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Structure of the Vacuum Singularity in Reggeon Field Theory

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ABSTRACT

We use the methods of the renormalization group to analyze the behavior of all Reggeon proper vertex functions in a Reggeon field theory when all angular momenta are near one or all Reggeon momenta are small. This behavior is governed by an infrared stable Gell-Mann - Low zero which arises when the triple Pomeron coupling is imaginary. A renormalized trajectory must be singular at t=0, and a variety of scaling laws for the vertex functions are obeyed. Coupling particles to the Reggeons and using the scaling laws we find to high accuracy that $\sigma_T(s) \sim A(\log s)^{1/6} \times [1 - B/(\log s)^{1/2} + \dots]$ where A factorizes.

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The Reggeon calculus developed by Gribov[1] several years ago provides a constructive method to establish the contributions of multi-Reggeon cuts to two-to-two amplitudes which automatically satisfy the discontinuity relations[2] across those cuts. In this note we will indicate how one may use renormalization group techniques developed in the context of relativistic quantum field theory[3] to sum, in the Reggeon field theory, all the Reggeon cuts for the vacuum trajectory with $\alpha(0)=1$ in the neighborhood of small Pomeron energy (E=1 - 1) or small Pomeron momenta, $\vec{k}$, $t=-|\vec{k}|^2$.

We study a model of particular physical interest: linear trajectories for non-interacting Pomerons and a triple Pomeron coupling only. Our methods are clearly applicable to a much richer class of Reggeon field theories; many of these are being considered now.

We proceed by choosing the non-interacting Pomeron to have the energy momentum relation

$$E = 1 - \alpha(\vec{k}) = \alpha_0' \vec{k}^2, \tag{1}$$

where the intercept $\alpha_0(0)=1$. This represents the Pomeron as a non-relativistic quasi-particle with no energy gap. The action which yields (1) is

$$A_0 = \int d^D x\, dt \left\{ \frac{i}{2} \psi^\dagger \overleftrightarrow{\partial_t} \psi - \alpha_0' \nabla\psi^\dagger \cdot \nabla\psi \right\} \tag{2}$$

with $\psi(\vec{x},t)$ the Reggeon field in D space and one time dimension. Physics takes place at D=2, but it will be both convenient and instructive to have D at our disposal. We choose the interaction Lagrangian

$$\mathcal{L}_I = -\frac{i r_0}{2} \left\{ \psi^\dagger \psi^2 + (\psi^\dagger)^2 \psi \right\}, \quad r_0 \text{ real}, \tag{3}$$

where r_0 is the bare triple Pomeron coupling. The imaginary nature of this coupling follows from Gribov's [1] signature analysis of Reggeon graphs.

The quantities of interest to us are the renormalized proper vertex functions for n incoming and m outgoing Pomerons. The unrenormalized functions $\Gamma_U^{(n,m)}$ depend on the E_i and $\vec{k}_i$ of the Pomerons and the parameters α_0', r_0, and a possible cutoff Λ. The renormalized functions $\Gamma_R^{(n,m)}$ depend on renormalized quantities α', r, and a renormalized intercept $\alpha(0)$. Choosing $\alpha(0)=1$, as we do, corresponds to a massless theory, and we thus need a parameter to give us a normalization point for the $\Gamma_R^{(n,m)}$. To stay away from cuts we normalize at zero momenta $\vec{k}_i$, but $E_i \alpha -E_N$, with $E_N > 0$. In particular we choose to normalize the vertex functions by the following conventions:

$$\Gamma_R^{(1,1)}(E, \vec{k}^2, r, \alpha', E_N)\Big|_{\substack{E=0 \\ \vec{k}^2=0}} = 0. \tag{4}$$

Since $\Gamma_R^{(1,1)}$ is the inverse propagator, this guarantees that renormalized Pomeron singularities, whatever their analytic nature, occur at $l=1$, $t=0$. Also we require:

$$\frac{\partial}{\partial E}\, i\Gamma_R^{(1,1)}(E, \vec{k}^2, r, \alpha', E_N)\Big|_{\substack{E=-E_N \\ \vec{k}^2=0}} = 1, \tag{5}$$

$$\frac{\partial}{\partial \vec{k}^2}\, i\Gamma_R^{(1,1)}(E, \vec{k}^2, r, \alpha', E_N)\Big|_{\substack{E=-E_N \\ \vec{k}^2=0}} = -\alpha'(E_N), \tag{6}$$

and

$$\Gamma_R^{(1,2)}(E_1,\vec{k}_1,E_2,\vec{k}_2,E_3,\vec{k}_3,r,\alpha',E_N)\Big|_{\substack{E_2=E_3=-\frac{E_N}{2}=\frac{E_1}{2}\\ \vec{k}_i=0}} = \frac{r(E_N)}{(2\pi)^{\frac{D+1}{2}}} . \qquad (7)$$

These conditions define the renormalized quantities α' and r in terms of which all $\Gamma_R^{(n,m)}$ will be parametrized.

There is one more useful observation. Taking into account that $\vec{x}$ and t are dimensionally distinct in this non-relativistic theory, we find it useful to eliminate $r(E_N)$ in terms of the dimensionless coupling

$$g(E_N) = r(E_N)\, E_N^{D/4-1}\big/(\alpha'(E_N))^{D/4} . \qquad (8)$$

The special role that D=4 will play emerges here.

The renormalized and unrenormalized vertex functions are related by

$$\Gamma_R^{(n,m)}(E_i,\vec{k}_i,\alpha',g,E_N) = (Z)^{\frac{n+m}{2}}\, \Gamma_U^{(n,m)}(E_i,k_i,\alpha_0',r_0,\Lambda). \qquad (9)$$

Noting that Γ_U is independent of E_N yields the crucial equation of the renormalization group

$$\left\{E_N\frac{\partial}{\partial E_N} + \beta(g)\frac{\partial}{\partial g} + \zeta(\alpha',g)\frac{\partial}{\partial\alpha'} - \frac{n+m}{2}\gamma(g)\right]\Gamma_R^{(n,m)}(E_i,\vec{k}_i,g,\alpha',E_N) = 0, \qquad (10)$$

where
$$\beta(g) = E_N\frac{\partial}{\partial E_N}g(E_N)\Big|_{\alpha_0',r_0,\Lambda \text{ fixed}} , \qquad (11)$$

$$\zeta(\alpha',g) = E_N\frac{\partial}{\partial E_N}\alpha'(E_N)\Big|_{\alpha_0',r_0,\Lambda \text{ fixed}} , \qquad (12)$$

and
$$\gamma(g) = E_N\frac{\partial}{\partial E_N}\log Z\Big|_{\alpha_0',r_0,\Lambda \text{ fixed}} . \qquad (13)$$

Ordinary dimensional analysis allows us to turn (10) into an equation for $\Gamma_R^{(n,m)}(\xi E_i, \vec{k}_i, g, \alpha', E_N)$

$$\left\{\xi \frac{\partial}{\partial \xi} - \beta(g)\frac{\partial}{\partial g} + [\alpha' - \zeta(\alpha', g)]\frac{\partial}{\partial \alpha'} + \left[\frac{n+m}{2}\gamma(g) - 1\right]\right\}\Gamma_R^{(n,m)}(E_i, \vec{k}_i, g, \alpha', E_N) = 0, \quad (14)$$

whose solution

$$\Gamma_R^{(n,m)}(\xi E_i, \vec{k}_i, g, \alpha', E_N) =$$
$$\Gamma_R^{(n,m)}(E_i, \vec{k}_i, \tilde{g}(-t), \tilde{\alpha}'(-t), E_N) \exp \int_{-t}^{0} dt' \left[1 - \left(\frac{n+m}{2}\right)\gamma(\tilde{g}(t'))\right], \quad t = \log \xi, \quad (15)$$

determines Γ_R in terms of effective slope and coupling parameters which satisfy

$$\frac{d\tilde{g}(t)}{dt} = -\beta(\tilde{g}(t)), \quad (16)$$

$$\frac{1}{\tilde{\alpha}'(t)}\frac{d\tilde{\alpha}'(t)}{dt} = 1 - \zeta(\tilde{\alpha}'(t), \tilde{g}(t))/\tilde{\alpha}'(t) = z(\tilde{g}(t)). \quad (17)$$

If we were to know β, ζ, and γ, then we could study the Γ_R as the E_i vary for fixed $\vec{k}_i$. Alas, this is tantamount to solving the full field theory. So we are only able to know these functions in perturbation theory in g. We shall proceed by studying (15), (16), and (17) using β, ζ, and γ in lowest order perturbation theory. We find

$$\gamma(g) = -2Kg^2, \quad (18)$$

$$\zeta(\alpha', g) = -\alpha' K g^2, \quad (19)$$

$$\beta(g) = -\left(\frac{4-D}{4}\right)g + \left(\tilde{K} + \frac{D}{4}K\right)g^3, \quad (20)$$

where K and $\tilde{K}$ are positive constants for $2 \leq D \leq 4$. From (20) we see that $\beta(g)$ has a zero at

$$g_1 = \left[\frac{4-D}{4\tilde{K}+DK}\right]^{1/2} \tag{21}$$

and $\beta'(g_1)>0$. The general analysis[3] tells us that this zero governs the $\xi \to 0$ (infrared) behavior of Γ_R .

This is a key result: $\beta(g)$ has an infrared stable zero which for $D \approx 4$ occurs at small renormalized coupling. This suggests a perturbation theory in $\varepsilon = 4-D$ akin to the ε-expansion[3] of statistical mechanics. The imaginary character of the triple Pomeron coupling is crucial in this. The functions ζ and γ are to order ε

$$\gamma = -\varepsilon/12, \tag{22}$$

$$\zeta/\alpha' = -\varepsilon/24 \tag{23}$$

indicating that even at D=2, where $\varepsilon = 2$, we are keeping terms in an expansion in small parameters.

Given a zero at g_1 we can use dimensional analysis again to determine the general form of Γ_R allowed for E_i small, fixed $\vec{k}_i$. We find in this regime

$$\Gamma_R^{(n,m)}(E_i, \vec{k}_i, g, \alpha', E_N) =$$

$$C_\gamma E_N \left[\frac{E_N}{C_\alpha \alpha'}\right]^{\frac{D}{4}(2-n-m)} \left(\frac{-E}{E_N}\right)^{1+z(g_1)\frac{D}{4}(2-n-m) - \frac{n+m}{2}\gamma(g_1)} \times \Phi_{n,m}\left(\frac{E_i}{E}, \left(\frac{-E}{E_N}\right)^{-z(g_1)} \frac{\vec{k}_i \cdot \vec{k}_j}{E_N} \alpha' C_\alpha, g_1\right) \tag{24}$$

with $E = \sum_{i=1}^{n} E_i$ and

where C_α and C_γ are two constants which each equal one at $g = g_1$, and $\Phi_{n,m}$ remains undetermined at this stage.

Now we have our first important result. If $\Gamma_R^{(1,1)}$ has a zero which moves with $\vec{k}^2$, it must yield a trajectory

$$\alpha(t) = 1 + (t)^{1/z(g_1)} \times f(g, E_N, \alpha'), \tag{25}$$

with f undetermined. In general $\alpha(t)$ will not be analytic at t=0. In our ε perturbation theory $\mathfrak{z} = 1 + \varepsilon/24$, so the trajectory function vanishes less rapidly than the non-interacting linear trajectory. In a naive fashion this eliminates the necessity for the vanishing at t=0 of the triple Pomeron coupling measured in inclusive processes.[4] Nevertheless, if we examine $\Gamma_R^{(1,2)}$, the renormalized triple Pomeron vertex, it does vanish, albeit non-linearly, as $E_i \to 0$, $\vec{k}_i \to 0$. All discussions proceeding from an analytic vanishing of this vertex function would seem to merit re-examination.

If we carry out the ε perturbation expansion, we can determine the function $\phi_{1,1}$ in (24), for example, as a power series in ε by comparison with the renormalized propagator evaluated to second order in g. Writing $\phi_{1,1}(\rho,\varepsilon)$ where

$$\rho = \left(-E/E_N\right)^{-\mathfrak{z}(\varepsilon)} \; C_\alpha \alpha' \vec{k}^2 / E_N , \tag{26}$$

we find

$$-i\,\phi_{1,1}(\rho,\varepsilon) = 1 + \rho + \frac{\varepsilon}{12}\left[1 + \rho/2\right]\left\{\log\left(1 + \rho/2\right) - 1\right\} + O(\varepsilon^2), \tag{27}$$

yielding a Pomeron pole with trajectory

$$\alpha(t) = 1 + E_N \left\{\frac{C_\alpha \alpha' t}{1 - \frac{\varepsilon}{24}(1 + \log 2)}\right\}^{1/1+\varepsilon/24} , \tag{28}$$

for $t > 0$.

Now we wish to couple particles into the theory. We do this by allowing two particles to emit n Reggeons with a strength N_n. The contribution to the particle partial wave amplitude $F(E,\vec{q})$ coming from n Reggeons emitted, interacting in all possible ways, and producing m Reggeons which are then absorbed is

$$I_{n,m}(E,\vec{q}) = N_n N_m \int d^D k_1 \cdots d^D k_{n+m}\, dE_1 \cdots dE_{n+m}\, \delta\Big(\sum_{i=1}^{n} E_i - E\Big)\, \delta^D\Big(\sum_{i=1}^{n} \vec{k}_i - \vec{q}\Big) \times$$
$$\times\, \delta\Big(\sum_{i=n+1}^{n+m} E_i - E\Big)\, \delta^D\Big(\sum_{i=n+1}^{n+m} \vec{k}_i - \vec{q}\Big)\, G_R^{(n,m)}(E_1, \vec{k}_1, \cdots E_{n+m}, \vec{k}_{n+m}) \qquad (29)$$

with $G_R^{(n,m)}(E_1, \vec{k}_1, \cdots E_{n+m}, \vec{k}_{n+m})$ the full renormalized Green's function. Using the scaling properties above we discover

$$I_{n,m}(E,\vec{q}) = E^{-1+\gamma(g_1)}\, E^{(n+m-2)[\gamma(g_1)/2 + D/4\, \zeta(g_1)]} \times$$
$$F_{n,m}\big(|\vec{q}|^2 / E^{\zeta(g_1)}\big), \qquad (30)$$

yielding an elastic amplitude

$$T_{el}(s,t) = s\, (\log s)^{-\gamma(g_1)} \times$$
$$\times \sum_{n,m} (\log s)^{-(n+m-2)[\gamma(g_1)/2 + D/4\, \zeta(g_1)]}\, \tilde{F}_{n,m}\big(t\,(\log s)^{\zeta(g_1)}\big). \qquad (31)$$

On utilizing (22) and (23) and evaluating at D=2, we have an expansion in powers of $(\log s)^{-p}$, $p \approx \frac{1}{2}$,

$$T_{el}(s,t) = s\,(\log s)^{1/6}\Big[\tilde{F}_{1,1}\big(t(\log s)^{13/12}\big) + (\log s)^{-1/2}\, \tilde{F}_{1,2}\big(t(\log s)^{13/12}\big) +$$
$$O\big((\log s)^{-1}\big)\ \Big], \qquad (32)$$

and a total cross section for $A+B \rightarrow$ anything

$$\sigma_T^{AB}(s) = \beta_A \beta_B (\log s)^{1/6}\, \big[1 - (\text{constant})/(\log s)^{1/2} + \cdots\big], \qquad (33)$$

where we have noted that the leading term factorizes.

The results presented here will be extensively elaborated on in a paper now in preparation. Let us comment on the achievements of this work. Our results are reminiscent of the Gribov-Migdal "strong coupling" solution of the interacting Pomeron problem.[5] However, there are significant differences. Our $\Gamma_R^{(1,1)}$ vanishes faster than linearly in E at $\vec{k}^2=0$, whereas theirs vanishes slower. We have a positive total cross section; they do not. The strongest conclusion we can draw from our work is that a "weak coupling" solution to the Pomeron problem would seem to be ruled out. That is, the Pomeron trajectory cannot be linear near t=0 when Pomeron interactions are taken into account. This renders all decoupling theorems[6] for the Pomeron of little interest. The most amusing possibility suggested by our procedures is a constructive perturbation expansion in the dimensions of (Reggeon) space around D=4 which yields the various proper vertex functions to high accuracy.

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6. See the lectures of R. C. Brower, C. E. deTar, and J. H. Weis at the Santa Cruz Particle Physics Summer School 1973 for an in depth review of these matters.