

Galileon Fifth Forces

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Abstract. The Galileon is a proposal to explain the current accelerated expansion of the universe through a scalar field with non-linear dynamics. The form of these non-linearities is protected by the Galileon symmetry and the theory is ghost free. I review the way in which the Galileon scalar field hides from experimental probes of gravity dynamically, showing that the theory is less constrained than previously supposed. I discuss the implications of this for quantum strong coupling scale.

1. Introduction

The past decade has provided compelling evidence that the expansion of the universe is accelerating. The simplest explanation for this is the presence of a cosmological constant term in the Einstein equations governing the evolution of the universe

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} + \Lambda g_{\mu\nu}, \quad (1)$$

where the effective cosmological constant which drives the accelerated expansion of the universe is the sum of a bare cosmological constant and a contribution from the energy density of the vacuum

$$\Lambda_{eff} = \Lambda + 8\pi G\langle\rho_{vac}\rangle. \quad (2)$$

Field theory estimates of the vacuum energy density indicate that it scales as the fourth power of the energy cut-off of the theory. This leads to the cosmological constant problem: the scale predicted for Λ is far larger than the observed value $\Lambda \sim 10^{-3}$ eV.

It is therefore worth considering whether there is an alternative explanation for the accelerated expansion of the universe. In particular whether our failure to explain observations of the current expansion of the universe indicates a failure of Einstein gravity on the largest scales in the universe. One example of such an alternative explanation is the DGP model [1] where the universe is confined to a 3-brane embedded in a 5D Minkowski bulk. In this scenario gravity appears four-dimensional below a length scale L , and five-dimensional at larger distances. The DGP model can describe an FRW universe which gets accelerated at late times by the leaking of gravity into the fifth dimension without the need for a cosmological constant (or a dynamical dark energy field) on the brane. However it has been shown that the self accelerating branch of the DGP model contains negative energy classical solutions known as ghosts and is therefore unstable [2].

Are there other modifications of gravity which could describe the accelerated expansion of the universe, without the problems encountered by the DGP model? Such a theory would have to describe significant modifications of gravity on the largest observable scales in the universe but negligible modifications of gravity within the solar system to be in agreement with observations. By necessity therefore, it must be non linear. To avoid the problems of the DGP model it must also be free of negative energy solutions - which requires that the equations of motion are second order. These two requirements led to the construction of the Galileon model by Nicolis, Rattazzi and Trincherini [3], which we now proceed to describe.

2. The Galileon

Consider modifications of general relativity where locally, and for weak fields, the deviations can be written in terms of a light scalar field, π . The Galileon model assumes that the theory obeys a covariant generalisation of the Galileon shift symmetry

$$\pi(x) \rightarrow \pi(x) + b_\mu x^\mu + c. \quad (3)$$

The coupling between π and other matter fields is assumed to be linear, and the back reaction of the energy density of the scalar field is neglected.

There are then only five possible Lagrangians which obey the Galileon symmetry and give rise to second order equations of motion [3]:

$$\mathcal{L}_1 = \pi, \quad (4)$$

$$\mathcal{L}_2 = -\frac{1}{2}\partial\pi \cdot \partial\pi, \quad (5)$$

$$\mathcal{L}_3 = -\frac{1}{2}(\Box\pi)\partial\pi \cdot \partial\pi, \quad (6)$$

$$\mathcal{L}_4 = -\frac{1}{4}[(\Box\pi)^2\partial\pi \cdot \partial\pi - 2(\Box\pi)\partial\pi \cdot \Pi \cdot \partial\pi - (\Pi \cdot \Pi)(\partial\pi \cdot \partial\pi) + 2\partial\pi \cdot \Pi \cdot \Pi \cdot \partial\pi], \quad (7)$$

$$\begin{aligned} \mathcal{L}_5 = & -\frac{1}{5}[(\Box\pi)^3\partial\pi \cdot \partial\pi - 3(\Box\pi)^2\partial\pi \cdot \Pi \cdot \partial\pi - 3\Box\pi(\Pi \cdot \Pi)(\partial\pi \cdot \partial\pi) + 6(\Box\pi)\partial\pi \cdot \Pi \cdot \Pi \cdot \partial\pi \\ & + 2(\Pi \cdot \Pi \cdot \Pi)(\partial\pi \cdot \partial\pi) + 3(\Pi \cdot \Pi)\partial\pi \cdot \Pi \cdot \partial\pi - 6\partial\pi \cdot \Pi \cdot \Pi \cdot \Pi \cdot \partial\pi], \end{aligned} \quad (8)$$

where $\Pi^\mu_\nu = \partial^\mu \partial_\nu \pi$ and $\cdot$ indicates a contraction of indices. The most general Galileon Lagrangian is a linear combination of the above with arbitrary constant coefficients

$$\mathcal{L}_\pi = \sum_{i=1}^5 c_i \mathcal{L}_i. \quad (9)$$

The Galileon symmetry prevents other non-linear terms from being generated by quantum fluctuations. A five dimensional theory has been constructed which gives rise to the Galileon model in an appropriate four-dimensional limit [4]. Then the constant coefficients c_i are related to brane tensions, and the coefficients of the curvature invariants in the higher dimensional Lagrangian. Previously it has been suggested that observational constraints force all of the c_i to be at the Planck scale. Here we show that this is not the case, broadening the spectrum of Galileon phenomenology.

The Galileon theory can describe the current accelerated expansion of the universe as there is a solution to the Galileon equations of motion which corresponds to de Sitter space, even if all other sources are absent [3]

$$\pi_{dS}(x) = -\frac{1}{4}H^2 x_\mu x^\mu. \quad (10)$$

This is a self accelerating solution, analogous to those found for the DGP model. But unlike the DGP model the Galileon scenario is free of ghost-like instabilities. The existence of this self acceleration solution imposes one algebraic constraint on the five coefficients c_i :

$$c_1 - 2c_2H^2 + 3c_3H^4 - 3c_4H^6 + \frac{3}{2}c_5H^8 = 0. \quad (11)$$

3. Galileon Forces

The existence of new light scalar fields as an explanation for the accelerated expansion of the universe has long been known to be problematic; such fields typically mediate new long range fifth forces. Laboratory and solar system tests of gravity tightly constrain the existence of such forces, and a light canonical scalar field can only exist if it couples to matter with a strength many orders of magnitude weaker than the gravitational force. However non-linearities were included in the Galileon model precisely to try to avoid this problem. We now turn to how these non-linearities help to screen the effects of the scalar field within the solar system.

In the presence of matter π is perturbed from its background value $\pi \rightarrow \pi_{dS} + \pi$, and it is these perturbations which potentially mediate strong forces between massive objects. The force due to the Galileon field can be shown to be proportional to the gradient of the scalar field profile around a massive object [5], therefore in order to know the strength of the Galileon force, we must first determine the field profile generated by massive objects.

The spherically symmetric, static equation of motion for the Galileon field near an object of mass M is

$$d_2 \left(\frac{\pi'}{r} \right) + 2d_3 \left(\frac{\pi'}{r} \right)^2 + 2d_4 \left(\frac{\pi'}{r} \right)^3 = \frac{M}{4\pi r^3}, \quad (12)$$

where the d_i are linear combinations of the coefficients c_i . Stable solutions to this equation exist if $d_2 > 0$, $d_4 \geq 0$, $d_3 \geq \sqrt{(3/2)d_2d_4}$, $d_5 < 0$ [3]. If the non-linearities are absent $d_3 = d_4 = d_5 = 0$ we recover the linear solution

$$\pi(r) = \pi_0 - \frac{M}{4\pi d_2 r}, \quad (13)$$

and the ratio of the Galileon force to the Newtonian gravitational one is

$$\frac{F_\pi}{F_N} = \frac{2M_P^2}{d_2}. \quad (14)$$

As previously discussed, we see that the scalar force can only be suppressed by tuning the parameter d_2 to be much larger than the Planck scale. However in the full Galileon theory the force can be made substantially weaker when the non-linearities become important.

The equation of motion (13) can be rewritten

$$g + \left(\frac{R_1}{r} \right)^3 g^2 + \left(\frac{R_2}{r} \right)^6 g^3 = 1 \quad (15)$$

where $\pi'/r = (M/4\pi r^3 d_2)g(r)$ and $R_1^3 = d_3 M/2\pi d_2^2$, $R_2^6 = M^2 d_4/8\pi^2 d_2^3$. Then the Galileon force is dynamically suppressed whenever $g(r) < 1$. It's clear that whenever d_3 or d_4 is non zero g is forced to tend to zero in the interior of any massive object. The radius within which the non-linearities become important and g becomes less than unity is known as the Vainshtein radius

$$r_* \sim \max\{R_1, R_2\}. \quad (16)$$

Unlike in the original massive gravity scenario discussed by Vainshtein, or the DGP model, the Vainshtein radius of an object is not fixed in the Galileon model. There remains enough freedom in the model to make the Vainshtein radius of any object arbitrarily large or small.

3.1. Within the Vainshtein Radius

There are two regimes of behaviour of the field within the Vainshtein radius, depending on which of the nonlinearities dominate in (13), forming concentric shells around the source object. We call these region A and region B.

In region A: $\alpha R_2 \ll r \ll R_1$, where $\alpha + (R_2/R_1)^3 < 1$. Here the ratio of the Galileon to gravitational force is

$$\frac{F_\pi}{F_N} = \frac{M_P^2}{4\pi d_2} \left(\frac{r}{R_1} \right)^{3/2} = \frac{M_P^2}{2} \left(\frac{1}{2\pi d_3 M} \right)^{1/2} r^{3/2}. \quad (17)$$

In region B: $0 < r \ll \alpha R_2$. Here the ratio of the Galileon to gravitational force is

$$\frac{F_\pi}{F_N} = \frac{M_P^2}{4\pi d_2} \left(\frac{r}{R_2} \right)^2 = \frac{M_P^2}{2} \left(\frac{1}{\pi d_4 M^2} \right)^{1/3} r^2. \quad (18)$$

The force inside the Vainshtein radius is always independent of d_2 and therefore we can vary the position of the Vainshtein radius and the strength of the Galileon force inside that Vainshtein radius independently. In particular note that, as the position of the Vainshtein radius is arbitrary, we do not need to impose that the Galileon force is weaker than gravity everywhere inside the Vainshtein radius in order to be in agreement with observational constraints.

To ensure that the Galileon field satisfies the observational constraints from any gravitational experiment dynamically, and without fine tuning of the parameters, we must follow a two step procedure. Firstly we must ensure that a gravitational experiment lies within its own Vainshtein radius, so that the scalar force is dynamically suppressed within the experiment. This imposes a constraint on either R_1 or R_2 . Secondly we must impose that the Galileon force within the experiment is sufficiently weak to evade detection. This imposes a constraint on d_3 or d_4 respectively.

Determination of the full set of constraints imposed on the Galileon model by gravitational experiments remains for future work. Here we make only the first step along this path by requiring that gravitationally bound objects should not feel significant deviations from general relativity. This requires that the Vainshtein radius of a gravitationally bound object must be bigger than its size, and that within the bound radius the Galileon force is suppressed compared to the gravitational one. This imposes two constraints on the parameters of the model:

$$d_2 \lesssim M_P^2, \quad (19)$$

$$d_3 \gtrsim 10^{118}. \quad (20)$$

4. The Strong Interaction Scale

As well as determining the strength of the Galileon force the couplings c_i , or equivalently d_i , also control the energy scale at which quantum fluctuations of the Galileon field becomes strongly self interacting. If this scale is too low, within reach of current experiments, we might worry that we do not have sufficient control over the theory to predict whether the theory really is in agreement with observations.

The Lagrangian for a canonically normalised perturbation ϕ about the de Sitter background $\pi \rightarrow \pi_{dS} + \phi$ can be written schematically as

$$\mathcal{L}_\phi = -(\partial\phi \cdot \partial\phi) \left(\frac{1}{2} + \frac{1}{2} \frac{d_3}{d_2^{3/2}} [P] + \frac{1}{4} \frac{d_4}{d_2^2} [P]^2 + \frac{1}{5} \frac{d_5}{d_2^{5/2}} [P]^3 \right) \quad (21)$$

where $[P]$ denotes polynomials in $\partial\partial\phi$ with the required contractions of indices. Inspection of this Lagrangian might lead us to conclude that strong interactions become important at potentially

low scales, for example $d_2^{1/2}/d_3^{1/3}$. However this would be misleading as no experiment can ever be done without the presence of matter. Instead we measure fluctuations in the neighbourhood of massive objects and we should compute the strong coupling scale within an object's Vainshtein radius.

Within the Vainshtein radius renormalisations from the background alter the strong coupling scale. For example if $d_4 = d_5 = 0$ we find that the Lagrangian for a perturbation is

$$\mathcal{L}_\phi = Z_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - \frac{d_3}{2} (\Box \phi) \partial \phi \cdot \partial \phi, \quad (22)$$

where $Z_{\mu\nu}$ can be diagonalised with all eigenvalues at the scale

$$Z \sim -\frac{d_2}{2} \left[1 - \frac{1}{2} \left(\frac{R_1}{r} \right)^{3/2} \right]. \quad (23)$$

Combining this with the previously found constraints on d_2 and d_3 , (20) and (21), we find that the quantum cut off at the surface of the earth is

$$\tilde{\Lambda}(x) \sim \frac{Z^{1/2}}{d_3^{1/3}} \lesssim \frac{1}{1 \text{ cm}} \quad (24)$$

This is problematic for the Galileon model as gravitational experiments at the surface of the earth have probed gravitational forces on distance scales shorter than this would allow. However this problem is alleviated if the other non-linearities also play a role. For example, allowing all the d_i to be non-zero, if the surface of the earth lies within region B, defined above, then the strong coupling scale is again determined by a diagonalisable matrix $Z_{\mu\nu}$, where the eigenvalues are now

$$Z_r \sim 3d_2 \left(\frac{R_2}{r} \right)^2, \quad (25)$$

$$Z_t \sim 3d_2 \left(\frac{R_2}{r} \right)^2 - \frac{d_5 d_2^{3/2}}{\sqrt{2} d_4^{3/2}}, \quad (26)$$

$$Z_\Omega \sim -d_2 + \frac{2d_3^2}{3d_4}, \quad (27)$$

which can all be made sufficiently large that the quantum strong interaction scale can be put well beyond the reach of experiments.

5. Conclusions

The Galileon is a scalar field which couples to the fields of the Standard Model. It has a background self accelerating solution which explains observations of the current accelerated expansion of the universe without the need for a cosmological constant. Its dynamics are non-linear but protected by the Galileon symmetry, and the equations of motion for the field are second order so that the theory is ghost free.

The non-linearities of the theory allow the field to hide from fifth-force experiments without fine tuning the parameters of the theory. The Galileon field effectively becomes weakly coupled in the presence of matter, and so hides from gravitational experiments dynamically. The size of the Vainshtein radius, which controls when the non-linearities become important, is arbitrary in the Galileon model, and remains to be constrained by experiment. In contrast to previous assumptions this means that we are not required to set all the parameters of the theory at the

Planck scale. It also means that the strong coupling scale of the theory can be made arbitrarily high, and therefore placed out of reach of experiments.

In summary the Galileon model is less constrained by experiment than was previously supposed, and much of its phenomenology remains to be explored!

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References

- [1] Dvali G R, Gabadadze G and Porrati M 2000 *Phys. Lett.* **B485** 208–214 (*Preprint hep-th/0005016*)
- [2] Luty M A, Porrati M and Rattazzi R 2003 *JHEP* **09** 029 (*Preprint hep-th/0303116*)
- [3] Nicolis A, Rattazzi R and Trincherini E 2009 *Phys. Rev.* **D79** 064036 (*Preprint 0811.2197*)
- [4] de Rham C and Tolley A J 2010 *JCAP* **1005** 015 (*Preprint 1003.5917*)
- [5] Burrage C and Seery D 2010 *JCAP* **1008** 011 (*Preprint 1005.1927*)