The study of the nonleptonic two body B decays involving a light tensor meson in final states

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In the perturbative QCD approach based on k_T factorization, we study the nonleptonic two body $B_{u,d,s,c}$ decays involving a light tensor meson in final states. The emission diagram with a tensor meson produced from vacuum is vanished. While contributions from the so-called hard scattering emission diagrams and annihilation type diagrams are important and calculable in the perturbative QCD approach. The branching rations of most decays are in the range of 10^{-4} to 10^{-8}, which are bigger by 1 or 2 orders of magnitude than the predictions given by naive factorization. We also give the predictions for the CP asymmetry parameters, some of which are large enough for the future experiments to measure. For decays with a vector meson and a tensor meson in final states, we predict a large percentage of transverse polarization contributions for many of these decays.

1 Introduction

The two body hadronic decays of B meson are important, since they can provide constraints of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, a test of the QCD factorization, information on the decay mechanism and the final state interaction, and also a good place to study CP violation and new physics signal. Most of the studies concentrate on the $B \to PP$, PV and VV decays, since they are easy to be measured at experiments. Recently, more and more experimental measurements about B decays with a light tensor meson in the final states have been obtained[1]. Inspired by these experiments, many theoretical studies on the $B_{u,d,s,c}$ decays involving a light tensor meson have been done based on naive factorization[2,3,4,5], QCD factorization (QCDF)[6,7] and perturbative QCD (PQCD) approach [8,9,10,11].

The tensor meson emission diagrams are prohibited, because the amplitude proportional to the matrix element $\langle T \mid j^\mu \mid 0 \rangle$ vanishes from Lorentz covariance considerations, where j^μ denotes the $(V \pm A)$ current or $(S \pm P)$ density[6,7]. Thus, these decay modes with a tensor meson emitted are prohibited in naive factorization. On the other hand, the naive factorization can not give creditable predictions for those color-suppressed or penguin dominant decays. What is more, the naive factorization can not deal with the pure annihilation type decays. The recent developed QCDF approach [6,7] and the PQCD factorization approach [9,10,11] overcome these shortcomings by including the large hard scattering contributions and the annihilation type contributions to give more reliable predictions for these decays, especially those with tensor meson emitted or pure annihilation type decays. It is worth of mentioning that the annihilation type diagrams can be perturbatively calculated without parametrization in PQCD approach [12,13]. Only the PQCD approach have successfully predicted the pure annihilation type decays $B_s \to \pi^+ \pi^-$ [14,15] and $B^0 \to D_s^- K^+$ [12,16], which have been confirmed by experiments later [17,1].

B meson decays into tensor mesons are of prime interest in several aspects. The branching ratios and CP asymmetries are helpful to inspect those different theoretical calculations. Moreover, from our computations, polarizations of the final state mesons in B decays into tensor mesons and vector

mesons are also beyond the naive hierarchy, which is similar to some decays to two light vector mesons and shed light on the helicity structure of the electroweak interactions.

2 FORMALISM

It is well known that the key step to predict two-body hadronic $B_{(s,c)}$ decays is calculating the hadronic transition matrix elements:

$$\mathcal{M} \propto \langle h_1 h_2 | \mathcal{H}_{eff} | B_{(s,c)} \rangle \tag{1}$$

with the weak effective Hamiltonian $\mathcal{H}_{eff}$ written as [18]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{u(c)b}^* V_{u(c)X} \left[C_1(\mu) O_1^q(\mu) + C_2(\mu) O_2^q(\mu) \right] - V_{tb}^* V_{tX} \left[\sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\}, \tag{2}$$

with the CKM matrix elements $V_{u(c)b(X)}$ and $V_{tb(X)}$ ($X = d, s$). $c_i(\mu)$ are the effective Wilson coefficients at the renormalization scale μ, whose expression can be found at ref.[19]. The local four-quark operators include the current-current (tree) operators $O_{1,2}$, the QCD penguin operators $O_{3,4,5,6}$ and the electroweak penguin operators $O_{7,8,9,10}$.

In the B meson rest frame, the light final state mesons with large momenta are moving fast on the light cone. The spectator quark in B meson is soft in the initial state, while collinear in the final state. There must be a hard gluon to kick the soft spectator quark into a collinear and energetic one. Thus the process is perturbatively calculable. However, when doing calculations, end-point singularity occurs. We have to keep the intrinsic transverse momentum k_T of valence quarks in the denominator to kill the divergence. With this additional energy scale k_T, the double logarithms appears, which should be resummed through the renormalization group equation to result in the Sudakov form factor. This form factor effectively suppresses the end-point contribution of distribution amplitude of mesons in the small transverse momentum region, which makes the calculation in the PQCD approach reliable and consistent.

Finally, based on the factorization, the decay amplitude can be described as the convolution of the the Wilson coefficients $C(t)$, the hard scattering kernel and the light-cone wave functions $\Phi_{h_i,B}$ of mesons [20],

$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3$$
$$\times Tr \left[C(t) \Phi_B(x_1, b_1) \Phi_{h_2}(x_2, b_2) \Phi_{h_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right], \tag{3}$$

where Tr denotes the trace over Dirac and colour indices, b_i is the conjugate variable of quark's transverse momentum k_{iT}, x_i is the momentum fractions of valence quarks and t is the largest energy scale in the hard part $H(x_i, b_i, t)$. The jet function $S_t(x_i)$ smears the end-point singularities on x_i, which is from the threshold resummation of the double logarithms $\ln^2 x_i$ [21].

The wave function for a generic tensor meson are defined by [8]

$$\Phi_T^L = \frac{1}{\sqrt{6}} \left[m_T \slashed{\epsilon}_{\bullet L}^* \phi_T(x) + \slashed{\epsilon}_{\bullet L}^* \slashed{P} \phi_T^t(x) + m_T^2 \frac{\epsilon_\bullet \cdot v}{P \cdot v} \phi_T^s(x) \right]$$
$$\Phi_T^\perp = \frac{1}{\sqrt{6}} \left[m_T \slashed{\epsilon}_{\bullet \perp}^* \phi_T^v(x) + \slashed{\epsilon}_{\bullet \perp}^* \slashed{P} \phi_T^T(x) + m_T i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_{\bullet \perp}^{*\nu} n^\rho v^\sigma \phi_T^a(x) \right], \tag{4}$$

with the vector $\epsilon_{\bullet\mu} \equiv \frac{\epsilon_{\mu\nu} v^\nu}{P \cdot v}$ related to the polarization tensor. The twist-2 and twist-3 distribution amplitudes are given only in the leading order as asymptotic form for simplicity. Based on the Bose statistics, the light-cone distribution amplitudes of the tensor meson are antisymmetric under the interchange of momentum fractions of the quark and anti-quark in the SU(3) limit (i.e. $x \leftrightarrow 1-x$) [6,7], which is consistent with the fact that $< 0 \mid j^\mu \mid T >= 0$, where j^μ is the $(V \pm A)$ or $(S \pm P)$ current.

All other meson wave functions $B_{u,d,s,c}$ and π, K etc. are adopted the same as in other pQCD papers [12,13,16,22,23], since we have argued that they should be universal for all decay channels.

3 RESULTS AND DISCUSSIONS

The predicted branching ratios of penguin-dominated and color-suppressed decays in PQCD [9] are larger than those of naive factorization [2,3,4], but are close to the QCDF predictions [7], which is caused by the fact that the naive factorization can not evaluate the contributions from penguin operators well. On the other hand, the B to tensor form factor calculated in PQCD approach is larger than that used in QCDF [7], thus for the tree-dominated modes such as $a_2^- \pi^+$ and $f_2 \pi^+$, the predicted branching ratios are bigger than QCDF predictions [7].

For $B^+ \to K_2^{*0} \pi^+$ and $B^0 \to K_2^{*+} \pi^-$ decays, the predictions in naive factorization approach [2] are 0 indeed, because the emitted meson is the tensor meson K_2^*. In PQCD approach, the expected hard scattering contributions are suppressed by the small Wilson coefficients. Thus the dominant contribution is from the chiral enhanced annihilation type contributions. While in QCDF, the dominant contribution comes from hard scattering diagrams with quark loop corrections, which is next-to-leading order and not considered in this work. The branching ratios of $a_2^+ \pi^-$ and $a_2^+ \pi^0$ modes are highly suppressed relative to $a_2^- \pi^+$ and $a_2^0 \pi^+$, respectively, because the expected dominant contribution of $B \to a_2^+ \pi^0 (a_2^+ \pi^-)$ is the color favored tensor emission diagram, while the dominant contribution for the other two channels is the color enhanced diagram with pion emission.

The direct CP asymmetries for some $B \to PT$ decays are also predicted [9]. Although some channels have very large direct CP asymmetries, they are difficult for experiments due to the small branching ratios. We recommend the experimenters to search for the direct CP asymmetry in the channels like $B^+ \to f_2 K^+$, $B^0 \to a_2^- K^+$, $B^+ \to a_2^+ \eta'$ and $B^+ \to f_2 \pi^+$, for they have both large branching ratios and direct CP asymmetry parameters.

We have also studied the two-body hadronic $B_s \to PT$ decays in the PQCD approach and given the predictions about branching ratios and CP observables [24]. For $\Delta S = 0$ decays, the B_s^0 ($\bar{B}_s^0$) meson decays to the final state f ($\bar{f}$), but not to $\bar{f}$ (f) with $f \neq \bar{f}$. Only the $B_s^0 \to \pi^+ K_2^{*-}$ decay has a sizable branching ratio due to the color enhanced emission amplitude T. Because the factorizable emission contributions are suppressed, the branching ratios of those color-suppressed modes, such as $B_s^0 \to \bar{K}^0 a_2^0$, $\bar{K}^0 f_2$ and $\bar{K}^0 f_2'$, are similar with their PV partners [15]. While for the color-favored $B_s^0 \to K^- a_2^+$ decay, the branching ratio 1.50×10^{-7} is much smaller than the $B_s^0 \to K^- \rho^+$ one, 1.78×10^{-5}, because the factorizable emission amplitude, which is dominant in $B_s^0 \to K^- \rho^+$, is forbidden in naive factorization with a tensor meson emission. Combing those predictions with the $B \to PT$ ones, we find large U-spin asymmetries in some decay modes, such as $B_s^0 \to \pi^+ K_2^{*-}$ and $B_s^0 \to K^- a_2^+$ [24].

There are two categories of decays with one charmed meson in the final states [10]. The $B_{(s)} \to \bar{D}^{(*)} T$ decays governed by the $\bar{b} \to \bar{c}$ transition, while the $B_{(s)} \to D^{(*)} T$ decays governed by the $\bar{b} \to \bar{u}$ transition. Clearly, there is a large enhancement of CKM matrix elements $|V_{cb}/V_{ub}|^2$ for the the former kinds of decays, especially for those without a strange quark in the four-quark operators. The branching ratios of $B_{(s)} \to \bar{D}^{(*)} T$ decays are thus larger than those of $B_{(s)} \to D^{(*)} T$ decays [10]. For most of the $B_{(s)} \to D^{(*)} T$ decays, the branching ratios are at the order of 10^{-6} or 10^{-7}; while for the $B_{(s)} \to \bar{D}^{(*)} T$ decays, the branching ratios are at the order 10^{-4} or 10^{-5}. With a charm quark in the final state, all these decays are governed by tree level current-current operators, without penguin operator contribution. Since the direct CP asymmetry is proportional to the interference between two different contributions. all these decays have no direct CP asymmetries.

For $B \to D^*(\bar{D}^*) T$ decays, we also calculate the percentage of transverse polarizations [10]. For those color suppressed $B \to \bar{D}^* T$ decays with the $\bar{D}^*$ emitted, the $\bar{c}$ is right-handed while the u quark is left-handed, because they are produced through $(V - A)$ current. Thus, the $\bar{D}^*$ meson is longitudinally polarized. But the massive $\bar{c}$ quark can flip easily from right handed to left handed. As a result, the polarization of the $\bar{D}^*$ meson becomes transverse with $\lambda = -1$. On the other hand, because of the additional contribution of orbital angular momentum, the recoiled tensor meson can also be transversely polarized with $\lambda = -1$ easily. So the transverse polarization fractions can be as large as 70%. While for color suppressed $B \to D^* T$ decays with D^* meson emitted, the D^* meson can also be transversely polarized, but with $\lambda = +1$. According to the angular momentum conservation, the recoiled tensor meson must also be transversely polarized with $\lambda = +1$. This calls for the tensor meson

getting contributions from both orbital angular momentum and spin, which is symmetric. Because the distribution amplitude of tensor meson is anti-symmetric, the total wave function is anti-symmetric, which is forbidden by Bose statistics. So the transverse polarization of final states is suppressed, with the percentage at the range of 20% to 30%.

For the W annihilation type $B_{(s)} \to D^*T$ decays, we also find very large transverse polarizations up to 80% . The light quark and unti-quark produced through hard gluon are left-handed or right-handed with equal opportunity. The c quark is left-handed, and then the D^* meson can be longitudinally polarized, or be transversely polarized with $\lambda = -1$. For the tensor meson, the anti-quark from weak interaction is right-handed; while the quark produced from hard gluon can be either left-handed or right-handed. When taking into account the additional orbital angular momentum, the tensor meson can be longitudinally polarized or transversely polarized with $\lambda = -1$. So the transverse contributions can become so large with interference from other diagrams. On the other hand, the W exchange diagrams can not contribute large transverse contributions, which is consistent with the argument in $B \to D^*V$ decays in refs.[16,25].

The B_c meson is unique, since it is a heavy quarkonium with two different flavors. Either the heavy b quark or the c quark can decay individually. And the W annihilation diagram decay of the B_c meson is also important due to the large CKM matrix element V_{cb}. Since the c quark in the $B_c \to D^{(*)}T$ decays is the spectator quark, these decays are dominated by the $B_c \to D^{(*)}$ transition form factors with a tensor meson emitted from vacuum, which are prohibited in naive factorization. To our knowledge, these decays are never considered in theoretical papers due to this difficulty of factorization. The annihilation amplitudes will be dominant in these $B_c \to D^{(*)}T$ decays, because they are proportional to the large CKM matrix elements $V_{cb}V^*_{cs(d)}$ instead of $V_{ub}V^*_{us(d)}$ in the emission diagrams.[11]

The LHC experiment, specifically the LHCb, can produce around 5×10^{10} B_c events each year[27]. The B_c decays with a decay rate at the level of 10^{-6} can be detected with a good precision at LHC experiments [28]. So some of these $B_c \to D^{(*)}T$ decays can be observed in the experiments. For example, $B_c \to D^{(*)+}K_2^{*0}$, the branching ratio is at the order of $10^{-5}(10^{-4})$. Taking into account the branching ratios of D^+ and K_2^{*0} decays with charged final states (10% ($D^+ \to K^-\pi^+\pi^+$)[29] and 25% ($\mathcal{B}(K_2^{*0} \to K\pi) = (49.9 \pm 1.2)\%$) respectively) and assuming a total efficiency of 1% [29], one can expect about dozens of events every year. For $B_c \to D_s f_2'$ with branching ratio 4×10^{-5}, taking into account the branching ratios of D_s^+ and f_2' decays with charged final states (6% ($D_s^+ \to K^+K^-\pi^+$) and 45% ($\mathcal{B}(f_2' \to K\bar{K}) = (88.7 \pm 2.2)\%$) respectively) and assuming a total efficiency of 1%, one can expect about one hundred events every year.

4   Summary

We have studied the $B_{u,d,s,c}$ decays involving a light tensor meson in final states within the framework of perturbative QCD approach. We calculate the contributions of different diagrams, especially the hard scattering and annihilation type diagrams, which are important to explain the large experimental data and the large transverse polarization fractions. For some decays with tensor meson emitted or pure annihilation type decays, we give the predictions for the first time. For those penguin dominant decays and color-suppressed decays, we give larger and more reliable predictions, which agree the experimental data better. For those color suppressed $B_{(s)} \to \bar{D}^*T$ decays, the transversely polarized contributions from hard scattering diagrams are very large. For those W annihilation type $B_{(c)} \to D^*T$ decays, the transverse polarized contributions from factorizable annihilation diagrams are as large as 80%.

References

1. J. Beringer *et al.*,(Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
2. C. S. Kim, B. H. Lim and S. Oh, Phys. Rev. D **67**, 014002 (2003).
3. J. H. Muñoz and N. Quintero, J. Phys. G **36**, 095004 (2009).
4. N. Sharma, R. Dhir and R. C. Verma, Phys. Rev. D **83**, 014007 (2011).
5. C. S. Kim, B. H. Lim and S. Oh, Phys. Rev. D **67**, 014011 (2003).
6. H. Y. Cheng, Y. Koike and K. C. Yang, Phys. Rev. D **82**, 054019 (2010).
7. Hai-Yang Cheng and Kwei-Chou Yang, Phys. Rev. D **83**, 034001 (2011).
8. Wei Wang, Phys. Rev. D **83**, 014008 (2011).
9. Z. T. Zou, X. Yu and C. D. Lü, Phys. Rev. D **86**, 094015 (2012).
10. Z. T. Zou, X. Yu and C. D. Lü, Phys. Rev. D **86**, 094001 (2012); Z. T. Zou, Zhou Rui and C. D. Lü, Chin. Phys. C **37**, 013103 (2013).
11. Z. T. Zou, X. Yu and C. D. Lü, Phys. Rev. D **87**, 074027 (2013).
12. C. D. Lu, K. Ukai, Eur. Phys. J. C **28**, 305 (2003).
13. Y. Li, C. D. Lu, J. Phys. G **29**, 2115 (2003); High Energy Phys. & Nucl. Phys. **27**, 1062 (2003).
14. Y. Li, C. D. Lü, Z. J. Xiao and X. Q. Yu, Phys. Rev. D **70**, 034009 (2004).
15. A. Ali *et al.*, Phys. Rev. D **76**, 074018 (2007).
16. R. H. Li, C. D. Lu and H. Zou, Phys. Rev. D **78** 014018 (2008).
17. M. J. Morello *et al.* (CDF Collaboration), CDF public note Report No.10498,2011.
18. G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
19. C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, (2001) 074009.
20. C. H. Chang and H. N. Li, Phys. Rev. D **55**, 5577 (1997); T. W. Teh and H. N. Li, Phys. Rev. D **56**, 1615 (1997).
21. H. N. Li, Phys. Rev. D **66**, 094010 (2002).
22. Y.Y.Keum, H. N. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).
23. H. N. Li, Prog. Part. Phys. **51**, 85 (2003), and reference therein.
24. Qin Qin, Zhi-Tian Zou, Xin Yu, Hsiang-nan Li and Cai-Dian Lü, arXiv:1401.1028.
25. H. Zou, R. H. Li, X. X. Wang and C. D. Lü, J. Phys. G **37**, 015002 (2010).
26. Zhou Rui, Z. T. Zou and C. D. Lü, Phys. Rev. D **86**, 074008 (2012).
27. N. Brambilla *et al.* (Quarkonium Working Group), Eur. Phys. J. C **71**, 1534 (2011).
28. S. Descotes-Genon, J. He, E. Kou and P. Robbe, Phys. Rev. D **80**, 114031 (2009).
29. Y. N. Gao, J. B. He, Patrick Robbe, Marie-Hélène Schune, Z. W. Yang, Chin. Phys. Lett 27, 061302 (2010).