defined analytic expressions having a definite number of adjustable parameters. This means, first, that our results for the phases are analytic functions that can be incorporated analytically into other calculations. Furthermore, our knowledge of the number of parameters permits a meaningful statistical analysis, as the number of degrees of freedom is known. In the Yale work a succession of correction functions were used to modify the phases to conform with the data. This gives neither a well-defined analytic expression nor an unambiguous way of counting the number of constants used in the fitting process. Our work provides us with a statistical standard to which later work with dispersion theoretic energy dependences can be compared.

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FORM OF ONE-PION EXCHANGE POTENTIAL AND CHARGE INDEPENDENCE OF NUCLEON-NUCLEON INTERACTION (*)

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(presented by G. Breit)

This report is a continuation of previously made tests [1] of charge independence (CI) and of the form of the one pion exchange potential (0PEP) [2]. The method makes use of phase parameter fits obtained on the assumption that the (0PEP) applies to high L and J. The procedure of searching for the other phase parameters has been described [3,4] previously. The CI tests, the preliminary results of which are given below, are concerned only with the 0PEP group of phase parameters in contrast with those in [1]. The value of the pion-nucleon coupling constant referred to as g_0^2 in [5] has been varied from its normal value 14 which has been used in the searches for the non-0PEP phase parameters as in [1]. The difference from [1] consists in the employment of results of improved searches, the inclusion of data that became available since the publication of [3] and [4] and the inclusion of nucleon magnetic moment corrections [6] in formulae used for the calculation of experimental quantities from phase parameters.

The form of the 0PEP was tested by generalizing the 0PEP in two of the three ways used in [2] as follows

$$\delta_b = \delta^{1\pi} + \frac{14}{g_0^2} q_b \delta^{1\pi}_{(\vec{\sigma}\,\vec{\sigma})} = \frac{g_0^2}{14}\delta_0^{1\pi} + q_b \delta^{1\pi}_{0(\vec{\sigma}\,\vec{\sigma})}\,, \qquad (1)$$

$$\delta_0 = \delta^{1\pi} + \frac{14}{g_0^2} q_c \delta_c^{1\pi} = \delta^{1\pi} + q_c \delta_{co}^{1\pi}\,. \qquad (2)$$

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Here the δ represents schematically the phase shifts and coupling parameters, the subscript $\sigma\sigma$ denotes the part of the 0PEP δ which is caused by the spin-spin part of 0PEP, the subscript 0 indicates that the quantity as calculated for $g_0^2 = 14$, q_b, q_c are dimensionless parameters, having value 0 for 0PEP, subscripts c refer to a central potential the presence of which is also being tested. It is taken to have the form

$$V_c^{(2)} = \frac{1}{3}g^2\left(\frac{\hbar\mu}{2Mc}\right)^2 e^{-\mu r}/r\,, \qquad g^2 = \hbar c g_0^2\,. \quad (3)$$

This is the potential which would be obtained for $(\tau_1\tau_2) = 1$ and $(\sigma_1\sigma_2) = 1$ omitting S_{12} and is $-1/3$ of the singlet even potential corresponding to 0PEP. Variations of q_b tests the correctness of the proportions of the tensor and spin-spin parts of the 0PEP, variation of q_c for the presence of a central potential with the same range parameter $1/\mu$ as that of the theoretical 0PEP. In addition to varying the q for the nominal value $g_0^2 = 14$, this quantity was given several values and the q varied again; in this way the two dimensional minimum of weighted mean square deviation $D(q, g_0^2)$ was determined. Estimates of uncertainty in q and g_0^2 were made as described in the second reference in [2]. In all of the present work on pp data the pion mass was taken as $m(\pi^0)$. In some of the older work [1] $m(\pi^+)$ was used for $p-p$ and $n-p$ but in the later $n-p$ fits in [1,3,4] and here the combinations [3] $2\delta(m_{\pi+})-\delta(m_{\pi 0})$ for $T=1$ and of $(1/3)[2\delta(m_{\pi+})+\delta(m_{\pi 0})]$ for $T=0$ were employed. The values in the new CI tests for $p-p$ are as in Table I.

TABLE I

Values of g_0^2 from p—p fit YLAM (*)

K_4	Other L, J in 0PE	Magnetic moment effects	g_0^2
0PE	$L>3$	Included	13.7±0.8
non-0PE	$L>3$	Included	14.7±0.9
non-0PE	$L>3$	Omitted	15.5±1.0

(*) Error limits by graphical method [1] including factor $D^{1/2}$. They are believed to be somewhat too low.

Values of g_0^2 obtained from $n-p$ measurements which, in addition to new published data, included recent high accuracy large angle cross section material of Ashmore *et al.* [7] at 350 MeV are shown in Table II. These numbers may be compared with $g_0^2 = 14.3\pm1.0$ obtained in [7] by pole extrapolation. Comparison of values in Table II with those in Table I indicates no systematic difference between $p-p$ and $n-p$ values and supports CI. The small accuracy of value for the π^0 part in the $n-p$ case is caused by the dominance of cross section effects which emphasize charge exchange. The first value in Table I is affected by the impropriety of the inclusion of K_4 in the 0PE group as known from evidence on the TPE [8].

Values of (q, g_0^2) pairs obtained for $p-p$ and $n-p$ data are summarized in Tables III and IV.

Search YLAN3M″ differs from YLAN3M-350 through the omission of the 350 MeV cross section data; for it K_5 was used in the 0PE group. Compa-

TABLE II

Values of g_0^2 including effects of 350 MeV n—p and other later data, 0PEP for $L>4$ but K_5 released in g_0^2 variation.(**)

(Search YLAN3M—350, 393 data)

Values of τ	0,1	0(*)	1(*)		
Values of e_π^2	0,1	0,1	0,1	0(*)	1(*)
	14.1±0.3	13.7±0.6	14.9±1.2	15.9±2.7	14.4±0.4

(*) Last four columns are for simultaneous variations in pairs.

(**) All tabulated error limits are obtained graphically including factor $D^{1/2}$. Correlation of errors of g_0^2 with other uncertainties increases error limits by factor ~2.7.

rison of q_c values in Tables III, IV, V, VI shows a tendency for q_c to become $\neq 0$ if the 0PE group goes to too low L. This has been noticed previously [2] and interpreted [8] as evidence of TPE. The error estimates for the q are believed however somewhat too low on account of correlation effects mentioned in a foot note to Table II. In view of this as well as the spread in values of the q obtained in different ways, there is no reason from the tests reported on to regard the values of the q_b as being not zero within the uncertainty of the tests and similarly for q_c when too low L are not in the 0PE group.

An uncertain part of the analysis presented is the calculation of the error limits. These are strongly affected by the correlation of errors of the quantity to be determined (g_0^2 or q) and those of the searched phase parameters. This relationship is present in the standard employment of the error matrix but, since the searching for parameters is partly affected by judgment used regarding the choice of energy dependent fitting functions, such a purely formal procedure appeared unsatisfactory. The information supplied by the error matrix was therefore used only partially and more weight was attached to estimates made by the following methods.

The first consists in setting g_0^2 at a deliberately wrong value such as 10 or 18, searching for best phase parameters with the same degree of precision as for the probable g_0^2, computing the mean square deviation D for the end result of the search and employing the theoretical relationship between this D and the value of D for the search with $g_0^2 \cong 14$, D^0. A value for the desired factor is obtained from D and D^0. Fixing the phase parameters at the value obtained for a wrong g_0^2 the best g_0^2 for minimizing D is determined and D recomputed. From $(g_0^2)'$, the shifted g_0^2, and its present value, another in practice similar estimate of the factor is obtained. The consistency of the assumption that the searches are of the same quality is tested by the comparison of the three values of D which ideally related to the three values of g_0^2 by a simple formula. The whole procedure is repeated by presetting g_0^2 at another value on the other side of the probable g_0^2.

The second method makes use of tests for the statistical significance of searching for some of the parameters rather than including them in the 0PE group. The degree to which the search is overparametrized is thus determined and through the employment of the error matrix for part of the data the effect of overparametrization on the uncertainty estimate is ascertained.

The two methods gave approximately the same results on the basis of which statements regarding the error limits have been made above.

TABLE III

Values of (q, g_0^2) pairs from p—p search YLAM; K_4 in 0PEP groups; magnetic moment effects included.

Search	Case	q	g_0^2
YLAM	b	-0.20 ± 0.10	15.5 ± 1.0
	c	0.35 ± 0.30	14.0 ± 0.8

TABLE IV

Values of (q, g_0^2) pairs from n—p search YLAN3M; K_4 in 0PEP group; magnetic moment effects included; old data only.

Case	q	g_0^2
b	-0.43 ± 0.15	16.1 ± 1.8
c	-3.3 ± 1.2	10.1 ± 0.8

TABLE V

Values of q for n—p search YLAN3M—350 and YLAN3M″. Magnetic moment effects included.

Case	$3M-350$	$3M''$
b	0.07 ± 0.04	-0.27 ± 0.15
c	-0.44 ± 1.1	-1.44 ± 0.7

TABLE VI

Values of q_b and q_c determined from YLAM; magnetic moment effects and new data included; K_4 in 0PEP group.

Search	Assumed g_0^2	q_b	q_c
YLAM	14	-0.09 ± 0.05	0.34 ± 0.13

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DISCUSSION

MORAVCSIK: I would like to make a comment that I should have made when I gave the talk. I think that probably one of the most useful aspects of the work is that we have complete sets of predictions for measurable observables at all energies, at all angles, and we have graphs of these observables so that, in case any of the many groups planning a new nucleon-nucleon scattering experiment would like to have this, we would be glad to send it to them. I think it is useful to get an orientation of what magnitude you get, and it might also help to pick out those experiments which are the most interesting ones from the point of view of further determination of the nucleon-nucleon interaction.

CHEW: I want to point out that Balazs has developed a general set of effective range formulae which can be used in programmes such as described by Moravcsik and also in pion-nucleon, *K*-nucleon or any other scattering problems where you want to make an energy dependent phase-shift analysis. These formulae have the following properties: they can be used for two body channels of any partial waves, they are based on an analysis of the location of unphysical or inelastic cuts, on the range of the physical region that you want to represent, and on the desired accuracy of representation. He claims to be able to establish on an " a priori " basis, how many parameters you must take in the effective range formula in order to achieve a pre-assigned accuracy over a pre-assigned physical range without any detailed knowledge of the unphysical discontinuities. These formulae were the basis for part of the dynamical $\pi-\pi$ calculation that I described yesterday, but they were the non-controversial part. The controversy arises over whether you have the ability to establish the effective range parameters on theoretical grounds from crossing relations, not on whether the formulas are themselves accurate.

BREIT: I would also like to mention the names of the people concerned with the work on which I reported, although, of course, they are on the programme: Hull, Ruppel, McDonald and I have been concerned in this work and, just as in the case of the Livermore Group, we have a number of quantities available in the form of curves which we will be glad to furnish to experimental groups and, of course, both Livermore and Yale have been furnishing such graphs during the last few years.

EDEN: I would like to ask whether there has been any thought given to the analysis of phase shifts by Regge poles? The 0PEP is equivalent to a set of Regge poles along the negative integers in the complex *L*-plane. The two-pion exchange potential could be similarly analysed; they would both give energy independent location of the Regge poles but the Regge pole analysis should, I think, throw some light on the energy dependence of phase shifts. For example, one knows from the Yukawa analysis the kind of way that Regge poles should move, and one could see whether the sort of approximations implied by 0PEP and 2PEP are in any way equivalent to the energy dependence that one would expect.

BREIT: In connection with these Regge pole possibilities, just to introduce Regge poles without some prediction regarding their strengths, variation with energy would amount to another parametrization, it would seem to me. Are you in a position to make a prediction regarding what you expect?

EDEN: As far as the 0PEP is concerned or the 2PEP, one knows the strength of the Regge poles involved. There is an infinite series of these poles along negative integers and I do not think that any additional parameters are included there. It is simply a different way of stating the same problem.

BREIT: But don't you have to bring in the $\pi-\pi$ interaction and make some hypothesis about it?

EDEN: The $\pi-\pi$ interaction would come into the two-pion analysis.

BREIT: So that in some way one has got to bring in the physical processes. Would not that take one essentially into the first paper we heard?

EDEN: Yes, the point that I had in mind is that it gives another method of determining approximations involved: the Regge poles are, roughly speaking, rather directly related to the phase shifts and therefore one would expect the nearby Regge poles to be more important than the far-off ones.

BREIT: Well, is it not this way? The Regge poles give an analysis in terms of partial waves and in these calculations the contributions due to the one-pion exchange...

PEIERLS: This is a question which perhaps should be discussed at the next conference when we have had time to think more about this suggestion.

LOW: I have a question for Dr. Moravcsik: the accuracy of the determination of the coupling constant suggests the possibility of determining the three pion coupling constants, there are the positive and the two neutral coupling constants. Have you considered this problem?

MORAVCSIK: I should mention that the coupling constant we are determining, is the coupling of the neutral π to proton, since in proton-proton scattering the only pion that is being exchanged is π^0. If I understood your question right it is whether it is possible to compare this with coupling constant say, of π^+ to proton. I think that this might be possible if the *np* data were extensive and accurate enough, so that the presumably small differences in the coupling constant could be detected. We have do neno work on *n-p* data so far, but maybe Professor Breit has some comment on it.

BREIT: I thought that the paper presented dealt with answers to that question. The tables in my paper give values of the coupling constant which appear not to differ within the error of determination from the values obtained with the same methods from $p-p$ scattering.

CALCULATION OF PION-NUCLEON PHASE-SHIFTS FROM DISPERSION RELATIONS

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(presented by G. Höhler)

The small pion-nucleon scattering phase-shifts have been calculated by Chew, Goldberger, Low and Nambu [1], using relativistic dispersion relations and the data of the first resonance. The authors introduced several approximations without going into the details of their validity. In an earlier paper [2] we have pointed out that a more accurate evaluation is necessary at least for $\mathrm{Re}\, f_s^{(-)} = (\sin 2\alpha_1 - \sin 2\alpha_3)/6q$ (q = pion momentum). The corrections which follow from an exact treatment of the recoil effects and of the projection on the partial amplitudes are shown in Fig. 1. α_{33} has been taken from the results of the phase shift analysis ($\mathrm{Re}\, f_s^{(-)}$ is changed considerably if the effective range approximation is used above 200 MeV).

The difference Δ between the improved CGLN approximation and the experimental data is almost a constant up to 300 MeV (Fig. 1). At threshold some information on the neglects of the CGLN theory is found by a comparison of our improved CGLN formula with the unsubtracted dispersion relation for forward scattering. It turns out that $\Delta(0)$ is given by an integral over $\sigma_+ - \sigma_- - 2\sigma_{33}/3$ and the main

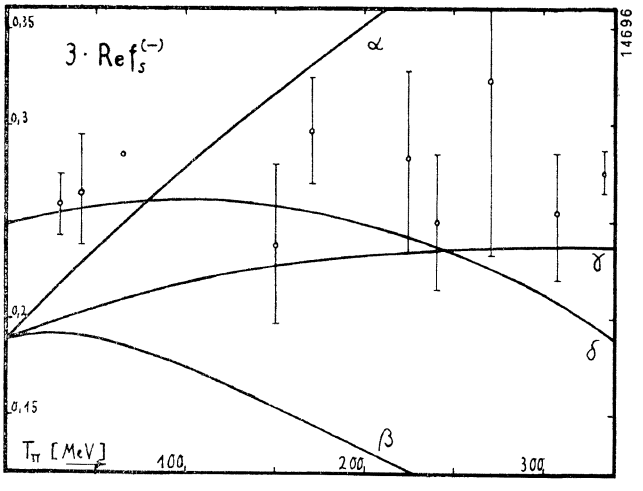

Fig. 1 3 Re $f_s^{(-)}$ as a function of the pion lab. energy. α, β, γ: CGLN term (α: formula (3.20) CGLN; β: exact treatment of recoil CGLN (3.19); γ: exact treatment of recoil and projection) δ: result of the present paper.

contributions come from the 2nd and 3rd maximum of the total $\pi-p$ cross section.

At the present time it is not possible to improve the CGLN approach by treating the 2nd and 3rd resonance