FLUCTUATIONS AND VORTICES IN EFFECTIVE THEORIES OF ANYONIC SUPERCONDUCTIVITY

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We study in detail the low-lying excitations and vortices in the broken symmetry phase of Landau-Ginzburg effective theories of Anyonic Superconductivity. These theories are described by a complex scalar field interacting with a U(1) gauge field. The gauge field action incorporates a Chern-Simons term. We find in the topologically trivial sector that the angular momentum has an anomaly and the low energy fluctuations are described by a charge-neutral massive *boson*. The vortices are charged and carry angular momentum. We investigate in detail the profiles for magnetic field, charge, electric field and order parameter. We find that the magnetic field nucleates away from the origin, and the charge and electric field distribution are localized within a ring away from the origin. We conjecture on the possibility of roton-like excitations in the spectrum.

There is presently much interest in the understanding of the superconducting properties of a system of particles with fractional statistics[1,2]. We study the Landau-Ginzburg effective theory as derived by Banks and Lykken[3] and Fradkin[4]. Here we will briefly report on our work[5,6]. This effective theory is described by a complex scalar field interacting with a U(1) gauge field and is obtained from the underlying microscopic theory in the long-wavelength limit after some duality transformations. The U(1) gauge field is the usual electromagnetic field, but after integrating out the statistical interaction in the microscopic theory, there appears a Chern-Simons term for the electromagnetic field in the effective long- wavelength theory. Finally the long-wavelength effective theory is described by the lagrangian

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}F_{\mu\nu}^2 + \frac{\theta}{2}\epsilon^{\mu\nu\lambda}\partial_\mu A_\nu A_\lambda + \\ &\quad (D_\mu\varphi)^\dagger(D^\mu\varphi) - V(\varphi^\dagger\varphi) \\ (D_\mu\varphi) &= \partial_\mu\varphi + ieA_\mu \\ V(\varphi^\dagger\varphi) &= \lambda(\varphi^\dagger\varphi - \rho_o)^2 \end{aligned} \tag{1}$$

The parameter θ is determined by the statistics of the anyons in the underlying microscopic theory, and the parameter λ is phenomenological and requires knowledge of the dynamics and spectrum of the microscopic theory. We will study the case of general anyon statistics and leave θ as a parameter. Let us first of all study the spectrum of low-lying excitations in the broken symmetry phase in the topologically trivial sector. To simplify our discussion we will decouple the Higgs mode and consider only the phase fluctuations of the order parameter. Furthermore since we are interested in the long- wavelength limit, we will neglect for the moment the $F_{\mu\nu}^2$ term since it is higher order in derivatives. The effective Lagrangian density describing the long-wavelength physics of the phase fluctuations (Goldstone modes) is

$$\mathcal{L} = \frac{\theta}{2}\epsilon^{\mu\nu\lambda}\partial_\mu A_\nu A_\lambda + \frac{1}{2}(\partial_\mu\phi - eA_\mu)^2 \tag{2}$$

where ϕ is the phase of the order parameter φ and we absorbed some normalization factors. We now proceed to quantize the low-lying modes of the theory. A consistent quantization first requires an analysis of the Hamiltonian structure of the Chern-Simons part of the above Lagrangian[5]. After a close look at the Lagrangian equations of motion one finds that the correct symplectic structure determines the following equal time commutator for the quantum theory

$$[A^i(x), A^j(y)] = \frac{i}{\theta}\epsilon^{ij}\delta^2(x-y) \tag{3}$$

The Hamiltonian density, Gauss' law operator G and magnetic field B are

$$\mathcal{H} = \frac{1}{2}\Pi_\phi^2 + \frac{1}{2}(\nabla\phi - e\mathbf{A})^2 - A_0 G \tag{4}$$

$$G = \theta B - e\Pi_\phi \tag{5}$$

$$\Pi_\phi = \dot{\phi} - eA_0$$

$$B = \epsilon^{ij}\partial_i A_j$$

By splitting the gauge field into longitudinal and transverse components $A_i(x) = \partial_i\eta + \epsilon_{ij}\partial_j\chi$ and using eq.(3) one finds the equal time commutators

$$\begin{aligned} &[\theta B(x), \eta(y)] = i\delta^2(x-y) \\ &[B(x), B(y)] = [\eta(x), \eta(y)] = 0, \ etc \end{aligned} \tag{6}$$

By choosing the representation in which both η and ϕ are diagonal, Gauss' law constraint on the wave functionals is solved by wave functionals of the form[5]

$$\Psi[\eta, \phi] = \Psi[\eta - \frac{1}{e}\phi] \tag{7}$$

The combination $\eta - \phi/e$ is *gauge invariant*. In the physical subspace (the wavefunctionals are annihilated by G) the last term in the Hamiltonian (4) vanishes. Let us introduce the canonical pair $\rho = \eta - \phi/e$, $\Pi_\rho = (\Pi_\eta - e\Pi_\phi)/2$, and pass to a discrete Fourier representation in a large volume V. Isolating the zero momentum modes and identifying the charge opeator Q with the zero momentum component of Π_ϕ we find on the physical subspace that the Hamiltonian (4) becomes[5]

$$H_{phys} = \frac{Q^2}{2V} + \sum_{k\neq 0}\frac{1}{2}[\ \Pi_\rho(k)\Pi_\rho(-k)\left(\frac{1}{e^2}+\frac{e^2}{k^2\theta^2}\right) + \frac{e^2k^2}{2}\rho(k)\rho(-k)] \tag{8}$$

Performing the Bogoliubov *canonical* transformation

$$\Pi_\sigma(k) = \Pi_\rho(k)\left(\frac{1}{e^2}+\frac{e^2}{k^2\theta^2}\right)^{1/2}$$
$$\sigma(k) = \rho(k)\left(\frac{1}{e^2}+\frac{e^2}{k^2\theta^2}\right)^{-1/2} \tag{9}$$

These are the fields that diagonalize the Hamiltonian

$$H = \frac{Q^2}{2V} + \sum_{k\neq 0}\frac{1}{2}\ [\Pi_\sigma(k)\Pi\sigma(-k) \left(k^2+\frac{e^4}{\theta^2}\right)\sigma(k)\sigma(-k)] \tag{10}$$

The low-lying spectrum is that of a charge neutral free massive boson. The gauge invariant angular momentum is obtained from the commutator of the generator of boosts constructed from the Hamiltonian (4) as required by the spin- statistics theorem. It is equivalent to the one obtained from the symmetric energy-momentum tensor. In the physical subspace it becomes

$$L = \int d^2x x_i\epsilon^{ij}\Pi_\sigma(x)\partial_j\sigma(x) + \frac{Q^2}{4\pi\theta} \tag{11}$$

The last term is the anomaly found by Hagen[7] and it may not be arbitrarily removed because the angular momentum generator is uniquely defined by the Poincare algebra. In the broken symmetry phase, the charge operator Q does not annihilate the vacuum, therefore the angular momentum is not a sharp operator and has strong fluctuations. We now investigate the vortices of the theory. The vortex solutions with n-units of flux are obtained as static solutions to the equations of motion obtained from the Lagrangian density (1) with the following ansatze:

$$\varphi(r,\vartheta) = \varphi(r)e^{in\theta},\ A_o(r,\vartheta) = A(r)$$
$$A_i(r,\vartheta) = \epsilon_{ij}r_j\frac{\Phi(r)}{r^2} \tag{12}$$

The equations of motion take on a very simple form by introducing the mass scales

$$M_H = 2\sqrt{\lambda}\rho_o,\ M_g = \sqrt{2}e\rho_o \tag{13}$$

and the dimensionless ratios and functions

$$\alpha = \frac{M_g^2}{M_H^2},\ \beta = \frac{\theta}{M_H},\ x = M_H r$$
$$f(x) = \frac{\varphi(r)}{\rho_o},\ h(x) = e\Phi(r),\ a(x) = \frac{eA(r)}{M_H} \tag{14}$$

For $\theta = 0$ (conventional superconductors), α is the stiffness parameter that determines type I or type II superconductivity[8]. The equations of motion become

$$a^2f + f'' + \frac{f'}{x} - \frac{(n-h)^2}{x^2}f - \frac{1}{2}f(f^2-1) = 0 \tag{15}$$
$$a'' + \frac{a'}{x} + \beta\frac{h'}{x} - \alpha af^2 = 0 \tag{16}$$
$$h'' - \frac{h'}{x} + \beta a'x + \alpha(n-h)f^2 = 0 \tag{17}$$

The regular solutions obey the boundary conditions as $x \to 0$

$$f \to 0,\ h \to 0,\ a \to constant \tag{18}$$

and as $x \to \infty$

$$f \to 1,\ h \to n,\ a \to 0 \tag{19}$$

With these boundary conditions we find that the vortices have charge Q and flux $\Phi = \int d^2rB(r)$ given by

$$\Phi = \frac{-2\pi n}{e},\ Q = \frac{-2\pi n\theta}{e} \tag{20}$$

The asymptotic solution as $x \to \infty$ reveals the presence of two masses[9] (in the units determined by the mass scales introduced above)

$$f(x) = 1 - f_1K_0(x) \tag{21}$$
$$a(x) = a_1K_0(m_+x) + a_2K_0(m_-x) \tag{22}$$
$$h(x) = 1 + a_1xK_1(m_+x) - a_2xK_1(m_-x) \tag{23}$$
$$m_\pm^2 = \alpha + \frac{\beta^2}{2}[1 \pm [1 + \frac{4\alpha}{\beta^2}]^{\frac{1}{2}}] \tag{24}$$

Above f_1, a_1, a_2 are integration constants. As $x \to 0$ the regular solutions are expressed as a power series expansion in x. We find

$$f(x) \sim f_0x + \cdots \tag{25}$$
$$a(x) \sim a_0 - \frac{\beta B_0}{4}x^2 + \cdots \tag{26}$$
$$h(x) \sim \frac{B_0}{2}x^2 + \cdots \tag{27}$$

Where again f_0, a_0, B_0 are integration constants. The set of integration constants is determined by integrating the differential equations and requiring regularity of the solutions both at infinity and at the origin. The constant B_0 is the magnetic field at the origin. For $\theta = 0$ ($\beta = 0$) the solutions are the Abrikosov-Nielsen-Olesen vortices with $a(x) = 0$. The solutions may be studied near the Abrikosov-Nielsen-Olesen (A-N-O) limit by using perturbation theory in β. One finds that there is *only one solution evolving smoothly from this limit* and that the magnetic field diminishes continuously from the origin. Furthermore the magnetic field at the origin diminishes as β increases, and finally in the Chern-Simons limit $\beta \to \infty$, $\alpha \to \infty$, $\alpha/\beta = fixed$, the magnetic field vanishes at the origin. An analytical study of the solutions in the asymptotic regions shows that the charge density and electric field vanish at the origin, rise reaching a maximun and fall-off exponentially forming a ring around the origin.

We solved the differential equations numerically in a wide range of parameters from the A-N-O limit to the Chern-Simons limit[6] for an $n = 1$ vortex. Figure 1 shows the profile for negative of the magnetic field $b(x) = h'/x$ for $\alpha = 1$, $\beta = 0,\ 0.3,\ 0.6$, and Chern-Simons limit. Figure 2 shows a profile for the order parameter $f(x)$. The profile changes very slightly within the whole range between A-N-O and Chern-Simons limits. Figure 3 shows the dimensionless charge density $\rho(x) = a(x)f^2(x)$, and Figure 4 shows the profile for the electric field $e = a'(x)$ for the same range of parameters as Figure 1. It is remarkable that for $\beta \neq 0$ the equations of motion in the unbroken phase do not admit a solution with non-zero magnetic field, whereas in the ordered phase the magnetic field nucleates for all values of the statistical parameter. However, near the Chern-Simons limit, the magnetic field, charge density and electric field are localized within a ring away from the origin. The total (gauge invariant) angular momentum is

$$L = \frac{Q^2}{4\pi\theta} \tag{28}$$

and coincides with the anomalous term for the angular momentum (11) for topologically trivial sector. The angular momentum density and magnetic moment density are again localized within a ring away from the origin[6]. These vortices are charged and have angular momentum. However in the broken symmetry phase there cannot be localized charged states because of screening. We then conjecture that because both charge and angular momentum have large fluctuations, the ground state will have vortex-antivortex pairs. But since these are screened, as they move through the medium they will drag a backflow current with them that will screen their charge at long distances. These excitations will then be reminiscent of the roton excitations in superfluid He. These excitations should manifest themselves as a minimum in the dispersion relation of the fully quantized vortex plus broken symmetry fluctuations.

Acknowledgements: This work was partially supported by N. S. F. through Grant PHY-8921311.

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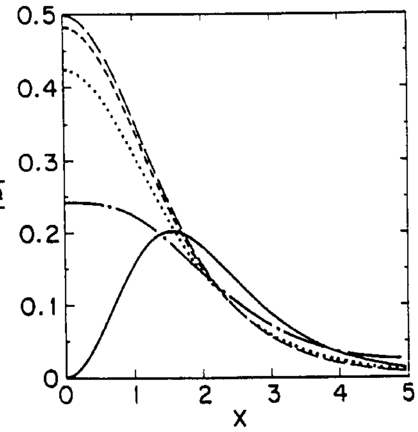

Figure 1

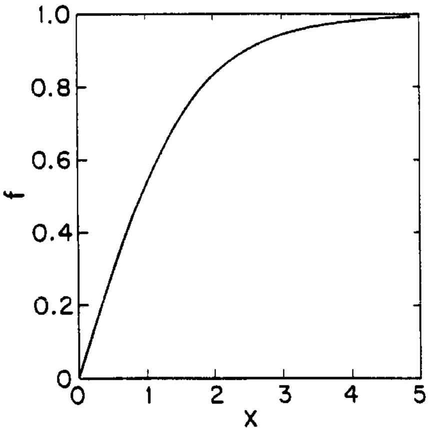

Figure 2

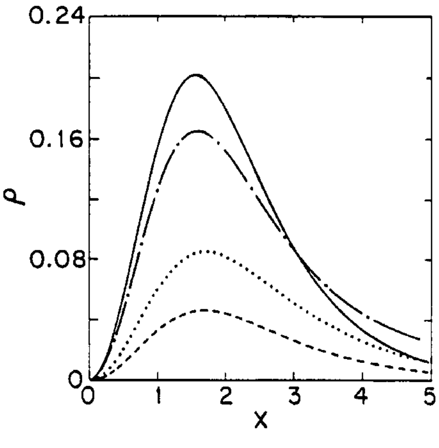

Figure 3

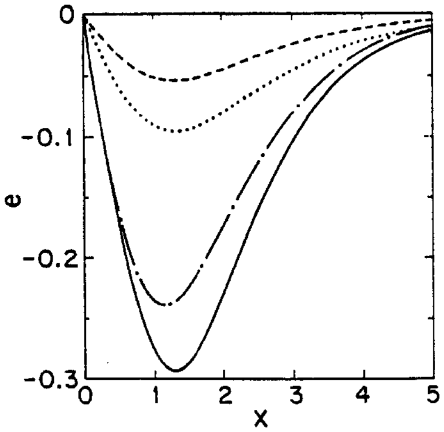

Figure 4