?</u>

To implement this purpose we will should follow some steps:

1- Define the four fields at different spacetime positions,

$$A_i(x_a) \text{ where } a = 1, \cdots, 4$$

in such a way we come up with non-local action.

2- Replace the momenta k_a in the amplitude by differentiation with respect to the appropriate coordinate in the effective action

$$k_a^j \longrightarrow -i\frac{\partial}{\partial x_j^a} \qquad (a = 1, 2, 3, 4).$$

3- Later on, multiply the resulting expression by delta functions and integrate over the x_a to come up with local action.4- Take the symmetric derivative operator

$$\mathfrak{g}(\partial_1 \cdot \partial_2 + \partial_3 \cdot \partial_4 \, , \, \partial_1 \cdot \partial_4 + \partial_2 \cdot \partial_3 \, , \, \partial_1 \cdot \partial_3 + \partial_2 \cdot \partial_4)$$

and introduce it into the whole expression of the complete action

$$\begin{aligned} S_{\text{eff}}^{abel}[A_j] = -\frac{1}{24}\int d^{10}x \Big\{ \prod_{a=1}^{4} d^{10}x_a \delta^{10}(x - x_a) \Big\} \times \\ \times \mathfrak{g}(\partial_1 \cdot \partial_2 + \partial_3 \cdot \partial_4 \, , \, \partial_1 \cdot \partial_4 + \partial_2 \cdot \partial_3 \, , \, \partial_1 \cdot \partial_3 + \partial_2 \cdot \partial_4) \times \\ \times t_{ijklmnpq} F^{ij}(x_1) F^{kl}(x_2) F^{mn}(x_3) F^{pq}(x_4). \end{aligned} \tag{4.9}$$

where the integrals over x_i are independent of each other to make the action well defined.
This is the action which reproduces the complete four massless string modes amplitude. And due to the absence of the cubic vertex interaction, this action only gives the one particle irreducible diagram Appendix A.2.2.

- How can one reproduce $\mathscr{A}^{(4)}$ from $S_{\text{eff}}^{abel}[A_j]$

Basing on the definition of Appendix A.2.2, we can calculate firstly the 4-point vertex in the spacetime position space

$$V^{(4)}_{ijkl}(x_1,x_2,x_3,x_4) = \frac{\delta^4 S^{abel}_{\text{eff}}[A_j]}{\delta A^i(x_1)A^j(x_2)A^k(x_3)A^l(x_4)}\Big|_{A_j=0}$$
$$= -4!2^4\frac{1}{24}(g\alpha')^2\int d^{10}y\Big\{\prod_{a=1}^{4} d^{10}y_a\delta(y-y_a)\Big\}\times$$
$$\times\,\mathfrak{g}(\partial_1,\partial_2,\partial_3,\partial_4)\,t_{minjkplq}\,\partial_1^m\,\delta(y_1-x_1)\times$$
$$\times\,\partial_2^n\,\delta(y_2-x_2)\partial_3^p\,\delta(y_3-x_3)\partial_4^q\,\delta(y_4-x_4).$$

To get this result, one should rename the dummy indices, and use in parallel the symmetry property of $\mathfrak{g}$. The factor of 2^4 is due to the substitution of F_{ij}, and the factor 4! arises from the distributive property of the functional derivative.

In going to momentum space we get

$$-\frac{1}{16(g\alpha')^2}(2\pi)^{10}\delta^{10}(k_1+k_2+k_3+k_4)V^{(4)}_{ijkl}(k_1,k_2,k_3,k_4) =$$

$$= t_{minjkplq}\int d^{10}y\Big\{\prod_a d^{10}y_a d^{10}x_a\delta(y_a-y)e^{ik_a}\cdot y_a\Big\}\times$$
$$\times\,\mathfrak{g}(\partial_1,\partial_2,\partial_3,\partial_4)\partial_1^m\,\delta(y_2-x_1)\partial_2^n\,\delta(y_2-x_2)\times$$
$$\times\,\partial_3^p\,\delta(y_3-x_3)\partial_4^q\,\delta(y_4-x_4)$$

$$= t_{minjkplq}\int d^{10}y\Big\{\prod_a d^{10}y_a\delta(y_a-y)\Big\}\times$$
$$\times\,\mathfrak{g}(\partial_1,\partial_2,\partial_3,\partial_4)\partial_1^m\partial_2^n\partial_3^p\partial_4^q\Big\{\prod_b\int d^{10}x_b e^{ik_b\cdot x_b}\delta(y_b-x_b)\Big\}$$

$$= t_{minjkplq}\int d^{10}y\Big\{\prod_a d^{10}y_a\delta(y_a-y)\Big\}\times$$
$$\times\,\mathfrak{g}(-i\partial_1,-i\partial_2,-i\partial_3,-i\partial_4)\partial_1^m\partial_2^n\partial_3^p\partial_4^q\Big\{\prod_a e^{ik_a\cdot y_a}\Big\}$$

$$= t_{minjkplq}\int d^{10}y\Big\{\prod_a d^{10}y_a\delta(y_a-y)e^{ik_a}\cdot y_a\Big\}\mathfrak{g}(k_1,k_2,k_3,k_4)k_1^m k_2^n k_3^p k_4^q$$

$$= t_{minjkplq}\mathfrak{g}(k_1, k_2, k_3, k_4)k_1^m k_2^n k_3^p k_4^q (2\pi)^{10}\delta^{10}(k_1 + k_2 + k_3 + k_4) \tag{4.10}$$

That is it.

4.1.2 Abelian open superstring 4-point effective action

In a similar way, we can construct the complete tree level 4-point open superstring effective action. Basing on [9, 10], we can write the leading sector of this action as

$$S^{abel}_{(2,0)} = \frac{1}{8}(2\pi\alpha')^2 \int d^{10}x \Big(\mathrm{tr}F^4 - \frac{1}{4}(\mathrm{tr}F^2)^2 + 2iF_{ij}F_{ik}\bar{\chi}\gamma_j\partial_k\chi - \\ - iF_{ij}F_{kl}\bar{\chi}\gamma_{ijk}\partial_k\chi - \frac{1}{3}\bar{\chi}\gamma_i\partial_j\chi\bar{\chi}\gamma_i\partial_j\chi\Big). \tag{4.11}$$

This sector of the 4-point effective action is responsible for reproducing the leading order contribution $\mathscr{A}^{(4)}_{\alpha'^2}$ to $\mathscr{A}^{(4)}$ with different forms of (K), corresponding respectively to 4 boson, (2 fermions and 2 bosons) and 4 fermions see Appendix A.2.3. Due to the factorization of the string theory tree amplitude, supersymmetry of the full effective action can be easily established. Let's first guess the form of the full effective action which means that the effective action contains higher derivative corrections as well as fermionic terms are included,

$$\begin{aligned} S^{abel}_{\text{eff}}[A_j, \chi] = -g^2\alpha'^2 \int \Big\{\prod_{a=1}^{4} d^{10}x_a \delta^{(10)}(x - x_a)\Big\} \times \\ &\mathfrak{g}(\partial_1\cdot\partial_2 + \partial_3\cdot\partial_4,\ \partial_1\cdot\partial_4 + \partial_2\cdot\partial_3,\ \partial_1\cdot\partial_3 + \partial_2\cdot\partial_4 \times \\ &\times [F_{ij}(x_1)F^{jk}(x_2)F_{kl}(x_3)F^{li}(x_4) - \\ &\frac{1}{4}F_{ij}(x_1)F^{ij}(x_2)F_{kl}(x_3)F^{kl}(x_4) + \\ &2i\bar{\chi}(x_1)\gamma^j\partial_k\chi(x_2)F_{ji}(x_3)F^{ik}(x_4) - \\ &i\bar{\chi}(x_1)\gamma^{ijk}\partial^l\chi_(x_2)F_{ij}(x_3)F_{kl}(x_4) - \\ &\frac{1}{3}\bar{\chi}(x_1)\gamma_i\partial_j\chi(x_2)\bar{\chi}(x_3)\gamma^i\partial^j\chi(x_4)]. \end{aligned} \tag{4.12}$$

Similarly to string theory case, It is easy here to see that (4.12) can reproduce the complete open superstring tree amplitude $\mathscr{A}^{(4)}$ with the three different types of (K) Appendix A.2.3.

In order to prove that (4.12) is supersymmetry, we base on the fact that The ($\mathfrak{g}$), *i.e.* the momenta factors, is converted to symmetric differential operator acts on (K), we can denote that one can follow the same Noether[2] procedures as the ones necessary to prove that (4.11) is supersymmetric. To see the real difference between the supersymmetry of local action and one of the non-local action, we consider as an example the supersymmetric transformation of the first term in (4.11). We have

$$\begin{aligned}\delta(\mathrm{tr}F^4) = \delta F_{ij}F^{jk}F_{kl}F^{li} + F_{ij}\delta F^{jk}F_{kl}F^{li} + \\ + F_{ij}F^{jk}\delta F_{kl}F^{li} + F_{ij}F^{jk}F_{kl}\delta F^{li}\end{aligned} \tag{4.13}$$

because of the fact of locality, this variation becomes

$$\delta(\mathrm{tr}F^4) = 4F_{ij}F^{jk}F_{kl}\delta F^{li} \tag{4.14}$$

which is considered as a required step for proving the supersymmetry. By contrast, Due to the non-locality of (4.12), (4.16) is not directly manifest, and it is feasible using the symmetry property of $\mathfrak{g}$. In addition to what we did in the local case, we just have to perform a partial integrations to demonstrate the supersymmetry of the non-local action. for instance, in the course of our calculation in the local case, we met the following total derivative term

$$\partial_i(F^{ij}\mathrm{tr}F^2\bar{\epsilon}\gamma_j\chi).$$

This term in the non-local case arises as

$$\Big(\frac{\partial}{\partial x_1^i} + \frac{\partial}{\partial x_2^i} + \frac{\partial}{\partial x_3^i} + \frac{\partial}{\partial x_4^i}\Big)F_{ij}(x_1)F^{kl}(x_2)F_{kl}(x_3)\bar{\epsilon}\gamma_j\chi(x_4).$$

[2]For more details concerning the Noether method, [6, 12] could be useful.

The $\sum_a \partial/\partial x_a^i$ term could be taken out of the x_a integration as a total derivative

$$\begin{aligned}
&\int d^{10}x\Big\{\prod_a d^{10}x_a\delta^{10}(x-x_a)\Big\}\\
&\times\,\mathfrak{g}(\partial_1\cdot\partial_2+\partial_3\cdot\partial_4,\ \partial_1\cdot\partial_4+\partial_2\cdot\partial_3,\ \partial_1\cdot\partial_3+\partial_2\cdot\partial_4)\times\\
&\times\,\Big(\sum_b\frac{\partial}{\partial x_b^i}\Big)F_{ij}(x_1)F^{kl}(x_2)F_{kl}(x_3)\bar{\epsilon}\gamma_j\chi(x_4)=\\
&=\int d^{10}x\frac{\partial}{\partial x}\Big\{\prod_a d^{10}x_a\delta^{10}(x-x_a)\Big\}\\
&\times\,\mathfrak{g}(\partial_1\cdot\partial_2+\partial_3\cdot\partial_4,\ \partial_1\cdot\partial_4+\partial_2\cdot\partial_3,\ \partial_1\cdot\partial_3+\partial_2\cdot\partial_4)\times\\
&\times\,F_{ij}(x_1)F^{kl}(x_2)F_{kl}(x_3)\bar{\epsilon}\gamma_j\chi(x_4).
\end{aligned}\tag{4.15}$$

We end up with the fact that the supersymmetry of the non-local action is a direct consequence of the supersymmetry of the local one.

4.1.3 Derivative expansion of 4-point effective action

We have seen above that the properties of $(\mathfrak{g})$ play in general the essential role to prove that the 4-point effective action is supersymmetric. And it is noticeable that $(\mathfrak{g})$ could be *any* symmetric differential operator $\Lambda(\partial_1,\partial_2,\partial_3,\partial_4)$. Our purpose first is to build the *most* general Lorentz invariant expression of $\Lambda(\partial_1,\partial_2,\partial_3,\partial_4)$, which satisfies the following properties

a- This expression has to be symmetric in k_a.

b- It must be regular as k_a goes to zero.

c- This expression should have only $k_a\cdot k_b$ combinations and may have their products.

d- Using the conservation law of momenta and the on-shell condition, the final expression of $\Lambda(\partial_1,\partial_2,\partial_3,\partial_4)$ can be written polynomially as combinations of $s, t,$ and u.

In the first stage, let's study the derivative expansion of the tree level open string effective action (4.9). Any polynomial expression has the properties mentioned above takes the following form

$$\sum_{p\leq q\leq r} \alpha'^{p+q+r} c_{p,q,r} O(p,q,r), \tag{4.16}$$

with $c_{p,q,r}$ are constants and

$$O(p,q,r) = s^p t^q u^r + s^p t^r u^q + s^r t^p u^q + s^r t^q u^p + s^q t^r u^p + s^q t^p u^r \tag{4.17}$$

Choose

$$M(n) = s^n + t^n + u^n \qquad \text{and} \qquad N = stu.$$

The Substitution of $M(n)$ and N in $O(p,q,r)$ leads to

$$O(p,q,r) = N^p(M(q-p)M(r-p) - M(q+r-2p)). \tag{4.18}$$

we get from

$$M(1)M(n-1) = 0$$

that

$$M(n) = \frac{1}{2}M(2)M(n-2) + NM(n-3).$$

As a conclusion, (4.16) can be expressed in powers of $M = M(2)$ and N,

$$\sum_{k,l} \alpha'^{2k+2l} f_{k,l} M^k N^l, \tag{4.19}$$

where $f_{k,l}$ are constants. The possible independent combinations number $\mathscr{L}_{M,N}(r)$ of M and Q, at order α'^r in the relation above

$$\mathscr{L}_{M,N}(r) = \begin{cases} \left[\dfrac{r}{6}\right] + 1, & \text{if } r \neq 6 \times \left[\dfrac{r}{6}\right] + 1 \\ \left[\dfrac{r}{6}\right], & \text{if } r = 6 \times \left[\dfrac{r}{6}\right] + 1, \end{cases}$$

where $[x]$ means the large integer smaller than x.

In the second stage, for any given order r in α', we realized that the number of independent supersymmetric contributions to the 4-point tree level open string is equal to $\mathscr{L}_{M,N}(r)$. the construction of the contributions to the tree level open superstring 4-point action (4.12) at any desired order in α', follows from the α' expansion of $\mathfrak{g}(\partial_1 \cdot \partial_2 + \partial_3 \cdot \partial_4,\, \partial_1 \cdot \partial_4 + \partial_2 \cdot \partial_3,\, \partial_1 \cdot \partial_3 + \partial_2 \cdot \partial_4)$ Appendix A.2.4.

In the end of this section, we can say that the string theory scattering amplitude approach seems to be a promising method to derive the string α' infinite series corrections to the Maxwell theory in its bosonic as well as in its supersymmetry generalization. In this thesis, we have only discussed the 4-point sector of the complete effective action, since the general wisdom of the complete scattering amplitudes in open superstring theory has maximally gone up to 5-point superstring amplitude [4] which vanishes in the abelian case (without Chan-Paton factors).

4.2 Nonabelian tree level open superstring effective action

In fact, several successful methods have attempted to derive the α' infinite series contribution to the Super Yang-Mills (SYM) action. Nevertheless, these approaches were used up to α'^4 order [13, 14], and one of the shortcomings of these methods is, all of them required in some moment some support form the open superstring tree amplitudes to fix some undetermined coefficients. In the previous work we derived the terms of the abelian effective action which are only quartic in the fields. In this section we generalize this previous result to nonabelian case.

4.2.1 Nonabelian open superstring 4-point effective action

The ten-dimensional complete tree level 4-point effective action of nonabelian open superstring theory can be written in term of two sectors

$$S_{\text{eff}}^{(4)nonabel.}[A_j, \chi_\alpha] = S_{SYM} + S_{\alpha' corr.}^{(4)nonabel.}. \tag{4.20}$$

The first sector is the ten-dimensional the super Yang-Mills theory, it can be written as

$$S_{SYM} = \int d^{10}x \text{tr}\Big[-\frac{1}{4}F_{ij}F^{ij} + \frac{i}{2}\bar{\chi}\gamma^i D_j \chi\Big], \tag{4.21}$$

S_{SYM} reproduces the on shell tree 4-point amplitudes $\mathscr{A}_{\alpha'^0}^{(4)nonabel.}$ in Super Yang-Mills theory which can be directly obtained from the zero order in α' of the complete 4-point scattering tree amplitudes of non abelian open superstring $\mathscr{A}^{(4)nonabel.}$.

The second sector represents the action which contains the infinite α' superstring contributions to the Super Yang-Mills action. Such sector in non-local context, takes the following form

$$\begin{aligned}
S^{(4)nonabel.}_{\alpha' corr.} = & -\frac{1}{2}g^2\alpha'^2 \int d^{10}\Big\{ \prod_{b=1}^{4} d^{10}x_b \delta^{10}(x-x_b)\Big\} \times \\
& \times \mathfrak{g}^{a_1a_2a_3a_4}(D_1\cdot D_2 + D_3\cdot D_4\,,\, D_1\cdot D_4 + \\
& + D_2\cdot D_3\,,\, D_1\cdot D_3 + D_2\cdot D_4) \times \\
& \times \Big[F_{a_1ij}(x_1)F^{jk}_{a_2}(x_2)F_{a3kl}(x_3)F^{li}_{a_4}(x_4) - \\
& \frac{1}{4}F_{a_1ij}(x_1)F^{ij}_{a_2}(x_2)F_{a_3kl}(x_3)F^{kl}_{a_4}(x_4) + \\
& 2i\bar{\chi}_{a_1}(x_1)\gamma^j D_k\chi_{a_2}(x_2)F_{a_3ji}(x_3)F^{ik}_{a_4}(x_4) - \\
& i\bar{\chi}_{a_1}(x_1)\gamma^{ijk}D^l\chi_{a_2}(x_2)F_{a_3ij}(x_3)F_{a_4kl}(x_4) - \\
& \frac{1}{3}\bar{\chi}_{a_1}(x_1)\gamma_i D_j\chi_{a_2}(x_2)\bar{\chi}_{a_3}(x_3)\gamma^i D^j\chi_{a_4}(x_4)\Big], \quad (4.22)
\end{aligned}$$

where the covariant derivative and non abelian field strength are defined in Appendix B.1.1.

$S^{(4)nonabel.}_{\alpha' corr.}$ reproduces indeed the $\mathscr{A}^{(4)nonabel.}_{\alpha' corr.}$ of the complete $\mathscr{A}^{(4)nonabl.}$.

- Remarks on $\mathfrak{g}^{a_1a_2a_3a_4}$.

a- In contrast to abelian case, ($\mathfrak{g}$) carries color indices which are due to the Chan-Paton factors at the free ends of the interacting open superstrings.

b- $\mathfrak{g}^{a_1a_2a_3a_4}(s,t,u)$ is manifestly symmetric in the pairs (k_m, a_m).

c- ($\mathfrak{g}^{a_1a_2a_3a_4}$), as an extra momentum factors in the open superstring four point amplitudes, is qualified to be converted into appropriate covariant differential operator

$$\mathfrak{g}^{a_1a_2a_3a_4}(D_1\cdot D_2 + D_3\cdot D_4\,,\, D_1\cdot D_4 + D_2\cdot D_3\,,\, D_1\cdot D_3 + D_2\cdot D_4)$$

which could act on the fields in $S^{(4)nonabel.}_{\alpha' corr.}$.

d- In non abelian open superstring amplitudes $\mathscr{A}^{(4)nonabel.}_{\alpha' corr.}$, ($\mathfrak{g}^{a_1a_2a_3a_4}$)

behaves as

$$\mathfrak{g}^{a_1a_2a_3a_4}(s,t,u) = \Big\{[\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4}) + \mathrm{tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_3}\lambda^{a_2})]\omega(s,t) + [\mathrm{tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_2}\lambda^{a_4}) + \mathrm{tr}(\lambda^{a_4}\lambda^{a_2}\lambda^{a_3}\lambda^{a_1})]\omega(t,u) + [\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_4}\lambda^{a_3}) + \mathrm{tr}(\lambda^{a_3}\lambda^{a_4}\lambda^{a_2}\lambda^{a_1})]\omega(u,s)]\Big\} \tag{4.23}$$

where

$$\omega(s,t) = \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} - \frac{1}{\alpha'^2 st}$$

here we have subtracted the poles because we already considered it in the expression of $\mathrm{A}^{(4)nonabel.}_{\alpha'0}$. It turns out that $\mathfrak{g}^{a_1a_2a_3a_4}$ is regular as k_b goes to zero.

e- The α' expansion of $\mathfrak{g}^{a_1a_2a_3a_4}$ (see Appendix A.2.4) in (4.22) leads to the infinite α' series superstring corrections to the Super Yang-Mills action.

- How to reproduce $A^{(4)nonabel.}$ from $S^{(4)nonabel.}_{\text{eff}}$?

In fact, the complete 4-point tree amplitudes in nonabelian open superstring string theory can be expressed as sum of two contributions reproduced respectively by S_{SYM} and $S^{(4)nonabel.}_{\alpha' corr.}$

$$\mathscr{A}^{(4)nonabel.} = \mathscr{A}^{(4)nonabel.}_{\alpha'0} + \mathscr{A}^{(4)nonabel.}_{\alpha' corr.} \tag{4.24}$$

The zero order in α' 4-point amplitudes contribution behaves as

$$\mathscr{A}^{(4)nonabel.}_{\alpha'0} = 8ig^2(2\pi)^{10}\delta^{10}(k_1+k_2+k_3+k_4)\omega^{a_1a_2a_3a_4}_{\alpha^0}(k_1,k_2,k_3,k_4)K \tag{4.25}$$

with

$$\omega^{a_1a_2a_3a_4}_{\alpha^0}(k_1,k_2,k_3,k_4) = [\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4}) + \mathrm{tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_3}\lambda^{a_2})]\frac{1}{st} + [\mathrm{tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_2}\lambda^{a_4}) + \mathrm{tr}(\lambda^{a_4}\lambda^{a_2}\lambda^{a_3}\lambda^{a_1})]\frac{1}{ut} + [\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_4}\lambda^{a_3}) + \mathrm{tr}(\lambda^{a_3}\lambda^{a_4}\lambda^{a_2}\lambda^{a_1})]\frac{1}{us},$$

and where (K) is one of the kinematic factors of Appendix A.2.3 associated to the corresponding scattering process. And all kinds of these amplitudes can easily derived from S_{SYM}. The S_{SYM} gives rise to one-particle irreducible diagrams and one-particle reducible ones which arise only in this zero order α' contribution of the whole effective action. And the derivation of the 4-gluons amplitudes of these diagrams are based in the field theory calculation already done in Appendix B.1. Afterwards, a very short algebraic manipulation leads to the $\mathscr{A}^{(4)nonabel.}_{\alpha'^0}$.

Let's move now to the α' contribution of the Open superstring amplitude $\mathscr{A}^{(4)nonabel.}_{\alpha' corr.}$. This contribution can be expressed as

$$\mathscr{A}^{(4)nonabel.}_{\alpha' corr.} = 8i(g\alpha')^2(2\pi)^2\delta^{10}(k_1+k_2+k_3+k_4)\mathfrak{g}^{a_1a_2a_3a_4}(k_1,k_2,k_3,k_4)K \tag{4.26}$$

where (K) exactly the same kinematic factors given in Appendix A.2.3.

Since the poles are isolated to be in the zero order α' contribution (S_{SYM}) of the whole effective action, we can follow closely the steps done for reproducing the 4-point amplitude from the abelian effective to derive $\mathscr{A}^{(4)nonabel.}_{\alpha' corr.}$ from $S^{(4)nonabel.}_{\alpha' corr.}$ which also is the generator of 1-particle irreducible diagrams.

Therefore, to reproduce $\mathscr{A}^{(4)nonabel.}_{\alpha' corr.}$, one needs firstly to substitute (4.22) in $(A.15)$, afterwards, derives similarly to the abelian case all the 4-particle vertices[3] defined in Appendix A.2.2., and sticks them in $(A.17)$.

We eventually come up with the complete on shell tree level 4-point amplitude of non abelian open superstring theory

$$\begin{aligned}\mathscr{A}^{(4)nonabel.} &= 8ig^2(\alpha')^2(2\pi)^{10}\delta^{10}(k_1+k_2+k_3+k_4)\\ &\times \left\{\frac{\Gamma(-\alpha's)\Gamma(-\alpha't)}{\Gamma(1-\alpha's-\alpha't)}[\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_3}\lambda^{a_2})]\right.\\ &+\frac{\Gamma(-\alpha't)\Gamma(-\alpha'u)}{\Gamma(1-\alpha't-\alpha'u)}[\mathrm{tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_2}\lambda^{a_3})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_2}\lambda^{a_4})]\\ &\left.+\frac{\Gamma(-\alpha'u)\Gamma(-\alpha's)}{\Gamma(1-\alpha'u-\alpha's)}[\mathrm{tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_4}\lambda^{a_2})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_4}\lambda^{k)a_3})]\right\}\\ &\times K\end{aligned} \tag{4.27}$$

where (K) is the same kinematic factor mentioned above.

[3] A detailed derivation of theses vertices is available in [15, 16].

As an application of 4-point abelian effective action and its nonabelian generalization, we refer the reader to [15, 16] where one can find construction of some higher order terms in the 4-point effective actions.

Chapter 5

conclusion

5.1 Born-Infeld theory

Appendix A

Dimensional Analysis and Definitions

A.1 Dimensions

We prefer here to make our analysis in arbitrary dimension d. As is customary in high energy physics, natural units are used, *i.e.*

$$c = \hbar = 1$$

and all dimensions are given in terms of mass (inverse length dimension). Our metric convention is

$$\eta^{\mu\nu} = \text{diag}(-, +, +, \cdots, +.) \tag{A.1}$$

The Regge slope α' has in this system

$$[\alpha'] = (mass)^{-2}.$$

and The string tension is

$$T = \frac{1}{2\pi\alpha'}.$$

To facilitate dimensional analysis, it is preferable to collect the dimensions of all frequently used fields and constants of this thesis in the following table

Fields and Constants	Dimensions
$F_{\mu\nu}$(x)	$d/2$
$A_\mu(x)$	$d/2-1$
k_μ	1
ζ_μ	$(d-1)/2$
V_k^M	$M(1-d/2)+d$
g	$-1/2(d-4)$
$\delta^d(k)$	$-d$
$\delta^d(x)$	d
$\mathscr{A}^{(M)}$	$M/2$

where V_k^M is a M-massless bosons vertex interaction discussed in the next section.

We denote here that the dimension of $\mathscr{A}^{(M)}$ is related to that of the polarization vector ζ_μ of external state.

Proof

We start our proof with evaluating the dimension of $V^(M)$.

In space-time.

The $(A.11)$ of the next section leads to

$$\mathrm{dim}V_x^{(M)} = -dM + M\Big(\frac{d}{2}-1\Big) = 0 \Rightarrow$$

$$\mathrm{dim}V_x^{(M)} = M\Big(\frac{d}{2}+1\Big). \tag{A.2}$$

Example:

The 4-point correlation function in $d=4$ has

$$\mathrm{dim}[\langle 0 \mid T(A(x_1)A(x_2)A(x_3)A(x_4)) \mid 0\rangle] = 4.$$

This correlation function can also be expressed as

$$T(A(x_1)A(x_2)A(x_3)A(x_4)) \mid 0\rangle = 4 \times 1/k^2 \times V_x^{(4)}$$

where $\Delta \simeq 1/k^2$'s are the external propagators of the 4-point vertex.

It comes out that $\dim V_x^{(4)} = 12$ which can be directly calculted from $(A.2)$.

In momentum space

The expression $(A.12)$ of next section, and after the substitution of $V_x^{(M)}$ gives

$$V_k^{(M)} = d\Big(1 - \frac{M}{2}\Big) + M. \tag{A.3}$$

Let's study now the dimensions of $\mathscr{A}^{(M)}$ for two different dimensions of ζ.

- $\dim\zeta = d/2 - 1$:

The general structure of M-boson interaction $(A.13)$ has the following dimension

$$\begin{aligned}\dim\mathscr{A}^{(M)} &= -d + M\dim\zeta + \dim V_k^{(M)} \\ &= -d + M\Big(\frac{d}{2} - 1\Big) + d\Big(1 - \frac{M}{2}\Big) + M = 0.\end{aligned} \tag{A.4}$$

The 4-point amplitude $\mathscr{A}^{(4)}$ suggests that the general structure of $\mathscr{A}^{(M)}$ behaves as

$$\mathscr{A}^{(M)} = \delta^{(d)}(k_1 \cdots k_M)(g\alpha')^{M-2}\zeta^M k^M f(\alpha'^{1/2}k) \tag{A.5}$$

where $f(\alpha'^{1/2}k)$ is dimensionless factor which in general, its α' expansion is responsible for derivative corrections of the effective action.

Therefore,

$$\dim\mathscr{A}^{(M)} = -d - \frac{d}{2}(M-2) + M\Big(\frac{d}{2} - 1\Big) + M = 0.$$

Eventually, for this given dimension of ζ the amplitude has to be dimensionless.

In fact, we can rewrite $\mathscr{A}^{(M)}$ as power of α' which will be useful in evaluating amplitudes of higher value in M, and this might be achieved by making in the amplitude two dimensional combinations

$$\alpha'^{1/2(d/2-1)} \qquad \text{and} \qquad \alpha'^{1/2\,k}.$$

Hence,

$$\mathscr{A}^{(M)} = \delta^{(d)}(k_1 \cdots k_M) g^{M-2} \alpha'^r \left(\alpha'^{\frac{1}{2}(\frac{d}{2}-1)}\right)^M (\alpha'^{\frac{1}{2}k})^M, \qquad \text{(A.6)}$$

and r to be determined.

We have $\mathscr{A}^{(M)}$ is dimensionless, so the relation

$$0 = -d - \frac{1}{2}(M-2)(d-4) - 2r$$

leads to

$$r = \frac{n}{4}(4-d) - 2. \qquad \text{(A.7)}$$

- dim$\zeta = (d-1)/2$:

By Following the same calculations as in the first case , one should find that

$$\dim \mathscr{A}^{(M)} = \frac{M}{2}, \qquad \text{(A.8)}$$

which is consistent with the dimension which could be evaluated evaluated from (3.18) for d=10.

Finally, it turns out that the dimension of the amplitude in open superstring depends *only* on the dimension of the polarization vector ζ of external string state.

A.2 Definitions

A.2.1 t_8 tensor

The miraculous t_8 tensor[1], characteristic of the 4 massless vector boson scattering amplitude, is antisymmetric in the pairs $(ij), (kl), \cdots$, and is symmetric under such of pairs, these properties of t_8 lead to the following equivalent relations

$$g(1,2,3,4) t_{ijklmnpq} S_1^{ij} S_2^{kl} S_3^{mn} S_4^{pq} = g(1,2,3,4)(S_{1ab} S_2^{bc} S_{3cd} S_4^{da} - \\ - \frac{1}{4} S_{1ab} S_2^{ab} S_{3cd} S_4^{cd}) \qquad \text{(A.9)}$$

[1]An explicit expression for it may be found in [5]

where $g(1,2,3,4)$ is any symmetric operator in (1,2,3,4) which act on index I of an antisymmetric tensor S_I^{ab} for $I = 1,2,3,4$.

Similarly we have

$$\begin{aligned} t_{ijklmnpq} S_1^{ij} S_2^{kl} S_3^{mn} S_4^{pq} = & -2(\mathrm{tr} S_1 S_2 \mathrm{tr} S_3 S_4 + \mathrm{tr} S_1 S_3 \mathrm{tr} S_2 S_4 + \\ & + \mathrm{tr} S_1 S_4 \mathrm{tr} S_2 S_3) + 8(\mathrm{tr} S_1 S_2 S_3 S_4 + \\ & + \mathrm{tr} S_1 S_3 S_2 S_4 + \mathrm{tr} S_1 S_3 S_4 S_2). \end{aligned} \quad (A.10)$$

A.2.2 Effective action

One of the nice features of the tree level low energy effective theory, that the action of this theory is the generator of one particle irreducible diagrams, in a sense, in ten dimensional space-time we have

- Abelian string effective action

$$\begin{aligned} S_{\text{eff}}^{abel}[A_j] = \sum_M \frac{1}{M!} \int & d^{10}x_1 \cdots d^{10}x_M V_{j_1 \cdots j_M}^{(M)}(x_1, \cdots, x_M) \times \\ & \times A^{j_1}(x_1) \cdots A^{j_M}(x_M), \end{aligned} \quad (A.11)$$

thus, it comes out that the M-point vertex in the target space, is

$$V_{j_1 \cdots j_M}^{(M)}(x_1, \cdots, x_M) = \left. \frac{\delta^M S_{\text{eff}}^{abel}[A_j]}{\delta A^{j_1}(x_1) \cdots A^{j_M}(x_M)} \right|_{A_j=0}.$$

And in the momentum space it can be written as

$$\begin{aligned} (2\pi)^2 \delta^{10}(k_1 + \cdots k_M) V_{j_1 \cdots j_M}^{(M)}(k_1, \cdots, k_M) = \int & \prod_{i=1}^M d^{10}x_i e^{ik_i \cdot x_i} \times \\ & \times V_{j_1 \cdots j_M}^{(M)}(x_1 \cdots x_M). \end{aligned} \quad (A.12)$$

The $V_k^{(M)}$ vertex interaction (M-point function) gives to the the S-matrix the following contribution

$$\mathscr{A}^{(M)} = i(2\pi)\delta^{10}(k_1 + \cdots + k_M) \zeta_1^{j_1} \cdots \zeta_M^{j_M} V_{j_1 \cdots j_M}^{(M)}(k_1 \cdots k_M). \quad (A.13)$$

- Nonabelian superstring effective action

In this case, since we have discussed in this thesis only the superstring effective action involving up to quadratic fermionic field terms, we restrict our definition of the nonabelian superstring effective action to the 4-point effective action one, in a sense we take

$$S^{nonabel.}_{\alpha' corr.}[A_j,\chi_\alpha] = S^{(4)}_{\alpha' corr.}[A_j,\chi_\alpha] + S^{(M>4)}_{\alpha' corr.}[A_j,\chi_\alpha].$$

where $S^{(4)}_{\alpha' corr.}[A_j,\chi_\alpha]$, is the nonabelan 4-point open superstring effective action. Hence we have

$$\begin{aligned} S^{(4)}_{\alpha' corr.}[A_j,\chi_\alpha] = \int \prod_{i=1}^{4} \Big[& \{\frac{1}{4!} V^{(4)a_1a_2a_3a_4}_{j_1j_2j_3j_4}(x_1,x_2,x_3,x_4)\times \\ & \times A^{j_1}_{a_1}(x_1)A^{j_2}_{a_2}(x_2)A^{j_3}_{a_3}(x_3)A^{j_4}_{a_4}(x_4)\} + \\ & + \{\frac{1}{2!2!} V^{(4)a_1a_2a_3a_4}_{j_1j_2\alpha_3\alpha_4}(x_1,x_2,x_3,x_4)\times \\ & \times A^{j_1}_{a_1}(x_1)A^{j_2}_{a_2}(x_2)\chi^{\alpha_3}_{a_3}(x_3)\chi^{\alpha_4}_{a_4}(x_4)\} + \\ & + \{\frac{1}{4!} V^{(4)a_1a_2a_3a_4}_{\alpha_1\alpha_2\alpha_3\alpha_4}(x_1,x_2,x_3,x_4)\times \\ & \times \chi^{\alpha_1}_{a_1}(x_1)\chi^{\alpha_2}_{a_2}(x_2)\chi^{\alpha_3}_{a_3}(x_3)\chi^{\alpha_4}_{a_4}(x_4)\}\Big] \end{aligned} \quad \text{(A.14)}$$

Where

$$\begin{aligned} V^{(4)a_1a_2a_3a_4}_{j_1j_2j_3j_4}(x_1,x_2,x_3,x_4) &= \frac{\delta^4 S^{nonabel.}_{\alpha' corr.}[A_j,\chi_\alpha]}{\delta A^{j_1}_{a_1}(x_1)\delta A^{j_2}_{a_2}(x_2)\delta A^{j_3}_{a_3}(x_3)\delta A^{j_4}_{a_4}(x_4)}\Bigg|_{A^j_a,\chi^\alpha_a=0} \\ V^{(4)a_1a_2a_3a_4}_{j_1j_2\alpha_3\alpha_4}(x_1,x_2,x_3,x_4) &= \frac{\delta^4 S^{nonabel.}_{\alpha' corr.}[A_j,\chi_\alpha]}{\delta A^{j_1}_{a_1}(x_1)\delta A^{j_2}_{a_2}(x_2)\delta \chi^{\alpha_3}_{a_3}(x_3)\delta \chi^{\alpha_4}_{a_4}(x_4)}\Bigg|_{A^j_a,\chi^\alpha_a=0} \\ V^{(4)a_1a_2a_3a_4}_{\alpha_1\alpha_2\alpha_3\alpha_4}(x_1,x_2,x_3,x_4) &= \frac{\delta^4 S^{nonabel.}_{\alpha' corr.}[A_j,\chi_\alpha]}{\delta \chi^{\alpha_1}_{a_1}(x_1)\delta \chi^{\alpha_2}_{a_2}(x_2)\delta \chi^{\alpha_3}_{a_3}(x_3)\delta \chi^{\alpha_4}_{a_4}(x_4)}\Bigg|_{A^j_a,\chi^\alpha_a=0} \\ &. \end{aligned} \quad \text{(A.15)}$$

These 4-point functions can be written in the momentum space as

$$\begin{aligned} (2\pi)^{10}\delta^{(10)}(k_1+k_2+k_3+k_4)V^{(4)a_1a_2a_3a_4}_{s_1s_2s_3s_4}(k_1,k_2,k_3,k_4) = \\ \int \prod_{i=1}^{4} d^{10}x_i e^{ik_i\cdot x_i} V^{(4)a_1a_2a_3a_4}_{s_1s_2s_3s_4}(x_1,x_2,x_3,x_4). \end{aligned} \quad \text{(A.16)}$$

Our convention here of $s_i{}^2$ means the i-th vector index or the i-th spinor index.

Finally, the scattering amplitude corresponding of these 4-point functions are

$$\mathscr{A}^{(4)nonabel.}_{\alpha' corr.} = -i(2\pi)^{10}(k_1+k_2+k_3+k_4)\Lambda_1^{s_1}\Lambda_2^{s_2}\Lambda_3^{s_3}\Lambda_4^{s_4} \times \times V^{(4)a_1a_2a_3a_4}_{s_1s_2s_3s_4}(k_1,k_2,k_3,k_4), \tag{A.17}$$

with $\Lambda_i^{s_i}$ is the i-th massless boson polarization vector or i-th fermion wave function.

A.2.3 4-particle kinematic factor (K)

Here, in this part, we write down the the kinematic terms (K) of $\mathscr{A}^{(4)}$ for

▶ four bosons

$$K_{(4\,bosons)} = t^{ijklmnpq}\zeta_i^1 k_j^1 \zeta_k^2 k_l^2 \zeta_m^3 k_n^3 \zeta_p^4 k_q^4. \tag{A.18}$$

It is obvious that $K_{(4bosons)}$is symmetric under the interchange of the external particles.

▶ 2 bosons and 2 fermions

Due to the momentum conservation and the mass-shell condition, and, basing on [5, 11], the kinematic term of this case takes the following form

$$K_{(2\,bosons,\,2\,fermions)} = -\frac{s}{8}\Big[\bar{v}_2\not\zeta_3(\not k_4+\not k_1)\not\zeta_4 v_1\Big] - \frac{t}{8}\Big[2(\bar{v}_2\not\zeta_3 v_1)(k_3\cdot\zeta_4) + + 2(\bar{v}_2\not k_4 v_1)(\zeta_3\cdot\zeta_4)2(\bar{v}_2\not\zeta_4 v_1)(k_4\cdot\zeta_3)\Big]. \tag{A.19}$$

this factor is antisymmetric under interchange of the two fermions and is symmetric under interchage of the the bosons.

▶ 4 bosons

[2] throughout the thesis, we use for spinor components the indices $\longrightarrow \alpha,\beta,\gamma\cdots$, and for vector components $\longrightarrow i,j,k,l,\mu,\nu,\rho\cdots$.

Under the same circumstances of the previous case, this kinematic term comes out to be

$$K_{(4\,fermions)} = -\frac{s}{8}\bar{v}_1\gamma_\mu v_4\bar{v}_2\gamma^\mu v_3 + \frac{t}{8}\bar{v}_1\gamma_\mu v_2\bar{v}_4\gamma^\mu v_3. \tag{A.20}$$

We observe in this case that $K_{(4\,fermions)}$ is completly antisymmetric under any fermionic particle interchange.

A.2.4 α' expansion

► $\mathfrak{g}(k_1, k_2, k_3, k_4)$ α' expansion

To establish the $\mathfrak{g}(k_1, k_2, k_3, k_4)$ α' expansion, we use the Taylor expansion of $\ln\Gamma(1+z)$[3]

$$\ln\Gamma(1+z) = -\gamma z + \sum_{r=2}^{\infty}(-1)^r\zeta r\frac{z^r}{r}, \qquad \text{with } -1 \leq z \leq 1 \tag{A.21}$$

In (A.21), γ is the Euler-Mascheroni constant and $\zeta(r)$ is the Ziemann zeta function. This leads to

$$\alpha'^2 h(s,t) = \frac{1}{st}\exp\Big\{\sum_{r=2}^{\infty}\alpha'^r\frac{\zeta(r)}{r}(s^r + t^r - (s+t)^r)\Big\}. \tag{A.22}$$

We take here the first terms of $(A.22)$, expressed in M and N

$$\begin{aligned}\mathfrak{g}(k_1,k_2,k_3,k_4) = & -\frac{1}{2}\pi^2 - \frac{1}{48}\alpha'^2\pi^4 M - \frac{1}{2}\alpha'^3\zeta(3)N - \frac{1}{960}\alpha'^4\pi^6 M^2 - \\ & - \frac{1}{48}\alpha'^5\pi^2 \times (\pi^2\zeta(3) + 12\zeta(5))MN - \\ & - \frac{1}{967680}\alpha'^6(51\pi^8 M^3 + 8\pi^2(31\pi^6 + 30240\zeta(3)^2)N^2) + \\ & \cdots \end{aligned} \tag{A.23}$$

► $\mathfrak{g}^{a_1a_2a_3a_4}(k_1, k_2, k_3, k_4)$ α' expansion

[3]More about this function may be found in G.Arfken book: *Mathematical methods for physics* .

Appendix B

Useful Calculations

In this appendix, we first evaluate the double x integration up to a certain order in α' and put some comments on the evaluation of the triple one, later on, we discuss our attempt toward the simplification of $A(1, \cdots, M)$ (3.19), at last, we derive the possible Feynman diagrams of the six point function.

B.1 x integration

B.1.1 The double integral

As we mentioned, the 5-point function open superstring amplitude involves kinematic factors (α' and k_i dependence) which are double integrals over x_2 and x_3. These kinematic factors are related to each other in such a away that evaluating one of them makes the others feasible to be computed.

Therefore, let us take for instance,

$$L = \int_0^1 dx_3 \, x_3^{2\alpha'\alpha_{13}-1}(1-x_3)^{2\alpha'\alpha_{34}} \times \int_0^{x_3} dx_2 \, x_2^{2\alpha'\alpha_{12}-1}(1-x_2)^{2\alpha'\alpha_{24}-1}(x_3-x_2)^{2\alpha'\alpha_{23}}. \quad \text{(B.1)}$$

where

$$\alpha_{ij} = k_i \cdot k_j \qquad \text{for } i,j = 1,2,3,4.$$

We perform the change $x_2 = u \cdot x_3$ of variables when x_3 acts as a constant in the inner integral, thus,

$$L = \int_0^1 dx_3\, x_3^{2\alpha' r-1}(1-x_3)^{2\alpha'\alpha_{34}} \int_0^1 du\, u^{2\alpha'\alpha_{12}-1}(1-u)^{2\alpha'\alpha_{23}}(1-ux_3)^{2\alpha'\alpha_{24}-1}, \qquad \text{(B.2)}$$

with

$$r = \alpha_{12} + \alpha_{13} + \alpha_{23}.$$

To evaluate this integral up to certain order in α' we consider

$$(1-ux_3)^{\alpha'\alpha_{24}-1} = \sum_{n=0}^{\infty} \frac{(-2\alpha'\alpha_{24}-1)_n}{n!} x^n y^n \qquad \text{(B.3)}$$

where we define the Poch-Hammer symbol $(a)_n$ by;

$$(a)_n = a(a+1)(a+2)\cdots(a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}. \qquad \text{(B.4)}$$

Inserting $(B.3)$ in $(B.2)$ leads to

$$L = \sum_{n=0}^{\infty} \frac{(-2\alpha'\alpha_{24}-1)_n}{n!} \int_0^1 dx_3\, x_3^{2\alpha' r+n-1} \times (1-x_3)^{2\alpha'\alpha_{34}} \int_0^1 du\, u^{2\alpha'\alpha_{12}+n-1}(1-u)^{2\alpha'\alpha_{23}}. \qquad \text{(B.5)}$$

Using the definition of the Euler Beta function;

$$B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx,$$

we rewrite $(B.5)$ as

$$L = \sum_{n=0}^{\infty} \frac{(-2\alpha'\alpha_{24}-1)_n}{n!} \times B(2\alpha' r+n, 2\alpha'\alpha_{34}+1)\cdot B(2\alpha'\alpha_{12}+n, 2\alpha'\alpha_{23}+1). \text{(B.6)}$$

Let's now expand $(B.6)$ up to $\mathcal{O}(\alpha')$,

- <u>For n=0</u>

The α' expansion for this value of n is

$$
\begin{aligned}
L_{(n=0)} = &\frac{1}{4\alpha' r\cdot\alpha_{12}} - \frac{\pi^2}{6}\Big(\frac{\alpha_{23}}{r}+\frac{\alpha_{34}}{\alpha_{12}}\Big) \\
&+\Big(2\frac{\alpha_{23}\cdot\alpha_{12}}{r}+2\frac{\alpha_{23}^2}{r}+\frac{r\cdot\alpha_{34}}{\alpha_{12}}+\frac{\alpha_{34}^2}{\alpha_{12}}\Big)\zeta(3)\alpha'+\mathcal{O}\big(\alpha'^2\big)
\end{aligned}
\tag{B.7}
$$

- For n=1

L behaves as,

$$
L_{(n=1)} = -1+2(r+\alpha_{12}+\alpha_{23}-\alpha_{24}+\alpha_{34})\alpha'+\mathcal{O}\big(\alpha'^2\big) \tag{B.8}
$$

- For $n>1$

We obtain

$$
L_{(n>1)} = \frac{1}{3}\alpha_{24}(18-\pi^2-6\zeta(3))\alpha'+\mathcal{O}\big(\alpha'^2\big)\,. \tag{B.9}
$$

We finally have that

$$
\begin{aligned}
L = &\frac{1}{(2\alpha')^2}\Big[\frac{1}{r\cdot\alpha_{12}}\Big]-\Big[1+\frac{\pi^2}{6}\Big(\frac{\alpha_{34}}{\alpha_{12}}+\frac{\alpha_{23}}{r}\Big)\Big] \\
&+\Big[\alpha_{24}\Big(4-\frac{\pi^2}{3}\Big)+2(r+\alpha_{12}+\alpha_{23}+\alpha_{34})+ \\
&\Big(-2\alpha_{24}+2\frac{\alpha_{12}\cdot\alpha_{23}}{r}+\frac{r\cdot\alpha_{34}}{\alpha_{12}}+2\frac{\alpha_{23}^2}{r}+\frac{\alpha_{34}^2}{\alpha_{12}}\Big)\zeta(3)\Big]\alpha'+\mathcal{O}\big(\alpha'^2\big)\,.
\end{aligned}
\tag{B.10}
$$

When looking for expansion of the higher orders[1] in α' of the kinematic factors, some care must be taken with some of the double integrals, because they need to be regularized. A matter of quite interest is to have an α' series for all kinematic factors, since this allows to have the superstring corrections to the Yang-Mills five tree amplitude $\mathscr{A}^{(5)}$.

B.1.2 Toward the eveluation of the triple integral

For the six point function, as we mentioned, we have different types of terms multiplied by several kinematic factors which are triple integrals

[1] We already checked it for the second order in α'.

over x_2, x_3 and x_4 following the order

$$0 < x_2 < x_3 < x_4 < 1.$$

These triple integrals are slightly different to each other in such a way that, similarly to the double integrals, we evaluate one of these integrals and the others could be calculated by following the same steps, and by using the possible relations which could link these terms one to another.

Let's consider as an example, the triple integral of the first combination of the first type of terms

$$(\zeta_1 \cdot \zeta_2)(\zeta_3 \cdot \zeta_4)(\zeta_5 \cdot \zeta_6)(k_3 \cdot k_2)(k_1 \cdot k_4) \times \{\text{Triple integral L}\},$$

where L behaves as

$$\begin{aligned} L = \int & dx_2 dx_3 dx_4 (1-x_4)^{2\alpha' k_5 \cdot k_4} (1-x_3)^{2\alpha' k_5 \cdot k_3} \times \\ & (1-x_2)^{2\alpha' k_5 \cdot k_2} (x_4 - x_3)^{2\alpha' k_4 \cdot k_3 - 1} (x_4 - x_2)^{2\alpha' k_4 \cdot k_2} (x_4)^{2\alpha' k_4 \cdot k_1 - 1} \\ & \times (x_3 - x_2)^{2\alpha' k_3 \cdot k_2 - 1} (x_3)^{2\alpha' k_3 \cdot k_2 - 1} (x_3)^{2\alpha' k_3 \cdot k_1} (x_2)^{2\alpha' k_2 \cdot k_1 - 1}. \end{aligned} \tag{B.11}$$

Now, we perform the following change of variables

$$x_2 = u \cdot x_3, \qquad x_3 = u \cdot x_4 \tag{B.12}$$

where x_3 and x_4 act as constants in the inner integrals.

then, $(B.11)$ can be written as

$$\begin{aligned} L = \int_0^1 & du\, (u)^{2\alpha' r - 1} (1-u)^{2\alpha' \alpha_{43} - 1} \int_0^1 dv\, (v)^{2\alpha' \alpha_{21} - 1} (1-v)^{2\alpha' \alpha_{32} - 1} \\ & \int_0^1 dx_4\, x_{(4)}^{2\alpha' s - 2} (1-x_4)^{2\alpha' \alpha_{54}} (1-uv)^{2\alpha' \alpha_{42}} (1-ux_4)^{2\alpha' \alpha_{53}} \times \\ & (1-uvx_4)^{2\alpha' \alpha_{52}}, \end{aligned} \tag{B.13}$$

where

$$k_i \cdot k_j = \alpha_{ij}, \qquad \text{for } i, j = 1, 2, 3, 4, 5.$$

And

$$\begin{aligned} r &= a_{32} + a_{31} + a_{12} \\ s &= a_{32} + a_{31} + a_{21} + a_{41} + a_{42} + a_{43}. \end{aligned} \tag{B.14}$$

We have,

$$
\begin{aligned}
(1-uvx_4)^{2\alpha'\alpha_{52}} &= \sum_{n=0}^{\infty} \frac{(-2\alpha'\alpha_{52})_n}{n!} u^n v^n x_4^n \\
(1-uv)^{2\alpha' a_{42}} &= \sum_{p=0}^{\infty} \frac{(-2\alpha'\alpha_{42})_p}{p!} u^p v^p \\
(1-ux_4)^{2\alpha'\alpha_{53}} &= \sum_{q=0}^{\infty} \frac{(-2\alpha'\alpha_{53})_q}{q!} u^q x_4^q
\end{aligned} \qquad \text{(B.15)}
$$

where $(a)_n$ is the Poch-Hammer symbol defined before. We obtain,

$$
\begin{aligned}
L = & \sum_{n=0}^{\infty}\sum_{p=1}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{(-2\alpha'\alpha_{52})_n}{n!} \frac{(-2\alpha'\alpha_{42})_p}{p!} \frac{(-2\alpha'\alpha_{53})_q}{q!} \times \\
& \int_0^1 du\,(u)^{2\alpha' r+n+p+q-1}(1-u)^{2\alpha'\alpha_{43}-1} \int_0^1 dv\,(v)^{2\alpha'\alpha_{21}+n+p-1} \times \\
& (1-v)^{2\alpha'\alpha_{32}-1} \int_0^1 dx_4\,(x_4)^{2\alpha' s+n+q-2}(1-x_4)^{2\alpha'\alpha_{54}}
\end{aligned} \qquad \text{(B.16)}
$$

Using the definition of the Euler Beta function leads to

$$
\begin{aligned}
L = & \sum_{n=0}^{\infty}\sum_{p=1}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{(-2\alpha'\alpha_{52})_n}{n!} \frac{(-2\alpha'\alpha_{42})_p}{p!} \frac{(-2\alpha'\alpha_{53})_q}{q!} \times \\
& B(2\alpha' r+n+p+q, 2\alpha'\alpha_{43}) B(2\alpha'\alpha_{21}+n+p, 2\alpha'\alpha_{32}) \times \\
& B(2\alpha' s+n+q-1, 2\alpha'\alpha_{54}+1).
\end{aligned} \qquad \text{(B.17)}
$$

In fact, the evaluation of the triple integral is intricate because it seems complicated to determine the source of singularities . Nevetheless, after the α' expansion up to $\mathcal{O}(\alpha')$ term, we can observe from the possible Feynman digrams which could be derived from the non abelian effective action that

1) The answer of the simple integral in the four point tree amplitude expression (in non abelian open superstring theory) involves Gamma functions which could be proportional to *one* Euler Beta fuction, the leading term of this Beta function is α'^{-1} contribution associated to the *one* propagator in diagram (a) figure B.1 in the end of this Appendix.

2) The complete calculation of the five point tree amplitude in nonabelian Open superstring theory leads to double integral evaluated as a multiplication of *two* Beta functions, the leading term of this double integral is the α'^{-2} contribution associated with the *two* propagators of diagram (c) figure B.1.

3) In our opinion, the triple integral in the nonabelian open superstring six point tree amplitude can be calculated as a multiplication of three Beta functions, and the leading term in this case is α'^{-3} contribution associated with the *three* propagators of diagrams (f) and (k) figure B.1.

B.2 Toward the simplification of $A(1, \cdots, M)$

we basically consider,

$$f_M(\zeta, k, \theta, \phi) = \sum_{i \neq j}^{M} \frac{(\theta_i - \theta_j)\phi_i(\zeta_i \cdot k_j)(2\alpha')^{11/4} - 1/2\phi_i\phi_j(\zeta_i \cdot \zeta_j)(2\alpha')^{9/2}}{x_i - x_j - \theta_i\theta_j}. \tag{B.18}$$

This function can be reexpressed as

$$f_M(\zeta, k, \theta, \phi) = \sum_{i=1}^{M} \phi_i a_i + \sum_{i \neq j}^{M} \phi_i\phi_j b_{ij}. \tag{B.19}$$

with

$$a_i = \sum_{j}^{M} \frac{(\theta_i - \theta_j)(2\alpha')^{11/4}}{x_i - x_j - \theta_i\theta_j} \quad \text{, and} \quad b_{ij} = \frac{-1/2\phi_i\phi_j(\zeta_i\zeta_j)(2\alpha')^{9/2}}{x_i - x_j - \theta_i\theta_j}. \tag{B.20}$$

Then, the exponential of F_M leads to

$$\exp\left[f_M(\zeta, k, \theta, \phi)\right] = \prod_{i=1}^{M} \exp\left(\phi_i a_i\right) \times \prod_{i \neq j}^{M} \exp\left(\phi_i\phi_j b_{ij}\right) \tag{B.21}$$

Writing out $(B.21)$ gives

$$\exp\left[f_M(\zeta, k, \theta, \phi)\right] = (1 + \phi_1 a_1)\cdots(1 + \phi_M a_M)(1 + \phi_1\phi_2 b_{12})\cdots \times (1 + \phi_{M-1}\phi_M b_{M-1\,M}). \tag{B.22}$$

Now, this expression of (3.19) looks ready to be integrated over ϕ_i, so,

$$\begin{aligned}\int d\phi_1 \cdots d\phi_M = & \; a_1 \cdots a_M + permutations + \\ & a_1 \cdots a_{M-2} b_{M-1\,M} + permutations + \\ & \vdots \\ & b_{12} \cdots b_{M-1\,M} + permutations. \end{aligned} \tag{B.23}$$

At this stage, we recall the gauge invariance, and impose it on (3.19) by making the polarization vector of one of the external string states replaces the momentum which corresponds to the same state,

$$\zeta_i \to k_i \;\; \text{for one} \;\; i = 1, \cdots M.$$

We eventually come up with several relations among the terms existing in (B.23) in such a way that one term could be evaluated, and the remainder could be calculated by using these relations.

B.3 Feynman diagrams

In order to find the leading term of the α' infinite series of the 6-point tree amplitude in open syperstring theory, we try to compute all the possible Feynman diagrams (fig.B.1,D3) which could be derived from 6-point tree level open superstring effective action that we have not obtained yet. Here, we must distinguish two cases;

B.3.1 Abelian case

Since the cubic interaction and the five point vertex vanish in the abelian case, we have only two different diagrams;

1) The 1-particle reducible diagram (g) of figure B.1 which is responsible for the singularity of the $\mathscr{A}^{(6)}_{abel.}$ because of the intermediate state. This diagram is a combination of two quartic 1PI vertices. So, up to $\mathcal{O}(\alpha'^4)$ term it can be expressed in on-shell and physical conditions as

$$V_{j_1 \cdots j_6}(1, \cdots, 6)_{qq} =$$

$$= \frac{V^{(2)m}_{j_1 j_2 j_3}(k_1, k_2, k_3, k_4 + k_5 + k_6) V^{(2)}_{m j_4 j_5 j_6}(-k_4 - k_5 - k_6, k_4, k_5, k_6)}{2(k_4 \cdot k_5 + k_4 \cdot k_6 + k_5 \cdot k_6)} + \mathcal{O}(\alpha'^5) \tag{B.24}$$

where "qq" stands for quartic interaction and $V^{(i)}$ represents the α'^i contribution to the vertex.

2) The on- shell 1-particle irreducible (l) diagram which is generated by the abelian 6-point effective action, it should behave up to the same order in α' as

$$V_{j_1\cdots j_6}(1,\cdots,6) = V^{(4)}_{j_1\cdots j_6}(k_1,\cdots,k_6) + \mathcal{O}\big(\alpha'^5\big)\,. \tag{B.25}$$

We conclude that $A(1,\cdots,6)$ of $\mathscr{A}^{(6)}_{abel.}$ up to $\mathcal{O}(\alpha'^5)$ has the form

$$A(1,\cdots,6) = \zeta_1^{j_1}\cdots\zeta_6^{j_6}\Big[\sum_{perm.} V_{j_1\cdots j_6}(1,\cdots,6)_{qq} + V_{j_1\cdots j_6}(1,\cdots,6)\Big] + \mathcal{O}\big(\alpha'^5\big) \tag{B.26}$$

where the $\sum_{perm.}$ is an algebraic sum over all the possible permutations of the sets

$$\{\zeta_1,k_1\}\cdots\{\zeta_6,k_6\}.$$

We eventually denote that the leading term of the tree amplitude in abelian open superstring is of the *fourth* order in α'.

B.3.2 Nonabelian case

In this case, in addition to (g) and (l) Feynman diagrams we have the diagrams (f),(h), (m) and (k) in figure B.1. Moreover, as for the permutations of the external lines, in addition to the lorentz indices, we have to take the color indices into account. The Feynman rules of (f) and (k) can take respectively the following form

$$V_{j_1\cdots j_6}(1,\cdots,6)_{cccc} = \frac{V^m_{j_1j_2}V_{mnl}V^l_{j_3j_4}V^n_{j_5j_6}}{prop.\times prop.\times prop.} \tag{B.27}$$

and

$$V_{j_1\cdots j_6}(1,\cdots,6)_{cccc} = \frac{V^m_{j_1j_2}V_{mj_6n}V^n_{j_5l}V^l_{j_3j_4}}{prop.\times prop.\times prop.} \tag{B.28}$$

where ($cccc$) symbol stands for the four cubic vertices. These diagrams do not have any α' correction because the cubic interaction basically doesn't depend on α'.

Now, The diagram (g) with color indices up to $\mathcal{O}(\alpha'^3)$ term leads to

$$V_{j_1\cdots j_6}(1,\cdots,6)_{qq} =$$

$$= \Big[\frac{V^{(0)m}_{j_1j_2j_3}(k_1,k_2,k_3,k_4+k_5+k_6)V^{(0)}_{mj_4j_5j_6}(-k_4-k_5-k_6,k_4,k_5,k_6)}{(k_4+k_5+k_6)^2}\Big]$$
$$+\Big[\frac{V^{(2)m}_{j_1j_2j_3}(k_1,k_2,k_3,k_4+k_5+k_6)V^{(0)}_{mj_4j_5j_6}(-k_4-k_5-k_6,k_4,k_5,k_6)}{(k_4+k_5+k_6)^2}\Big]$$
$$+\Big[\frac{V^{(0)m}_{j_1j_2j_3}(k_1,k_2,k_3,k_4+k_5+k_6)V^{(2)}_{mj_4j_5j_6}(-k_4-k_5-k_6,k_4,k_5,k_6)}{(k_4+k_5+k_6)^2}\Big]$$
$$\vdots$$
$$+\mathcal{O}\big(\alpha'^4\big)\,. \tag{B.29}$$

The diagram (h) can be expressed in the physical conditions as a combination of one five point vertex and one cubic vertex. Up to $\mathcal{O}(\alpha'^3)$ term we have

$$V_{j_1\cdots j_6}(1,\cdot,6)_{fc} =$$

$$= \frac{V^{(2)m}_{j_1\cdots j_4}(k_1,k_2,k_3,k_4,k_5+k_6)V_{mj_5j_6}(-k_5-k_6,k_5,k_6)}{2k_5\cdot k_6}$$
$$+\frac{V^{(3)m}_{j_1\cdots j_4}(k_1,k_2,k_3,k_4,k_5+k_6)V_{mj_5j_6}(-k_5-k_6,k_5,k_6)}{2k_5\cdot k_6}$$
$$+\mathcal{O}\big(\alpha'^4\big) \tag{B.30}$$

where "f" represents the five point vertex. Afterwards, we have next the one particle irreducible one (l) which in the non abelian case can be written up to $\mathcal{O}(\alpha'^3)$ as

$$V_{j_1\cdot j_6}(1,\cdots,6) = V^{(2)}_{j_1\cdots j_6}(k_1,\cdots k_6)+V^{(2)}_{j_1\cdots j_6}(k_1,\cdots k_6)+\mathcal{O}\big(\alpha'^4\big)\,. \tag{B.31}$$

At last, the diagram (m) is considered as a combination of one quartic and two cubic interactions, up to $\mathcal{O}(\alpha'^3)$, this Feynman diagram has

$$V_{j_1\cdots j_6}(1,\cdots,6)_{qcc} = \Big[\frac{V^{(0)m}_{j_1j_2j_3}V_{mj_6n}V^n_{j_4j_5}}{prop.\times prop.}\Big] + \Big[\frac{V^{(2)m}_{j_1j_2j_3}V_{mj_6n}V^n_{j_4j_5}}{prop.\times prop.}\Big] + \Big[\frac{V^{(3)m}_{j_1j_2j_3}V_{mj_6n}V^n_{j_4j_5}}{prop.\times prop.}\Big] + \mathcal{O}\big(\alpha'^4\big)\,. \tag{B.32}$$

It turns out that $A(1,\cdots,6)$ of $\mathscr{A}^{(6)}_{nonabel.}$ up to the third order in α' has the following form

$$A(1,\cdots,6) = \zeta^{j_1}\cdots\zeta^{j_6}\Big\{\sum_{perm.}\Big[V_{j_1\cdots j_6}(1,\cdots,6)_{cccc} + V_{j_1\cdots j_6}(1,\cdots,6)_{qq} + V_{j_1\cdots j_6}(1,\cdots,6)_{fc} + V_{j_1\cdots j_6}(1,\cdots,6)_{qcc}\Big] + V_{j_1\cdots j_6}(1,\cdots,6)\Big\} + \mathcal{O}\big(\alpha'^4\big) \tag{B.33}$$

where $\sum_{perm.}$ is the sum over all possible permutations of the sets

$$\{\zeta_1, k_1, a_1\}\cdots\{\zeta_6, k_6, a_6\}.$$

we conclude that the leading term of the six point tree amplitude in the non abelian open superstring theory is of order zero in α'.

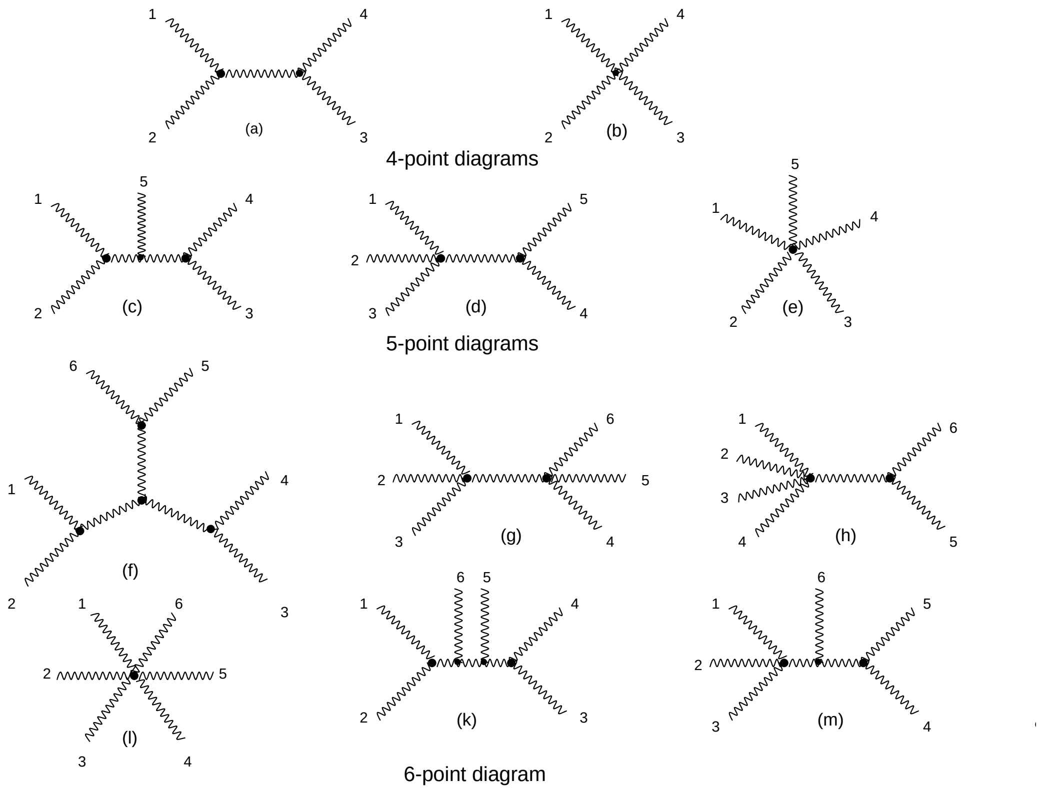

Figure B.1: Feynman diagrams

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