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Rotation curve for the Milky Way galaxy in conformal gravity

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Abstract. Galactic rotation curves have proven to be the testing ground for dark matter bounds in galaxies, and our own Milky Way is one of many large spiral galaxies that must follow the same models. Over the last decade, the rotation of the Milky Way galaxy has been studied and extended by many authors. Since the work of conformal gravity has now successfully fit the rotation curves of almost 140 galaxies, we present here the fit to our own Milky Way. However, the Milky Way is not just an ordinary galaxy to append to our list, but instead provides a robust test of a fundamental difference of conformal gravity rotation curves versus standard cold dark matter models. It was shown by Mannheim and O'Brien that in conformal gravity, the presence of a quadratic potential causes the rotation curve to eventually fall off after its flat portion. This effect can currently be seen in only a select few galaxies whose rotation curve is studied well beyond a few multiples of the optical galactic scale length. Due to the recent work of Sofue et al and Kundu et al, the rotation curve of the Milky Way has now been studied to a degree where we can test the predicted fall off in the conformal gravity rotation curve. We find that – like the other galaxies already studied in conformal gravity – we obtain amazing agreement with rotational data and the prediction includes the eventual fall off at large distances from the galactic center.

1. Introduction

Since its inception over seventy five years ago, the concept of dark matter has been widely accepted as the explanation for the missing mass problem in spiral galaxies. Many current large scale collaborations have been created to search for dark matter with little notable success. In the current paradigm, the parameter space of cold dark matter is being constrained to higher and higher degrees, but physical searches have turned up with null or conflicting results. Most notably, the recent Cryogenic Dark Matter Search (CDMS) [1] faced the same unfortunate conclusions as many of its predecessors. As the community further pushes technology to probe deeper into the possible parameter space of dark matter, the lack of pure observable evidence and reliance on the rotation curves of galaxies has opened the door for many alternative theories to standard Einstein Gravity. Many of these alternative theories attempt to solve the rotation curve problem without the need for invoking dark matter. Although many of these theories have had success in fitting the rotation curves of spiral galaxies in the past, such as Modified Newtonian Dynamics (MOND) [2], Scalar Tensor Vector Gravity (STVG) [3], and more recently the Luminous Convolution Model (LCM) [4], still many of these theories struggle with the universality of galactic rotational dynamics. However, recently one of the authors has fit a



diverse set of over 130 rotation curves of various galaxies using Conformal Gravity (CG) as an alternative gravitational model [5]. Mannheim and O'Brien have successfully fit the most recent data of the THINGS survey [6] as well as the Ursa Major galaxies of Verheijen et al [7]. Their research continued with less studied low surface brightness galaxies of Kim et al [8] and a very recent survey of dwarf galaxies by Swaters [9]. To further diversify the studies performed by Mannheim and O'Brien, they have fit conformal gravity to the three Tidal Dwarf Galaxies (TDG) of NGC 5291 with an astonishing degree of success [10]. We note that the TDG galaxies are a unique testing ground of gravitational physics in the sense that they offer an observable rotation curve [2] despite the fact that they should not be in existence due to the parameter space defined by Λ CDM.

With these surveys fully populating the spectrum of studied galactic rotation curves, Conformal Gravity emerges as an alternative gravitational theory that can universally explain the rotation curve problem, and unlike theories such as MOND the creation of CG was not formulated to address the rotation curve problem alone. For the discussion of this paper, the outstanding difference between conformal gravity and the other alternative gravitational theories is the presence of a quadratic potential, which neither forces rotation curves to forever remain flat or allows rotation curves to infinitely rise. Instead as shown in [11] the presence of the quadratic term begins to compete at large distance scales and eventually forces the stable orbits of particles in the presence of the gravitational potential of the galaxy to terminate. Hence, extremely large galaxies, viz. ones whose outermost data points are well beyond the optical scale lengths, serve as a perfect testing ground for this prediction of the conformal theory. In this paper, we discuss how the latest rotation curve data of the Milky Way galaxy provides us with a great comparison of various rotation curve models, with conformal gravity being able to once again explain the rotation observed without invoking copious dark matter. Moreover, due to the shape of the rotation curve of the Milky Way galaxy, conformal gravity's predication of an eventual fall off of the rotational velocity can be illuminated and discussed.

2. The Milky Way Galaxy Data

The Milky Way has long been a galaxy studied by astronomers. For many years the rotation curve of the Milky Way has been known, but was not of particular interest to the missing mass problem due to its relatively low density of data points, and its early termination of optical data. The standard rotation curve can be obtained as seen in [12] where the observable parameters are: the distance to the galactic center, R , the adopted distance from the Sun to the galactic center, R_{S0} , the relative velocity of the Local Standard of Rest, V_{LSR} , and the angular direction of the tracer stars, (l, b) . The rotation velocity, V_{ROT} , is then explained by the formula

$$V_{ROT} = \frac{R}{R_{S0}} \left(\frac{V_{LSR}}{\sin l \cdot \cos b} - 220 \right). \quad (1)$$

With the advent of recent observational techniques, many new types of radio tracers can be used to increase the validity and density of the Milky Way rotation curve. In 2009, Sofue [13] published a unified rotation curve of the Milky Way galaxy synthesizing data from over eight different tracer sources (including the aforementioned [12]) to build one of the densest rotation curves ever produced, consisting of 610 data points. The vast data points observed by Sofue highlight the structure of the Milky Way, including a bulge in the inner region, and extend the data to 20 kpc.

The work of Sofue allowed for the structure of the Milky Way for $R \sim 20$ kpc [13] to be well established. The high density of points for $R < 20$ kpc can easily be seen in Fig. 1(a). The need to extend the rotation velocity past the disk region is expressed throughout the current literature [12]. Xue et al was able to then extend the Milky Way rotation curve to approximately 60 kpc [14]. Recent work by Kundu et al [15] has extended the rotation curve from the local

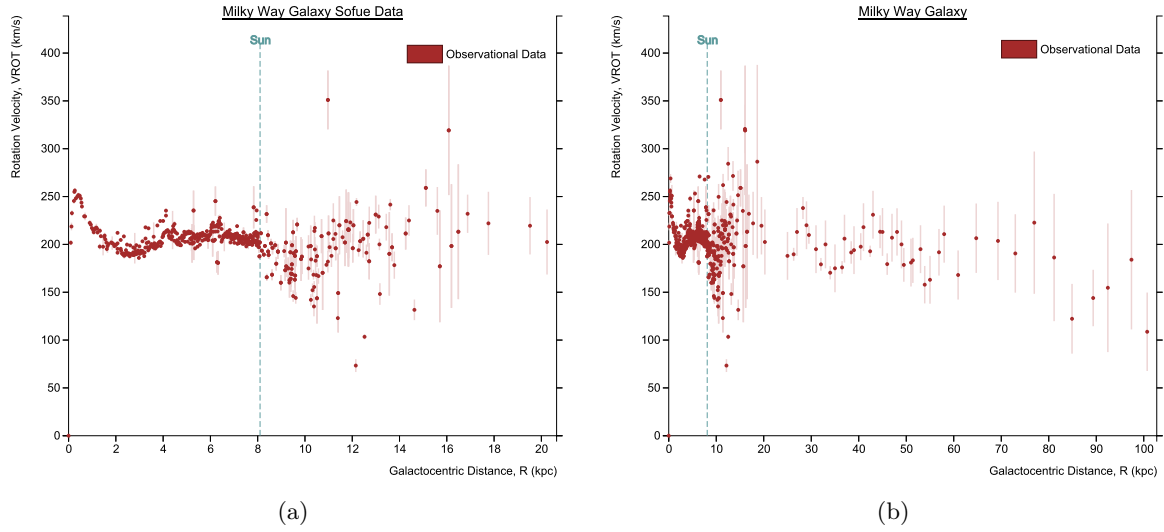


Figure 1. (a) The dense number of observations taken by Sofue. Note that the uncertainty increases when tracing objects away from the galactic center. (b) The full synthesized data set of the rotation curve.

disk region to the outer regions of the galaxy, continuing far past the galactic disk using Blue Horizontal Branch tracers (BHB) to an astonishing distance. They note that the rotation curve is slowly declining as R increases. Here we synthesized the data sets from [13], [14] and [15] which accumulate to a total of 636 observed data points for the rotation curve as shown in Fig. 1(b). The falloff of the observed rotation velocity serves as a testing ground for the predictions of competing theories, and separates the conformal theory in its unique prediction of eventual falloff.

3. Models of the Milky Way rotation curve

3.1. General Relativity Prediction

Similar to all other observed rotation curves, the Milky Way suffers from the same missing mass problem. The prediction set forth by general relativity (GR) can be found by starting with a single point mass solution to the Einstein field equations, and then modeling the galaxy as a collection of point masses arranged in a disk in superposition. We assume for simplicity that the disk is infinitely thin, and the distribution of mass falls exponentially as

$$\Sigma(R) = \Sigma_0 e^{-\frac{R}{R_0}} \quad (2)$$

where R_0 is the luminous scale length, and Σ_0 is the central density. Upon integrating over the disk in cylindrical coordinates, one arrives at the familiar

$$v_{GR}(R) = \sqrt{\frac{N^* \beta^* c^2 R^2}{2R_0^3} \left[I_0\left(\frac{R}{2R_0}\right) K_0\left(\frac{R}{2R_0}\right) - I_1\left(\frac{R}{2R_0}\right) K_1\left(\frac{R}{2R_0}\right) \right]}, \quad (3)$$

where I_0 , I_1 , K_0 , and K_1 are Bessel functions. This is the well established Freeman curve, and is assumed that each parameter is fixed. The only free parameter in this equation then is the overall number of stars,

$$N^* = \frac{M_{disk}}{M_\odot}. \quad (4)$$

Although this is a free parameter for fitting purposes, it is physically bounded by the preservation of the mass to light ratio to be on the order of unity.

3.2. *Lambda Cold Dark Matter (Λ CDM) Model*

To solve the missing mass problem, one can assume that since the velocity is a function of R and M , then more mass at the appropriate locations from the galactic center could rectify the GR prediction to match the data and hence cold dark matter can be introduced. In order to make the data match the prediction of equation (3), we can assume the total rotational velocity would be given as

$$v_{total}(R) = \sqrt{v_{GR}^2 + v_{dark}^2}. \quad (5)$$

The question then arises as how to put the dark matter into the galaxy as to match the prediction, but not force the inner region to then overshoot in turn. Following the prescription described in [6] the dark matter contribution can take the form

$$v_{dark}(R) = \sqrt{4\pi\beta^*c^2\sigma_0 \left[1 - \frac{r_0}{R} \arctan\left(\frac{R}{r_0}\right)\right]} \quad (6)$$

where σ_0 is the dark matter density and r_0 is the dark matter halo radius. Since we have not yet physically observed dark matter, these two parameters have to be fit to the data using a χ^2 test against v_{total} . The two free parameters effectively shape the modeled rotation curve, and when coupled with the free parameter of the total luminous mass, M_{disk} , the theory has three total free parameters. It should be noted that although this is only two extra free parameters for the Milky Way, this process must be arbitrarily done for any studied galaxy, making the number of fitting parameters for a given sample twice the number of galaxies studied. Due to the nature of equation (6), the dark matter contribution for a given galaxy can cause the rotation curves to become asymptotically flat, hence can never predict an overall fall off. As described by the work in [11], we believe it is this very observation that can lead to overall departures from dark matter fitting in the largest of the studied galaxies. Since the Milky Way data is now the largest diameter galaxy we have studied, it makes our own galaxy an ideal test of conformal gravity against dark matter.

3.3. *Conformal Gravity Prediction*

The conformal gravity theory originally derived from Weyl, was later re-studied by Mannheim and Kazanas [5]. Conformal gravity retains a completely covariant metric theory of gravity but also includes the feature of local conformal invariance, where the action is unchanged due to local transformations $g_{\mu\nu}(x) \rightarrow e^{2\alpha(x)}g_{\mu\nu}(x)$ with local phase $\alpha(x)$. It should be noted that although it is an alternative theory of gravity, since it is still a metric theory of gravity, many of the familiar properties of General Relativity such as curvature of space and time, and the coupling of electromagnetic fields to gravity (bending of light) are retained. Moreover, the conformal theory was not originally studied to solve the rotation curve problem, but instead one may use the rotation curve problem as a testing ground for the overall theory. Conformal gravity assumes a scalar action which is described by

$$I_W = -\alpha_g \int d^4x (-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} = -2\alpha_g \int d^4x (-g)^{1/2} [R_{\mu\kappa} R^{\mu\kappa} - (1/3)(R^\alpha_\alpha)^2] \quad (7)$$

where

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2} (g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu}) + \frac{1}{6} R^\alpha_\alpha (g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu}), \quad (8)$$

$$4\alpha_g W^{\mu\nu} = 4\alpha_g \left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa} R_{\lambda\kappa} \right] = 4\alpha_g \left[W_{(2)}^{\mu\nu} - \frac{1}{3} W_{(1)}^{\mu\nu} \right] = T^{\mu\nu}, \quad (9)$$

and

$$\begin{aligned} W_{(1)}^{\mu\nu} &= 2g^{\mu\nu} (R^\alpha_\alpha)^{;\beta}_{;\beta} - 2(R^\alpha_\alpha)^{;\mu;\nu} - 2R^\alpha_\alpha R^{\mu\nu} + \frac{1}{2} g^{\mu\nu} (R^\alpha_\alpha)^2, \\ W_{(2)}^{\mu\nu} &= \frac{1}{2} g^{\mu\nu} (R^\alpha_\alpha)^{;\beta}_{;\beta} + R^{\mu\nu;\beta}_{;\beta} - R^{\mu\beta;\nu}_{;\beta} - R^{\nu\beta;\mu}_{;\beta} - 2R^{\mu\beta} R^\nu_\beta + \frac{1}{2} g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta}. \end{aligned} \quad (10)$$

Since the goal is to construct a galaxy in the same procedure as done in the GR prediction, we note that $W^{\mu\nu}$ vanishes when $R^{\mu\nu}$ vanishes. However, Mannheim and Kazanas noted that the vanishing solutions may not be the only solutions to the fourth order theory. Hence they solved the metric outside a static, spherically symmetric source of radius a . They found, as described in [5], that in the conformal theory the exact line element is given by $ds^2 = -B(r)dt^2 + B(r)^{-1}dr^2 + r^2 d\Omega_2$ where the exterior metric coefficient $B(r > a)$ is given by

$$B(r > a) = 1 - \frac{2\beta}{r} + \gamma r - kr^2. \quad (11)$$

We can immediately see the departures from the GR prediction, where as the two new factors γ and k arise due to the fact that the conformal theory is fourth order and thus must contain two additional terms. It should also be noted that when γ and k are small, we return the exact Schwarzschild solution. For a more rigorous treatment of the derivation see [6]. Now that we have the ‘‘Schwarzschild like’’ solution for conformal gravity, we can effectively follow the procedure above by noting that a galaxy is a disk with exponential density falloff in the radial direction. The only other issue that conformal gravity needs to account for is local vs. global effects. Since the theory is fourth order in construction, we no longer possess the power of a global guess law. Hence, the integration must be made both locally and globally [6] which gives rise to the total rotational prediction of the galaxy as

$$v_{CG}(R) = \sqrt{v_{GR}^2 + \frac{N^* \gamma^* c^2 R^2}{2R_0} I_1 \left(\frac{R}{2R_0} \right) K_1 \left(\frac{R}{2R_0} \right) + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R} \quad (12)$$

where the integration constants are set as

$$\begin{aligned} \gamma^* &= 5.42e^{-41} \text{cm}^{-1}, \\ \gamma_0 &= 3.06e^{-30} \text{cm}^{-1}, \\ \kappa &= 9.54e^{-54} \text{cm}^{-2}. \end{aligned}$$

As can be seen in equation (11) and (12), the presence of the linear and quadratic potential terms are negligible on solar system scales, but would begin to dominate at galactic scales. The key feature of the solution is the requirement that the k term be negative [11] which forces the quadratic term to compete and eventually dominate over the linear term. The result is the termination of stable orbits in the galaxy and the ultimate fall off of a galactic rotation curve very far from the center. It is this feature that was tested in a sample of the fourteen largest studies galaxies [11], and due to the recent Milky Way data described above, can now be tested in our own galaxy.

4. Milky Way as a test of Conformal Gravity

4.1. Input Parameters for the Milky Way

For the Milky Way galaxy, we use the same set of input parameters as per a normal rotation curve as discussed in [6]. The gas mass of the Milky Way is adopted from McGaugh 2008 [16] as $M_g = 1.18 * 10^{10} M_\odot$ which is inclusive of the HI as well as helium. For the scale length, as will be shown in the figures, we adopt the range of scale lengths from Porcel 1997 [17] as $R_0 = 2.1 \pm 0.3 \text{ kpc}$. To be able to best compare our fits with the MOND fits of [16], we will plot the Milky Way galaxy for the range of scale lengths from $2.0 \leq R_0 \leq 2.5$. The luminosity is adopted from Spruit [18] as $L = 1.62 * 10^{10} L_\odot$ and will be used to compute the mass to light ratios. For the gas scale length we adopt the same fitting procedure as in [6] where we use a gas scale length four times the optical scale length. Lastly, we implement a bulge contribution for the noted galactic bulge as in [6] such that,

$$v_{bulge}(R) = \sqrt{\frac{2N_b^* \beta^* c^2}{\pi R} \int_0^{R/t} dz z^2 K_0(z)}. \quad (13)$$

The total contribution of rotation for the Milky Way galaxy in the conformal theory is then given by:

$$v_{tCG}(R) = \sqrt{v_{CG}^2 + v_{gas}^2 + v_{bulge}^2} \quad (14)$$

It should be noted that for comparison, the gas contribution and bulge contribution are also added to the other respective theories in the final fits that will be presented.

4.2. Fits

Here we present the fits for equations (3), (6), and (14). To produce the fits we have made use of a new computational tool called the Rotation Curve Modeler (RoCM) which was developed recently by the two authors [19]. The specific input and output parameters are listed in Table 1. We see that due to the synthesized rotation curve shown in Fig. 2, the outermost points are the ones where the predictions of conformal gravity can truly be tested. As seen in all of the plots, the essence of the inner most points including the bulge are well described by general relativity, Λ CDM, and conformal gravity alike. In all of the fits, general relativity falls far too quickly to account for the data as described in the missing mass problem. However, due to the nature of the dark matter equation (6), the outermost points of the rotation curve prediction will continue to remain flat even as the data begins to show a falloff. The falloff for each point of the Milky Way is predicted and captured by the conformal theory.

<i>Plot</i>	R_0 (kpc)	M_{disk} ($10^{10} M_\odot$)	M/L (M_\odot/L_\odot)
Fig. 2(a)	2.0	5.4738	3.38
Fig. 2(b)	2.1	5.189	3.2
Fig. 2(c)	2.2	5.1738	3.19
Fig. 2(d)	2.3	5.2831	3.26
Fig. 2(e)	2.4	5.3262	3.29
Fig. 2(f)	2.5	5.2122	3.22

Table 1. List of parameters used to fit the rotation curve with varying scale length, R_0 , and disk mass, M_{disk} , to satisfy an acceptable mass to light ratio, M/L .

As another subject of comparison, we include in each figure, the dotted vertical line which is representative of where our sun lies in the Milky Way galaxy. For the dark matter theory,

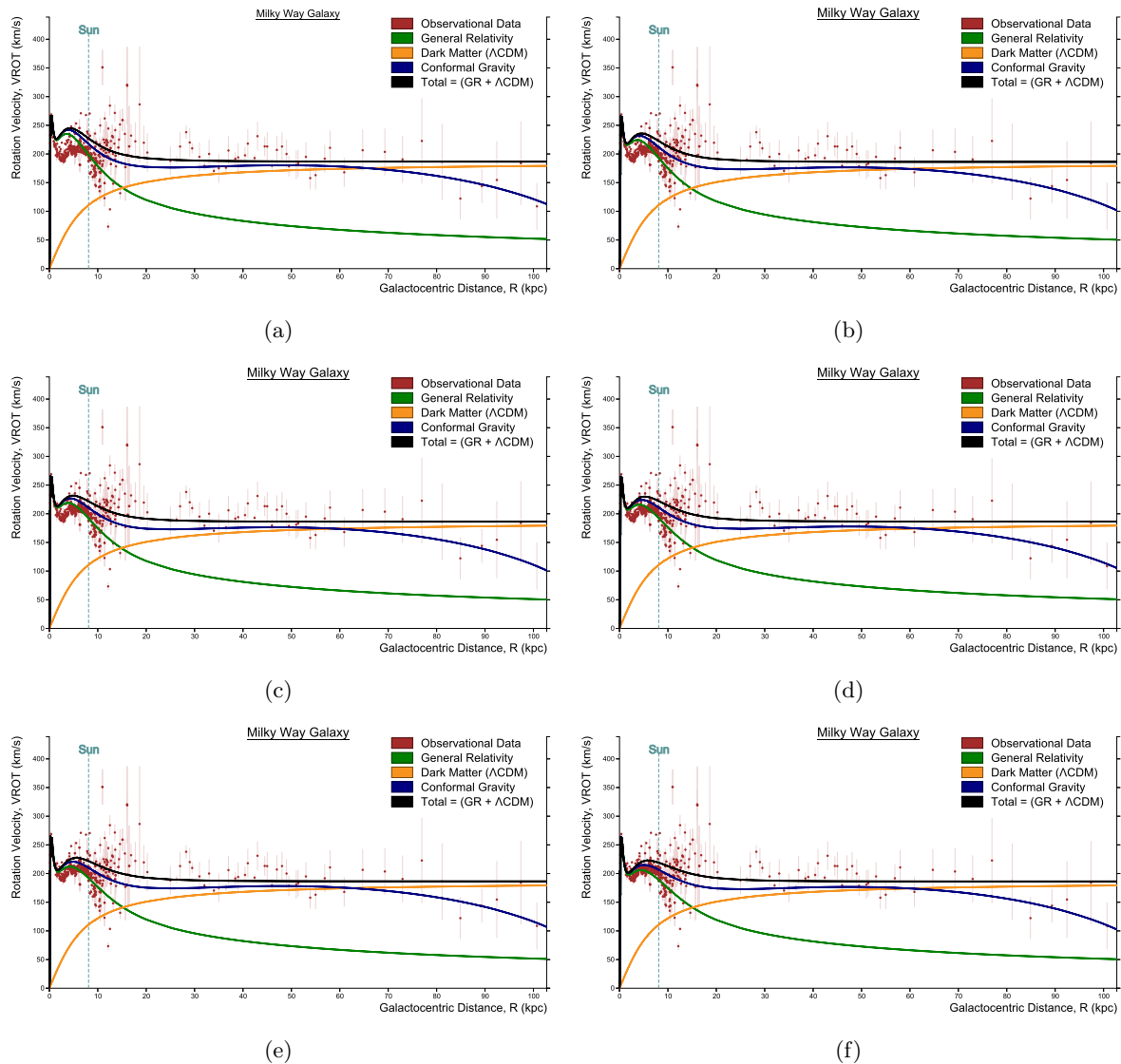


Figure 2. The scale length ranging from 2.0-2.5 kpc. These fits were produced using the RoCM software [19], and the colored predictions are described in the legend

we see that the fits are roughly generated by a 40 – 50 percent dark matter contribution. This poses a problem for nearby neighborhood dark matter searches which have returned null results, such as the recent work illustrated in [20]. The conformal theory once again can establish the predicted rotation even at the location of our sun with only baryonic matter. It should also be noted that we chose to plot the six figures of various scale length since MOND [16] seems to prefer only a smaller scale length for the Milky Way galaxy. In our fits, it's shown that the conformal theory is much less sensitive to the scale length. Instead, conformal gravity provides a model of the galaxy at any of the accepted scale lengths without the need for copious amounts of dark matter while still preserving a physically acceptable mass to light ratio.

5. Analysis

In order to better capture the overall power of the conformal gravity fit versus the standard theory or Λ CDM we produce a density plot of the velocity discrepancy. Here we take a point

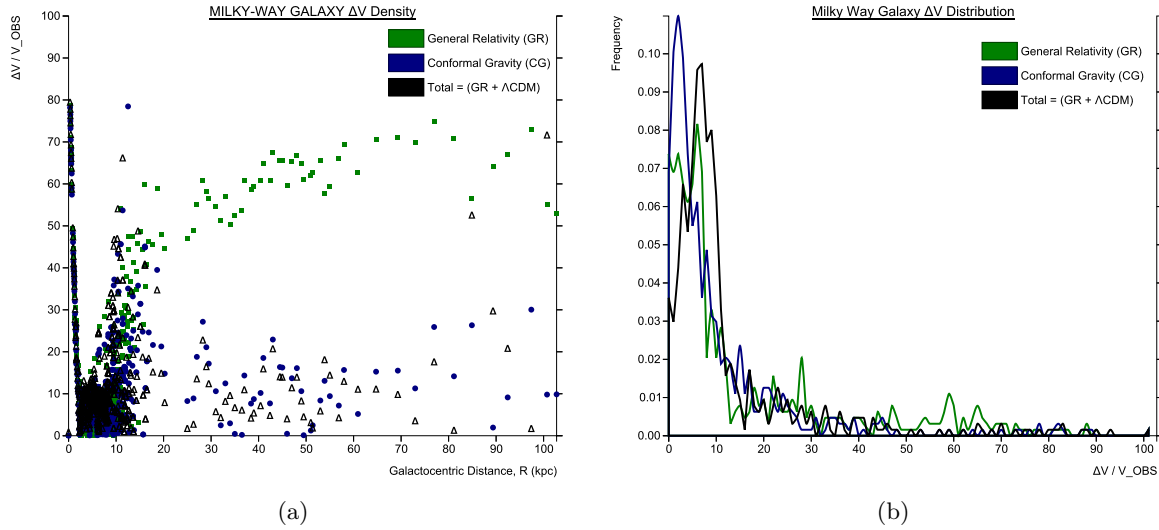


Figure 3. (a) The density plot of the velocity discrepancy relative to the galactocentric distance. (b) The frequency distribution of the velocity discrepancy which illustrates the aggregate comparison of each model to the observational data without the dependence of the galactocentric distance. Note that the vertical axis represents the frequency of the binned $\Delta V/V_{obs}$.

by point analysis of the 600+ points surveyed for the Milky Way and for each point compute $\Delta V/V_{obs}$ for each of the theories, where $\Delta V = |V_{obs} - V_{theory}|$. This gives us a more concrete comparison than a pure eyeball argument as to which prediction fits the data more precisely. The statistical fit is given in Fig. 3(a) and is shown for the Milky Way at scale length of $R_0 = 2.1$. This particular R_0 was chosen since it is the central value given in the literature. We see that much like the eyeball comparison of the fits, conformal gravity yields similar desirable statistics without the need for invoking dark matter.

To eliminate the dependency of the galactocentric distance from our analysis, we've generated a distribution of the velocity discrepancy for the Milky Way galaxy in Fig. 3(b). To clarify the comparison of each theory relative to the observations, the distribution can be used as an overall measure for the quality of the models. When comparing the distribution of conformal gravity vs. Λ CDM, it's straightforward to understand that conformal gravity predicts the observations to a higher degree. Each plot in Fig. 3 was generated by the RoCM tool [19] and can be produced for any arbitrary galaxy, and its data bin can be output. Future analysis will focus on comparing sets of galaxies with the density and distribution of the velocity discrepancy in order to achieve an aggregate analysis of each model. This can be accomplished using the power of RoCM and will result in a direct comparison of various models over entire sets of data.

6. Conclusion

In this work we presented the various rotation models of the synthesized data of the Milky Way galaxy. We leveraged the new tool developed by Moss and O'Brien [19] to simultaneously fit and analyze the rotation curve data for the Milky Way in the standard theory, conformal gravity, and the standard dark matter theory. We find that the Milky Way can now be added to the large galaxy survey of [11] as one of the largest galactic rotation curves currently in the literature. We find that like the other large galaxies in [11] the Milky Way can be fit by conformal to a high degree of precision without the need for dark matter. Further analysis will be to revisit the work of [11] with the current modeling power of [19] in order to build an overall velocity

discrepancy analysis of an unbiased survey of galaxies without reference to particular distances as in Fig. 3(b). The authors are pleased to note that the tool [19] used in analyzing the data has currently been made open to the public domain at www.wit.edu/rotationcurve.

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