Conformal Inflation, Modulated Reheating, and WMAP5

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Abstract

We investigate density perturbations generated through modulated reheating while inflation is driven by a conformally coupled scalar field. A large running of the spectral index is obtained, which reflects the basic nature of conformal inflation that higher-order time derivatives of the Hubble parameter during inflation are not necessarily small. This feature may allow us to distinguish between conformal inflation models and standard minimally coupled ones. We also investigate how the resulting fluctuations are modified when there is a deviation from an exact conformal coupling between the inflaton and gravity. Finally, we apply our results to the warped brane inflation model and see that observational bounds from the WMAP5 data suggest a blue tilted density perturbation spectrum. The discussion here is based on the paper [1].

1 Introduction

Ever since the idea of cosmic inflation was proposed, inflationary model building has largely focused on slow-rolling scalar fields minimally coupled to gravity as candidate inflatons. However, recently it was pointed out in [2] that conformally coupled scalar fields are also capable of accelerating the universe. Since the existence of such conformally coupled fields are rather common in models from string theory, it is of great interest to explore the possibility of conformally coupled fields driving inflation. In this light, we also come up with a new question of how we can distinguish between these "conformal inflation" models and standard minimally coupled ones.

The aim of our work is to focus on density perturbations and seek for distinctive features conformal inflation might have left. A conformally coupled inflaton itself cannot be responsible for generating primordial fluctuations, but instead string theory suggests alternative scenarios such as modulated reheating [3, 4]. Thence, we consider the case where modulated reheating generates density perturbations while inflation is driven by an almost conformally coupled inflaton. We study the scale dependence of the generated density perturbations.

We also investigate how the resulting density perturbations are modified when there is a deviation from an exact conformal coupling between the inflaton and gravity. The result will allow us to impose constraints on the inflaton's coupling from observational data. As a concrete example, our generic results are applied to the well-studied warped brane inflation model [5].

2 Review of Conformal Inflation

Here we give a brief review of conformal inflation. Consider the action

$$S = \int dx^4 \sqrt{-g} \left[\frac{M_p^2}{2}\mathcal{R} - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{\xi}{2}\mathcal{R}\phi^2 \right] \tag{1}$$

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where $\mathcal{R}$ is the scalar curvature and ξ is the non-minimal coupling to gravity. Choosing a flat FRW background $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$ and also introducing $\pi \equiv \dot{\phi} + H\phi$, the Friedmann equation is

$$M_p^2 H^2 = \frac{1}{3}V + \frac{1}{6}\pi^2 + \left(\xi - \frac{1}{6}\right)(2H\phi\pi - H^2\phi^2), \tag{2}$$

and the equation of motion of ϕ is

$$\dot{\pi} + 2H\pi + V' + 6\left(\xi - \frac{1}{6}\right)(\dot{H} + 2H^2)\phi = 0. \tag{3}$$

The conformal case $\xi = 1/6$ was investigated in [2], where inflation was realized while the equations (2) and (3) could be approximated to

$$M_p^2 H^2 \simeq \frac{1}{3}V, \qquad cH\pi \simeq -V'. \tag{4}$$

Here, c is a dimensionless constant[3]. Let us define three "flatness parameters" as

$$\epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \tilde{\epsilon} \equiv \frac{\phi V'}{2V}, \qquad \eta_c \equiv \eta + \frac{c}{3}\left(\frac{V''\phi}{V'} + c - 2\right), \tag{5}$$

where $\eta \equiv M_p^2 V''/V$. When $|c| \sim \mathcal{O}(1)$, one can check that the necessary conditions for (4) and $|\dot{H}/H^2| \ll 1$ are simply

$$\epsilon \ll 1, \quad |\tilde{\epsilon}| \ll 1, \quad |\eta_c| \ll 1. \tag{6}$$

The proportionality constant c is chosen such that it is the largest constant to minimize $|\eta_c|/c^2$ [1].

It is clear from (2) and (4) that π^2/V is small during inflation. Especially when the potential is flat enough to satisfy $|M_p^2 V'| \ll |c\phi V|$, then $\dot{\phi} \approx -H\phi$, which suggests that the inflaton is rapidly rolling towards its origin.

3 Density Perturbations from Modulated Reheating

We analyze the case where density perturbations are generated through modulated reheating [3, 4] after conformal inflation. The scale dependence of the perturbations can be expressed by expansion in terms of the flatness parameters. Furthermore, we take account of the inflaton's slight deviation from a conformal coupling $(\xi - 1/6)$. This procedure allows us to deal with a wide variety of situations, e.g., when there exists additional corrections to the action which ruins the exact conformal coupling, when the frame where the inflaton ϕ is conformally coupled to gravity differs from the frame where the light modulus χ is minimally coupled. In [1] the resulting spectral index and its running were shown to take the form,

$$\begin{aligned} n_s - 1 = \frac{d\ln H^2}{d\ln k} &= 2\frac{\dot{H}}{H^2}\left(1 + \frac{\dot{H}}{H^2}\right)^{-1} \\ &= -2\tilde{\epsilon} - \left(\frac{12}{c^2} + \kappa^2\right)\epsilon + \mathcal{O}(\epsilon^{3/2}, \epsilon\eta_c) + \left(\xi - \frac{1}{6}\right)\left\{2\kappa^2 + \mathcal{O}(\epsilon^{1/2})\right\} + \mathcal{O}\left(\left(\xi - \frac{1}{6}\right)^2\right), \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{dn_s}{d\ln k} &= 2\left(1 + \frac{\dot{H}}{H^2}\right)^{-3}\frac{1}{H}\left(\frac{\dot{H}}{H^2}\right)^{\cdot} \\ &= 2\left(3 - c + \frac{3}{c}\eta_c\right)\tilde{\epsilon} + \left(\frac{6(8-3c)}{c^2} + (7-3c)\kappa^2\right)\epsilon + \mathcal{O}(\epsilon^{3/2}, \epsilon\eta_c) \\ &\quad + \left(\xi - \frac{1}{6}\right)\left\{-4\kappa^2 + \mathcal{O}(\epsilon^{1/2})\right\} + \mathcal{O}\left(\left(\xi - \frac{1}{6}\right)^2\right), \end{aligned} \tag{8}$$

[3]Our parameterization of c differs from that of [2] by $c_{\text{ours}} = c_{[2]} + 2$.

where the right hand sides should be estimated at the moment of horizon crossing $k = aH$. We immediately see that $|dn_s/d\ln k|$ can become large, comparable to $|n_s - 1|$. This is due to the fact that the derivatives of the flatness parameters do not necessarily become smaller than the parameters themselves. Since this is a generic feature of conformal inflation, a large running is expected even if we consider mechanisms other than modulated reheating for generating fluctuations (e.g. curvaton models). Also, our results indicate that $(\xi - 1/6)$-corrections can dominantly determine the values of the cosmological observables unless $|\xi - 1/6|$ is smaller than the flatness parameters.

4 Application to Warped Brane Inflation

Let us now apply the results obtained in the previous section to a specific model. Here we consider the warped brane inflation model [5], where the universe experiences inflation while a D3-brane moves towards the tip of a flux compactified warped throat. The D3-brane is pulled by a stack of $\overline{\text{D3}}$-branes sitting at the tip. If the position of the D3-brane is a conformally coupled scalar (for a discussion on this issue, see e.g. [7]), then this model serves as a realization of conformal inflation.

Considering a throat whose geometry is $AdS_5 \times X_5$, the potential of the inflaton takes the form

$$V(\phi) = 2ph_0^4 T_3 \left(1 - \frac{h_0^4 T_3^2 R^4}{N\phi^4}\right). \tag{9}$$

Here, the inflaton is related to the radial position ρ of the D3 through $\phi = \sqrt{T_3}\rho$, p is the number of $\overline{\text{D3}}$s at the tip, $h_0 = \rho_0/R$ is the warping at the tip, $T_3 = 1/(2\pi)^3 g_s(\alpha')^2$ is the D3 tension, $R^4 = 2^2\pi^4 g_s(\alpha')^2 N/\text{Vol}(X_5)$ is the AdS radius of the throat, $\text{Vol}(X_5)$ is the dimensionless volume of the base space X_5, and $N(>1)$ is the 5-form charge.

The conformally coupled inflaton satisfies $\dot{\phi} \approx -\phi H$ during inflation, hence the number of e-foldings generated is $\mathcal{N} \approx \log(\rho_i/\rho_f) \approx -\log(h_0/\lambda_i)$, where we have set the initial position of the D3 by a dimensionless constant λ_i as $\rho_i = \lambda_i R$, and assumed inflation to end when the D3 approaches the tip $\rho_f \approx \rho_0$. This shows that in order to obtain enough e-foldings, the throat should be strongly warped $h_0/\lambda_i \ll 1$.

The scale dependence of the perturbations can be computed by (7) and (8). We parametrize the position of the D3 when the CMB scale was originally produced by $\rho_{\text{CMB}} = \lambda R$. Furthermore, for simplicity, we ignore the compactified bulk to which the throat is glued, hence $M_p^2 \simeq \frac{2}{(2\pi)^7 g_s^2(\alpha')^4}\int_{\rho_0}^R d\rho\, \text{Vol}(X_5)\frac{\rho^5}{h^4} \simeq \frac{\text{Vol}(X_5)R^6}{(2\pi)^7 g_s^2(\alpha')^4}$. Then under the assumption $h_0/\lambda \ll 1$, the flatness parameters and κ can be calculated (note that $c = 7$)

$$\epsilon \simeq \frac{2h_0^8}{\lambda^{10}N}, \quad \tilde{\epsilon} \simeq \frac{2h_0^4}{\lambda^4 N}, \quad \eta_c = \eta \simeq -\frac{5h_0^4}{\lambda^6}, \quad \kappa^2 \simeq \frac{4\lambda^2}{N}. \tag{10}$$

Since ϵ is extremely small compared to the other flatness parameters, the cosmological observables can be estimated as follows:

$$n_s - 1 \simeq -\frac{4}{N}\left\{\frac{h_0^4}{\lambda^4} - 2\lambda^2\left(\xi - \frac{1}{6}\right)\right\}, \qquad \frac{dn_s}{d\ln k} \simeq -\frac{16}{N}\left\{\frac{h_0^4}{\lambda^4} + \lambda^2\left(\xi - \frac{1}{6}\right)\right\}. \tag{11}$$

The 5-year WMAP+BAO+SN data gives bounds $n_s = 1.022^{+0.043}_{-0.042}$ (68% CL) and $dn_s/d\ln k = -0.032^{+0.021}_{-0.020}$ (68% CL) when tensor mode perturbations are negligible [6]. Since the h_0^4/λ^4 terms are too small to be constrained by the observational bounds, we ignore them. Then the observables are determined only by the inflaton's deviation from an exact conformal coupling,

$$n_s - 1 \simeq 8\frac{\lambda^2}{N}\left(\xi - \frac{1}{6}\right), \qquad \frac{dn_s}{d\ln k} \simeq -16\frac{\lambda^2}{N}\left(\xi - \frac{1}{6}\right). \tag{12}$$

It is easy to see that the spectral index and its running are related by

$$\frac{dn_s}{d\ln k} \simeq -2(n_s - 1). \tag{13}$$

Using the observational bound on $dn_s/d\ln k$ to constrain $\lambda^2(\xi-1/6)/N$,

$$0.001 \lesssim \frac{\lambda^2}{N}\left(\xi - \frac{1}{6}\right) \lesssim 0.003. \tag{14}$$

If we further make use of this bound to constrain the spectral index, we obtain

$$0.006 \lesssim n_s - 1 \lesssim 0.026. \tag{15}$$

Thus the 1σ observational bound on the running allows us to predict a blue tilt for the warped brane inflation model. Moreover, the values of the observables were dominantly determined by the inflaton's deviation from a conformal coupling. The bound (14) suggests that this deviation can be fairly large, e.g., $(\xi-1/6) \sim 10^{-1}$ when $\lambda \sim 1$, $N \sim 10^2$.

5 Conclusion

We have investigated the scale dependence of the density perturbations that are generated through modulated reheating after conformal inflation. We have written down the spectral index and its running in terms of the flatness parameters and the inflaton's deviation from an exact conformal coupling. The general results we have obtained are that (i) modulated reheating together with conformal inflation can produce a nearly scale-invariant spectrum, (ii) the running of the spectral index $|dn_s/d\ln k|$ turns out to be as large as $|n_s - 1|$. The latter result reflects the nonexistence of hierarchy among higher-order time derivatives of the Hubble parameter $d^n \ln H^2/dt^n H^n$ during conformal inflation. This is in strong contrast to standard minimal models where higher-order derivatives are suppressed by higher orders of the slow-roll parameters (and their derivatives). Hence this feature offers a chance of obtaining a smoking-gun signal for non-minimally coupled inflation models.

We also applied our results to the warped brane inflation model, where it was shown that observables were dominantly determined by the deviation from the conformal coupling. We have shown that the spectral index and its running are related by (13) for this model. Since the running is highly constrained by the WMAP5 data, a stringent bound on the coupling of the inflaton to gravity was obtained. Also, comparison with the WMAP5 data suggested a blue tilt of the spectrum.

While our analysis focused on general aspects of modulated reheating after conformal inflation, we have not presented the light modulus responsible for generating fluctuations in a concrete setup. In the warped brane inflation case, potential candidates for such light fields are angular positions of the D3($\overline{\text{D3}}$)-branes sitting in throats with angular isometries, and/or axions associated with shift symmetries of the Kähler potential. For further study, it is important to come up with an explicit realization of our mechanism based on fundamental theories. We leave this for future work.

One of the general lessons of our work is that a non-minimal coupling with gravity can drastically change the behavior of inflation. Large values for higher-order time derivatives of the Hubble parameter is a special nature of conformal inflation. Cosmological observations are imposing (not necessarily direct but) important constraints on such features even at the present stage.

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