SLAC-252
UC-34D
(T/E)

STUDIES IN TAU LEPTON PHENOMENOLOGY*

David Howard Miller
Stanford Linear Accelerator Center
Stanford University
Stanford, California 94305

August 1982

Prepared for the Department of Energy

under contract number DE-AC03-76SF00515

Printed in the United States of America. Available from National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Road, Springfield, VA 22161. Price: Printed Copy A05 ; Microfiche A01 .

*Ph.D. dissertation.

To my wife, Ellen; to my parents; and to those pitiable few who care more for truth than for social esteem.

"In questions of science the authority of a thousand is not worth the humble reasoning of a single individual."

... Galileo Galilei

Dialogue Concerning the Two Chief World Systems (1632)

ACKNOWLEDGMENTS

My thesis advisor, Frederick J. Gilman of the Stanford Linear Accelerator Center, coauthored the work presented in Chapter II. He also contributed substantially to the analyses of Chapter III. Finally, he listened patiently and made helpful comments on Chapter IV, as well as providing general advice and information.

I was once told by my advisor that a graduate student should expect to learn more from fellow graduate students and post-docs than from faculty members. I would like to thank all of the graduate students and post-docs at SLAC who have provided aid in my research or contributed to my general knowledge of physics — particularly Shahar ben-Menahem, Bruce Weeks and Subhash Gupts.

I would also like to thank the very helpful secretarial workers at SLAC, particularly Sharon Jensen and her assistants, and the SLAC publications/technical-typing department.

Most of all, I must thank my wife, who has patiently listened to monologues on leptons, hadrons, and nonintegrable parallel transport, and who has understood a bit more than she cares to admit.

Finally, I thank that tiny handful of people I have known at Stanford — most of whom were graduate students or post-docs though many were not physicists — who were more interested in finding the truth than in acquiring the marks of social success.

TABLE OF CONTENTS

CHAPTER Page

LIST OF TABLES

LIST OF FIGURES

STUDIES IN TAU LEPTON PHENOMENOLOGY

David Howard Miller, Ph. D.
Stanford University, 1982

ABSTRACT

This thesis consists of several studies in tau lepton phenomenology.

The first study calculates decay rates of the tau proceeding through the weak vector current coupling to hadrons using published data on electron-positron annihilation into hadrons along with the conserved-vector-current (CVC) principle and assuming the tau to be an exact analogue of the electron.

The second study considers a number of possible alternative multiplet assignments for the tau and possible accompanying neutral particles within the standard electroweak gauge theory. It is shown that most of the obvious alternatives to the standard-model multiplet assignments can be ruled out.

The final study considers the possibility that the tau and its accompanying neutrino both have spin 3/2. It is shown that this unlikely possibility is not cleanly ruled out by existing experimental data. A peculiar property of Rarita-Schwinger particles in the limit that their mass goes to zero is also discussed and placed in the context of the general theorems relating to massless particles.

CHAPTER I

INTRODUCTION TO τ LEPTON PHENOMENOLOGY

Although the quest to identify and understand the elementary constituents of matter goes back thousands of years, the first solid success in that endeavor came with the confirmation of the atomic theory during the first half of the nineteenth century. Towards the end of that century, the British physicist J. J. Thomson discovered the first subatomic particle, the electron.

The electron was not only the first subatomic particle to be identified but was also the first of the family of particles known as leptons to be discovered. A lepton is a particle which has spin 1/2 and which is not subject to the strong nuclear force.

In the middle third of the twentieth century, three more leptons were discovered: the muon, a charged particle which appears to be a twin to the electron except that it weighs a bit more than 200 times the electron's weight; the electron neutrino, an uncharged, apparently massless particle associated with the electron in weak decays; and the muon neutrino, also uncharged and apparently massless and associated with the muon exactly as the electron neutrino is associated with the electron. The charged leptons were found to be associated only with left-handed neutrinos; right-handed neutrinos, so far as is known, do not exist.

In 1970, the lepton family of particles thus consisted of two pairs of particles, one pair seemingly the exact duplicate of the other save for the differing charged lepton masses. These two pairs fit easily and in the same way into the then recently developed $SU(2) \otimes U(1)$ gauge theory of weak-electromagnetic interactions.[1] (Aside from the massless quanta of long-range forces — photons and gravitons — these four leptons are the lightest of known elementary particles, hence the designation "lepton" meaning "light".) All four of these leptons seem to be point Dirac particles without discernible internal structure.

In the mid-1970s in electron-positron annihilation experiments, a class of events with low multiplicity and charged leptons among the final particles was observed. It was eventually established beyond a reasonable doubt that most of these events were due to the pair production and then decay of a new charged particle weighing about 1.8 GeV which was named the τ.[2]

All experimental data were consistent with the τ's being, except for its heavy mass (nearly 17 times the muon mass), an identical twin to the electron and to the muon; hence, it was oxymoronically referred to as a "heavy lepton".

To say that the experimental data were apparently consistent with the τ's being a massive twin of the electron (i.e., a spin-1/2 point Dirac particle not subject to the strong interaction with its own associated neutrino exactly analogous to the electron neutrino) is not to say that this was necessarily the only possible picture consistent with the data.

It is of obvious interest to know with what degree of certainty one may assert, on the basis of the experimental data, that the τ must be a heavy twin of the electron, i.e., a so-called sequential lepton. Not only is this of interest for understanding the τ but it is also of interest in trying to make sense of leptons as a whole. The following chapters are phenomenological studies of the τ which try to answer this question.

The next chapter uses experimental data from electron-positron annihilation experiments combined with the conserved-vector-current principle to calculate the vector-isovector hadronic decay rates of the τ assuming the τ to be a massive twin of the electron.[3]

Chapter III considers a number of possible multiplet assignments within SU(2) ⊗ U(1) for the τ and possible accompanying neutral leptons. It is shown that experimental data rule out most of the obvious alternatives to the standard model. (While better experimental data[3,4] is now available than that used in Chapter III, the more accurate data does not change the conclusions or arguments of that chapter.)

Chapter IV considers the possibility that both the τ and its neutrino may have spin 3/2. It is shown that, contrary to the conclusions of a previous study, this possibility is not (and cannot easily be) ruled out by experiment. In showing this, it is necessary to briefly consider some peculiarities of massless spin-3/2 particles.

References for each chapter are listed at the end of that chapter. Chapter II has been previously published (F. J. Gilman and D. H. Miller, Phys. Rev. D17, 1846 (1978)) in a slightly different form as has been Chapter III (D. H. Miller, Phys. Rev. D23, 1158 (1981)).

REFERENCES

1. S. Weinberg, Phys. Rev. Lett 19, 1264 (1967); A. Salam in Elementary Particle Physics: Relativistic Groups and Analyticity (Nobel Symposium No. 8), ed., N. Svartholm (Almquist and Wiksell, Stockholm, 1968), p. 367.
2. M. L. Perl et al., Phys. Rev. Lett. 35, 1489 (1975); M. L. Perl et al., Phys. Lett. 63B, 466 (1976); G. J. Feldman et al., Phys. Rev. Lett. 38, 177 (1976).
3. Note that the currently accepted value for the τ mass is 1.782 GeV, slightly lower than the value accepted at the time Chapter II was published. For a general overview of experimental data concerning the τ, see M. L. Perl in Annual Review of Nuclear and Particle Science Vol. 30, eds., J. D. Jackson, H. E. Gove and R. F. Schwitters (Annual Reviews Inc., Palo Alto, 1980), p. 199, and references cited therein.
4. The τ lifetime has recently been measured to be $(4.6 \pm 2.9) \times 10^{-13}$ s, G. J. Feldman et al., Phys. Rev. Lett. 48, 66 (1982).

CHAPTER II

DECAYS OF A HEAVY LEPTON INVOLVING THE HADRONIC VECTOR CURRENT

1. Introduction

As noted in Chapter I, the experimental discovery[1-9] of the τ lepton in the mid-1970s raised the question of the τ's being or not being a sequential lepton, a twin to the electron. If the τ is a twin to the electron, it must couple to a neutrino, ν_τ, via the charged weak current. Assuming this to be the same charged weak current as that responsible for the leptonic and semileptonic decays of the "ordinary" particles, we must expect the decays $\tau^- \to \nu_\tau e^- \bar{\nu}_e$, $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$ and $\tau^- \to \nu_\tau + (\text{hadrons})^-$. This last decay, if pictured as occurring by production of a light quark pair which then dress themselves as hadrons, is naively expected (because of three colors) to occur at three times the rate of $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ or $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$.

These decays, $\tau^- \to \nu_\tau + (\text{hadrons})^-$, are of considerable interest; for, not only does one want to know for theoretical reasons if the naively calculated rate agrees with the observed sum over the physical hadronic channels, but also experimentally these modes and their detailed properties serve to clarify the existence and nature of the τ and of its couplings.

A number of individual modes (like $\tau^- \to \nu_\tau \pi^-$) can be calculated from other known quantities (the pion decay constant). The Cabibbo allowed decays through the hadronic vector current may be related to the total cross section for e^+e^- annihilation into hadrons through the isovector electromagnetic current. In the past, several calculations of $\tau \to \nu_\tau + (\text{hadrons})^-$ have been made combining known couplings to a few channels with estimates of others.[10,11,12]

In this paper we recalculate the decays through the hadronic vector current. We do this because previous partial calculations plus estimates can now be replaced by a direct integration of colliding beam data over the entire energy range relevant to τ decay. In the next section we recall the relevant formulas for $\Gamma(\tau^- \to \nu_\tau + (\text{hadrons})^-)$ through the hadronic vector current and show how the ratio of three-charged-prong to one-charged-prong decays can be calculated. Then in Section III we present the detailed input and output of the calculation assuming various masses for τ and ν_τ. Section IV is a discussion of our results and conclusions.

2. Heavy Lepton Decay Rates via the Hadronic Vector Current

The formula for the decay rate for $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ or $\nu_\tau \mu^- \bar{\nu}_\mu$, assuming the charged current has a $V \pm A$ form and is of universal strength at the $\tau - \nu_\tau$ vertex, is

$$\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right) = \frac{G^2 M_\tau^5}{192\pi^3} \quad . \qquad (1)$$

Here $G = 1.02 \times 10^{-5}/M_N^2$ is the weak coupling constant, and M_τ, the mass of the τ, is experimentally 1.9 ± 0.1 GeV.[3,8,9] We have assumed that all the final leptons may be taken as massless. With a massive neutrino the decay rate becomes

$$\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right) = \frac{G^2 M_\tau^5}{192\pi^3} F(\Delta) \quad , \qquad (2a)$$

with

$$F(\Delta) = 1 - 8\Delta^2 + 8\Delta^6 - \Delta^8 - 12\Delta^4 \ln \Delta^2 \quad , \qquad (2b)$$

and $\Delta = m_{\nu_\tau}/M_\tau$. The experimental upper bound on the neutrino mass, m_{ν_τ}, is 0.6 GeV.[3,8,9]

The corresponding decay rate for $\tau^- \to \nu_\tau + (\text{hadrons})^-$, proceeding through the action of the strangeness nonchanging hadronic vector current, is straightforward to calculate:[11]

$$\Gamma\left(\tau^- \to \nu_\tau + (\text{hadrons})^-\right)$$

$$= \frac{G^2\cos^2\theta_c}{96\pi^3 M_\tau^3} \int_0^{M_\tau^2} dQ_\tau^2 \left(M_\tau^2 - Q^2\right)^2 \left(M_\tau^2 + 2Q^2\right) \frac{\sigma^{(1)}_{e^+e^-}(Q^2)}{\sigma_{pt}(Q^2)} \quad , \qquad (3)$$

where $\cos\theta_c$ is the cosine of the Cabibbo angle, $\sigma^{(1)}_{e^+e^-}(Q^2)$ is the electron-positron cross section to annihilate into hadrons with total isospin one at $E^2_{c.m.} = Q^2$, and $\sigma_{pt}(Q^2) = 4\pi\alpha^2/3Q^2$ is the cross section for $e^+e^- \to \mu^+\mu^-$. The extension to the case of massive neutrinos is

$$\Gamma\left(\tau^- \to \nu_\tau + (\text{hadrons})^-\right)$$

$$= \frac{G^2\cos^2\theta_c}{96\pi^3 M_\tau^3} \int_0^{M_\tau^2} dQ^2 \left[M_\tau^4 + m_{\nu_\tau}^4 + Q^4 - 2m_{\nu_\tau}^2 M_\tau^2 - 2m_{\nu_\tau}^2 Q^2 - 2M_\tau^2 Q^2\right]^{1/2}$$
$$\times \left[M_\tau^4 + m_{\nu_\tau}^4 - 2Q^4 - 2m_{\nu_\tau}^2 M^2 + m_{\nu_\tau}^2 Q^2 + M_\tau^2 Q^2\right] \frac{\sigma_{e^+e^-}(Q^2)}{\sigma_{pt}(Q^2)} \qquad (4)$$

which reduces to Eq. (3) when $m_{\nu_\tau} = 0$.

The term involving the strangeness changing vector current which we have neglected is expected to be of order $\tan^2\theta_c \simeq 0.05$ relative to that which we are calculating. Furthermore, its main contribution, through

$\tau^- \to \nu_\tau + K^*(890)^-$, may be calculated separately, as we will do in Section IV. For the range of integration in Eqs. (3) or (4) of interest to us, purely multipion states very much dominate the final state hadron channels in electron-positron annihilation. The annihilation cross section into final states with total isospin one involves only those channels with even numbers of pions.

The $\pi\pi$ channel must be $\pi^+\pi^-$ in electron-positron annihilation and $\nu_\tau + \pi^0\pi^-$ in τ^- decay, and so it results in a single charged prong for the final τ^- decay products. The four-pion channel must be either $2\pi^+2\pi^-$ or $\pi^+\pi^-2\pi^0$ in colliding beams[13] and $\nu_\tau + \pi^+2\pi^-\pi^0$ or $\nu_\tau + \pi^-3\pi^0$ in τ^- decay. The four-pion states in colliding beams and τ^- decay are total $I_z = 0$ and -1 states, respectively, of the same total $I = 1$ state. This fact allows us to derive[14] a relation between the populations of the two charge states of four pions in colliding beams and the two charge states of four pions in τ^- decay. For any invariant mass, Q, of the four-pion system it is:

$$\frac{d\Gamma\left(\tau^- \to \nu_\tau + \pi^+2\pi^-\pi^0\right)}{d\Gamma\left(\tau^- \to \nu_\tau + \pi^-3\pi^0\right)} = 1 + 2\ \frac{\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)}{\sigma(e^+e^- \to 2\pi^+2\pi^-)} \quad . \tag{5}$$

Thus the proportion of four-charged pions out of all four pion final states in colliding beams tells us the proportion of three-charged-prong decays for $\tau^- \to \nu_\tau + (4\pi)^-$. The relative number of three-charged-prong to one-charged-prong decays arising from $\tau \to \nu_\tau + 2\pi$ and $\tau \to \nu_\tau + 4\pi$, which is of some interest experimentally, then can be settled completely from electron-positron annihilation data.

3. Experimental Input and Results

As input to Eq. (4) we need data on electron-positron annihilation into $\pi^+\pi^-$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$, ... in the center-of-mass energy range from the threshold to M_τ. For this purpose we have taken cross section data from experiments done at Orsay,[15,18,19] Novosibirsk,[16] and Frascati.[17,20] Our method has been to use what we considered to be the best data on a particular process in a given energy range. We have not made a statistical average of all available data. On occasion we have interpolated experimental data points to get a cross section at a desired energy. Our specific choice of data is as follows:

A. $e^+e^- \to \pi^+\pi^-$

From Q = 0.28 to 0.90 GeV we use the Orsay[15] fit (taking $\rho-\omega$ interference into account) to their data on $\left|F_\pi(Q^2)\right|^2$:

$$\left|F_\pi(Q^2)\right|^2 = \frac{F_0^2 M_\rho^2 \Gamma_\rho^2}{\left(M_\rho^2 - Q^2\right)^2 + M_\rho^2\Gamma_\rho^2\left(\frac{p}{p_0}\right)^6\left(\frac{M_\rho}{Q}\right)^2} , \qquad (6)$$

where Q is the total center-of-mass energy and p the pion momentum. For this fit the rho mass M_ρ = 0.7754 GeV, Γ_ρ = 0.1496 GeV, F_0 = 5.83 and p_0, the pion momentum at the rho mass, is 0.3615 GeV. The cross section for $e^+e^- \to \pi^+\pi^-$ is related to $\left|F_\pi(Q^2)\right|^2$ by

$$\sigma(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2}{3Q^2}\left(\frac{2p}{Q}\right)^3 \left|F_\pi(Q^2)\right|^2 . \qquad (7)$$

Between Q = 0.90 and 1.34 GeV we use Novosibirsk data[16] on $e^+e^- \to \pi^+\pi^-$, which are significantly above the ρ meson tail calculated from Eq. (6). Above 1.34 GeV the measurements[17] of $|F_\pi|^2$ are consistent, within rather large error bars, with Eq. (6) once again. We use this formula as input in this region, but in any case this domain makes a very small contribution to $\tau^- \to \nu_\tau + \pi^-\pi^0$.

B. $e^+e^- \to 2\pi^+2\pi^-$

Between Q = 0.90 and 1.34 GeV we use the data from Novosibirsk[16] along with the Orsay data[18] at 0.91, 0.99 and 1.076 GeV to guide us at the lower end. Above 1.34 GeV our input is based on data[19] from DCI at Orsay, as smoothed by a fit involving both interfering resonance and background contributions.

C. $e^+e^- \to \pi^+\pi^-2\pi^0$

Again we use the Novosibirsk data[16] for this channel up to 1.34 GeV, with earlier Orsay results[18] used to pin down the threshold behavior (0.9 to ~1.0 GeV). Above 1.34 GeV, we turn to the DCI data[19] on the sum of $\pi^+\pi^-2\pi^0$. $2\pi^+2\pi^-2\pi^0$, and $\pi^+\pi^-4\pi^0$. These join on well to the $\pi^+\pi^-2\pi^0$ Novosibirsk data[16] at the lower end.

D. $e^+e^- \to 6\pi$

As just noted, the six pion channels involving π^0's are taken into account along with $\pi^+\pi^-2\pi^0$ from using the DCI data above 1.34 GeV. Direct measurements[20] of $e^+e^- \to 3\pi^+3\pi^-$, as well as diffractive photoproduction,[21] show an effective threshold near 2 GeV.

The input cross sections are summarized in Tables I and II.

We estimate the total error in our calculation due to statistical and systematic errors in the input data to be about ±12%. The largest part of this comes from the $\pi^+\pi^-$ channel below 900 MeV, and is calculated from the statistical errors stated by the Orsay group on the parameters in Eq. (6) combined with their estimates of the systematic errors.[15] That the errors due to the $\pi^+\pi^-$ data dominate the total error is not because the intrinsic statistical or systematic errors in that experiment are particularly large — it is simply that the bulk of the answer comes from that source. Although we have assigned large systematic errors to the multipion data at higher energies, they do not make an important contribution to the overall errors because the magnitude of the multipion contributions is not large and we have added the errors from different channels and energy regions in quadrature.

It is convenient to state our results for $\Gamma(\tau \to \nu_\tau + (2n \text{ pions}))$ in terms of its magnitude relative to that for $\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$. For a τ mass of 1.9 GeV and massless ν_τ, we find a value for this ratio of 1.69. We expect a value of $1.5 \cos^2\theta_c = 1.43$ on the basis of the naive model where $\tau^- \to \nu_\tau + (2n \text{ pions})^-$ is due to $\tau^- \to \nu_\tau + \bar{u}d$, with light $\bar{u}$ and d quarks coming in three colors.[22] Our calculated value is within 20% of this naive result and is even closer to the result obtained with the logarithmic correction due to asymptotic freedom.[23] The contributions to the total result of 1.69 come from individual channels as follows: 1.12 from $\pi^+\pi^-$, 0.22 from $2\pi^+2\pi^-$, and 0.35 from $\pi^+\pi^-2\pi^0$ (plus the six pion channels involving π^0's).

The variation in $\Gamma(\tau^- \to \nu_\tau + (2 \text{ pions})^-)/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$ with M_τ is shown in Fig. 1 ($m_{\nu_\tau} = 0$). There is relatively little variation with M_τ as long as it is in the 1.5 to 2 GeV range.

Similarly, the decay width for nonzero values of m_{ν_τ} (with M_τ fixed at 1.9 GeV) is shown in Fig. 2. Here $\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$ is computed from Eq. (2) with $m_{\nu_\tau} \neq 0$. Only when the neutrino mass exceeds about 600 MeV does one see a fairly sizeable variation in the ratio $\Gamma(\tau \to \nu_\tau + (2n \text{ pions})^-)/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$.

Employing an integrated version of Eq. (5), we can calculate the ratio $\Gamma(\tau^- \to \nu_\tau + \pi^+2\pi^-\pi^o)/\Gamma(\tau^- \to \nu_\tau + \pi^+3\pi^o)$. For a nominal τ mass of 1.9 GeV and a massless τ neutrino this ratio is 4.18, if we assume that in our input data the six pion contribution is negligible compared to that from $\pi^+\pi^-2\pi^o$. In other words, under the same assumption ~ 81% of τ decays involving four pions have three charged prongs. Since $\Gamma(\tau^- \to \nu_\tau + \pi^+2\pi^-\pi^o)/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e) \approx 0.57$, we conclude that $\Gamma(\tau^- \to \nu_\tau + \pi^+2\pi^-\pi^o)/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e) \approx 0.46$

4. Discussion

Using data from electron-positron annihilation, we have calculated the decay rate for $\tau^- \to \nu_\tau + (2n \text{ pions})^-$ which proceeds through the hadronic vector weak current. There is in addition to what we have calculated a small contribution to τ decays coming from the strangeness changing vector current. This contribution is proportional to $\sin^2\theta_c$ and is likely dominated by the $K^*(890)$ in the same way that the ρ dominates the strangeness nonchanging contribution. Assuming $\Gamma(\tau^- \to \nu_\tau + K^*) = \tan^2\theta_c \Gamma(\tau^- \to \nu_\tau + \rho^-)$, we estimate $\Gamma(\tau^- \to \nu_\tau + K^{*-})/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e) = 0.05$.

So, the sum of the τ decay widths to $\nu_\tau + e^-\bar{\nu}_e$, $\nu_\tau + \mu^-\bar{\nu}_\mu$, ν_τ + (2n pions)$^-$, and $\nu_\tau + K^*(890)^-$ is (1 + 0.98 + 1.69 + 0.05) $\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e) = 3.72\ \Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$. This is a lower bound on the total width, and hence we have an upper bound on the branching ratio into $\nu_\tau + e^-\bar{\nu}_e$:

$$BR\left(\tau^- \to \nu_\tau + e^-\bar{\nu}_e\right) \leq \frac{1}{3.72} \quad 0.27 \quad .$$

While this bound applies for M_τ = 1.9 GeV and $m_{\nu_\tau} = 0$, the results of the last section show that it is not sensitive to variations in these masses by several hundred MeV.

The experimental measurements[9] of $BR(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$ are all smaller than our bound and typically less than about 0.2. Most measurements lie in the range 0.15 to 0.20. Since the bound would be saturated if the only τ decays were into $\nu_\tau + e^-\bar{\nu}_e$, $\nu_\tau + \mu^-\bar{\nu}_\mu$ and ν_τ + hadrons through the hadronic axial-vector weak current, we conclude that there must exist other decays. Of course, one does expect decays into ν_τ + hadrons through the hadronic axial-vector weak current. Using our calculation for the vector current contribution we compute that the width for $\tau^- \to \nu_\tau$ + (hadrons)$^-$ arising from the axial-vector current is 2.95, 2.16 and 1.28 times $\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$ for values of the $\tau^- \to \nu_\tau + e^-\bar{\nu}_e$ branching ratio of 0.15, 0.17 and 0.20, respectively.

Part of these decays through the axial-vector current can be calculated from known quantities: $\Gamma(\tau^- \to \nu_\tau\pi^-)$ and $\Gamma(\tau^- \to \nu_\tau K^-)$ just involve the additional knowledge of the pion and kaon decay constants. For M_τ = 1.9 GeV and $M_{\nu_\tau} = 0$ one finds

$$\frac{\Gamma(\tau^- \to \nu_\tau \pi^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 0.54$$

and

$$\frac{\Gamma(\tau^- \to \nu_\tau K^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 0.03 \quad .$$

Assuming that $\nu_\tau \pi^-$ and $\nu_\tau K^-$ decays occur at the predicted rate, we have seen that the total width for $\tau^- \to \nu_\tau$ + (hadrons)$^-$ proceeding through the axial-vector current is much larger. Three must be decays through the axial-vector current other than $\nu_\tau \pi^-$ and $\nu_\tau K^-$. Specifically, taking our calculation of the vector current decays and those through the axial-vector current involving only a π^- or K^-, we still have 2.38, 1.59 and 0.71 times $\Gamma(\tau^- \to \nu_\tau + e^- \bar{\nu}_e)$ for the decay widths $\tau^- \to \nu_\tau$ + (hadrons $\neq \pi^-, K^-$) through the axial-vector weak current when $BR(\tau^- \to \nu_\tau + e^- \bar{\nu}_e)$ is 0.15, 0.17 and 0.20, respectively.

There have been reports[8,9,24] of the decay $\tau^- \to \nu_\tau A_1^- \to \nu_\tau (3\pi)^-$ at roughly the level we are deducing here.[25] Establishing this and the other semihadronic modes of the τ are important; for, if $\tau^- \to \nu_\tau (3\pi)^-$, $\tau^- \to \nu_\tau \pi^-$, and the decays through the vector current do not all occur at the rates discussed above, then the weak current involved in τ decays is not the one responsible for all other weak decays observed until now.

REFERENCES

1. M. L. Perl et al., Phys. Rev. Lett. 35, 1489 (1975); Phys. Lett. 63B, 466 (1976); G. J. Feldman et al., Phys. Rev. Lett. 38, 177 (1976).

2. M. Cavalli-Sforza et al., Phys. Rev Lett. 36, 588 (1976).

3. H. Meyer in Deeper Pathways in High Energy Physics, A. Perlmutter and L. Scott, eds. (Plenum Press, New York, 1977), p. 351; J. Burmester et al., Phys. Lett. 68B, 297 and 301 (1977); G. Knies in Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies, F. Gutbrod, ed. (DESY, Hamburg, 1977), p. 93.

4. R. Brandelik et al., Phys. Lett. 70B, 125 (1977); S. Yamada in Proceedings of the 1977 Interantional Symposium on Lepton and Photon Interactions at High Energies, F. Gutbrod, ed. (DESY, Hamburg, 1977), p. 69.

5. A. Barbaro-Galtieri et al., Phys. Rev. Lett. 39, 1058 (1977); A. Barbaro-Galtieri in Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies, F. Gutbrod, ed. (DESY, Hamburg, 1977), p. 21.

6. J. Kirkby in Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies, F. Gutbrod, ed. (DESY, Hamburg, 1977), p. 3.

7. J. G. Smith et al., Phys. Rev. D18, 1 (1978).

8. M. L. Perl et al., Phys. Lett. 70B, 487 (1977).

9. A review is found in M. L. Perl in Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies, F. Gutbrod, ed. (DESY, Hamburg, 1977),p. 145.

10. H. B. Thacker and J. J. Sakurai, Phys. Lett. 36B, 103 (1971).

11. Y. S. Tsai, Phys. Rev. D4, 2821 (1971).

12. K. Gaemers and R. Raitio, Phys. Rev. D14, 1262 (1976).

13. The $4\pi^0$ channel is forbidden by charge conjugation invariance.

14. This relation may be derived in a particularly simple manner using the method of L. M. Shmushkevich, Dokl. Akad. Nauk SSSR 103, 235 (1955) by demanding equal π^+,π^- and π^0 populations when one averages over $I_Z = -1$, 0 and +1 states with equal weight.

15. D. Benaksas et al., Phys. Lett. 39B, 289 (1972).

16. V. Sidorov, in Proceedings of the XVII International Conference on High Energy Physics, Tbilisi, July, 1976 (Joint Institute for Nuclear Research, Dubna, 1977), Vol. II, p. B13.

17. V. Alles-Borelli et al., Phys. Lett. 40B, 433 (1972); G. Barbiellini et al., Lett. Nuovo Cimento 6, 557 (1973); M. Bernardini et al., Phys. Lett. 44B, 393 and 46B, 261 (1973); D. Bollnin et al., Lett. Nuovo Cimento 14, 418 (1975).

18. G. Cosme et al., Phys. Lett. 40B, 685 (1972) and 63B, 349 (1976).

19. F. Laplanche in Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies, F. Gutbrod, ed. (DESY, Hamburg, 1977), p. 189.

20. B. Bartoli et al., Phys. Rev. D6, 2374 (1972).

21. See B. Knapp in Particles and Fields '76, H. Gordon and R. Peierls, eds. (Brookhaven National Laboratory, Upton, N.Y., 1977), p. A13.

22. The rate for $\tau^- \to \nu_\tau + u\bar{d}$ is three times that for $\tau^- \to \nu_\tau + e^-\bar{\nu}_e$, but half occurs through the vector and half through the axial-vector weak current of the quarks.

23. C. S. Lam and T. M. Yan, Phys. Rev. D16, 703 (1977).

24. J. Jaros, SLAC seminar, October, 1977, and private communication.

25. The experimental values reported, assuming the A_1 decays to $\pi\rho$, are $BR(\tau^- \to \nu_\tau + A_1) = 0.11 \pm 0.04 \pm 0.03$ from Ref. 3 and 0.20 ± 0.08 from Ref. 24. Our range for $BR(\tau^- \to \nu_\tau + (\text{hadrons} \neq \pi^-, K^-))$ through the axial-vector current is 0.14 to 0.36.

TABLE I

Input data from 0.91 to 1.33 GeV

Q(GeV)	(A)	(B)	(C)
0.91	133	0.0	0.0
0.93	115	0.0	0.0
0.95	90.3	0.0	2.0
0.97	68.9	0.0	4.0
0.99	58.7	1.0	6.0
1.01	62.3	1.5	8.0
1.03	49.3	2.0	10.0
1.05	40.6	3.9	7.6
1.07	40.9	3.0	25.5
1.09	41.9	7.0	25.7
1.11	20.0	5.0	18.7
1.13	32.5	5.2	35.1
1.15	37.5	9.6	20.9
1.17	21.1	10.2	29.8
1.19	19.0	10.4	22.5
1.21	21.9	11.7	35.1
1.23	17.2	12.7	30.1
1.25	14.2	15.8	37.1
1.27	10.1	13.6	41.6
1.29	5.7	17.1	19.4
1.31	7.6	19.0	22.6
1.33	4.8	20.0	36.0

(A) = $\sigma(e^+e^- \to \pi^+\pi^-)$ (nb), Ref. 16.

(B) = $\sigma(e^+e^- \to 2\pi^+2\pi^-)$ (nb), Refs. 16, 18.

(C) = $\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)$ (nb), Refs. 16, 18.

TABLE II

Input data from 1.35 to 1.95 GeV

Q(GeV)	(A)	(B)
1.35	23.9	33.7
1.40	28.8	38.7
1.45	36.5	44.2
1.50	45.6	57.3
1.55	54.0	62.3
1.60	40.0	64.8
1.65	34.0	54.7
1.70	30.9	79.2
1.75	26.0	62.3
1.80	19.6	44.2
1.85	16.8	35.8
1.90	15.4	33.7
1.95	14.2	32.5

(A) $\sigma(e^+e^- \to 2\pi^+2\pi^-)$(nb), Ref. 19.

(B) $\sigma(e^+e^- \to \pi^+\pi^-2\pi^o, 2\pi^+2\pi^-2\pi^o, \pi^+\pi^-4\pi^o)$(nb), Ref. 19.

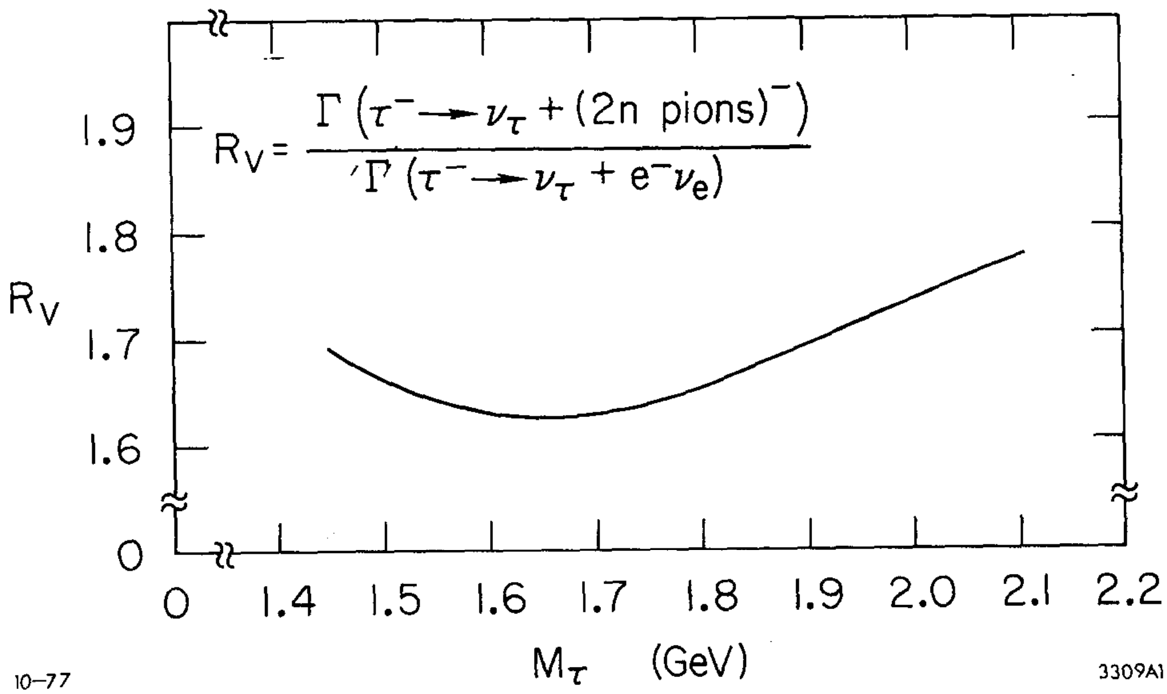

Fig. 1. The ratio, R_V, of the width for $\tau^- \to \nu_\tau + (2n \text{ pions})^-$ proceeding through the hadronic vector weak current, to that for $\tau \to \nu_\tau + e^-\bar{\nu}_e$ as a function of M_τ; $m_{\nu_\tau} = 0$.

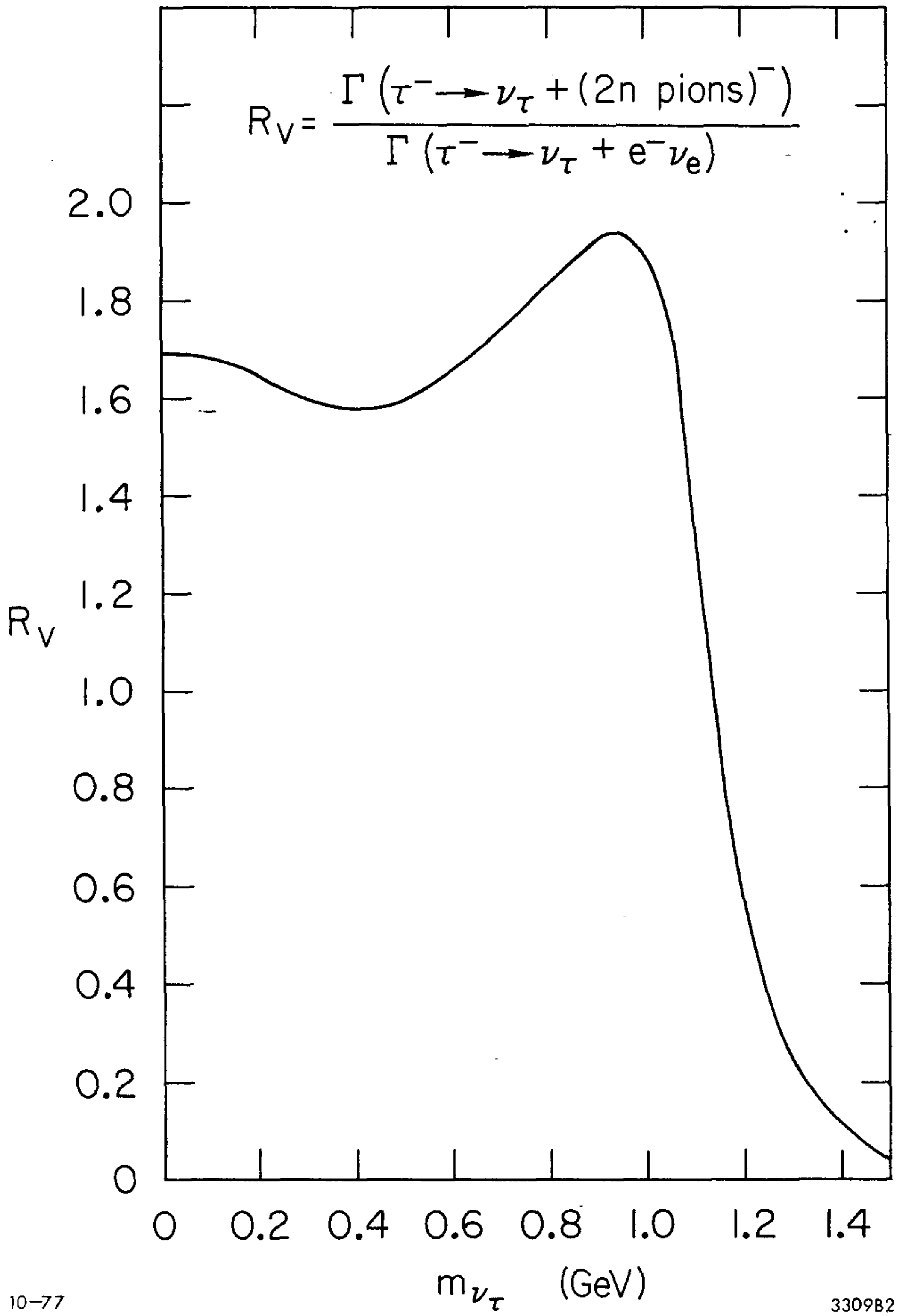

Fig. 2. The dependence of the ratio R_V, as in Fig. 1, on m_{ν_τ} for $M_\tau = 1.9$ GeV. $\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$ is computed from Eq. (2) with the appropriate value of m_{ν_τ}.

CHAPTER III

NONSTANDARD ASSIGNMENTS OF THE τ LEPTON WITHIN $SU(2) \otimes U(1)$

1. Introduction

In the previous chapter, we calculated certain decay rates for the τ lepton[1] on the assumption that the τ was a sequential lepton which fit into the standard SU(2) ⊗ U(1) model just like the electron—i.e., with a weak SU(2) assignment to a right-handed singlet and a left-handed doublet comprised of the τ and a massless neutrino ν_τ.

Although this standard multiplet assignment might be preferred on aesthetic grounds to possible alternatives, one cannot *a priori* rule out alternative multiplet assignments: for example, assignments which involve a heavy neutral partner of the τ, assignments which place the τ in a right-handed doublet, et cetera. In this chapter we systematically consider a number of the more obvious alternative multiplet structures within SU(2) ⊗ U(1) and show that many of these alternatives are inconsistent with currently available experimental data.

Nearly all of these alternatives would be trivially ruled out if mass eigenstates were necessarily identical to weak eigenstates. However, this need not be so; there can be Cabibbo-like mixing in the leptonic sector analogous to the well-known Cabibbo mixing among quarks. Experimental data place constraints on these leptonic Cabibbo angles: for example, an upper limit on the τ lifetime (equivalent to a lower limit on the τ decay width) implies that the τ must have some minimum coupling to a light neutral lepton. Because Cabibbo mixing is constrained to be unitary, mixing involving the τ generally affects mixing of the μ and e multiplets also. Therefore, μ and e physics measurements also place relevant constraints on the leptonic mixing. For example, in multiplet assignments which do not have the GIM mechanism, the experimental limits on $\mu - e$ neutral currents restrict the allowed mixing. In order to fully rule out a proposed multiplet structure, it is necessary to show that it is ruled out for any values of the mixing angles. One does this generally by showing that the various constraints on the mixing due to τ physics and due to $\mu - e$ physics are inconsistent.

The experimentally established facts about the τ which we primarily use for this purpose are:

(1) The τ lifetime is less than 1.4×10^{-12} seconds.[4] This fact, combined with the τ mass[5] of $1782 \pm^{3}_{4}$ MeV and branching ratios for $\tau \to \nu e \bar{\nu}$ and/or $\tau \to \nu\pi$, implies a lower bound on the strength of the τ to ν coupling.

(2) The Michel ρ parameter for the e^- energy spectrum in $\tau \to \nu e \bar{\nu}$ equals[6] 0.72 ± 0.15. This value was deduced taking into account radiative corrections, so as to make it directly comparable to the (nonradiatively corrected) theoretical value which, e.g., is 0.75 in the standard model with a purely V-A current connecting the τ and the ν.

(3) Muon neutrinos, ν_μ, couple to the τ with a strength (coupling squared) which is at most[7] 2.5% of the ν_μ to μ^- coupling.

(4) An upper limit[8] on the sum of branching ratios $B(\tau^- \to e^- e^+ e^-) + B(\tau^- \to e^- \mu^+ \mu^-) + B(\tau^- \to \mu^- e^+ e^-) + B(\tau^- \to \mu^- \mu^+ \mu^-)$ is 0.017.

(5) An upper limit[6] on the mass of the ν in $\tau \to \nu e \bar{\nu}$ is 250 MeV.

As for μ and e physics, our arguments rely particularly on:

(1) The strength of the $\nu_\mu - e^-$ coupling is at most[7] 0.3% of that for ν_μ to μ^-.

(2) $\mu - e$ universality is known to hold to a few percent. Based on the measured $\pi \to \mu\bar{\nu}$ and $\pi \to e\bar{\nu}$ rates, the e to ν exceeds the μ to ν coupling strength[9] by $3.2 \pm 1.9\%$.

(3) Based on the lack of μ to e conversion on nuclei,[10] $\mu - e$ neutral currents are at most 1.2×10^{-8} of full strength neutral currents.[11]

(4) The Michel ρ parameter in μ decay[12] is 0.7518 ± 0.0026. Therefore both e^- and μ^- couple with better than 99% left-handed chirality.

In the following section, we discuss eight models which possess a nonstandard multiplet structure and in which all neutral leptons are either massless or more massive than the τ. We show that only one of these alternative models, a slight variation on the standard model, is consistent with experiment. Then, in the last section we discuss briefly other kinds of models which can be ruled out with present or soon to be available data and state our conclusions.

2. Alternative Models

By allowing complete freedom in the choice of the weak-electromagnetic gauge group, representations of that group, and yet undiscovered charged and neutral leptons, one can produce an infinity of different leptonic models. To avoid this unmanageable situation, one restricts one's attention to a limited number of structures selected on the basis of aesthetic criteria such as simplicity.

In this section we consider models within the standard $SU(2) \otimes U(1)$ gauge group (with gauge bosons W, Z, γ) and with all leptons in SU(2) doublets or singlets. For the e, μ, ν_e and ν_μ we assume the standard multiplet structure

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L ; \quad (e)_R ; \quad (\mu)_R$$

of left-handed weak doublets and right-handed singlets. The left-handed assignments have long been established experimentally. Placing the e^- in a right-handed singlet rather than a doublet is also required, particularly by the polarized electron-nucleon assymetry measurements.[13] Although the assignment of μ_R^- to a singlet is not yet conclusively established by experiment, with the right-handed μ^- in a right-handed doublet experiments would place sharp limits on some of the resulting mixing angles. The right-handed singlet assignment is chosen on the grounds of simplicity.

All leptons are treated as spin-1/2 point Dirac particles. The e, μ and τ (and their antiparticles) are assumed to be the only charged leptons. We take the τ^- to be a lepton and the τ^+ to be an antilepton rather than the other way around.

The different models we consider then differ in their neutral lepton content. We do not add neutral leptons in SU(2) singlets beyond necessity. More precisely, we only include singlet neutral leptons when necessary to allow for a mass for a neutral present in a doublet. There are then nine cases with the τ and new neutral leptons, the mass of which is either zero or greater than the τ mass.

We now proceed to discuss all these models and to briefly give the arguments which show the status of the various models vis-à-vis experiment. In the following we use primes (e', τ', et cetera) to indicate weak eigenstates; unprimed symbols (e, τ, et cetera) to denote mass eigenstates. The symbol ν refers to massless neutral leptons, while N refers to leptons with $m_N \geq m_\tau$.

A. The standard model

The multiplet structure is:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L ; \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L ; \quad (e)_R ; \quad (\mu)_R ; \quad (\tau)_R \ .$$

The neutral leptons are taken as massless and need not have right-handed components. Since the neutrals have the same mass, without loss of generality, one can set all mixing angles to zero; i.e., the weak eigenstates can be defined to be equal to the mass eigenstates. The standard model is consistent with all well-established experimental facts of τ and $\mu - e$ physics.

B. Superfluous heavy neutral model

The multiplet structure is:

$$\begin{pmatrix} \nu'_e \\ e \end{pmatrix}_L ; \quad \begin{pmatrix} \nu'_\mu \\ \mu \end{pmatrix}_L ; \quad \begin{pmatrix} \nu'_\tau \\ \tau \end{pmatrix}_L ; \quad (N'_\tau)_L ; \quad (e'')_R ; \quad (\mu'')_R ; \quad \begin{pmatrix} N_\tau \\ \tau'' \end{pmatrix}_R \ .$$

With all mixing angles negligible or zero, this model differes from the standard model only in the presence of the right-handed (N_τ, τ) doublet. Neutral currents involving the τ will be purely vector as a consequence, and with no mixing this is the handle by which this model eventually might be ruled out.

In general there will be mixing. If one sets intergenerational Higgs couplings (e.g., $\overline{e}\tau\phi$) to zero, only ν_τ and N_τ will mix: $e'' = e$,

$\mu'' = \mu$, $\tau'' = \tau$, $\nu'_e = \nu_e$, $\nu'_\mu = \nu_\mu$. If we further restrict ourselves to singlet and doublet Higgs bosons, the mixing among left-chirality neutrals is

$$\nu'_{\tau L} = \left(\frac{m_\tau}{m_{N_\tau}}\right) N_{\tau L} + \left(1 - \frac{m_\tau^2}{m_{N_\tau}^2}\right)^{1/2} \nu_{\tau L} \quad . \tag{1}$$

For m_{N_τ} close to m_τ, the $\tau - \nu_\tau$ coupling strength is reduced and the lifetime gets longer, with the experimental limit becoming relevant. But with $m_{N_\tau} \to \infty$ (or triplet Higgs), the mixing becomes negligible. This situation cannot be ruled out by current experiment.

C. Economy model

The multiplet structure is:

$$\begin{pmatrix} \nu_e \\ e' \end{pmatrix}_L \; ; \; \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L \; ; \quad (\tau')_L \; ; \; (e)_R \; ; \; (\mu)_R \; ; \; (\tau)_R \quad .$$

The name, "economy model," is due to Cabibbo[14] and refers to the lack of new neutral leptons. If all mixing angles equal zero, the model is trivially ruled out as then the τ does not decay.

In general, however, the τ can mix into the μ and e doublets on the left:

$$\begin{aligned} e'_L &\cong \left(1 - \frac{\varepsilon_e^2}{2}\right) e_L + \varepsilon_e \tau_L \\ \mu'_L &\cong \left(1 - \frac{\varepsilon_\mu^2}{2}\right) \mu_L + \varepsilon_\mu \tau_L - \varepsilon_e \varepsilon_\mu e_L \quad , \end{aligned} \tag{2}$$

and thereby is allowed to decay by coupling to ν_e and ν_μ. Here and in later models, we have expanded the exact expressions to lowest significant order in ε_e, ε_μ; we also ignore possible complex phases which do not affect the phenomenology we are considering. We have found that more careful analysis which avoids these approximations yields the same results.

With mixing there are $\tau-\mu$ and $\tau-e$ neutral currents, and their consequences in terms of $\Gamma(\tau\to\nu e\bar{\nu})\neq\Gamma(\tau\to\nu\mu\bar{\nu})$ and $\tau\to$ three charged leptons were used by Altarelli et al.,[11] and Horn and Ross,[15] respectively, to rule out the model. Here we simply note that the width for the purely charged current process $\tau\to\nu\pi$ would be[16]

$$\begin{aligned}\Gamma(\tau\to\nu\pi) &= \left(\varepsilon_e^2+\varepsilon_\mu^2\right)\frac{\left(f_\pi\cos\theta_c G_F\right)^2}{16\pi}\, m_\tau^3\left(1-\frac{m_\pi^2}{m_\tau^2}\right)^2 \\ &= \frac{\varepsilon_e^2+\varepsilon_\mu^2}{2.64\times10^{-12}\ \text{sec}}\end{aligned} \tag{3}$$

and the measured[17] $B(\tau\to\nu\pi) = 11.7\pm2.2\%$ and lifetime limit[4] yield

$$\varepsilon_e^2+\varepsilon_\mu^2 > 0.18 \quad . \tag{4}$$

But the limit on $\mu-e$ conversion[10] requires that:

$$\varepsilon_e^2\varepsilon_\mu^2 < 1.2\times10^{-8} \tag{5}$$

while $\mu-e$ universality[9,17] implies:

$$\varepsilon_\mu^2-\varepsilon_e^2 = 0.032\pm0.019 \quad . \tag{6}$$

Therefore

$$\varepsilon_e^2 + \varepsilon_\mu^2 = \left[\left(\varepsilon_\mu^2 - \varepsilon_e^2\right)^2 + 4\varepsilon_e^2\varepsilon_\mu^2\right]^{1/2} < 0.070 \quad , \qquad (7)$$

contradicting Eq. (4). The model is inconsistent with experiment for any values of the mixing angles.

D. Left-handed heavy neutral model

The multiplet structure is:

$$\begin{pmatrix} \nu_e \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L ; \begin{pmatrix} N_\tau \\ \tau' \end{pmatrix}_L ; (e)_R ; (\mu)_R ; (\tau)_R ; \left(N_\tau\right)_R \quad .$$

As in the economy model, the τ can decay only if there is mixing. We define ε_e, ε_μ as in Eq. (2). Equations (4) and (6) still hold but we no longer have a constraint on $\varepsilon_e^2\varepsilon_\mu^2$ as this model possesses a GIM mechanism[3] which prevents lepton-family-changing neutral currents. Still, the experimental limit[7] on ν_μ production of τ requires:

$$\varepsilon_\mu^2 \le 0.025 \quad , \qquad (8)$$

which when combined with Eq. (6) implies[18]

$$\varepsilon_e^2 + \varepsilon_\mu^2 = -\left(\varepsilon_\mu^2 - \varepsilon_e^2\right) + 2\varepsilon_\mu^2 < 0.056 \qquad (9)$$

again contradicting Eq. (4) and ruling out the model.[19]

E. Right handed τ doublet model

The multiplet assignment is:

$$\begin{pmatrix} \nu_e \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L ; (\tau')_L ; (e'')_R ; (\mu'')_R ; \begin{pmatrix} \nu_\tau \\ \tau'' \end{pmatrix}_R .$$

In the absence of mixing this model is consistent with all experimental facts except that the right-handed charged current coupling of τ to ν_τ makes the Michel parameter $\rho = 0$ in contrast to the experimental[6] value 0.72 ± 0.15. The question is whether the mixing on the left can be made large enough within the other experimental constraints to get an acceptable value of ρ.

We parametrize e', μ' and τ' as in Eq. (2) with parameters $\varepsilon_{\mu L}$ and ε_{eL}, and similarly on the right for e'', μ'' and τ'' with $\varepsilon_{\mu R}$ and ε_{eR}. The limit from $\mu - e$ conversion[10] now implies

$$\left(\varepsilon_{eL}\varepsilon_{\mu L}\right)^2 + \left(\varepsilon_{eR}\varepsilon_{\mu R}\right)^2 \leq 1.2 \times 10^{-8} \quad . \tag{10}$$

Furthermore, the ρ parameter measurements from μ decay indicate that

$$\varepsilon^2_{eR} + \varepsilon^2_{\mu R} < 0.01 \quad . \tag{11}$$

The restriction from $\mu - e$ universality is instead of Eq. (6)

$$\varepsilon^2_{\mu L} - \varepsilon^2_{eL} = 0.032 \pm 0.019 \pm 0.01 \quad , \tag{12}$$

with the extra ± 0.01 due to possible $\mu - \nu_\tau$ or $e - \nu_\tau$ right-handed couplings. Combining[18] Eqs. (10) and (12)

$$\varepsilon^2_{\mu L} + \varepsilon^2_{eL} = \left[\left(\varepsilon^2_{\mu L} - \varepsilon^2_{eL}\right)^2 + 4\varepsilon^2_{\mu L}\varepsilon^2_{eL}\right]^{1/2} \leq 0.08 \quad . \tag{13}$$

The ρ parameter in $\tau \to \nu e \bar{\nu}$, including neutral current contributions, is

$$\rho = \frac{3}{4} \frac{\frac{3}{4}\varepsilon^2_{eL} + \varepsilon^2_{\mu L} + \frac{3}{4}\varepsilon^2_{eR}}{\frac{3}{4}\varepsilon^2_{eL} + \varepsilon^2_{\mu L} + \frac{3}{4}\varepsilon^2_{eR} + \left(1 - \varepsilon^2_{eR} - \varepsilon^2_{\mu R}\right)} < 0.066 \quad . \tag{14}$$

Equation (14) is still inconsistent with experiment,[6] and the model is ruled out in general.

F. Ambidextrous τ model

The multiplet structure is:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L ; \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L ; (e'')_R ; (\mu'')_R ; \begin{pmatrix} \nu_\tau \\ \tau'' \end{pmatrix}_R \quad .$$

Withoug mixing, the $\tau - \nu_\tau$ current is pure vector, resulting in a ρ parameter of 0.375. Even with mixing, the τ'' must be more than 99% τ because of the constraints on right-handed currents in muon decay. Neutral current contributions on the right are negligible compared to charged current contributions and therefore in $\tau \to \nu e \bar{\nu}$,

$$\rho = \frac{3}{4} \frac{1}{1 + \left(1 - \varepsilon^2_{eR} - \varepsilon^2_{\mu R}\right)} \leq 0.38 \quad , \tag{15}$$

which is more than two standard deviations from the measured[5] $\rho = 0.72 \pm 0.15$ and the model is ruled out in general.

Note that if we limit ourselves to singlet and doublet Higgs we have a mass relationship

$$m_{\nu_\tau}^2 = \varepsilon_{eR}^2 m_e^2 + \varepsilon_{\mu R}^2 m_\mu^2 + \left(1 - \varepsilon_{eR}^2 - \varepsilon_{\mu R}^2\right) m_\tau^2 \quad . \tag{16}$$

This follows from the Higgs coupling to $\overline{\ell}_L \tau_R$ being equal to that for $\overline{\nu}_{\ell L} \nu_{\tau R}$ for $\ell = e, \mu$ or τ. If $m_{\nu\tau} << m_\tau$, then the chirality constraint in $\mu \to \nu e \overline{\nu}$ demands small $(\varepsilon_{eR})^2$ and $(\varepsilon_{\mu R})^2$. But then Eq. (16) makes $m_{\nu_\tau}^2 \approx m_\tau^2$, contradicting our assumption of small m_{ν_τ}. Therefore, with singlet and doublet Higgs the requirement that $m_{\nu_\tau} << m_\mu$ cannot be met.

G. Heavy ambidextrous model

The multiplet structure is:

$$\begin{pmatrix} \nu_e \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L ; \begin{pmatrix} N_\tau \\ \tau' \end{pmatrix}_L ; \quad (e'')_R \; ; \quad (\mu'')_R \; ; \begin{pmatrix} N_\tau \\ \tau'' \end{pmatrix}_R \quad .$$

The argument used to rule out the heavy neutral model ("D" above) also rules out this model.

H. Backward heavy neutral model

The multiplet structure is:

$$\begin{pmatrix} \nu_e' \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu' \\ \mu' \end{pmatrix}_L ; \quad (\tau')_L \; ; \quad (N_\tau')_L \; ; \; (e'')_R \; ; \; (\mu'')_R \begin{pmatrix} N_\tau \\ \tau'' \end{pmatrix}_R \quad .$$

We define the mixing of the charged leptons as before and parametrize the neutrals on the left by

$$\nu'_{eL} \approx \left(1 - \frac{\delta_e^2}{2}\right)\nu_{eL} + \delta_e N_{\tau L} - \delta_e \delta_\mu \nu_{\mu L} \quad ,$$

$$\nu'_{\mu L} \approx \left(1 - \frac{\delta_\mu^2}{2}\right)\nu_{\mu L} + \delta_\mu N_{\tau L} \quad . \tag{17}$$

The mixing among left-handed neutrals prevents us from ruling out this model as we did the economy model. For, $\mu - e$ universality can be enforced even with very unequal mixing of the e and μ on the left by introducing compensating mixing ($\delta_e^2 - \delta_\mu^2 = \varepsilon_{\mu L}^2 - \varepsilon_{eL}^2$) of the neutral leptons, and then we lack a restriction on $\varepsilon_{eL}^2 - \varepsilon_{\mu L}^2$. In general the massless components of ν'_{eL} and $\nu'_{\mu L}$ are not even orthogonal, but the experimental limit of 0.3% on ν_μ production of electrons[7] requires

$$\left(\delta_e \delta_\mu\right)^2 < 0.003 \quad , \tag{18}$$

so that the nonorthogonality is very small.

We still have that the $\tau \to \nu\pi$ charged current decay only occurs through charged lepton mixing on the left and as in Eq. (4)

$$\varepsilon_{eL}^2 + \varepsilon_{\mu L}^2 > 0.18 \quad .$$

The limit[7] on ν_μ production of the τ implies $\varepsilon_{\mu L}^2 < 0.025$, so $\varepsilon_{eL}^2 > 0.155$. Then the very small limit on $\mu - e$ conversion, $\varepsilon_{eL}^2 \varepsilon_{\mu L}^2 < 1.2 \times 10^{-8}$, forces

$$\varepsilon_{\mu L}^2 < 10^{-7}$$

so that the τ mixes on the left almost entirely with the electron.

Now the process $\tau \to \nu e \bar{\nu}$ in this model can be either $\tau \to e \nu_\mu \bar{\nu}_\mu$ (through neutral currents alone because the τ and μ don't mix to the 10^{-7} level on the left) or $\tau \to \nu_e e \bar{\nu}_e$ (through charged or neutral currents). Relative to the standard model the rate for the first process, $\tau \to e\nu_\mu\bar{\nu}_\mu$, is

$$\left(\frac{\varepsilon_{eL}^2}{4} + \frac{\varepsilon_{eR}^2}{4}\right)\left(1 - \delta_\mu^2\right) \quad ,$$

while for the second process, $\tau \to \nu_e e\bar{\nu}_e$, the rate is

$$\left(\frac{\varepsilon_{eL}^2}{4} + \frac{\varepsilon_{eR}^2}{4}\right)\left(1 - \delta_e^2\right) \quad .$$

The rate for the neutral current process $\tau \to e\mu\bar{\mu}$ for the left-handed muons alone is, in the same units,

$$\left(\frac{\varepsilon_{eL}^2}{4} + \frac{\varepsilon_{eR}^2}{4}\right)\left(1 - 2\sin^2\theta_W\right)^2 \quad .$$

Therefore

$$\frac{\Gamma(\tau \to e\mu\bar{\mu})}{\Gamma(\tau \to e\nu\bar{\nu})} \geq \frac{\left(1 - 2\sin^2\theta_W\right)^2}{2 - \delta_e^2 - \delta_\mu^2} \geq 0.25 \qquad (20)$$

using[20] $\sin^2\theta_W < 0.25$. Experimentally this last ratio[8] is $< (3.3 \times 10^{-4}) / (16.5 \pm 1.5 \times 10^{-2}) = 0.03$, and the model is ruled out in general.

I. Heavy left-light right model

The multiplet structure is:

$$\begin{pmatrix}\nu_e \\ e'\end{pmatrix}_L ; \begin{pmatrix}\nu_\mu \\ \mu'\end{pmatrix}_L ; \begin{pmatrix}N_\tau \\ \tau'\end{pmatrix}_L ; (e'')_R ; (\mu'')_R ; \begin{pmatrix}\nu''_\tau \\ \tau''\end{pmatrix}_R ; N''_{\tau R} \quad .$$

Without mixing, the model involves a purely right-handed $\tau - \nu_\tau$ coupling and therefore a predicted ρ parameter in $\tau \to \nu e\bar{\nu}$ which disagrees with experiment. There can be little (< 1%) coupling of the e and/or μ on the right to ν_τ through mixing because of the ρ parameter in μ decay. On the left, $\mu - e$ universality and the limits on ν_μ production of τ then imply[21]

$$\varepsilon_{eL}^2 + \varepsilon_{\mu L}^2 = 2\varepsilon_{\mu L}^2 - \left(\varepsilon_{\mu L}^2 - \varepsilon_{eL}^2\right) \le 0.05 - (0.032 \pm 0.019 \pm 0.01) \le 0.066 \quad . \tag{21}$$

But the lifetime limit and branching ratio for $\tau \to \nu\pi$ require that the total coupling strength of the τ to a neutrino be > 18% of full strength. The coupling strength on the right must then be > 11.4% of full strength.

The fact that the charged-current coupling of the τ to a neutrino is predominantly right-handed cannot yet be directly translated into a limit on the ρ parameter in $\tau \to \nu e \bar{\nu}$; for both neutral and charged currents are involved in this process. Fortunately, the contribution to $\tau \to \nu e \bar{\nu}$ from neutral currents (which all involve a right-handed τ) and from the charged current contribution to $\tau_R \to e\nu_\tau\bar{\nu}_\tau$ can be related to $\tau \to e\mu\mu$:[22]

$$\Gamma\left(\tau_R \to e_R\nu_e\bar{\nu}_e\right) \quad \Gamma\left(\tau_R \to e_R\bar{\nu}_\mu\nu_\mu\right) \quad \Gamma\left(\tau_R \to e_R\nu_\tau\bar{\nu}_\tau\right)$$

$$\le \frac{3}{\left(1 - 2\sin^2\theta_W\right)^2} \Gamma\left(\tau_R \to e_R\mu_L\bar{\mu}_L\right) \le 12\Gamma(\tau \to e\mu\bar{\mu}) \le 4 \times 10^{-3}\Gamma(\tau \to \text{all}) \quad . \tag{22}$$

These contributions to $\tau \to \nu e \bar{\nu}$ then can be neglected. We are left with the charged current induced processes $\tau_R \to \nu_R e_L \bar{\nu}_L$ and $\tau_L \to \nu_L e_L \bar{\nu}_L$, for which the bounds on charged current couplings derived above imply that

$$\rho \leq \frac{3}{4} \frac{0.066}{0.066 + 0.114} = 0.275 \quad , \qquad (23)$$

in contradiction with the measured $\rho = 0.72 \pm 0.15$.

3. Discussion of More General Cases

The nine models above are the only models which meet the criteria stated at the beginning of Section 2. We now relax some of these criteria, producing more general classes of models. We will not present detailed arguments for each of these models, but will simply discuss results and the key facts that lead to them.

The multiplet structure for the electron and muon sectors remains as before. For the sake of brevity from here on we only list the τ sector multiplet structure.

A. Intermediate mass neutrals

If we relax the constraint that the neutral leptons associated with the τ be either zero-mass neutrinos or have a mass greater than that of the τ, we have seven additional models with the following multiplet structures (L^o denotes an intermediate mass neutral lepton, $0 < m_{L^o} < m_\tau$):

$$1'. \quad \begin{pmatrix} L^o \\ \tau' \end{pmatrix}_L \; ; \quad (\tau')_R \; ; \quad (L^o)_R$$

$$2'. \quad (\tau')_L \; ; \quad (L^{o\prime})_L \; ; \quad \begin{pmatrix} L^o \\ \tau' \end{pmatrix}_R$$

3'. $\begin{pmatrix} L^o \\ \tau' \end{pmatrix}_L$; $\begin{pmatrix} L^o \\ \tau'' \end{pmatrix}_R$

4'. $\begin{pmatrix} L^o \\ \tau' \end{pmatrix}_L$; $\begin{pmatrix} \nu''_\tau \\ \tau'' \end{pmatrix}_R$: $(L^{o''})_R$

5'. $\begin{pmatrix} \nu'_\tau \\ \tau \end{pmatrix}_L$; $(L^{o'})_L$; $\begin{pmatrix} L^o \\ \tau'' \end{pmatrix}_R$

6'. $\begin{pmatrix} L^{o'} \\ \tau \end{pmatrix}_L$; $(N'_\tau)_L$; $\begin{pmatrix} N''_\tau \\ \tau'' \end{pmatrix}_R$; $(L^{o''})_R$

7'. $\begin{pmatrix} N'_\tau \\ \tau \end{pmatrix}_L$; $(L^{o'})_L$; $\begin{pmatrix} L^{o''} \\ \tau'' \end{pmatrix}_R$; $(N''_\tau)_R$.

Strictly speaking we cannot use the chirality constraints or mass limits derived from $\tau \to \nu e \bar{\nu}$ as they exist in published form to rule out these models. The chirality constraint was derived on the basis of the assumption that the τ couples only to a massless neutral lepton in $\tau \to \nu e \bar{\nu}$ —— an assumption violated in these models. Similarly, the mass limit, $m_{L^o} \leq 250$ MeV, was derived under the assumption that the τ couples to only one neutral lepton lighter than the τ; in these models, on the contrary, the τ can, via mixing, couple both to L^o and to ν_e, ν_μ or ν_τ.

However, we believe on the basis of qualitative heuristic arguments that an appropriate reanalysis of the raw unpublished data would produce constraints sufficient to rule out most of these models for most values of m_{L^o}. For example, we expect the chirality constraint to be stronger for higher neutral lepton masses. The published value of the ρ parameter corresponds to a "hard" electron energy spectrum. Both adding in a right-handed (V+A) chirality component to the $\tau - L^0$ current and raising the mass of L^0 tend to "soften" this spectrum. Thus, the higher m_{L^0} is, the less (V+A) component one can include and still produce the "hard" spectrum observed experimentally.

If m_{L^0} is sufficiently close to m_τ, the phase space for any decay involving L^0 will be negligible and the model will be functionally equivalent to a model in which L^0 is replaced by a heavy N_τ. For example, in model 1', as in the left-handed heavy neutral model, the coupling to ν_e and ν_μ is constrained to be < 0.056. If m_{L^0} > 1.0 GeV, the phase space for $\tau \to L^0 e^- \bar{\nu}_e$ is less than 10% of the phase space for the zero-mass case. The total rate for $\tau \to \nu e^- \bar{\nu}_e$ plus $\tau \to L^0 e^- \bar{\nu}_e$ would then be inconsistent with the experimental limit on the lifetime.[4] A similar argument for model 3' shows that m_{L^0} > 1100 MeV in that model.

In the limit that m_{L^0} is very close to m_τ, 1', 2', 3' and 4' are ruled out. Models 6' and 7' are allowed in this limit if one is willing to accept an apparent $G_{F\ \mathrm{lepton}}$ substantially less than the standard-model $G_{F\ \mathrm{quark}}$ (one could avoid this discrepancy by introducing an appropriate nonstandard multiplet structure in the quark sector also). In this limit, model 5' is functionally equivalent to the superfluous heavy neutral model and is therefore allowed.

If m_{L^0} is very close to zero, the models are functionally equivalent to models obtained by replacing L^0 by a massless neutrino. Cases 1' and 6' will then be allowed; all others are inconsistent with experiment.

One should also note that, unless mixing is appropriately restricted, L^0 can decay (into, e.g., $e^+e^-\nu_e$) if $m_{L^0} > 2m_e$ in these models. This provides further constraints on these models. Other authors have used astrophysical considerations to constrain the number of neutrinos and their masses.[23]

B. Models with extra neutrals

To all the above models one can add extra neutral leptons, which must be in singlets if one does not add extra charged leptons. If one adds massless neutrinos to the economy model, right-handed τ doublet model, or ambidextrous τ model, they are still ruled out. In these models the mixing with the extra neutrinos may be defined away by a redefinition of the neutrino mass eigenstates. The backward heavy neutral model with extra neutrinos can also be ruled out. The other five models with extra neutrinos reduce to the standard model or to the superfluous heavy neutral model for appropriate values of the mixing angles and are therefore allowed.

If instead one adds more heavy neutral leptons ($m \geq m_\tau$), the economy model, backward heavy neutral model and right-handed τ doublet model are still ruled out. The left-handed heavy neutral model, heavy ambidextrous model and heavy left-light right model with extra heavy neutrals can be made consistent with experimental lepton data by appropriate mixing with the heavy neutrals to preserve μ - e universality; however, the apparent $G_{F\ \text{lepton}}$ will differ substantially from $G_{F\ \text{quark}}$. The other three models with extra heavy neutrals are allowed for appropriate values of the mixing angles.

If one adds both neutrinos and neutrals with mass $\geq m_\tau$, the economy model, right-handed τ model and backward heavy neutral model will be ruled out; the other six models will be allowed.

If one adds neutrals with intermediate mass, one has the difficulties discussed above.

C. Anomalous τ lepton number

If there is no mixing among lepton generations, it is a matter of convention whether the τ^+ or τ^- be considered lepton or antilepton. With intergenerational mixing, however, the distinction is real: is it the neutral partner of the τ^- or the τ^+ which mixes with ν_e, ν_μ?

For example, in the left-handed heavy neutral model, the multiplet structure would be

$$\begin{pmatrix} \tau^+ \\ N_\tau \end{pmatrix}_R \; ; \quad (\tau^+)_L \; ; \quad (N_\tau)_L \quad .$$

τ^+ could not mix with e^- or μ^-, and mixing among the neutral leptons would violate GIM (since I_3 differs for N_τ and ν_e or ν_μ). (Note that we require τ^+ in a right-hand doublet so that τ^- will be in a left-hand doublet.)

If one similarly reverses the τ lepton number in our canonical nine models, the conclusions will not change — all but the standard model and the superfluous heavy neutral model will be ruled out.

D. Other possibilities

Placing the μ in a right-handed doublet with a heavy neutral partner alters some of the above results: for appropriate mixing angles, the ambidextrous τ model would be allowed. The left-handed heavy neutral model, heavy ambidextrous model and heavy left-light right model would also be allowed if one will accept $G_{F\ \mathrm{lepton}} \neq G_{F\ \mathrm{quark}}$. (The chirality structure of the $\mu - \mu$ neutral current, and hence the right-handed assignment of the μ will be probed by currently planned experiments.)

One need not restrict oneself to $SU(2) \otimes U(1)$ doublets and singlets, e.g., one could assign the τ to:

$$\begin{pmatrix} L^+ \\ \nu_\tau \\ \tau^- \end{pmatrix}_L \; ; \; \left(L^+\right)_R \; ; \; \left(\tau^-\right)_R \quad .$$

Or one could abandon the conventional $SU(2) \otimes U(1)$ gauge-theoretic framework altogether.

Although these may be real possibilities, we will not consider them here. We have also not discussed the interesting phenomena encompassed in "neutrino oscillations."

4. Conclusion

We have discussed the standard model and eight plausible variations involving the τ $SU(2) \otimes U(1)$ multiplet structure and have shown that all but two of these models are inconsistent with experiment.

Until $\tau - \tau$ neutral currents are measured, it will apparently not be possible to discriminate between these two possibilities.

We have briefly discussed wider classes of models beyond the nine canonical models. For the most part, these models appear to be ruled out except when they are essentially equivalent to our two allowed canonical models.

While existing information from experiment does not uniquely require the standard model, it does rule out the bulk of the simple alternatives and justifies a strong prejudice in favor of the standard model.

FOOTNOTES AND REFERENCES

1. M. L. Perl et al., Phys. Rev. Lett. 35, 1489 (1975); M. L. Perl et al., Phys. Lett. 63B, 466 (1976); G. J. Feldman et al., Phys. Rev. Lett. 38, 177 (1976).

2. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam in Elementary Particle Physics: Relativistic Groups and Analyticity (Nobel Symposium No. 8), ed., N. Svartholm (Almquist and Wiksell, Stockholm, 1968), p. 367.

3. S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970).

4. TASSO Collaboration, R. Brandelik et al., Phys. Lett. 92B, 199 (1980).

5. W. Bacino et al., (DELCO), Phys. Rev. Lett. 41, 13 (1978); J. Kirkby in Proceedings of the Ninth International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, 1979, eds. T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia, Illinois, 1980), p. 107.

6. W. Bacino et al., Phys. Rev. Lett. 42, 749 (1979).

7. A. M. Cnops et al., Phys. Rev. Lett. 40, 144 (1978).

8. K. Hayes, Ph.D. thesis, Stanford University, 1980 (unpublished).

9. E. di Capua et al., Phys. Rev. 133, B1333 (1964); D. Bryman and C. Piccioto, Phys. Rev. D11, 1337 (1975).

10. D. A. Bryman, M. Blecher, K. Gotow and R. J. Powers, Phys. Rev. Lett. 28, 1469 (1972).

11. G. Altarelli, N. Cabibbo, L. Maiani and R. Petronzio, Phys. Lett. 67B, 463 (1977). We used Eqs. (12),(13) in Altarelli *et al.*; however, rather than take $\sin^2\theta_W \sim 0.3$, we have conservatively taken $\sin^2\theta_W \geq 0.20$ which weakens the bound.

12. Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

13. C. Y. Prescott *et al.*, Phys. Lett. 84B, 524 (1979).

14. N. Cabibbo-in *The Ways of Subnuclear Physics*, ed. A. Zichichi (Plenum Press, New York, 1979), p. 691.

15. D. Horn and G. Ross, Phys. Lett. 67B, 460 (1977).

16. One could also use $\tau \rightarrow \rho\nu$ or $\tau \rightarrow \mu\bar{\nu}\nu$ or $\tau \rightarrow e\bar{\nu}\nu$ to convert the lifetime limit to a limit on the $\nu - \tau$ coupling at this point rather than $\tau \rightarrow \pi\nu$. However, these alternatives would be less clean: e.g., $\tau \rightarrow e\bar{\nu}\nu$ can involve both charged and neutral currents.

17. C. A. Blocker, Ph.D. thesis, Lawrence Berkeley Laboratory LBL-10801 (Berkeley, 1980).

18. Here and elsewhere, we conservatively take two standard deviations from the central value in deriving the inequality.

19. This result has been previously reported by F. J. Gilman in *Proceedings of the Kyoto Summer Institute for Particle Physics*, eds. M. Ida, T. Kamae and M. Kobayashi (Research Institute for Fundamental Physics, Kyoto University, Kyoto, 1978), p. 85, reporting on work by D. H. Miller and F. J. Gilman; and by G. Altarelli at the 1978 Tokyo Conference (G. Altarelli in *Proceedings of the 19th International Conference on High*

Energy Physics, eds. S. Homma, M. Kavaguchi and H. Miyazawa (Physical Society of Pana, 1979), p. 411).

20. P. Langacker et al., in Neutrino '79, eds. A. Haatuft and C. Jarlskog (University of Bergen, Bergen, 1980), vol. 1, p. 276.

21. As in Eq. (12), the extra 0.01 is due to possible (but necessarily small because of the constraint from the μ-decay ρ parameter) e - ν or μ — ν right-handed couplings.

22. $\tau_R \to e_R \nu_\tau \bar{\nu}_\tau$ involves both charged and neutral currents which interfere destructively so as to produce the same total rate as would be produced by the neutral current amplitude alone.

23. E. W. Kolb and T. Goldman, Phys. Rev. Lett. 43, 897 (1979), and references cited therein.

CHAPTER IV

τ DECAYS WITH SPIN-3/2 τ AND ν_τ AND PECULIARITIES OF MASSLESS RARITA-SCHWINGER PARTICLES

1. Introduction

The discussions of the τ lepton[1] in the previous two chapters stayed within the general framework of the standard $SU(2) \otimes U(1)$ electroweak gauge theory[2] with the τ being a spin-1/2 point Dirac particle. We have seen that the experimental evidence is consistent with this picture with the τ in a right-handed SU(2) singlet and the τ and a massless ν_τ in a left-handed doublet.

A priori, there are a number of possible alternatives to this conventional picture: para- or ortho-leptons, nonstandard multiplet assignments of τ and ν_τ, et cetera.[3] In this paper, we consider the possibility that τ and ν_τ both have spin 3/2.

The possibility that τ and/or ν_τ have spin 3/2 has been raised in the past.[4] For example, the initial difficulty in observing the decay mode $\tau \to \pi\nu_\tau$ produced the suggestion that ν_τ has spin 3/2 and τ has spin 1/2: with a massless ν_τ restricted to helicities of $\pm 3/2$, $\tau \to \pi\nu_\tau$ would then be strictly forbidden by helicity conservation. (Subsequent observation of $\tau \to \pi\nu_\tau$ therefore rules out this possibility.)

Tsai[6] has argued that if the τ has spin 3/2, the behavior of the cross section $\sigma(e^+e^- \to \tau^+\tau^-)$ would be inconsistent with experiment. However, Kane and Raby[7] have suggested possible subterfuges by which nature might evade Tsai's argument; they therefore hold that the possibility that τ and ν_τ both have spin 3/2 remains open.

Alles[8] claims to dispose of this possibility by showing that spin-3/2 τ and ν_τ imply branching ratios and an electron energy spectrum in $\tau \to \nu_\tau e^- \bar{\nu}_e$ that are inconsistent with experiment; however, as

Kane and Raby have pointed out, Alles fails to consider the most general V,A current that can be constructed from spin-3/2 τ and ν_τ.

In this paper, we assume, as does Alles, that the τ decay amplitude is of the current-current form $J^\mu_{(\tau - \nu_\tau)} \cdot J_{\mu(\text{other})}$, where $J_{\mu(\text{other})}$ is the standard V,A current which has been observed in other weak-interaction processes involving e, μ, hadrons, et cetera. Unlike Alles, we consider the most general form for $J^\mu_{(\tau - \nu_\tau)}$ which is consistent with proper Lorentz invariance for τ and ν_τ spins of 3/2.

The nonexistence of a renormalizable field theory for fundamental point-like spin-3/2 particles might be thought to rule out consideration of spin-3/2 leptons. However, as Kane and Raby suggest, τ and ν_τ might be composite particles with spins of 3/2; then, the fundamental constituent particles which make up the spin-3/2 τ and ν_τ could have spins less than 3/2. The fundamental interaction involving these constituent particles would not then involve spin-3/2 particles and could therefore be renormalizable. Of course, even though the fundamental theory would be renormalizable, the effective low-energy form of the interaction involving the composite spin-3/2 particles would not necessarily be renormalizable. However, one would still expect that the low-energy phenomenological amplitudes involving the spin-3/2 composite particles could be expressed in a current-current form with one current involving only the spin-3/2 composite τ and ν_τ and the other current involving the other particles participating in the reaction. An analogous situation presumably occurs in the weak decay $\Delta^+ \to \Delta^{++} e^- \bar{\nu}_e$. Although the fundamental (renormalizable) interaction presumably involves spin-1/2 quarks, one expects the phenomenological amplitude to be of the form $J^{\mu}_{(\Delta^{++}-\Delta^{+})} \cdot J_{\mu(e-\nu_e)}$ where $J^{\mu}_{(\Delta^{++}-\Delta^{+})}$ is constructed of Rarita-Schwinger spinors[9] representing the two spin-3/2 particles.

Since spin-3/2 τ and ν_τ might well be composite, one must allow nonconstant form factors, analogues of a Pauli anomalous magnetic moment term, et cetera, in $J^{\mu}_{(\tau-\nu_\tau)}$. Just as the p - n weak current is not the simple V-A current of point particles, so one should not expect $J^{\mu}_{(\tau-\nu_\tau)}$ for composite particles to have the simplest conceivable form. (In fact, for spin-3/2 particles, it is difficult to decide which current is the "simplest conceivable.")

Allowing the most general $J^{\mu}_{(\tau-\nu_\tau)}$ with arbitrary form-factors, we find that Alles' conclusions ruling out spin-3/2 τ and ν_τ cannot be sustained: τ decays involving spin-3/2 τ and ν_τ can be made indistinguishable from the spin-1/2 case so long as one does not measure the τ or ν_τ spin or helicity.

Before discussing the general V,A currents for spin-3/2 τ and ν_τ and their applications to τ decay in Section 4, we first discuss in the next section an apparent discontinuity in the $M_{\nu_\tau} \to 0$ limit of certain currents (and total rates) involving a spin-3/2 ν_τ. This discontinuity is related to the problem of the helicity states allowed to a massless particle. In Section 3, this problem is reviewed with emphasis on two theorems due to Wigner and Weinberg. It is concluded that one may eliminate the discontinuity discussed in Section 2 by allowing states corresponding to nonmaximal helicities of a massless ν_τ. In light of this possibility, in Section 4 we discuss τ-decays both in the case that ν_τ is restricted to maximal helicities and in the case that all four helicities are involved.

2. An Apparent Discontinuity as $M_{\nu_\tau} \to 0$

In calculating weak decay amplitudes for the τ, Alles assumes the $\tau - \nu_\tau$ current to be

$$J^\mu_{(\tau - \nu_\tau)} = \frac{2ia}{M_\tau} \bar{u}^{\mu\beta}\left(1 - y\gamma_5\right) u_\beta \quad , \tag{1}$$

where $u^{\mu\beta}$ is the curl of the standard Rarita-Schwinger spinor corresponding to a particle of four-momentum k:

$$u^{\mu\beta} = k^\mu u^\beta - k^\beta u^\mu \quad . \tag{2}$$

The quantities a and y are (arbitrary) constants. Wtih $M_{\nu_\tau} = 0$ and restricting the ν_τ to have maximal helicity ($\left|\lambda_{\nu\tau}\right| = 3/2$), the rate for $\tau \to \nu_\tau e^- \bar{\nu}_e$ is:

$$\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right) = \frac{G_F^2\, a^2\, M_\tau^5}{216(2\pi)^3} \left(1 + y^2\right) \quad . \tag{3}$$

One might also attempt to calculate this rate by first calculating the rate for a massive ν_τ and then taking the limit as $M_{\nu_\tau} \to 0$. Proceeding this way, one finds a rate

$$\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right) = \frac{17}{15} \frac{G_F^2\, a^2\, M_\tau^5}{216(2\pi)^3} \left(1 + y^2\right) \quad . \tag{4}$$

The limit as $M_{\nu_\tau} \to 0$ appears to be discontinuous.

The occurence of a discontinuity in the zero mass limit has a precedent elsewhere. It has been known for a decade that a theory with massive gravitons does not approach the standard zero mass theory in the limit that the graviton mass goes to zero.[10]

For spin-3/2 particles, the $M \to 0$ discontinuity has a straightforward origin. A Rarita-Schwinger spinor possesses both a Lorentz vector index and a Dirac spinor index. It can be conceived of as being a spin-1 field combined with a spin-1/2 field. The combination, of course, produces both total spin 1/2 and 3/2. Imposition of the standard condition

$$\gamma_\mu u^\mu = 0 \tag{5}$$

constrains u so that only the total-spin-3/2 portion remains. Writing out the helicity states of the resulting spin-3/2 field in terms of those of the spin-1 and spin-1/2 components, one finds that this condition insures that

$$\begin{aligned}
|3/2, \ 3/2\rangle &= |1, 1\rangle\, |1/2, 1/2\rangle \\
|3/2, \ 1/2\rangle &= \sqrt{2/3}\, |1, 0\rangle\, |1/2, 1/2\rangle + \sqrt{1/3}\, |1, 1\rangle\, |1/2, -1/2\rangle \\
|3/2, -1/2\rangle &= \sqrt{1/3}\, |1, -1\rangle\, |1/2, 1/2\rangle + \sqrt{2/3}\, |1, 0\rangle\, |1/2, -1/2\rangle \\
|3/2, -3/2\rangle &= |1 - 1\rangle\, |1/2 - 1/2\rangle \quad ,
\end{aligned} \tag{6}$$

which are nothing but the standard Clebsch-Gordan relations for combining spin 1/2 and spin 1 to form total spin 3/2.

As $M \to 0$, the longitudinal (helicity zero) vector contribution, $|1, 0\rangle$, to the $|3/2, \pm 1/2\rangle$ states has components which blow up; in a coordinate system where k^μ is $\begin{pmatrix} k \\ 0 \\ 0 \\ k \end{pmatrix}$ (note that the following are four-vectors not Dirac spinors),

$$|1,\ 0\rangle = \frac{1}{M}\begin{pmatrix} \sqrt{k + M^2} \\ 0 \\ 0 \\ k \end{pmatrix} \xrightarrow[M \to 0]{} \begin{pmatrix} \infty \\ 0 \\ 0 \\ \infty \end{pmatrix} ,$$

where $|1,\ 0\rangle$ is normalized to unity.

This of course also occurs for the electromagnetic field. There, the longitudinal contribution can be eliminated for the zero-mass photon by going from the field A^μ to the field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. In the case of the photon, by eliminating the longitudinal contribution ($\lambda = 0$), one leaves only maximal helicity states ($|\lambda| = 1$).

A similar use of the curl in the massless spin-3/2 case also removes the (infinite) longitudinal contribution associated with the vector index.* [11] However, unlike the electromagnetic case, the non-maximal helicity states $\left(|\lambda_{\nu_\tau}| = \pm 1/2\right)$ for spin 3/2 involve not only a longitudinal vector piece which is eliminated by the curl, but also [as shown in eq. (6)] a portion which is transverse in the vector index and which is not eliminated by the curl.

Therefore, use of the curl formalism for massless spin-3/2 Rarita-Schwinger particles, while it will eliminate the infinite longitudinal

---

* In electromagnetism, A^μ couples only to conserved currents and the longitudinal piece will therefore not contribute to matrix elements even if one uses A^μ rather than $F^{\mu\nu}$. Since u^μ need not couple to a conserved current, in the Rarita-Schwinger case, one must employ the curl formalism to ensure finite matrix elemets.

vector contribution, will not — unlike electromagnetism — completely eliminate the states with nonmaximal helicities.

As $M \to 0$, the contributions to $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ from these transverse-vector parts of the nonmaximal helicity ($|3/2, \pm 1/2\rangle$) states of ν_τ survive. As is verified by explicit calculation, it is these contributions which make the rate in Eq. (4) greater than the rate which is due solely to maximal helicities $\left(\left| \lambda_{\nu_\tau} \right| = 3/2 \right)$ and which is given by eq. (3).

According to the conventional wisdom of particle physics, only maximal helicities can exist for a particle the mass of which is strictly zero. Since, as $M \to 0$, nonmaximal helicities continue to contribute in the case under discussion, a discontinuity as $M \to 0$ appears unavoidable.

One cannot rule out *a priori* the possibility of such a discontinuity, but it is rather unsettling. For example, it implies that one could experimentally distinguish between the case of M_{ν_τ} finite but unbelievably small (e.g., $M_{\nu_\tau} = 10^{-1000}$ eV) and the case that M_{ν_τ} is strictly zero.

However, if it were possible for a massless particle to have a full set of helicity states rather than being restricted to maximal helicities, then it would be possible to avoid this discontinuity as $M \to 0$.

In the next section we review the general problem of the helicity states of a massless particle and conclude that one need not throw out the $\lambda = \pm 1/2$ states of ν_τ in the specific problem with which we are concerned when $M_{\nu_\tau} = 0$ and that therefore the discontinuity can be avoided.

Although our interest in the subject of the next section is motivated by the apparent discontinuity discussed in this section, our arguments in the next section rest solely on general considerations concerning massless particles. We do not claim that the goal of eliminating a discontinuity validates any of the following arguments.

3. Helicity States of a Massless Particle

In this section, we will review classical analyses concerning the helicity states of a massless particle and discuss their relevance to a massless spin-3/2 ν_τ.

The assumption that strictly massless particles must have only maximal helicity states rests on two theorems due to Wigner and Weinberg. Contrary to what one might expect, there are certain circumstances, including that of a massless spin-3/2 Rarita-Schwinger particle, in which these two theorems do not suffice to rule out the possibility of there being a full set of states corresponding to the full range of helicities for a massless particle.

A. Wigner's theorem

Wigner's theorem is the familiar statement that the helicity of a massless particle is invariant under the restricted Lorentz group— helicities do not mix. A single helicity forms an irreducible representation of the little group.[*,12]

---

* We will take "Wigner's theorem" to refer <u>only</u> to this strict statement that different helicities of a massless particle may not mix — i.e., "Wigner's theorem" will <u>not</u> be used to refer to the restriction on the allowed helicities of a massless particle which is generally believed to be a corollary of this theorem.

It might be thought that the requirement that a massless particle have only maximal helicities is an immediate consequence of this theorem. For, Wigner and others have chosen to define "particle" as "an irreducible representation of the Lorentz group." With this definition, it of course follows that a massless "particle" has only a single helicity state. A massless particle with a full set of helicity states would form a reducible representation and would therefore, by definition, not constitute a "particle" but rather a set of several distinct "particles."

However, this conclusion clearly conveys no information about the nature of the physical world beyond the information contained in the statement that if a massless particle does have several helicity states they will not mix under Lorentz transformations. In particular, this conclusion does not tell us whether or not there exists in nature a full set of states corresponding to all helicities of a massless particle. It merely informs us that if such a full set of helicity states exists, and if we choose to define the word "particle" in a certain manner, then we must talk about this set of helicity states in a certain way—i.e., as several "particles" rather than as a single "particle."

To believe that this line of argument reveals information about the allowed particle states which can exist in the real world is therefore to confuse physics with semantics. Obviously, the existence or nonexistence of certain states in nature does not depend on how one chooses to define the word "particle."

Wigner's definition of "particle" is of course convenient for some purposes, but it may prove rather inconvenient for other purposes. For example, when one is taking the massless limit of a finite-mass theory as in the previous section, it is natural to define "particle" in the strictly massless case to be the set of massless states, if it exists, which corresponds to the limit of the finite-mass states. With this definition of "particle" for the massless case, the question of whether or not a particle can possess a full set of massless helicity states can be settled not by definition but only by investigation: do nonmaximal helicity states decouple when $M = 0$?, will a full set of massless helicity states mix and violate Wigner's theorem?, et cetera.

We shall employ this definition, which differs from Wigner's, and which is more convenient for our purposes, throughout this paper.

It should now be clear that the conclusion that one must throw out nonmaximal helicities of a massless spin-3/2 ν_τ because massless particles must have maximal helicities and that, therefore, the $M \to 0$ discontinuity discussed in Section 2 is unavoidable is, in fact, an invalid conclusion resulting from a misunderstanding involving Wigner's definition of "particle."

Unfortunately, Wigner's definition seems somewhat prone to this sort of misunderstanding. For example, suppose an experimenter discovers a very light particle, so light that he is unable to determine whether its mass is strictly zero or is an extremely small but finite number. The experimenter might decide, wrongly of course, that if he can observe a full set of helicity states for the new particle, he will have proven that its mass must not be zero. Similarly, if an experimenter is confident that a particle is strictly massless, he may falsely conclude that it would be fruitless to investigate whether states corresponding to nonmaximal helicities exist.

Thus, even where it is useful, Wigner's definition can be rather misleading and should be handled with care.*

Wigner's theorem does not then trivially rule out the possibility that states corresponding to nonmaximal helicities of a massless particle may exist. However, if such states exist, Wigner's theorem does require that they not mix under Lorentz transformations.

If the curl formalism for spin-3/2 massless particles were not used, different helicities would mix under restricted Lorentz transformations, violating Wigner's theorem. However, with employment of the curl formalism, mixing of helicities does not occur and the situation is in fact in accord with Wigner's theorem.

The same situation arises for a massless vector particle. If one does not use the curl formalism, different helicities mix in violation of Wigner's theorem. As in the Rarita-Schwinger case, the curl formalism ensures that different helicities do not mix.

However, in the massless vector case, the curl also eliminates the nonmaximal helicity state. This does not occur, as we have emphasized, in the massless Rarita-Schwinger case.

---

* We apologize to the reader who is quite immune to misuse of Wigner's definition and who views the preceding discussion as overemphasizing a trivial and obvious point. However, a majority of the established particle theorists with whom we discussed the result of Section 2 did misapply Wigner's definition to this specific problem with which we are concerned; hence, we thought it necessary to discuss this matter in some detail.

In the Rarita-Schwinger case, the curl formalism allows nonmaximal helicities to exist without violating Wigner's theorem. We conclude that for spin-3/2, Wigner's theorem is consistent with there being a full range of helicities for a massless spin-3/2 particle.

B. Weinberg's theorem

Weinberg's theorem[13] explicitly specifies which helicity states can exist for a massless particle in a given representation of the Lorentz group. First define

$$\vec{A} = \frac{\vec{J} + i\vec{K}}{2} \quad \text{and} \quad \vec{B} = \frac{\vec{J} - i\vec{K}}{2} \quad , \tag{7}$$

where $\vec{J}$ and $\vec{K}$ are the usual generators of rotations and boosts, respectively. Since $\vec{A}$ and $\vec{B}$ commute, and since each generates an SU(2) algebra, any representation of the Lorentz group can be specified in terms of its representation content with respect to $\vec{A}$ and $\vec{B}$ and can be labelled accordingly: (A,B). A Dirac spinor corresponds to (1/2, 0) + (0, 1/2). A four-vector behaves as (1/2, 1/2). A Rarita-Schwinger spinor, which combines a vector and a Dirac index, corresponds to $(1/2, 1/2) \otimes [(1/2, 0) + (0, 1/2)] = (1, 1/2) + (0, 1/2) + (1/2, 1) + (1/2, 0)$. Parts of these last representations are eliminated by the standard constraint [eq. (5)].

Weinberg's theorem is the statement that a massless particle in the representation (A,B) can only have a single helicity:

$$\lambda = B - A \quad . \tag{8}$$

At first glance, it therefore appears to vindicate the common belief that a massless particle cannot have a full range of helicities.

However, if one applies Weinberg's criterion to some specific examples, one finds that, in fact, it is not at all in agreement with the usual belief that massless particles have only maximal helicities.

For example, for a spin-1 particle field described by a four-vector (e.g., electromagnetism with the photon field A_μ), $(A,B) = (1/2, 1/2)$, so that Weinberg's criterion implies $\lambda = 1/2 - 1/2 = 0$. Thus, Weinberg's theorem requires that a massless vector field can only have a longitudinal component, that it can only have nonmaximal helicity!

Similarly, Weinberg's theorem demands that in the massless Rarita-Schwinger case $\left|\lambda_{\nu_\tau}\right| = 1/2$. Again, maximal helicities are forbidden. Thus, while Weinberg's theorem does seem to prevent a massless particle from having a full set of helicities, the helicity states allowed by Weinberg's theorem are not, in general, the maximal helicities. On the contrary, the theorem forbids maximal helicities for both vector and Rarita-Schwinger fields.

There is, of course, a loophole in these results.* The helicity states allowed by Eq. (8) (in both the vector and Rarita-Schwinger cases) are precisely those states which have infinite components when $M = 0$, with the standard normalizatin. If one normalizes the Rarita-Schwinger spinors (or the spin-1 vector representation) so that these components are finite, the other helicity states will indeed vanish as required by Weinberg's theorem.

---

* Weinberg was, of course, aware that a loophole existed, although he was concerned with a somewhat different facet of the problem than we are.

If, however, one chooses the standard normalization in which these components are infinite and then employs the curl formalism to eliminate the infinite contributions, one escapes Weinberg's theorem. Equation (8) was derived as a necessary condition to ensure that different helicities do not mix, but the curl formalism guarantees this even if $\lambda \neq B - A$. Thus, Weinberg's theorem does not restrict the allowed helicities of a massless Rarita-Schwinger particle if the curl formalism is employed.

C. Is "total spin" meaningful for massless particles?*

We have concluded that neither Wigner's theorem nor Weinberg's theorem requires one to throw out the $\lambda = \pm 1/2$ states which appear in the Rarita-Schwinger formalism for a massless spin-3/2 particle. We have pointed out that whether one views these helicity-1/2 states as comprising a separate particle from the helicity-3/2 states or merely as different states of the particle which also has helicity-3/2 states is a matter of convenience. Since all four helicity states of a massless spin-3/2 ν_τ correspond to the $M \to 0$ limit of a single finite-mass particle, it is convenient to refer to the four helicity states as comprising the same particle.

However, it is standard practice to identify the spin of a massless particle as $|\lambda|$. Standard practice would thus assign the $\lambda = \pm 1/2$ states a spin of 1/2 and the $\lambda = \pm 3/2$ states a spin of 3/2, which conforms nicely with Wigner's definition which defines these states as being separate particles.

---

* The discussion in this subsection is in response to queries raised by L. Wolfenstein.

If we view all four helicities as comprising one particle, however, we would assign them all a spin of 3/2.

For a massive particle, spin is a physically measurable quantity. If the same were true for a massless particle, one could (in principle) measure the total spin of the $\lambda = \pm 1/2$ states and prove either Wigner's definition or our own to be wrong: either the $\lambda = \pm 1/2$ states would have the same spin as the $\lambda = \pm 3/2$ states, or they would not.

In fact, total spin is apparently not a meaningful quantity for a massless particle. Obviously, one cannot go to the rest frame to measure $S_x^2 + S_y^2 + S_z^2$. The Pauli-Lubanski vector, $\Gamma^\mu = \varepsilon^{\mu\nu\rho\sigma} p_\nu M_{\rho\sigma}$, has magnitude $M^2|S|(|S|+1)$ which uniquely determines the spin $|S|$ — unless $M = 0$.

For a massive particle the transformation properties under boosts and rotations of a state of helicity λ depend not only on λ but also on $|S|$, and this allows one in principle to physically measure $|S|$. However, Wigner's theorem proves that for a massless particle the transformation properties depend only on λ and cannot therefore determine $|S|$.

The standard approach to coupling angular momenta of several particles requires knowledge of each particle's spin. Which Clebsch-Gordan table one uses depends on the magnitude of the spins of the particles one is considering. One expects this to carry over to the massless case; i.e., depending on whether one assigns a spin of 3/2 or 1/2 to our $\lambda = \pm 1/2$ states one expects to use a different set of Clebsch-Gordan coefficients to combine these states with other particles to form some composite angular momentum state.

This is indeed so. If Clebsch-Gordan coefficients are — in principle — physically measurable, the assignment of total spin to a massless particle would not be arbitrary. However, Clebsch-Gordan coefficients specify a particle's component of spin along a definite fixed direction, generally not the direction of the particle's motion. For a massive particle, the component of apin along a fixed direction can be physically measured by bringing the particle to rest. For a massless particle this cannot be done and the Clebsch-Gordan coefficients therefore cannot be measured physically.

For massless particles, the only physical approach to specifying the spin state is to give the helicity. If one couples the angular momenta of several particles in the helicity basis (a generalized Jacob-Wick approach), it can be proven that the helicity coefficients analogous to Clebsch-Gordan coefficients do not depend on the spin of any of the particles — whether the particles are massive <u>or</u> massless. (This result therefore does not depend on Wigner's theorem.)

Since only the helicity basis is physically meaningful for massless particles, the combining of angular momenta and the dependence of Clebsch-Gordan coefficients on the magnitude of the spin does not therefore allow one to give a physical meaning to the spin of a massless particle.

None of the obvious approaches to physically measuring the spin of a massless particle works. Indeed, Wigner's theorem probably rules out any such approach.

We conclude that neither Wigner's theorem nor Weinberg's theorem, nor considerations of the total spin of a massless particle, constrains the helicity states allowed for a massless Rarita-Schwinger field. It appears that when constructing a theory one can, if one chooses, assume that a massless Rarita-Schwinger field has all four helicity states: both $\lambda_{\nu_\tau} = \pm\ 3/2$ and $\lambda_{\nu_\tau} = \pm\ 1/2$. (Of course, whether nature in fact chooses to conform to such a theory is a question to be settled by experiment.) If one does choose to allow all four helicities when $M = 0$, the $M \to 0$ discontinuity in Section 2 disappears.*

4. V,A τ,ν_τ Currents and τ Decay

In Section 2 we showed that if we start with a theory with a massive neutrino and let $M_{\nu_\tau} \to 0$, all four helicities of the ν_τ continue to contribute; none totally decouples. Furthermore, we argued in Section 3 that, contrary to what one might expect, even when M_{ν_τ} is strictly zero one can, if one wishes, allow all four helicity states to exist.

Given these considerations, we will present results in this section based on the assumption that all four helicity states for ν_τ are present for $M_{\nu_\tau} = 0$. Of course, it is not necessary for all four helicity states to exist in the strictly massless case — it is possible to have only maximal helicity states present. We will therefore also discuss the results in this case.

---

* Weinberg and Witten have recently shown that a massless spin-3/2 particle cannot have a conserved Lorentz-covariant vector current or a conserved Lorentz-covariant stress-energy tensor.[14] As they point out, there are known theories which lack a Lorentz-covariant conserved vector current or conserved stress-energy tensor but which are nonetheless acceptable theories.

Of course, one can avoid Weinberg's and Witten's theorem entirely by simply giving ν_τ an arbitrarily tiny yet nonzero mass. Obviously, all four ν_τ helicities would then automatically exist.

More bizarre possibilities exist in the strictly massless case; e.g., one could have $\lambda_{\nu_\tau} = 3/2,\ 1/2,\ -1/2$ states existing but $\lambda_{\nu_\tau} = -3/2$ not existing. We will not discuss such possibilities.

For spin-3/2 τ and ν_τ, with arbitrary masses, there are in general seven independent pairs of V,A currents which can be formed from the τ and ν_τ Rarita-Schwinger spinors:

$$
\begin{aligned}
A_V^\lambda + aA_A^\lambda &= \bar{u}^{\lambda\beta}(\nu_\tau)\left(1+a\gamma_5\right) u_\beta(\tau) \\
B_V^\lambda + bB_A^\lambda &= \frac{\left(p^\lambda+k^\lambda\right)}{M_\tau^2}\ \bar{u}^{\alpha\beta}(\nu_\tau)\ p_\alpha\left(1+b\gamma_5\right) u_\beta(\tau) \\
C_V^\lambda + cC_A^\lambda &= \frac{\left(p^\lambda-k^\lambda\right)}{M_\tau^2}\ \bar{u}^{\alpha\beta}(\nu_\tau)\ p_\alpha\left(1+c\gamma_5\right) u_\beta(\tau) \\
D_V^\lambda + dD_A^\lambda &= \bar{u}^{\alpha\beta}(\nu_\tau)\ \frac{p_\alpha}{M_\tau}\left(1+d\gamma_5\right)\gamma^\lambda\ u_\beta(\tau) \qquad (9) \\
E_V^\lambda + eE_A^\lambda &= \frac{M_{\nu_\tau} M_\tau}{p\cdot k}\ \bar{u}^{\beta}(\nu_\tau)\left(1+e\gamma_5\right) u_\beta(\tau)\left(p^\lambda+k^\lambda\right) \\
F_V^\lambda + fF_A^\lambda &= \frac{M_{\nu_\tau} M_\tau}{p\cdot k}\ \bar{u}^{\beta}(\nu_\tau)\left(1+f\gamma_5\right) u_\beta(\tau)\left(p^\lambda-k^\lambda\right) \\
G_V^\lambda + gG_A^\lambda &= M_{\nu_\tau}\ \bar{u}^{\beta}(\nu_\tau)\left(1+g\gamma_5\right)\gamma^\lambda\ u_\beta(\tau) \quad .
\end{aligned}
$$

Here k^λ, p^λ is the four momentum of ν_τ, τ respectively. Other currents can be written in terms of these seven; e.g., by the Gordon decomposition, $(q = k - p)$

$$
i\ \frac{p_\alpha}{M_\tau}\ \bar{u}^{\alpha\beta}\ q_\nu\ \sigma^{\lambda\nu}\ u_\beta = M_\tau\ D_V^\lambda - M_\tau\ B_V^\lambda \quad . \qquad (10)
$$

The currents E^{λ}, F^{λ} and G^{λ} involve the parts of the $\lambda_{\nu_\tau} = \pm 1/2$ helicity states which satisfy Weinberg's criterion and the components of which become infinite as $M_{\nu_\tau} \to 0$. However, E^{λ}, F^{λ}, G^{λ} are constructed so as to approach a finite limit as $M_{\nu_\tau} \to 0$ even though the components of the spinors comprising E^{λ}, F^{λ}, G^{λ} blow up as $M_{\nu_\tau} \to 0$.

When ν_τ is strictly massless, E^{λ}, F^{λ}, G^{λ} are of the indeterminate form $0 \times \infty$ (assuming the standard normalization for u^{β}) and are hence undefined. For this reason, we will refrain from using these currents in our analysis.

If $M_{\nu_\tau} = 0$ and if one is restricted to maximal helicities (but not if all four ν_τ helicities are allowed), then

$$A_V^{\lambda} = D_V^{\lambda} \quad ; \qquad A_A^{\lambda} = D_A^{\lambda} \quad .$$

Therefore, in the maximal-helicity case there are only three pairs of independent currents: $A_{V,A}^{\lambda}$, $B_{V,A}^{\lambda}$ and $C_{V,A}^{\lambda}$. If all four helicity states of a massless ν_τ are allowed $D_{V,A}^{\lambda}$ must be included as a fourth pair of independent currents.

Alles assumes that

$$J^{\lambda}_{(\tau - \nu_\tau)} = K\left(A_V^{\lambda} + aA_A^{\lambda}\right) \quad ,$$

with K an arbitrary constant and shows that the ratio (assuming only maximal helicities for ν_τ)

$$\frac{\Gamma\left(\tau \to \pi\nu_\tau\right)}{\Gamma\left(\tau \to \nu_\tau e^- \overline{\nu}e\right)}$$

is 2.25 times the standard-model value. He concludes that the disagreement of this prediction with experiment definitely excludes the hypothesis that τ and ν_τ both have spin 3/2.

In fact, if one allows a more general form for $J^\lambda_{(\tau-\nu_\tau)}$, Alles' conclusion is false; for, let

$$J^\lambda_{(\tau-\nu_\tau)} = K \frac{p_\alpha}{M_\tau}\left[\bar{u}^{\alpha\beta}\gamma^\lambda u_\beta(\tau) + \kappa i \bar{u}^{\alpha\beta}\frac{q_\nu}{M_\tau}\sigma^{\lambda\nu}u_\beta\right]$$

$$= K\left[(1+\kappa)\, D^\lambda_V - \kappa B^\lambda_V\right] . \quad (11)$$

Since the current $\langle\pi|J^\lambda|0\rangle$ is proportional to q^λ, the Pauli term, $q_\nu\sigma^{\lambda\nu}$, does not contribute to $\Gamma(\tau\to\pi\nu_\tau)$ at all and $\Gamma(\tau\to\pi\nu_\tau)$ is independent of κ. In particular, $\Gamma(\tau\to\pi\nu_\tau)$ is finite as $\kappa\to\infty$. Since the Pauli term gives a nonzero contribution to $\Gamma(\tau\to\nu_\tau e\bar{\nu}_e)$, $\Gamma(\tau\to\nu_\tau e\bar{\nu}_e)$ will go to ∞ as $\kappa\to\infty$. Therefore, $\Gamma(\tau\to\pi\nu_\tau)/\Gamma(\tau\to\nu_\tau e^-\bar{\nu}_e)$ goes to 0 as $\kappa\to\infty$.

Since $\Gamma(\tau\to\pi\nu_\tau)/\Gamma(\tau\to\nu_e e^-\bar{\nu}_e)$ is 2.25 when $\kappa = 0$ and 0 when $\kappa=\infty$, and since it is a continuous function of κ, it follows that there exists a κ corresponding to any value of this ratio between 0 and 2.25. Since both the experimental value and the standard-model theoretical value for this ratio lie between 0 and 2.25, there does exist, contrary to Alles, a $J^\mu_{(\tau-\nu_\tau)}$ involving spin-3/2 τ and ν_τ which produces the desired value of $\Gamma(\tau\to\pi\nu_\tau)/\Gamma(\tau\to\nu_\tau e\bar{\nu}_e)$ with ν_τ restricted to maximal helicities.

This reasoning applies also when all four ν_τ helicities are allowed.*

The fact that $\Gamma(\tau \to \nu_\tau \pi)/\Gamma(\tau \to \nu_\tau e\bar{\nu}_e)$ can be adjusted so as to agree with experiment leaves open the possibility that other branching ratios, the e^- energy spectrum, et cetera, might not be similarly adjustable.

In fact, if one is willing to allow arbitrary form factors, there exists $J^\lambda_{(\tau-\nu_\tau)}$ such that the "lepton trace"

$$L^{\lambda\rho} = \sum_{\text{spins}} J^\lambda_{(\tau-\nu_\tau)} \left(J^\rho_{(\tau-\nu_\tau)}\right)^*$$

is identical to that for the spin-1/2 case. Such a $J^\lambda_{(\tau-\nu_\tau)}$ with the same $L^{\lambda\rho}$ as in the standard model will clearly reproduce the standard-model branching ratios and, in the τ rest frame, the standard-model energy spectra for unpolarized τ and undetected ν_τ helicity.

In the maximal helicity case, an appropriate $J^\lambda_{(\tau-\nu_\tau)}$ is

$$J^\lambda_{(\tau-\nu_\tau)} = \frac{M_\tau}{k\cdot p}\left(A^\lambda_V + A^\lambda_A\right) - \left(\frac{1}{2} - \frac{1}{2\sqrt{3}}\right)\frac{M^3_\tau}{(k\cdot p)^2}\left(B^\lambda_V - C^\lambda_V + B^\lambda_A - C^\lambda_A\right) . \quad (12)$$

When all four ν_τ helicities are allowed

$$J^\lambda_{(\tau-\nu_\tau)} = \frac{M_\tau}{k\cdot p}\left(D^\lambda_V + D^\lambda_A\right) - \left(\frac{1}{2} + \frac{1}{2\sqrt{5}}\right)\frac{M^3_\tau}{(k\cdot p)^2}\left(B^\lambda_V - C^\lambda_V\right) . \quad (13)$$

---

* When all four helicities are involved and $\kappa = 0$, $\Gamma(\tau \to \pi\nu_\tau)/\Gamma(\tau \to \nu_\tau e^-\bar{\nu}_e)$ is 75/34 ($\cong$ 2.21) rather than 2.25 of the standard-model value. Otherwise, the reasoning is unchanged.

For unpolarized τ and undetected ν_τ helicity, these currents will reproduce the branching ratios, energy spectra, angular distributions, et cetera, of the unpolarized standard-model spin-1/2 case.

The price one pays for achieving this mimicry of the standard model is the need to use some rather unaesthetic form factors. (Contrary to appearances, $J^\lambda_{(\tau-\nu_\tau)}$ does not, of course, blow up as $k^\lambda \to 0$.) However, as we argued in Section 1, the fact that spin-3/2 τ and ν_τ if they exist are probably composite combined with the uncertainty as to what is the "simplest" current for spin-3/2 τ and ν_τ compels one to accept the probability of nonconstant form factors. Unless one has a specific theory concerning these form factors, one cannot rule out the possibility that $J^\lambda_{(\tau-\nu_\tau)}$ is of the form given by eqs. (12) or (13).

It is of course impossible in general for a polarized spin-3/2 τ to reproduce the angular distributions produced by a spin-1/2 τ. Therefore, if one can produce fully polarized τ's, one could determine the τ spin, the results of this section notwithstanding.

The restriction of this section that the τ be totally unpolarized is somewhat more severe than one might suppose. For example, spin-3/2 $\tau^+\tau^-$ produced in e^+e^- annihilation would not, in general, be unpolarized — e.g., the helicities $\lambda = \pm 3/2$ and $\lambda = \pm 1/2$ might not be equally populated. This situation might produce not only angular correlations differing from the standard model but also energy spectra in the lab frame which differ from the standard-model spectra; for, if there is any correlation between the direction of the τ spin and the direction of the boost from the rest frame to the lab frame, then the energy spectra in the laboratory frame depend not only on the rest-frame spectra but also on the rest-frame angular distributions. The existence of such a correlation is equivalent to there being a differential population of the various τ helicity states (i.e., to there not being equal numbers of τ's of various helicities). Therefore, even if one takes $J^{\lambda}_{(\tau - \nu_\tau)}$ to be given by eqs. (12) or (13), if the τ helicities are differentially populated in e^+e^- annihilation the lab-frame energy spectra (integrated over angles) will not necessarily agree with the standard model predictions even though the <u>rest-frame</u> spectra (integrated over angles) <u>will</u> agree with the standard-model predictions.

Of course, it is *a priori* possible that the four helicities of spin-3/2 τ's produced in e^+e^- annihilation could be equally populated in which case lab-frame spectra would agree with the standard-model predictions. However, even if the different helicity states are equally populated, there must at least be a correlation between the τ^+ and τ^- helicities for spin-3/2 . This would probably produce correlations between τ^+ and τ^- energy spectra and angular distributions which differ from the standard model.

Therefore, detailed consideration[15] of the $\tau - \tau$ electromagnetic current for spin-3/2 τ would probably reveal either energy spectra or correlations between τ^+ and τ^- angular distributions or energy spectra which differ from the standard-model predictions and which might thus enable one to distinguish experimentally between τ spin of 1/2 and 3/2.

5. Conclusion

We have considered the possibility that τ and ν_τ both have spin 3/2. We have found that, contrary to the usual assumption, it is apparently not necessary for a massless spin-3/2 ν_τ to be restricted to maximal helicity. For unmeasured ν_τ helicity and unpolarized τ, it is possible for spin-3/2 τ and ν_τ to precisely mimic the standard-model decay rates and energy distributions of a spin-1/2 τ and ν_τ. Only in situations where one has some information about the τ polarization, as in the correlations that must exist for the τ^+,τ^- helicities in $e^+e^- \to \tau^+\tau^-$ might it be possible to rule out the possibility that τ and ν_τ both have spin 3/2. Although we share the general prejudice against spin-3/2 τ and ν_τ as unaesthetic and lacking in the simplicity of the standard model, we must conclude that existing theoretical and experimental analysis is not sufficient to strictly rule out the hypothesis that both τ and ν_τ have spin 3/2.

Acknowledgments

The author would like to thank A. S. Wightman of Princeton and R. Blankenbecler of SLAC for helpful comments and information.

REFERENCES

1. M. L. Perl et al., Phys. Rev. Lett 35, 1489 (1975);
M. L. Perl et al., Phys. Lett. 63B, 466 (1976);
G. J. Feldman et al., Phys. Rev. Lett. 38, 177 (1976).

2. S. Weinberg, Phys. Rev. Lett 19, 1264 (1967); A. Salam, in Elementary Particle Physics: Relativistic Groups and Analyticity (Nobel Symposium No. 8), ed. N. Svartholm (Almquist and Wiksell, Stockholm, 1968), p. 367.

3. D. H. Miller, Phys. Rev. D23, 1158 (1981); and M. L. Perl in New Phenomena in Lepton-Hadron Physics, Karlsruhe 1978 (New York, Plenum, 1979), p. 115, and references cited therein.

4. Y. Ahn, J. Kim and H. S. Song, Phys. Lett. 73B, 196 (1978);
J. Leite Lopes, J. A. Martins Simoes and D. Spehler, Phys. Rev. D23, 797 (1981).

5. C. P. Korthals-Altes and P. Mery, Phys. Lett. 72B, 375 (1978).

6. Y. S. Tsai, SLAC preprint SLAC-PUB-2105 (April 1978) and "Heavy Leptons," in Proceedings of the 1980 Guangzhou Conference on Theoretical Particle Physics (Science Press; Beijing, China, 1980).

7. G. L. Kane and S. Raby, Phys. Lett. 89B, 203 (1980).

8. W. Alles, Nuovo Cimento Lett. 25, 404 (1979).

9. W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).

10. L. H. Ford and H. Van Dam, Nucl. Phys. B169, 126 (1980);
H. Van Dam and M. Veltman, Nucl. Phys. B22, 297 (1970);
D. G. Boulware and S. Deser, Phys. Rev. D6, 3368 (1972);
H. Van Dam and M. Veltman, G.R.G. 3, 215 (1972).

11. S. Kusaka, Phys. Rev. 60, 61 (1941).

12. E. P. Wigner, Ann. Math. 40, 149 (1939), and Theoretical Physics (International Atomic Energy Agency, Vienna, 1963).

13. S. Weinberg, Phys. Rev. 134, B882 (1964).

14. S. Weinberg and E. Witten, Phys. Lett. 96B, 59 (1980).

15. Some aspects of the $\tau-\tau$ electromagnetic current for spin-3/2 τ are discussed in W. Alles and V. Alles Borelli, Nuovo Cimento 35A 125 (1976).

CHAPTER V

CONCLUSIONS

In the preceding three phenomenological studies of the τ lepton we have seen that, while many alternatives to the standard model of the τ as a sequential lepton have been ruled out and while experimental data is consistent with the τ's being a sequential lepton, other possibilities do still remain. Most of these possibilities are rather bizarre. For example, we think it quite implausible that τ and ν_τ have spin 3/2 as discussed in Chapter IV; nonetheless, it will be difficult to rigorously rule out this possibility (not least because of the absence of a good theory for spin-3/2 leptons).

Probably the least bizarre viable alternative to the standard model that we have discussed is the superfluous heavy neutral model of Chapter II. Observation of the structure of $\tau - \tau$ neutral currents could rule out this model.

A development to be hoped for in τ physics is the observation of the ν_τ (and a good determination of its mass). We know of no model consistent with experimental data and lacking a new light neutral lepton. It would therefore be extremely interesting if it turned out that ν_τ did not exist.

We believe that in all probability the τ is a massive identical twin of the electron, a sequential lepton. Nonetheless, it is worth keeping in mind, perhaps, that the case is not yet completely closed.