

The Color Glass Condensate: A classical effective theory of high energy QCD

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Abstract. The Color Glass Condensate is a theory of the dynamical properties of partons in the Regge limit of QCD: $x_{\text{Bj}} \rightarrow 0$, $Q^2 \gg \Lambda_{\text{QCD}}^2 = \text{fixed}$ and the center of mass energy squared $s \rightarrow \infty$. We provide a brief introduction to the theoretical ideas underlying the Color Glass Condensate. We also discuss how these ideas provide a unified framework to discuss both Deeply Inelastic Scattering (DIS), and hadronic collisions (from pp to AA) in QCD.

1. Introduction

The study of the properties of the strong interactions in the asymptotic Bjorken limit of momentum transfer squared $Q^2 \rightarrow \infty$, the center of mass energy squared $s \rightarrow \infty$, and the Bjorken variable $x_{\text{Bj}} \approx Q^2/s = \text{fixed}$ has proved to be one of the most creative ideas in theoretical physics [1]. Relatively little work has been done in the other high energy limit, namely, $x_{\text{Bj}} \rightarrow 0$, $s \rightarrow \infty$ and $Q^2 = \text{fixed}$. This limit of the strong interactions, which we shall call the Regge limit, was studied intensively in the 60's and indeed led eventually to string theory. The reason these studies fell into disfavor in the strong interactions was that there was no small parameter in these studies (in modern parlance, $Q^2 \leq \Lambda_{\text{QCD}}^2$).

With the advent of the collider era, we can now probe a wide window of physics where $s \gg Q^2 \gg \Lambda_{\text{QCD}}^2$. In fact, this "window" describes the bulk of the high energy cross-section. One therefore has finally the possibility of studying the properties of the Regge limit of the theory using *weak coupling methods*. In this limit, the hadron behaves like matter that's dense but weakly coupled-not dissimilar to much of condensed matter physics [2].

In Regge asymptotics, the number of partons increases rapidly due to QCD bremsstrahlung. This growth is described, in the leading logarithmic approximation in x , by the BFKL equation [3]. Since the typical size of the partons in this limit is of order $1/Q^2$, the hadron becomes closely packed when the number of partons is of order $R^2 Q^2$. In fact, this corresponds to an occupation number $f \sim 1/\alpha_s$. When the density of partons is of this order, repulsive many body "recombination" and screening effects compete with QCD Bremsstrahlung leading to a saturation of the number of partons in the hadron's wavefunction [4, 5, 6]. The saturation of partons of different sizes happens at different values of x . The scale at which this occurs is the saturation scale $Q_s(x)$ —a dynamically generated semi-hard scale that controls the dynamics of physics in this regime of QCD.

In the language of the Operator Product Regime (OPE), the line $Q \equiv Q_s(x)$ in the x - Q^2 plane denotes the regime beyond which (when approached from high Q^2) higher twist effects become important. Recall that the OPE is best formulated in the Bj-limit where higher twists

are power suppressed and can be forgotten. The opposite is true in the Regge limit. Since the number of twist operators grows (nearly) exponentially with the twist, the OPE quickly becomes unwieldy. Thus to describe physics in this regime we need a new organizing principle in QCD beyond the OPE.

In the following section, we shall describe an effective field theory formalism which provides such an organizing principle. We shall discuss key features of this theory which are best summarized by the label Color Glass Condensate (CGC). The quantum evolution of the theory at high energies is described by renormalization group equations. These have remarkable features which we shall outline. We next discuss the applications of this theory to both DIS and hadronic scattering. The theory predicts novel universal observables which can be measured in both. Finally, we discuss the consequence of this approach for a theoretical description of the quark gluon plasma.

2. A classical effective theory for high energy QCD

There is a formal Born-Oppenheimer separation between large x and small x modes [8] for a quantum field theory on the light cone. These are respectively the slow and fast modes in the effective theory. Thus on the time scale of the "wee" parton small x fields, the large x partons can be viewed as static charges. Since these are color charges, they cannot be integrated out of the theory but must be viewed as sources of color charge for the dynamical wee fields. With this dynamical principle in mind, one can write down an effective action for wee partons in QCD at high energies [7]. The generating functional of wee partons has the form

$$\mathcal{Z}[j] = \int [d\rho] W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA] \delta(A^+) e^{iS[A, \rho] - j \cdot A}}{\int^{\Lambda^+} [dA] \delta(A^+) e^{iS[A, \rho]}} \right\} \quad (1)$$

where the wee parton action has the form

$$S[A, \rho] = \frac{-1}{4} \int d^4x F_{\mu\nu}^2 + \frac{i}{N_c} \int d^2x_{\perp} dx^- \delta(x^-) \times \text{Tr} (\rho(x_{\perp}) U_{-\infty, \infty}[A^-]) . \quad (2)$$

In Eq. 1, ρ is a classical color charge density of the static sources and $W[\rho]$ is a weight functional of sources (which sit at momenta $k^+ > \Lambda^+$: note, $x = k^+ / P_{\text{hadron}}^+$). The sources are coupled to the dynamical wee gluon fields (which in turn sit at $k^+ < \Lambda^+$) via the gauge invariant term ¹ which is the first term on the RHS of Eq. 2. The second term in Eq. 2 is the QCD field strength tensor squared-thus the wee gluons are treated in full generality in this effective theory, formulated in the light cone gauge $A^+ = 0$. The source j is an external source-derivatives taken with respect to this source (with the source then put to zero) generate correlation functions in the effective theory.

The argument for why the sources are classical is subtle and follows from a coarse graining of the effective action to only include modes of interest. For large nuclei, or at small x , the wee partons couple to a large number of sources. For a large nucleus, it can be shown explicitly that this source density is classical [10]. Further, it was conjectured that the weight functional for a large nucleus was a Gaussian in the source density (corresponding to the quadratic Casimir operator) [7, 11]. This was shown explicitly recently to be the correct-albeit with a small correction proportional to the cubic Casimir operator which generates Odderon excitations [10]. For a large nucleus, the variance of the Gaussian distribution, the color charge squared per unit area μ_A^2 , proportional to $A^{1/3}$, is a large scale-and is the only scale in the effective action ². Thus

¹ An alternative form is obtained in Ref. [9]-it reproduces the BFKL equation more efficiently.

² μ_A^2 is simply related in the classical theory to the saturation scale Q_s^2 via the relation $Q_s^2 = \alpha_s N_c \mu_A^2 \ln(Q_s^2 / \Lambda_{\text{QCD}}^2)$

for $\mu_A^2 \gg \Lambda_{\text{QCD}}^2$, $\alpha_S(\mu_A^2) \ll 1$, and one can compute the properties of the theory in Eq. 1 in weak coupling.

The Yang-Mills equations can be solved analytically to obtain the classical field of the nucleus as a function of ρ : $A_{\text{cl.}}(\rho)$ [7, 11, 12]. From the generating functional in Eq. 1, one obtains for the two point correlator,

$$\langle AA \rangle = \int [d\rho] W_{\Lambda^+}[\rho] A_{\text{cl.}}(\rho) A_{\text{cl.}}(\rho). \quad (3)$$

From this expression, one can determine (for Gaussian sources) the occupation number $\phi = dN/\pi R^2/dk_\perp^2 dy$ of wee partons in the classical field of the nucleus. For $k_\perp \gg Q_s^2$, one has the Weizsäcker-Williams spectrum $\phi \sim Q_s^2/k_\perp^2$, while for $k_\perp \leq Q_s$, one has a resummation to all orders in $k_\perp$, which gives $\phi \sim \frac{1}{\alpha_S} \ln(Q_s/k_\perp)$. (The behavior at low $k_\perp$ can, more accurately, be represented as $\frac{1}{\alpha_S} \Gamma(0, z)$ where Γ is the incomplete Gamma function and $z = k_\perp^2/Q_s^2$.) A nice expression for the classical field of the nucleus containing these two limits is given in Ref. [13].

We are now in a position to discuss why a high energy hadron behaves like a Color Glass Condensate [2]. The "color" is obvious since the degrees of freedom, the partons, are colored. It is a glass because the stochastic sources (frozen on time scales much larger than the wee parton time scales) induce a stochastic (space-time dependent) coupling between the partons under quantum evolution (to be discussed in the next section)-this is analogous to a spin glass. Finally, the matter is a condensate since the wee partons have large occupation numbers (of order $1/\alpha_S$) and have momenta peaked about Q_s . As we will discuss, these properties are enhanced by quantum evolution in x . The classical field retains its structure-while the saturation scale grows: $Q_s(x') > Q_s(x)$ for $x' < x$.

Small fluctuations about the effective action in Eq. 2 were computed in Ref. [14]. These gave large corrections of order $\alpha_S \ln(1/x)$ which suggested that the Gaussian weight functional was fragile under quantum evolution of the sources. A Wilsonian renormalization group (RG) approach systematically treats these corrections [15]. The basic recipe is as follows. Begin with the generating functional in Eq. 1 at some Λ^+ , with an initial source distribution $W[\rho]$. Perform small fluctuations about the classical saddle point of the effective action, integrating out momentum modes in the region $\Lambda'^+ < k^+ < \Lambda^+$, ensuring that Λ'^+ is such that $\alpha_S \ln(\Lambda^+/\Lambda'^+) \ll 1$. The action reproduces itself at the new scale Λ'^+ , albeit with a charge density $\rho' = \rho + \delta\rho$, and $W_{\Lambda^+}[\rho] \rightarrow W_{\Lambda'^+}[\rho']$. The change of the weight functional $W[\rho]$ with x is described by the JIMWLK- non-linear RG equation [15] which we shall not write explicitly here.

The JIMWLK equations form an infinite hierarchy (analogous to the BBGKY hierarchy in statistical mechanics) of ordinary differential equations for the gluon correlators $\langle A_1 A_2 \cdots A_n \rangle_Y$, where $Y = \ln(1/x)$ is the rapidity. The expectation value of an operator $\mathcal{O}$ is defined to be

$$\langle \mathcal{O} \rangle_Y = \int [d\alpha] \mathcal{O}[\alpha] W_Y[\alpha], \quad (4)$$

where $\alpha = \frac{1}{\sqrt{2}} \rho$. The corresponding JIMWLK equation for this operator is

$$\frac{\partial \langle \mathcal{O}[\alpha] \rangle_Y}{\partial Y} = \left\langle \frac{1}{2} \int_{x_\perp, y_\perp} \frac{\delta}{\delta \alpha_Y^a(x_\perp)} \chi_{x_\perp, y_\perp}^{ab}[\alpha] \frac{\delta}{\delta \alpha_Y^b(y_\perp)} \mathcal{O}[\alpha] \right\rangle_Y. \quad (5)$$

χ here is a non-local object expressed in terms of path ordered (in rapidity) Wilson lines of α [2]. This equation is analogous to a (generalized) functional Fokker-Planck equation, where Y is the "time" and χ is a generalized diffusion coefficient. This equation illustrates the stochastic

properties of operators in the space of gauge fields at high energies. For the gluon density, which is proportional to a two-point function $\langle \alpha^a(x_\perp) \alpha^b(y_\perp) \rangle$, one recovers the BFKL equation in the limit of low parton densities. The theory is conformal so it is not inconceivable that the full hierarchy is exactly solvable. Preliminary numerical solutions exist [16] but much work remains in that direction. A mean field solution deep in the saturation regime [17] shows that the weight functional there is a non local Gaussian with a variance proportional to $k_\perp^2$ for $k_\perp^2 < Q_s^2$.

For large N_c and large Λ ($\alpha_s^2 \Lambda^{1/3} \gg 1$), the expectation value of the product of traces of Wilson lines factorizes into the product of the expectation values of the traces:

$$\langle \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle \longrightarrow \langle \text{Tr}(V_x V_z^\dagger) \rangle \langle \text{Tr}(V_z V_y^\dagger) \rangle, \quad (6)$$

where $V_x = \mathcal{P} \exp(\int dz^- \alpha^a(z^-, x_\perp) T^a)$. Here $\mathcal{P}$ denotes path ordering in x^- and T^a is the SU(3) generator in the adjoint representation. In Mueller's dipole picture³, the cross-section for a dipole scattering off a target can be expressed in terms of these 2-point dipole operators as [18]

$$\sigma_{q\bar{q}N}(x, r_\perp) = 2 \int d^2b \mathcal{N}_Y(x, r_\perp, b), \quad (7)$$

where $\mathcal{N}_Y$, the imaginary part of the forward scattering amplitude, is defined to be $\mathcal{N}_Y = 1 - \frac{1}{N_c} \langle \text{Tr}(V_x V_y^\dagger) \rangle_Y$. Note that the size of the dipole, $\vec{r}_\perp = \vec{x}_\perp - \vec{y}_\perp$ and $\vec{b} = (\vec{x}_\perp + \vec{y}_\perp)/2$. The JIMWLK equation for the two point Wilson correlator is identical in this large Λ , large N_c mean field limit to an equation derived independently by Balitsky and Kovchegov-the Balitsky-Kovchegov equation [20], which has the operator form

$$\frac{\partial \mathcal{N}_Y}{\partial Y} = \bar{\alpha}_S \mathcal{K}_{\text{BFKL}} \otimes \{ \mathcal{N}_Y - \mathcal{N}_Y^2 \}. \quad (8)$$

Here $\mathcal{K}_{\text{BFKL}}$ is the well known BFKL kernel. When $\mathcal{N} \ll 1$, the quadratic term is negligible and one has BFKL growth of the number of dipoles; when $\mathcal{N}$ is close to unity, the growth saturates. The approach to unity can be computed analytically [21]. The B-K equation is the simplest equation including both the Bremsstrahlung responsible for the rapid growth of amplitudes at small x as well as the repulsive many body effects that lead to a saturation of this growth.

A saturation condition, say $\mathcal{N} = 1/2$, determines the saturation scale. One obtains $Q_s^2 = Q_0^2 \exp(\lambda Y)$, where $\lambda = c\alpha_s$ with $c \approx 4.8$. The saturation condition affects the overall normalization of this scale but does not affect the power λ . In fixed coupling, the power λ is large and there are large pre-asymptotic corrections to this relation-which die off only slowly as a function of Y . BFKL running coupling effects change the behavior of the saturation scale completely-one goes smoothly at large Y to $Q_s^2 = Q_0^2 \exp(\sqrt{2b_0 c}(Y + Y_0))$ where b_0 is the coefficient of the one-loop QCD β -function. The state of the art computation of Q_s is the work of Triantafyllopoulos, who obtained Q_s by solving NLO-resummed BFKL in the presence of an absorptive boundary (which corresponds to the CGC) [22]. The pre-asymptotic effects are much smaller in this case and the coefficient $\lambda \approx 0.25$ is very close to the value extracted from saturation model fits to the HERA data [23]. No analytical solution of the BK equation exists in the entire kinematic region but there have been several numerical studies at both fixed and running coupling [25, 24, 26]. These studies suggest that the solutions have a soliton like structure and that the saturation scale has the behavior discussed here. Geometrical scaling of solutions is seen for a wide window in rapidities. Running coupling effects, as suggested, are important and make the results of the computations more physically plausible.

The soliton like structure is no accident, as was discovered by Munier and Peschanski [27]. They noticed that the BK-equation, in a diffusion approximation, bore a formal analogy to

³ See also Ref. [19].

the FKPP equation describing the propagation of unstable non-linear wavefronts in statistical mechanics [29]. In addition, the full BK-equation lies in the universality class of the FKPP equation. This enables one to extract the universal properties such as the leading pre-asymptotic terms in the expression for the saturation scale. It was realized [28] that a stochastic generalization of the FKPP equation-the sFKPP equation-could provide insights into impact parameter dependent fluctuations [30] in high energy QCD beyond the BK-equation. This is a very active area of research now, with several groups hunting for the Pomeron loops responsible for these fluctuations.

To summarize, the Color Glass Condensate is a weak coupling effective theory describing the properties of hadron wavefunctions in QCD at high energies. Renormalization group equations-the JIMWLK equations-describe the behavior of multi-parton correlations in the hadron wavefunction as a function of rapidity. The theory has stochastic features closely analogous to the propagation of unstable non-linear wave fronts in statistical mechanics. Recent work is focused on trying to understand possible corrections beyond JIMWLK at low parton densities-which may be responsible for Pomeron loops. We now turn to the applications of this theory to hadronic scattering.

3. Hadronic scattering and $k_\perp$ factorization in the Color Glass Condensate

At collider energies, a new window opens up where $\Lambda_{\text{QCD}}^2 \ll M^2 \ll s$. In principle, cross-sections in this window can be computed in the usual collinear factorization language-however, one needs to sum up large logarithmic corrections in s/M^2 . An alternative formalism is that of $k_\perp$ -factorization [31, 32], where one has a convolution of $k_\perp$ dependent “un-integrated” gluon distributions from the two hadrons with the hard scattering matrix. In this case, the in-coming partons from the wavefunctions have non-zero $k_\perp$. The rapidity dependence of the unintegrated distributions is given by the BFKL equation. However, unlike the structure functions, it has not been proven that these unintegrated distributions are universal functions.

At small x , both the collinear factorization and $k_\perp$ factorization limits can be understood in a systematic way in the framework of the Color Glass Condensate. Rather than a convolution of probabilities, one has instead a collision of classical gauge fields. The expectation value of an operator $\mathcal{O}$ can be computed as

$$\langle \mathcal{O} \rangle_Y = \int [d\rho_1] [d\rho_2] W_{x_1}[\rho_1] W_{x_2}[\rho_2] \mathcal{O}(\rho_1, \rho_2), \quad (9)$$

where $Y = \ln(1/x_F)$ and $x_F = x_1 - x_2$. All operators at small x can be computed in the background classical field of the nucleus at small x . Quantum information, to leading logarithms in x , is contained in the source functionals $W_{x_1(x_2)}[\rho_1(\rho_2)]$. The operator $\mathcal{O}$ can be expressed in terms of gauge fields $A^\mu[\rho_1, \rho_2](x)$.

Inclusive gluon production in the CGC is computed by solving the Yang-Mills equations $[D_\mu, F^{\mu\nu}]^a = J^{\nu,a}$, where $J^\nu = \rho_1 \delta(x^-) \delta^{\nu+} + \rho_2 \delta(x^+) \delta^{\nu-}$, with initial conditions given by the Yang-Mills fields of the two nuclei before the collision. These are obtained self-consistently by matching the solutions of the Yang-Mills equations on the light cone [33]. The initial conditions are determined by requiring that singular terms in the matching vanish. Since we have argued in Section 2 that we can *compute* the Yang-Mills fields in the nuclei before the collision, the classical problem is in principle completely solvable. Quantum corrections not enhanced by powers of $\alpha_S \ln(1/x)$ can be included systematically. The terms so enhanced are absorbed into the weight functionals $W[\rho_{1,2}]$. As we will now discuss, hadronic scattering in the CGC can be studied through a systematic power counting in the density of sources in powers of $\rho_{1,2}/k_{\perp,1,2}^2$.

3.1. Gluon and quark production in the dilute/pp regime: $(\rho_{p1}/k_\perp^2 \rho_{p2}/k_\perp^2 \ll 1)$

The power counting here is applicable either to a proton at small x , or to a nucleus (whose parton density at high energies is enhanced by $A^{1/3}$) at large transverse momenta. The relevant quantity here is Q_s , which, as one may recall, is enhanced both for large A and small x . As long as $k_\perp \gg Q_s \gg \Lambda_{\text{QCD}}$, one can consider the proton or nucleus as being dilute.

To lowest order in $\rho_{p1}/k_\perp^2$ and $\rho_{p2}/k_\perp^2$, inclusive gluon production was computed in the CGC framework in $A^\tau = 0$ gauge [33] and subsequently in the Lorentz gauge $\partial_\mu A^\mu = 0$ [34]. For $Q_s \ll k_\perp$, the inclusive cross-section is expressed as the product of two unintegrated ($k_\perp$ dependent) distributions times the matrix element for the scattering. This well known result for gluon production is substantially modified, as we shall discuss shortly, by high parton density effects in the nuclei.

$k_\perp$ factorization is a good assumption at large momenta for quark pair-production. This was worked out in the CGC approach by François Gelis and myself [35]. The result for inclusive quark pair production can be expressed in $k_\perp$ factorized form. The matrix element can be shown to be identical to the result derived in the $k_\perp$ -factorization approach [31, 32]-thereby establishing that well known results can be recovered in the limit of low parton densities.

3.2. Gluon and quark production in the semi-dense/pA region $(\rho_p/k_\perp^2 \ll 1 \rho_A/k_\perp^2 \sim 1)$.

The power counting here is best applicable to asymmetric systems such as proton-nucleus collisions, which naturally satisfies the power counting for a wide range of energies. Of course, as one goes to extremely high energies, it is conceivable that the parton density locally in the proton can become comparable to that in the nucleus. In the semi-dense/pA case, one solves the Yang-Mills equations $[D_\mu, F^{\mu\nu}] = J^\nu$ with the light cone sources $J^{\nu,a} = \delta^{\nu+} \delta(x^-) \rho_p^a(x_\perp) + \delta^{\nu-} \delta(x^+) \rho_A^a(x_\perp)$, to determine the gluon field produced-to lowest order in the proton source density and to all orders in the nuclear source density.

Inclusive gluon production in this framework was first computed by Kovchegov and Mueller [36] and shown to be $k_\perp$ factorizable in Ref. [37]. In Ref. [38], the gluon field produced in pA collisions was computed explicitly in Lorentz gauge $\partial_\mu A^\mu = 0$. Our result is exactly equivalent to that of Dumitru & McLerran in $A^\tau = 0$ gauge [43]. The well known ‘‘Cronin’’ effect is obtained in our formalism and can be simply understood in terms of the multiple scattering of a parton from the projectile with those in the target. The Cronin effect, its quantum evolution and comparison with experiment was discussed at this meeting in Yuri Kovchegov’s talk and will therefore not be discussed here.

Quark production in p/D-A collisions can be computed with the gauge field in Lorentz gauge [39]. The field is decomposed into the sum of ‘regular’ terms and ‘singular’ terms; the latter contain $\delta(x^+)$. The regular terms are the cases where a) a gluon from the proton interacts with the nucleus and produces a $q\bar{q}$ -pair outside, b) the gluon produces the pair which then scatters off the nucleus. Naively, these would appear to be the only possibilities in the high energy limit where the nucleus is a Lorentz contracted pancake. However, in the Lorentz gauge, one has terms identified with the singular terms in the gauge field which correspond to the case where the quark pair is both produced and re-scatters in the nucleus! ⁴. Unlike gluon production, neither quark pair-production nor single quark production is strictly $k_\perp$ factorizable. The pair production cross-section can however still be written in $k_\perp$ factorized form as a product of the unintegrated gluon distribution in the proton times a sum of terms with three unintegrated distributions, $\phi_{g,g}$, $\phi_{q\bar{q},g}$ and $\phi_{q\bar{q},q\bar{q}}$. These are respectively proportional to 2-point, 3-point and 4-point correlators of the Wilson lines we discussed previously. For instance, the distribution $\phi_{q\bar{q},g}$ is the product of fundamental Wilson lines coupled to a $q\bar{q}$ pair in the amplitude and adjoint Wilson lines coupled to a gluon in the complex conjugate amplitude. For large transverse momenta or large mass

⁴ Related work for single quark production has been discussed in Refs. [41, 42].

pairs, the 3-point and 4-point distributions collapse to the unintegrated gluon distribution, and we recover the previously discussed $k_\perp$ -factorized result for pair production in the dilute/pp-limit. Single quark distributions are straightforwardly obtained and depend only on the 2-point quark and gluon correlators and the 3-point correlators. For Gaussian sources, as in the MV-model, these 2-,3- and 4-point functions can be computed exactly as discussed in Ref. [39]. In Ref. [40], we used these results to explicitly study the magnitude of violation of $k_\perp$ -factorization.

Our results, coupled with the previous results for inclusive and diffractive distributions in DIS suggest that at small x dipole and multipole correlators of Wilson lines can be extracted in both DIS and hadronic collisions. These operators are gauge invariant and process independent. The renormalization group running of these operators will be a powerful and sensitive harbinger of new physics.

3.3. Gluon and quark production in the dense/AA region ($\rho_{A1}/k_\perp^2 = \rho_{A2}/k_\perp^2 \sim 1$).

In nucleus-nucleus collisions, $\rho_{1,2}/k_\perp^2 \sim 1$. Unlike gluon production in the pp and pA cases, $k_\perp$ -factorization breaks down in the AA-case [46, 45]. This is because the classical field comes in with a factor $1/g$ -thus each insertion on the gluon is of order $O(1)$. One cannot therefore factor the quantum evolution of the initial wavefunctions into unintegrated gluon distributions unlike the pA case.

Nevertheless, the problem of nuclear collisions is well defined in weak coupling and can be solved numerically [46, 47]. The numerical simulations thus far assume Gaussian initial conditions as in the MV model. These are good initial conditions for central Gold-Gold collisions at RHIC where the typical x is of order 10^{-2} . They are not good initial conditions at the LHC where the typical x at central rapidities will be at least an order of magnitude lower. In that case, one has to use solutions of JIMWLK RG equations [16]. The numerical lattice formalism of Ref. [46] is ideal for computing particle production in the forward light cone by matching the Wilson lines from each of the nuclei on the light cone.

We restrict ourselves to discussing numerical solutions with Gaussian initial conditions. The saturation scale Q_s (which is an input in the numerical solutions in this approximation) and the nuclear radius R are the only parameters in the problem. The energy and number respectively of gluons released in a heavy ion collision of identical nuclei can therefore be simply expressed as

$$\frac{1}{\pi R^2} \frac{dE}{d\eta} = \frac{c_E}{g^2} Q_s^3; \quad \frac{1}{\pi R^2} \frac{dN}{d\eta} = \frac{c_N}{g^2} Q_s^2, \quad (10)$$

where (up to 10% statistical uncertainty) we compute numerically $c_E = 0.25$ and $c_N = 0.3$. Here η is the space-time rapidity. The number distributions of gluons can also be computed in this approach. Remarkably, one finds that a) the number distribution is infrared finite, and b) the distribution is well fit by a massive Bose-Einstein distribution for $k_\perp/Q_s < 1.5$ GeV with a “temperature” of $\sim 0.47Q_s$ and by the perturbative distribution $Q_s^4/k_\perp^4$ for $k_\perp/Q_s > 1.5$.

The MV model when applied to heavy ion collisions correctly predicted the initial multiplicity at RHIC [46] and was remarkably successful in explaining rapidity distributions and the centrality dependence of multiplicities [48]. However, it soon became clear that the CGC alone was not sufficient to explain the RHIC data since a) it could not explain the RHIC v_2 data and b) it predicted a suppression in D-A collisions at RHIC which disagreed with the RHIC data [44]. This failure of the CGC (here meaning quantum evolution as opposed to the MV model-which has no evolution) strongly suggested that final state interactions are important at RHIC. This corroborates the remarkable success of hydrodynamic models. Why do predictions of bulk features-the multiplicity [46] and rapidity and centrality dependence [48] do so well then? If hydrodynamic behavior sets in early, and viscous effects are small, the bulk features from the

initial conditions will be preserved by hydrodynamic flow [49]. Many puzzles remain. We don't understand early thermalization or why viscous corrections are small.

The RHIC data on the multiplicity and transverse energy of produced hadrons combined with Eq. 10 place strong constraints on what Q_s can be. If Q_s is too small, we find, absurdly, that the initial transverse energy is less than the final measured transverse energy. If Q_s is too large, we find that the initial multiplicity of gluons is greater than the final multiplicity of hadrons. These constraints therefore allow us to place the bound that $1.3 < Q_s < 2$ GeV. This bound is consistent with an $A^{1/3}$ extrapolation of the Golec-Biernat–Wusthoff fit of Q_s^2 to the HERA data [23]. A simple extrapolation gives $Q_s \approx 1.4$ GeV.

The transition to the QGP from the CGC remains as an outstanding theoretical problem. Due to the rapid expansion of the system, the occupation number of modes falls well below one on time scales of order $1/Q_s$. A necessary condition for thermalization is that momentum distributions should be isotropic. The CGC initial conditions are very anisotropic with $\langle p_\perp \rangle \sim Q_s$ and $\langle p_z \rangle \sim 0$. How does this isotropization take place? All estimates of final state re-scattering of partons formed from the melting CGC give longer times than what the RHIC collisions seem to suggest [50].

Collective instabilities [51], analogous to the well known Weibel instabilities in plasma physics, can speed up thermalization [52, 53]. Starting from very anisotropic (CGC-like) initial conditions, these instabilities drive the system to isotropy on very short time scales, of order $1/Q_s$ in some estimates. An equally interesting problem is that of chemical equilibration. One would expect, in weak coupling, the production of quarks to be suppressed. However, since the CGC produces strong fields of order $1/g$, it could drive the system to chemical equilibrium. First steps have been taken to study this problem [54, 55] by numerically solving the Dirac equation in the background field of the two nuclei.

4. Open Issues in the CGC

The CGC is a framework to think about problems in high energy QCD. A topic of much excitement among theorists recently is whether there are contributions beyond the JIMWLK equations—in particular those that generate "Pomeron loops". We addressed the issue of $k_\perp$ -factorization and why "dipole" and "multipole" operators are relevant variables at high energies. Can one derive factorization theorems in this framework analogous to those derived previously for Collinear Factorization? Turning to phenomenology, we have the beginnings of a consistent phenomenological picture of the CGC and the QGP in D-A and A-A collisions. For this to become a quantitative science, we need to understand the problem of thermalization from first principles in QCD. It is a difficult task but by no means an impossible one.

Acknowledgements

This work was supported in part by DOE Contract No. DE-AC02-98CH10886 and by a research grant from the Alexander Von Humboldt Foundation. I would like to further acknowledge the kind hospitality of the Institute for theoretical physics-II at Hamburg University and the European Center for Theoretical Nuclear Physics and related areas (ECT*).

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