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Gödel metrics with chronology protection in Horndeski gravities

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ABSTRACT

Gödel universe, one of the most interesting exact solutions predicted by General Relativity, describes a homogeneous rotating universe containing naked closed time-like curves (CTCs). It was shown that such CTCs are the consequence of the null energy condition in General Relativity. In this paper, we show that the Gödel-type metrics with chronology protection can emerge in Einstein–Horndeski gravity. We construct such exact solutions also in Einstein–Horndeski–Maxwell and Einstein–Horndeski–Proca theories.

1. Introduction

Gödel metric [1], constructed in 1949, is one of the most interesting exact solutions predicted by General Relativity (GR). The original construction by Gödel requires a fine-tuning balance between a negative cosmological constant and the matter density of some uniform homogeneous pressureless perfect fluids or dusts. The Gödel universe is a direct product of a three-dimensional homogeneous rotating spacetime with a real line. This spacetime metric exhibits several peculiar features, such as the presence of naked closed time-like curves (CTCs) and the absence of globally spatial-like Cauchy surface.

The Gödel metric can be generalized by introducing a constant α, and the resulting metrics are referred as the Gödel type. Naked CTCs are present as long as $0 < \alpha < 1$, with $\alpha = 1/2$ corresponding to the original Gödel metric. (See section 2 for details.) In this paper, we consider only these four-dimensional *Gödel-type metrics*, and we believe that it is appropriate to call all these metrics simply as *Gödel metrics*. There have been continuing efforts in the GR community to construct and study the Gödel metrics, see, for example, [2–13,15,14,16–23] and references therein. Gödel metrics can also be embedded in strings and M-theory [9,21]. (In literature, the Gödel metric was generalized to include two parameters; however, as was demonstrated in [21], these metrics are related to the metric of the single-parameter α by coordinate transformations.)

The existence of naked CTCs in spacetime violates the chronology protection conjecture proposed by Hawking [24]. Since Gödel metrics can be supported by fundamental matter fields such as Maxwell and axion fields, microscopic Gödel metrics present challenges to the chronological protection. Gödel metrics are absent from CTCs when the parameter $\alpha \geq 1$. What is intriguing is that with the framework of Einstein gravity with minimally-coupled matter, the null energy condition implies that $0 < \alpha \leq 1$ [21]. Thus naked CTCs are unavoidable for Gödel metrics in Einstein gravity, unless $\alpha = 1$, which corresponds to simply a direct product of a real line and anti-de Sitter spacetime (AdS) in three dimensions. The general $\alpha > 1$ metrics with chronology protection requires modified gravities beyond Einstein gravity [7,13–16,18–20]. In particular, in the low-energy effective theory of strings with higher-derivative corrections, Gödel metrics with $\alpha > 1$ was constructed in [7]. However, this theory, when treated on its own, involves ghost modes. It is thus of interest to look for ghost-free theories beyond Einstein gravity that support Gödel metrics with $\alpha > 1$ so that the spacetime is absent from CTCs.

In this paper, we construct Gödel metrics in Einstein–Horndeski gravity. The Horndeski terms [25] involve a non-minimally coupled axionic scalar that enters the Lagrangian only through a covariant derivative. The theory is analogous to Gauss–Bonnet gravity where the linearized equations of motion are of two derivatives. Horndeski gravity has been deeply investigated in the context of cosmology, (see e.g. [26,27];) and also in the context of black holes [28–31], black hole thermodynamics [32,33], and holographic properties [34–37]. Recently new black holes were constructed that

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violate [38] the conjecture of the reverse isoperimetric inequality proposed in [39].

In the embedding of Gödel metrics in string and M-theory, an axion carrying a proper magnetic axion charge plays an important role [21]. It is thus natural to consider Horndeski gravity where an axion is a necessary component in the theory. Furthermore, as a modified theory to Einstein gravity, its null energy condition is modified, and hence Horndeski gravity may restore the chronological protection, and we find that it is indeed the case. Chronological protection in general classes of Horndeski gravities were also discussed in [40].

The paper is organized as follows. In section 2, we consider Gödel metrics with general α parameter and review that $\alpha \geq 1$ metrics are chronologically protected. We then demonstrate that such metrics can arise from Einstein–Horndeski gravity where the Horndeski axion carries a magnetic charge. In section 3, we obtain the Gödel metrics in Einstein–Horndeski–Maxwell and Einstein–Horndeski–Proca theories. We conclude the paper in section 4.

2. Gödel metrics in Horndeski theory

In this paper, we consider a class of metrics in four dimensions that take the form

$$ds^2 = \ell^2\left[-(dt + rd\phi)^2 + \alpha r^2 d\phi^2 + \frac{dr^2}{r^2} + dz^2\right], \tag{2.1}$$

where ℓ and $\alpha > 0$ are constants. The metric is homogeneous and a direct product of a real line $\mathbb{R}$ and a three-dimensional rotating spacetime which was called G_α in [21]. The G_α associates with the three dimensional metric of (t, ϕ, r) and the $\mathbb{R}$ associates with the coordinate z. When $\alpha = 1/2$, the original Gödel metric is recovered, corresponding to $\mathrm{G}_{1/2} \times \mathbb{R}$. The solution can be constructed by fine-tuning a negative cosmological constant against homogeneous pressureless perfect fluids [1]. When $\alpha = 1$, the G_α part of the metric is locally AdS_3, i.e. $\mathrm{G}_1 = \mathrm{AdS}_3$. In this paper, we find it appropriate to refer all the $\mathrm{G}_\alpha \times \mathbb{R}$ metrics (2.1) as Gödel metrics.

For $\alpha \geq 1$, t is the globally-defined time coordinate. This is no longer true when $\alpha < 1$, which is indicative of the existence of possible CTCs. To see this explicitly, one can make a coordinate transformation [21]

$$r = \cosh\hat{r} + \cos\hat{\phi}\,\sinh\hat{r}, \qquad r\phi = \frac{1}{\sqrt{\alpha}}\sin\hat{\phi}\,\sinh\hat{r},$$
$$\tan\big(\tfrac{1}{2}\hat{\phi} + \tfrac{1}{2}\sqrt{\alpha}\,(t - \hat{t})\big) = e^{-\hat{r}}\tan(\tfrac{1}{2}\hat{\phi}), \tag{2.2}$$

the Gödel metrics (2.1) become

$$ds^2 = \ell^2\Big[-\Big(d\hat{t} + \tfrac{2}{\sqrt{\alpha}}\sinh^2(\tfrac{1}{2}\hat{r})\,d\hat{\phi}\Big)^2 + \sinh^2\hat{r}\,d\hat{\phi}^2 + d\hat{r}^2 + dz^2\Big]. \tag{2.3}$$

Absence of a conic singularity at $\hat{r} = 0$ requires that $\hat{\phi}$ be periodic and the period be $\Delta\hat{\phi} = 2\pi$. It follows that for $\alpha < 1$ the spacetime develops negative $g_{\hat{\phi}\hat{\phi}}$ for sufficiently large r, indicating naked CTCs. Such CTCs are absent for Gödel metrics with $\alpha \geq 1$. In the previous related works [21], we found within the framework of Einstein gravity, the null energy condition requires that $\alpha \leq 1$, and hence the Gödel metrics in Einstein gravity necessary have naked CTCs.

It should be emphasized that the three-dimensional metric G_α is completely specified by its Ricci tensor. Thus, as was remarked in the introduction, the two-parameter Gödel metrics in literature can be all derived by some coordinate transformations from our G_α metric.

In this section, we show that solutions with $\alpha > 1$ can emerge in Einstein–Horndeski theory. Horndeski terms are a class of higher derivative polynomials constructed from Riemann tensors and axionic scalars [25]. The action of Einstein–Horndeski gravity at the lowest-order in four dimensions is given by

$$\mathcal{S} = \int \sqrt{-g}\,L\,d^4x,$$
$$L = \kappa(R - 2\Lambda) - \frac{1}{2}\left(\beta\,g_{\mu\nu} - \gamma\,G_{\mu\nu}\right)\nabla^\mu\chi\,\nabla^\nu\chi + L^{\mathrm{mat}}, \tag{2.4}$$

where κ, β and γ are coupling constants, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the Einstein tensor, χ is axionic scalar field, and L^{mat} is the Lagrangian of matter. When $\gamma = 0$, $\beta = 0$ and $\kappa = 1$, the theory reduces to Einstein theory with a cosmological constant. When the axion χ is constant, the Einstein theory is also recovered. The explicit Einstein and axion equations can be found in literature, see, e.g. [29,32]. For simplicity, we take $\kappa = 1$ throughout the paper.

Since the Gödel metric (2.1) is homogeneous, the coefficient m_0 in the kinetic term for the axion, namely $K = \frac{1}{2}m_0^2\,\dot{\chi}^2$, is constant, and it must be non-negative. In other words, the ghost-free condition requires that

$$m_0^2 = -\left(\beta\,\eta^{00} - \gamma\,G^{00}\right) = \frac{4\alpha\beta\ell^2 + (3 - 4\alpha)\gamma}{4\alpha\ell^2} \geq 0\,. \tag{2.5}$$

For the general Gödel metric (2.1), we follow the analogous construction of [21] and take the axion to be magnetic:

$$\chi = kz\,. \tag{2.6}$$

For pure Einstein–Horndeski theory without matter, we obtain the solution

$$\gamma = \frac{4\alpha\beta\ell^2}{4\alpha - 1}, \quad k = \sqrt{\frac{4\alpha - 1}{\alpha\beta}}, \quad \Lambda = -\frac{4\alpha - 1}{4\alpha\ell^2}\,. \tag{2.7}$$

Substituting the solution into the ghost-free condition (2.5), we find

$$m_0^2 = \frac{2\beta}{4\alpha - 1} > 0\,. \tag{2.8}$$

Together with the reality condition for constant k, we find two branches of solutions

$$\begin{cases}\alpha > \frac{1}{4}\\ \beta > 0\end{cases} \qquad \text{or} \qquad \begin{cases}0 < \alpha < \frac{1}{4}\\ \beta < 0\end{cases}\,. \tag{2.9}$$

Thus we see that the original ($\alpha = 1/2$) Gödel metric can emerge in Horndeski gravity; furthermore, in addition to the usual Gödel metrics with $\alpha < 1$, metrics with $\alpha > 1$ that maintain the chronological protection can also emerge.

3. Gödel metrics in Horndeski theory with matter

In this section, we generalized the Gödel metrics in Horndeski gravity by introducing matter fields, such as Maxwell and Proca fields. We also restrict the constant β to be positive only. In some cases, we can set $\beta = 1$ without loss of generality.

3.1. Maxwell field

The Lagrangian for the Maxwell field is given by

$$L^{\mathrm{mat}} = -\tfrac{1}{4}F^2\,, \qquad F = dA\,. \tag{3.1}$$

Assuming that the coordinate z is periodic, one can take the following ansatz [5]:

$$A = q\sin(wz)(dt + rd\phi)\,, \tag{3.2}$$

where w is a constant, inverse to the period of coordinate z. The solution is best described as $G_\alpha \times S^1$ rather than $G_\alpha \times \mathbb{R}$. For the metric (2.1), the solutions are given by

$$w = \frac{1}{\sqrt{\alpha}}\,,\qquad \Lambda = \frac{(1-2\alpha)(1-4\alpha)\gamma - 8\alpha^2\beta\ell^2}{8\alpha\,\ell^2\big((1-2\alpha)\gamma + 2\alpha\beta\ell^2\big)}\,, \tag{3.3}$$

$$q^2 = \frac{\ell^2(\alpha-1)\big((4\alpha-1)\gamma - 4\alpha\beta\ell^2\big)}{(1-2\alpha)\gamma + 2\alpha\beta\ell^2}\,,$$

$$k = \frac{\sqrt{2}\,\ell}{\sqrt{(1-2\alpha)\gamma + 2\alpha\beta\ell^2}}\,. \tag{3.4}$$

Solutions with $\gamma = 0$ were constructed in [5]. The reality conditions of (q, k) imply that

$$(\alpha-1)\big((4\alpha-1)\gamma - 4\alpha\beta\ell^2\big) \ge 0\,,\qquad (2\alpha-1)\gamma - 2\alpha\beta\ell^2 < 0\,. \tag{3.5}$$

When $\alpha = 1$, Maxwell field vanishes, reverting back to a special case discussed in the previous section. For $\alpha > 1$, we find that γ must lie within the range:

$$\alpha > 1: \qquad \frac{4\alpha\beta\ell^2}{4\alpha-1} \le \gamma < \frac{2\alpha\beta\ell^2}{2\alpha-1}\,. \tag{3.6}$$

Substituting the γ range into (2.5), we find that the ghost free condition is indeed satisfied, namely

$$0 < \frac{\beta}{2(2\alpha-1)} \le m_0^2 < \frac{2\beta}{4\alpha-1}\,. \tag{3.7}$$

Gödel metrics with $\alpha < 1$ can also arise. In this case, the reality conditions imply that

$$(4\alpha-1)\gamma - 4\alpha\beta\ell^2 \le 0\,,\qquad (2\alpha-1)\gamma - 2\alpha\beta\ell^2 < 0\,. \tag{3.8}$$

Together with the ghost-free condition (2.5), we find that $\alpha < 1$ solutions are also possible in Einstein–Horndeski–Maxwell gravity and the corresponding γ range is given by

$0 < \alpha \le \frac{1}{4}$	$\frac{1}{4} < \alpha < \frac{3}{4}$	$\frac{3}{4} \le \alpha < 1$
$\gamma \ge -\frac{4\alpha\beta\ell^2}{3-4\alpha}$	$-\frac{4\alpha\beta\ell^2}{3-4\alpha} \le \gamma \le \frac{4\alpha\beta\ell^2}{4\alpha-1}$	$\gamma \le \frac{4\alpha\beta\ell^2}{4\alpha-3}$

(3.9)

3.2. Including $\chi\, F \wedge F$

We can further add a topological term so that the matter content becomes,

$$L^{mat} = -\tfrac{1}{4}F^2 + \tfrac{1}{8}\chi\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\,. \tag{3.10}$$

The coordinate z is now treated as a real line, instead as a circle. Making an ansatz for the Maxwell potential [21]

$$A = qrd\phi\,, \tag{3.11}$$

we find that the solutions are given by

$$k = \frac{1}{\sqrt{\alpha}}\,,\quad \Lambda = -\frac{8\alpha^2\ell^2 + (2\alpha-1)\gamma}{16\alpha^2\ell^4}\,,$$

$$q^2 = \frac{(1-\alpha)\left(4\alpha\ell^2 - \gamma\right)}{2\alpha}\,,\quad \beta = 1 + \frac{(2\alpha-1)\gamma}{2\alpha\ell^2}\,. \tag{3.12}$$

Solutions with $\gamma = 0$ were constructed in [21]. We require that $\beta > 0$, and together with ghost-free condition (2.5) and reality condition, we find that the results are:

$0 < \alpha \le \frac{1}{4}$	$\frac{1}{4} < \alpha < \frac{3}{4}$	$\frac{3}{4} \le \alpha \le 1$	$\alpha > 1$
$-4\alpha\ell^2 \le \gamma < \frac{2\alpha\ell^2}{1-2\alpha}$	$\lvert\gamma\rvert \le 4\alpha\ell^2$	$-\frac{2\alpha\ell^2}{2\alpha-1} < \gamma \le 4\alpha\ell^2$	$\gamma \ge 4\alpha\ell^2$

(3.13)

3.3. Proca field

We now consider the Proca field, and the Lagrangian is given by

$$L^{\rm mat} = -\tfrac{1}{4}F^2 - \tfrac{1}{2}m^2A^2\,, \tag{3.14}$$

where $F = dA$. Taking the ansatz [21]

$$A = q(dt + rd\phi)\,, \tag{3.15}$$

we find general Gödel solutions with

$$m = \frac{1}{\sqrt{\alpha}\,\ell}\,,\quad \Lambda = \frac{(1-2\alpha)(1-4\alpha)\gamma + 4(1-3\alpha)\alpha\beta\ell^2}{4\alpha\ell^2(\gamma - 3\alpha\gamma + 4\alpha\beta\ell^2)}\,, \tag{3.16}$$

$$q^2 = \frac{\ell^2(\alpha-1)(\gamma(4\alpha-1) - 4\alpha\beta\ell^2)}{\gamma(1-3\alpha) + 4\alpha\beta\ell^2}\,,$$

$$k = 2\,\ell\sqrt{\frac{\alpha}{\gamma(1-3\alpha) + 4\alpha\beta\ell^2}}\,. \tag{3.17}$$

Solutions with $\gamma = 0$ were constructed in [21]. The reality condition, together with the ghost-free condition (2.5), restrict the parameter regions. For our choice of $\beta > 0$, we find that the chronology-protected ($\alpha > 1$) Gödel metrics exist in Einstein–Horndeski–Proca theory, and the Horndeski coupling constant γ lies in the following regions:

$1 < \alpha \le 2$	$\alpha > 2$
$\frac{4\alpha\beta\ell^2}{4\alpha-1} < \gamma < \frac{4\alpha\beta\ell^2}{3\alpha-1}$	$\frac{4\alpha\beta\ell^2}{4\alpha-1} < \gamma \le \frac{4\alpha\beta\ell^2}{4\alpha-3}$

(3.18)

Gödel metrics with $\alpha \le 1$ can also arise and the results for the constant γ range is identical to those in the Einstein–Maxwell theory discussed earlier.

4. Conclusions

In this paper, we considered a class of Gödel metrics (2.1) with a free parameter α. In Einstein gravity, the null energy condition imposes that $\alpha \le 1$, with $\alpha = 1$ corresponding to $AdS_3 \times \mathbb{R}$. Since Gödel metrics with $\alpha < 1$ have naked CTCs, the chronology in these universes are not protected.

We constructed Gödel metrics in Einstein–Horndeski theories, with or without additional matter that includes Maxwell and Proca fields. We find that Gödel metrics with $\alpha > 1$ that evade CTCs can also emerge, and consequently the corresponding chronology is protected. We determine the range of the Horndeski coupling constant γ so that the theories that admit the Gödel metrics are absent from ghost excitations. The full stability of these metrics however requires further investigation.

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