# THE RESURRECTION OF A FORGOTTEN SYMMETRY: DE BROGLIE'S SYMMETRY*

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The aim of the present talk is not historical, even if I am calling for historical facts in order to perform an assessment of the present situation in particle theory. What I want to show you is that a symmetry which played an important role in the birth of quantum theory has been neglected, forgotten and buried, without a ceremony or tears, at the age of three. Paradoxically, to-day, the majority of the physicists think that this symmetry is still alive and textbooks are mentioning it without any announcement of death. In listening to that, you must have the feeling that I am referring to some accident in the history of physics and, consequently, that the situation I want to talk about is not very dramatic; after all, it is the kind of things historians or philosophers are interested in, and we, physicists, we can ignore it. This is not true. This lack of symmetry had very serious consequences for the present status of quantum theory and we are faced with inconsistencies which have to be cured. To-day, I intend to tell you what these inconsistencies are and I will show you how to restore the initial symmetry and put back the coherence which has been lost.

## 1. Symmetry with or without group theory. The Einstein-Faraday symmetry.

There are essentially two ways in introducing symmetries in physics, even if the frontier between the two is not easy to draw.

The first one consists in examining the equations of some theory and in looking for its invariance group of transformations. This is, for instance, what was made by Henri Poincaré, when he examined Maxwell's equations and the transformations introduced by Hendrik Antoon Lorentz. He arrived at the so-called inhomogeneous Lorentz group, which is now known as the Poincaré group[1]. Later, it was shown independently by the British physicists Harry Bateman[2] and Ebenezer Cunningham[3] that Maxwell's equations in the vacuum were invariant under a fifteen dimensional Lie

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[1]This appellation was proposed by Eugene Wigner around 1961.

[2]H. Bateman, Proc. London Math. Soc., **8**, 223 (1910).

[3]E. Cunningham, Proc. London Math. Soc., **8**, 77 (1910)

group, the so-called conformal group[1]. Eugene Wigner was the one who completed the Poincaré group in adding parity and time reversal[2].

It is certainly true that, in his symmetry investigation of Maxwell's equations, Poincaré was very close to the discovery of special relativity. What is not very well known is that Albert Einstein's derivation of this theory was induced by another kind of symmetry approach[3]. Einstein was worrying about the fact that there were two different interpretations of the electromagnetic induction effect, depending on the way the objects were moving. If a circular wire is moving near a fixed magnet, the Laplace force obliges the electric charges of the wire to move and create a current. But, if the circular wire is now fixed and the magnet is moving, we have an induced electric field and the Laplace force is useless. This dissymetry between the two explanations was not accepted by Einstein once he realized that the phenomenon only depends on a symmetric quantity, namely the relative velocity. It was a single effect and he thought that a given phenomenon cannot suffer two completely opposite explanations. Any symmetry in a given effect must correspond to some symmetric element in the theory. According to Holton, this is the analysis which led Einstein to special remativity.

## 2. The de Broglie symmetry (1923).

Another "added symmetry" was proposed by Louis de Broglie in 1923, one year after the explanation of the Compton effect. It is interesting to underline that it was the time where physicists began to accept the light quanta of Einstein and, therefore, the so-called wave-corpuscle dualism for light. The symmetry idea of de Broglie was to propose the extension of the wave-corpuscle dualism to matter particles and, especially, to electrons. It must be emphasized that, at that time, nobody was able to understand really what was the deep meaning of this dualism. Since 1905, almost all physicists were aware that there was a difficult problem to solve: the necessity of reconciling the (discontinuous) theories of the Compton and the photoelectric effects with the (continous) Maxwell theory.

It is this de Broglie symmetry - the "democracy" of all particles - I am interested in. I want to show how it has been given up three years later, without saying it explicitly. Surprisingly, this death was unnoticed.

## 3. De Broglie's symmetry buried (1926).

The de Broglie symmetry consisted in extending the 1905 Einstein wave-particle dualism of light to all kinds of particles. However, the relationship between the wave aspect and the particle aspect was not very clear. The experiment of Clinton Joseph Davisson and Lester Germer in 1927 was considered with just reason as a success of de Broglie's idea, although it was not suggested by it. The first particle interpretation of the Schrödinger wave function is due to Max Born (1926). This statistical interpretation broke the de Broglie symmetry, since Maxwell's field could not be given such a meaning.

The reason why the Born suggestion is incompatible with the de Broglie symmetry is that it cannot be applied to the photon. This can be shown in many ways. Let me give you two non sophisticated manners of proving this fact. The first one is based on dimensional analysis. If one of the electromagnetic vectors (**A**, **E**, or **B**)

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[1]For historical facts, see José M. Sanchez-Ron, *The role played by symmetries in the introduction of relativity in Great Britain*, in M. G. Doncel, A. Hermann, L. Michel and A. Pais, Editors, *Symmetries in physics*, published by Servei de Publicacions, Universitat Autònoma de Barcelona, Spain (1987).
[2]E. P. Wigner, Annals of Mathematics, **40**, 149 (1939), reprinted in Y. S. Kim and W. W. Zachary, *Proceedings of the International Symposium on Spacetime Symmetries*, North Holland, Amsterdam (1989).
[3]G. Holton, *Thematic origins of scientific thought*, Harvard University Press (1973).

described the probability amplitude density to find a photon at some place in space, we must have a dimensionless integral of the form $\int |\lambda \mathbf{A}|^2 \, d^3\mathbf{x}$, where $\lambda$ is some factor which can be expressed with the aid of the fundamental constants $c$, $e$, $\hbar$. Unfortunately, dimensional analysis shows that this problem has no solution. Therefore the Born interpretation of the wave function (the counterpart of the electromagnetic wave) cannot apply to the electromagnetic wave.

The other elementary way of showing the difficulty concerning the photon consists in proving that the vector operator $i\frac{\partial}{\partial \mathbf{p}}$ cannot be diagonalized. Indeed, define

$$\mathbf{F} = \mathbf{B} - i\,\mathbf{E}$$

and suppose that we have the following eigenvalue equation

$$i\frac{\partial}{\partial p_i} F_j(\mathbf{p}) = x_i F_j(\mathbf{p}) \; (i, j = 1, 2, 3)$$

The Maxwell equation $\mathbf{p}.\mathbf{F}(\mathbf{p}) = 0$ (transversality condition) gives

$$i\frac{\partial}{\partial p_i}[\mathbf{p}.\mathbf{F}(\mathbf{p})] = i\frac{\partial}{\partial p_i}[\textstyle\sum p_j F_j(\mathbf{p})] = iF_i(\mathbf{p}) + i\displaystyle\sum_j p_j \frac{\partial F_j}{\partial p_i}$$

$$= iF_i(\mathbf{p}) + ix_i[\mathbf{p}.\mathbf{F}(\mathbf{p})] = 0$$

and, therefore, $\mathbf{F}(\mathbf{p}) = \mathbf{0}$.There is no localized state for the photon.

You could protest and say that $i\frac{\partial}{\partial \mathbf{p}}$ is perhaps not a good position operator for the photon. After all, the photon is a relativistic particle and the Schrödinger position operator is a non relativistic one. This is a good point, but I have two answers: first, my argument based on dimensional analysis is still valid; second, everybody knows that, in 1949, T.D. Newton and Eugene Wigner[1] (followed by Arthur Wightman[2], in 1962) have shown, from relativistic considerations, that there were no localized states for the photon. The reason is not due to the massless character of that particle but to the fact that it is a *spinning and massless* particle. In other words, it is due to i) the transverse character of the electromagnetic field (spin), ii) the impossibility of the value $\mathbf{p} = \mathbf{0}$ for a massless particle[3].

The N.W. (Newton and Wigner) result is well known but it is usually considered as a curiosity or a technical point and many physicists do not mind because they say that the photon is not a matter particle. But this does not solve the problem. To-day, particle physicists believe in a theory where the photon is a brother of the intermediate boson which itself has localized states in the N.W. sense. Nobody is

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[1]T.D. Newton and E.P. Wigner, Rev. Mod. Phys. 21, 400 (1949).
[2]A.S. Wightman, Rev. Mod. Phys. 34, 845 (1962).
[3]It is perhaps important to underline that the energy and the momentum of a massless particle obey the relation $E = |\mathbf{p}|$ and that *to be at rest* for a particle means really $\mathbf{p} = \mathbf{0}$, in contradistinction with what is said in almost all books on quantum mechanics, where the meaning of this expression is related with the speed. Everybody knows how the speed is defined in quantum mechanics: it is the group velocity of the wave associated with the given particle. According to the superposition principle, a photon is not necessarily in an energy-momentum state (another generally accepted idea!). Since the group velocity can have any value (less than or equal to $c$), a photon has states of *any velocity*, even the zero velocity!

worrying about the way a broken symmetry is accompanied by a broken de Broglie symmetry.

## 4. Restoring the de Broglie symmetry.

I guess that many people of the audience will object that, to-day, the wave-particle dualism is no longer the one I am describing but refers to the modern quantum field theory, where fields obey differential equations. Before examining this objection, I will tell you how we can restore the de Broglie symmetry in the context of quantum mechanics.

For that purpose, I have to recall you that the Born statistical interpretation of the wave function is no longer considered as an axiom of quantum mechanics. It became a consequence of one of the axioms of the theory of quantum measurement. More precisely, it concerns the measurement of the position operator **X**. Using the Dirac formalism, $|\psi(x)|^2$ is just the modulus squared matrix element $|\langle x|\psi\rangle|^2$, where $|x\rangle$ describes a localized state for which three commuting observables have been measured. This means that we could say that the Born idea, in its generalized formulation, is simply a probalistic statement about the *measurement of a complete set of commuting observables* for a system in a given state $|\psi\rangle$.

The interesting fact is that, if we do not require the commutativity of the coordinate operators X, Y, Z as one of the Wightman axioms, the photon can be given a position operator. From consistency arguments, it also follows that the spinning particles with mass must also have non commuting coordinate operators. For the moment, I only want to mention that the de Broglie symmetry can be restored at such a price and that for that price one gets some extra advantages.

## 5. The Bohr complementarity principle and the Einstein-Faraday symmetry.

I described at the beginning of this talk how Einstein worried about the double[1] explanation of the induction effect and how profitable was his investigation. To-day, there is still an experiment which is suffering *two* explanations. It is the electron slit experiment when a light source is used to detect through which slit the electrons go. If the source is intense, the interference pattern disappears completely. If we make the intensity of the source decreasing progressively, interference fringes reappear. The two distinct explanations I am referring to are the following ones:

a) If the source is such that the flux of photons per second is decreasing, there are more and more electrons which are not scattered by photons; they are not detected and, therefore, contribute to the interference pattern. This is a particle argument.

b) If the frequency of the source is decreasing, the image of the electron is not a point, but a spot of increasing radius which does not always permit to say through which slit the electron went. This is a wave argument.

Since we are free to choose a light source as we want, the reapparence of the interference pattern needs *both* wave and particle arguments and Bohr's complementary principle according to which an experiment is either of wave type or of particle type is not allowable. To have recourse to Bohr's principle is equivalent, from Einstein's point of view, to explain the induction effect with the help of some complementary principle where *wave* and *particle* are replaced respectively by *induced electric field* and *Laplace force* (or *vice versa*).

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[1]The word "double" is not the right one. The induction phenomenon occurs whatever are the speeds of the wire and of the magnet. At the time of Einstein's investigations, we needed a *continuous* set of explanations.

Fortunately, when one chooses the non commutative position operator I proposed for the photon[1], we get a *single* explanation for the slit electron experiment we have just discussed. This is for the following reason. When a photon is *almost* in an eigenstate of the momentum operator, say in direction z, we have the aproximative commutation relation

$$[x,y] \sim i \frac{p_z}{p^3}$$

which gives the uncertainty relation

$$\Delta x \, \Delta y \sim \lambda^2$$

the particle counterpart of the wave spot argument. This relation provides us with a nice and satisfactory explanation of the fact that a photon is more and more localizable when its frequency is larger and larger.

It is now time to give a résumé of the advantages of the new position operator. They are five in number. Up to now, we have examined the first three ones.

1. De Broglie's symmetry is restored. All particles (bosons and fermions) have a position operator *built with a unique procedure*. The only change is that one cannot measure simultaneously two components of the position operator for spinning particles.

2. Obviously, there is no localized state for a photon in the N.W. sense but now, without calling upon the complementary principle of Bohr, we have the following quantum situation: a photon is more and more localizable when its frequency is larger and larger.

3. The Bohr principle was not able to describe experiments which were intermediate between pure wave type and pure particle type. The typical experiment is the two slit experiment for an electron beam, when a light source of *variable* intensity is used to localize some electrons. We now have a *unique quantum* explanation of this experiment.

4. When the spin of an electron is neglected, the new position operator reduces to the ordinary Schrödinger one. In the nonrelativistic approximation (small momenta[2]), the potential in the Schrödinger equation is modified in such a way that the spin-orbit coupling rises automatically, with the correct factor.

5. The interference of light waves in the slit experiment can be given a particle interpretation. After all, each photon is known to face the two slits. Everybody will agree that such a situation corresponds to a quantum question concerning a *position*. Given a photon in the momentum state $\mathbf{p} = (0,0,p_z)$ at time zero, the presence of the two slits is equivalent to the question: *"is x equal to* $\pm a$*?"*. The answer is *"yes"*. After the measurement, the photon is in a reduced state. This new state $|\psi(t)\rangle$ permits to know the probability density $|\langle x|\psi(t)\rangle|^2$ at any time t and the integral $\int_0^\infty |\langle x|\psi(t)\rangle|^2 \, dt$ must give the function of x expected by the interference pattern.

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[1]H. Bacry, *Localizability and space in quantum physics*, Lecture Notes in Physics, vol. 308, Springer-Verlag (1988). See also H. Bacry, *The position operator revisited*, Ann. Inst. Henri Poincaré, 49,245 (1988) and*The notions of localizability and space, from Eugene Wigner to Alain Connes*, in Y.S. Kim and W. W. Zachary, *op. cit.*

[2]an approximation which is distinct from the Galilean approximation (*c* going to infinity).

Before examining the problem of quantum field theory, I have the following comments to add.

## 6. Comments on the new position operator.

1. I have shown in my book how the construction of the new position operator is related with the Schrödinger *zitterbewegung* for the electron. The motivation of Schrödinger is to-day forgotten. What he wanted to do is *to propose a new position operator for the Dirac electron.* As far as the Dirac equation governs the Hilbert space of the one electron states, the Schrödinger procedure is similar to mine. When I read the Schrödinger paper, I had the feeling that there was a kind of *zitterbewegung* for the photon behind my position operator. Recently, in his attempt towards a point description of a massless particle with helicity, Plyushchay[1] found a proof of it.

2. It can be shown that the difficulty about the lack of localized states for spinning massless particles is not only quantal but has its counterpart in classical theories. I refer you to my book and to two more recent articles[2] which concern the problem of the position of a *classical* massless particle. In one of these articles, Grigore wrote the rigorous classical counterpart of the Wightman axioms for localized states and is led to coordinates which are not (Poisson) commuting.

3. I remind you that all the difficulties related with the problem of localized states were due to the existence of the spin (for spinless particles, the N.W. conclusions are satisfactory). If we note that the unit of spin is the Planck constant, we can bring together this fact and the discreteness of the electric charge and conclude that in quantum physics, we do not have to "measure" angular momenta nor charges; we only to "count" them. In that sense, the experimental values of the two fundamental constants $\hbar$ and $e$ belong to *classical* physics. This is opposite to the ordinary statement made in all textbooks where $\hbar$ is considered to characterize quantum physics. We can deduce from that that position measurement of particles have a mysterious discrete character and, if position measurement has something to do with space, we probably have to find the real microstructure of our space before constructing a new quantum theory of fields.

4. As I explained in my book, I knew for more than twenty years that the difficulty about the lack of localized states of the photon was due to the Poincaré group itself[3]. To-day I can say more: the microstructure of the Minkowski space is unacceptable[4], even in the context of classical physics. What I mean is not that experiments oblige us to give up the Minkowski space, but that special relativity cannot be built with the aid of a continuous spacetime.

## 7. Why Minkowski's space is unacceptable.

It woud be too long to give a detailed discussion about the unacceptability of the Minkowski space in the classical theory of special relativity. Let me say in a few words about the main argument. The fact that Einstein used light rays (and clocks) to explain how to measure distances is in itself contradictory. Indeed, measuring distances between points implies that these points have a small expanse. It is easy that this expanse is of a few wavelengths of the light rays if we want the light rays to propagate

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[1]M.S. Plyushchay, *Massless point particle with rigidity*, Mod. Phys. Lett. A4, 837 (1989).
[2]D.R. Grigore, *Localizability and covariance in analytical mechanics*, Jour. Math. Phys. **30**, 2646 (1989). C. Duval, J. Elhadad and G.M. Tuynman, *Puk ánsky's condition and symplectic induction*, preprint (1990).
[3]See J.-M. Souriau, *Structure des systèmes dynamiques*, Dunod (1970).
[4]H. Bacry, *A contradiction in Special Relativity*, preprint (1989).

at group velocity c. Since the wavelength is not Lorentz invariant (Doppler effect), we cannot say that the points we are considering are small. There exist observers for which they are as large as we want. This means that Minkowski's space is good provided we restrict ourselves to small boosts and expanded points. It also has an important corollary: *the $x_\mu$ coordinates in Maxwell classical theory cannot have a sharp space-time interpretation.*

## 8. Quantum field theory and the de Broglie symmetry.

If we examine the evolution of theoretical physics between 1905 and 1934, we can say the following things:

1. There were two known fields, the Maxwell field and the gravitational field, both relativistic.
2. Among these two fields, only one has waves.
3. Since 1905, electromagnetic waves have also particle characters.
4. Since 1923, it is stated that electrons and protons (the only matter particles known) have also wave properties. This is the birth of de Broglie's symmetry.
5. Schrödinger writes the wave equation for these particles. It is a nonrelativistic equation. The Born interpretation of the wave function destroys the de Broglie symmetry.
6. In 1928, the Dirac equation replaces the Schrödinger one.
7. In introducing the meson in 1934, Yukawa introduces a new kind of democracy: *the field-particle duality.*

The de Broglie symmetry seems to have be restored by Yukawa. This is an illusion because the introduction of a field does not erase the dissymetry related with the measurement of the position for one particle states. Moreover, the notion of field has an ingredient which is unsatisfactory, namely the Minkowski space. Fortunately, Fock told us how to build fields without it. For each kind of field, we start with the one particle Hilbert space of states, a carrier space of an irreducible representation of the Poincaré group[1] for which the energy-momentum and angular momentum are natural observables. The direct sum of the symmetrized (or antisymmetrized) tensor products of this Hilbert space provides us with the Hilbert Fock space for this kind of field. Once the tensor product of all Fock spaces is taken, we have constructed in principle the space of all systems. The only thing which is left is a procedure to determine the Hamiltonian of the system we choose to study. Unfortunately, no procedure is known, except the Lagrangian one, but a Lagrangian needs some parameters and these parameters are the $x_\mu$, the so-called coordinates of the Minkowski space, *which do not have a space-time interpretation.*

For me, the solution of this difficulty must be found in the non commutative position operator. This means probably that we need a *non commutative space.*

## 9. Are we living in a non commutative space?

First, let me try to answer a question asked to Professor Manin to-day. Somebody in the audience wanted to understand what was a non commutative space. His handwaving expressed his confusion.

The problem of space is an old one. Greek thinkers were aware of the relationship between space and numbers. Everybody knows the difficulties they encountered with irrational numbers. The existence of a bijection between real numbers and the points of a straightline seemed more and more natural to philosophers and scientists. With Descartes, the isomorphism between our space and $\mathbb{R}^3$ became

[1]Wigner's work on the representations of the Poincaré group is an illustration of his belief in the de Broglie symmetry.

obvious, even if the concept of real numbers was not completely understood at that time. However, on the one hand, nobody is able to show you a point in space, on the other hand, I can speak of a lot of real numbers. The points are out of reach but we are idealizing them with the aid of real numbers, a mind construction. We are so accustomed to this surprising association, that we are not bringing that into question. However, as physicists, we cannot be satisfied by that: there are absolute scales (the size of an electron, for instance) in space, there are not in the set of real numbers, a set which looks the same at any scale! The most trivial fractal!

However, in order to investigate a space, the mathematicians made a new step, gave up the set of real numbers and used one of the many (commutative) algebras of functions on the space. They discovered continuous spaces, differential manifolds, etc. They replaced an abstract construction by another abstract construction. Nowadays, they are not more abstract than before. We are still not able to understand with our body the intimate structure of space but they are able to generalize geometrical computations in replacing the commutative algebra of functions by any *non commutative algebra*. We are not yet accustomed to this idea,... not yet... I said that the real line does not possess an intrinsic scale, neither does a commutative space, but a non commutative one? We could expect that our space will explain why the angular momentum is discrete.

The non commutative geometry was founded by Alain Connes. As claimed by his inventor, this new geometry was inspired by quantization itself. If the operators x, y, z do not commute, it is natural to imagine that we are living in a non commutative space. But Connes has other arguments in favour of such an idea. He made some attempts in the use of noncommutative spaces in quantum field theory[1] and very encouraging facts are an indication that we have to explore this kind of solution. The main argument is that Connes have shown that replacing the ordinary spacetime by a non commutative space is equivalent to introducing a cutoff: the ultraviolet divergences vanish...

If you want more details about the arguments I have mentioned in my talk, you can refer to my book and the general introduction in Connes' book. My hope is that I already convinced you that we are living in a non commutative space. If it is the case, I invite you to read the whole book of Alain Connes.

[1]A. Connes, *Géométrie non commutative*, InterEditions (Paris, 1990). A. Connes, *Essay on physics and non-commutative geometry*, preprint ((1989). A. Connes and J. Lott, *Particle models and noncommutative geometry*, preprint (1989).