# THEORETICAL SUMMARY

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## 1. Introduction

The excellent plenary talks presented at this conference have given an overview of the current situation in particle theory. In my opinion, two features stand out: while there is a healthy supply of competing ideas for the theory of the future, experimental information about the world "beyond the standard model" is at best ambiguous and perhaps non-existent. The anomalous collider results may be what we are looking for, but only time will tell.

The current state of affairs does not lend itself to a fifty minute summary and it will be impossible for me simply to summarize what has been said here. It does seem to me, however, that a number of interesting topics have not been touched on and I would like to devote my talk to a discussion of neglected but important ideas.

## 2. Search for Hidden Scales

Theory "predicts" new physics a a variety of high mass scales:

1. Post-Weak $10^{+3}$ Gev
2. Axion Scale $10^{+11}$ Gev
3. GUT Scale $10^{+14}$ Gev
4. Planck Mass $10^{+19}$ Gev

Finding experimental evidence for any of these is our most urgent task. So far (with the possible exception of the post-weak scale) no such evidence has appeared. New ideas on how to search for these deeply hidden scales would be most welcome.

There is an interesting theoretical development which may have some bearing on the search for the lowest of these new scales. The recently discovered triviality of pure $\phi^4$ theory[1] means that the Higgs sector of electroweak theory must have substructure at scale

$$\Lambda = 1 \text{ Tev}*\exp(1 \text{ Tev}/\text{Mhiggs})$$

In other words, the more massive the physical Higgs, the more accessible the structure underlying electroweak symmetry breaking.[2] If the Higgs is not found below 1 Tev, then

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some composite structure must be found at about the same mass. Bad news for the Higgs searchers is good news for the builders of supercolliders. Of course, if a Higgs were found at the more conventional mass of 10 Tev or so, the same theorem says that the Higgs field will look like a fundamental field all the way down to the Planck length!

Laboratory tests sensitive to the axion mass scale and beyond are not easy to come by. The cleanest and most familiar example is the proton decay experiment, which has so far yielded only negative results. I would like to bring to your attention a new proposal for detecting axion scale physics which provides a perfect example of the sort of test we theorists should, in my opinion, be looking for.

I assume that everyone is familiar with the basic features of the invisible axion.[3] It is a scalar field associated with a symmetry (used to eliminate strong CP-violation in QCD) which is spontaneously broken at an energy scale, $f_a$, expected to be of order $10^{+11}$ Gev. The axion mass and couplings to ordinary matter are given in terms of $f_a$ by

$$m_a^2 = m_\pi^2 \frac{f_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u+m_d)^2} \tag{1}$$

$$L_{eff} = \frac{a}{f_a} \frac{m_u m_d}{(m_u+m_d)} \{i(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) + \theta(\bar{u}u+dd)\} \tag{2}$$

where a is the axion field, $f_\pi$ is the pion decay constant, $m_u$ $m_d$ are the light quark masses and $\theta$ is a parameter determined by explicity CP violation in the weak interactions. Because of the theta term, there is a T-violating static axion-exchange potential between two nucleons,[4] one of them polarized with spin $\vec{s}$, of the following form:

$$V(r) = \text{const} \times \frac{\theta}{m_a r^2} (\hat{r}\cdot\vec{s}) (1+m_a r) e^{-m_a r} \tag{3}$$

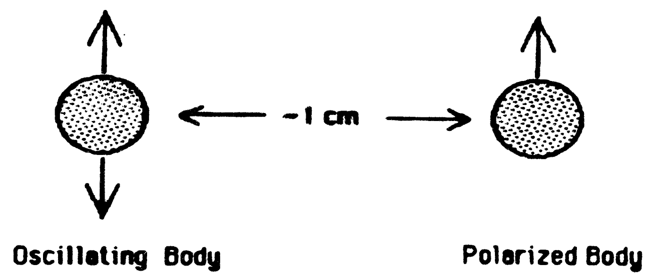


Figure 1: Schematic version of an axion Cavendish experiment.

The range of this potential is expected to be large enough ($1/m_a \sim 1$ cm) for macroscopic quantities of matter to act coherently on each other as described schematically in Fig. 1. The accelerations in this "Cavendish" experiment are small, but potentially detectable, especially since the T-violating nature of the force allows the cancellation of many noise backgrounds.

## 3. Cosmology

One way to explore super-high mass scales is to make use of Nature's own super-accelerator, the Big Bang. Since the Big Bang presumably excites all scales, however briefly, astrophysical searches for relics of high mass scales are possible. There are a number of exciting issues in this field, such as the role of inflation, the origin of galactic density perturbations, and the identity of dark matter. These have been brilliantly discussed in the plenary talk on cosmology, and I would like instead to emphasize the importance of non-standard approaches to the problem of detecting cosmological relics. A good example is the use of monopole catalysis and the existence of neutron stars to place stringent limits on the flux of magnetic monopoles in the universe.[5] Since I plan to return to the catalysis question later on, and since the observational results are so far all negative, I will not say anything more about this subject now.

Instead, I would like to bring to your attention an interesting proposal for detecting the expected cosmological flux of axions.[6] The basic point is that the axion field and the electromagnetic field have a simple coupling described by the Lagrangian

$$L(\gamma,a) = \frac{1}{2}\left(\bar{E}^2-\bar{B}^2\right) + \frac{2}{f_a}\,\bar{E}\cdot\bar{B} + L_o(a) \tag{4}$$

where a, $f_a$ and $m_a$ are as defined before, E and B are the electromagnetic field strengths and $L_o$ is the free axion Lagrangian. This Lagrangian leads to the following modified Maxwell equations:

$$\bar{\nabla}\cdot\bar{B} = 0 \qquad\qquad \nabla\times\bar{E} + \frac{1}{c}\,\dot{\bar{B}} = 0$$

$$\bar{\nabla}\cdot\bar{E} = -\bar{B}\cdot\bar{\nabla}\left(\frac{a}{f_a}\right) \qquad\qquad \nabla\times\bar{B} - \frac{1}{c}\,\dot{\bar{E}} = \frac{\dot{a}}{f_a}\,\bar{B} - \bar{E}\times\frac{\bar{\nabla}a}{f_a} \tag{5}$$

They imply that, in the presence of static electric or magnetic fields, a fluctuating axion field generates fluctuating electromagnetic currents and charge densities. These will, in turn, via Maxwell's equations, generate fluctuating electric fields which one might hope to detect.

The scenario for electromagnetic detection of the cosmic axion flux is presented schematically in Fig. 2: one looks for small voltage fluctuations across condenser plates immersed in a static magnetic field. the signal scales with $a/f_a$, where a is the rms amplitude of the axion field. The requirement that the total energy in axions be less that the closure density of the universe[7] restricts this parameter to be less than $10^{-21}$! This is small indeed, but apparently not small enough to rule out a detection scheme of this kind.

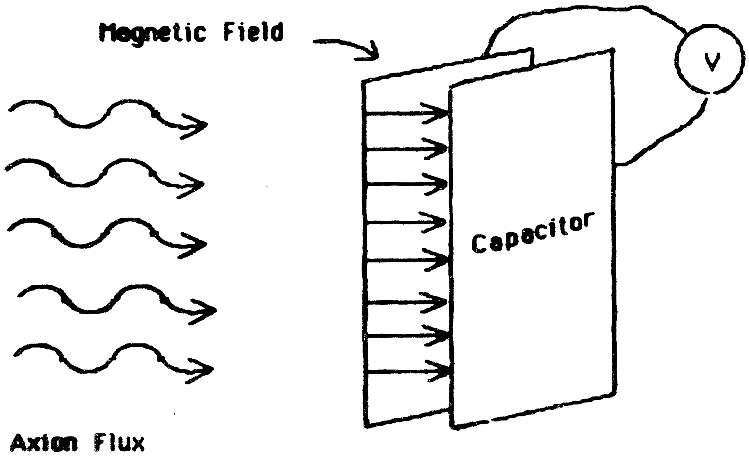


Figure 2: Schematic version of a cosmological axion flux detector.

4. Progress in Field Theory

There have been some interesting advances in our understanding of quantum field theory and I would like to single out two which seem to me to be of special interest: rigorous results for phenomenologically relevant theories and advances in the understanding of θ-vacuum physics of QCD.

The most spectacular rigorous result is the proof of the triviality of scalar field theory in four dimensions: If $\phi^4$ field theory is constructed with a cutoff $\Lambda$ and the renormalized coupling $\lambda_R$ is defined at a physical scale $\mu$, then it can be shown that

$$\lambda_R < \frac{\text{const}}{\ell n(\Lambda/\mu)} \tag{6}$$

In the limit of arbitrarily high cutoff scale, the renormalized coupling goes to zero, indicating that the continuum limit of scalar field theory is necessarily a free theory. Because of the role of the Higgs scalar field in electroweak physics, this result has important consequences which we have already discussed.

There are a number of interesting new results in lattice gauge theory: It has been proven that there is no deconfining phase transition in pure SU2 lattice gauge theory.[8] In the same theory, it has been proven that there is a temperature above which confinement is undone.[9] For a lattice gauge theory with fermions it has been proven that there is a temperature above which chiral symmetry is unbroken.[10] These results confirm the conventional wisdom, and are therefore very important but as yet (unlike the previously cited theorem about scalar field theory) say nothing directly about the continuum limit: all estimates of dimensional quantities such as phase transition temperatures are expressed in terms of the cutoff scale rather than some physical renormalization scale. If a way can be found to remedy this, rigorous results will be on the verge of having direct phenomenological usefulness.

A very interesting set of results has been obtained by exploiting the positivity of the functional measure. The path integral for QCD with F flavors of Dirac fermions is obtained by integrating over the measure

$$d\mu = [DA_\mu] \exp[-\frac{1}{g^2} \int d^4x F^2] [\det(D_A+m)]^F \tag{7}$$

The fermion determinant can be written as a product over the eigenvalues of the Dirac operator

$$\det(D_A) = \prod_n (i\lambda_n+m)(-i\lambda_n+m) > 0 \tag{8}$$

Because the eigenvalues come in real pairs of opposite sign, the fermion determinant is manifestly positive. Consequently, the whole measure is positive, and from this fact some important consequences can be shown to follow: There is no spontaneous breaking of global vector symmetries like isospin;[11] spontaneous breaking of chiral symmetry must elect the Goldstone mode.[12] Although these results confirm conventional wisdom, the ability to prove them is a major step forward.

There has been major progress in understanding the $\theta$-vacuum of QCD. Theta is a hidden parameter of QCD, not visible in perturbation theory.[13] Although its physical value is very small, our ability to predict what happens for arbitrary values of $\theta$ is a test of our mastery of the crucial nonperturbative aspects of the theory. The Euclidean partition function of QCD, including the effect of $\theta$, is written as

$$Z(\theta) = \int [DA_\mu] \exp \int d^4x \, [-\frac{1}{g^2} \mathrm{tr}(F\cdot F) + \frac{i\theta}{32\pi^2} \mathrm{tr}(F\tilde{F})] \tag{9}$$

The path integral decomposes into a sum over discrete sectors with integer topological charge

$$Z(\theta) = \sum_n e^{in\theta} \int [DA_\mu]_n \, e^{-1/g^2 \int d^4x \mathrm{tr}(F^2)} \tag{10}$$

where topological charge is defined by

$$\text{Topological charge} = \frac{1}{32\pi^2} \int d^4x \, \mathrm{tr}(F\tilde{F}) = n \tag{11}$$

and takes on integer values.

In the semiclassical approximation the sectors with different n are built by simple superposition out of the basic instanton (n=1) and anti-instanton (n=-1) solutions. The result for the partition function itself is

$$Z(\theta) = \exp\,[-VTE(\theta)] \tag{12}$$

where VT is the space-time volume and

$$E(\theta) = -\cos\theta\ \Big[\int_{o} \frac{d\lambda}{\lambda^5}\ \Big(\frac{8\pi^2}{g^2(\lambda)}\Big)^P\ e^{-8\pi^2/g^2(\lambda)}\Big] \tag{13}$$

where $\lambda$ is the instanton scale size and $g(\lambda)$ is the corresponding value of the running coupling. The key result is the simple cosine dependence on theta which is the reflection of a specific nonperturbative structure. All the physical properties of the theory will depend on $\theta$ and in this approximation one finds that correlation lengths tend to increase with $\theta$.[14]

The only way to determine the actual $\theta$ dependence of QCD is to perform a lattice Monte Carlo calculation. That would require us to calculate

$$Z(\theta) = \int \prod_{\ell} DU_{\ell}\ \exp\,\Big[-\frac{1}{g^2}\sum \mathrm{tr}\ \big(\prod_{p} U_{\ell}\big) + i\theta\ Q(\{U_{\ell}\})\Big] \tag{14}$$

where the U's are the lattice links and $Q(\{U\})$ is the topological charge assigned to the particular lattice configuration. Since topology is not a natural lattice concept, the whole trick lies in choosing Q.[15] It must have the right continuum limit, should be invariant to small changes in the link variables (in order to reflect the key fact that topological effects are completely absent in perturbation theory), and should be easy to compute.

A good such definition has recently been found[16] and the appropriate lattice Monte Carlo calculations done.[17] The essential results can be summarized by saying that all the predictions of the semiclassical model (most notably that $E(\theta) \sim \cos\theta$) are verified. This suggests that the basic physics of the instanton semiclassical model is correct and raises the hope that one might use lattice input to normalize the semi-analytic instanton model in order to get reliable quantitative predictions from it.

5. Anomalies

It has been known for a long time that the gauge or global symmetries of a field theory may be smaller than those of the Lagrangian from which it is derived.[18] Such a circumstance is known as an anomaly and arises from nonperturbative aspects of quantum physics. This is an ancient subject, but one in which a number of exciting new developments have recently occurred. I will concentrate on two of them: the revival of chiral

Lagrangians as an effective theory of hadrons and the discovery of a new class of gravity anomalies.

To appreciate the first of these developments, it is necessary to review some of the properties of old-fashioned chiral Lagrangians. They are constructed to summarize the soft-pion theorems of spontaneously broken chiral SU(2)×SU(2) flavor symmetry. A typical such Lagrangian is

$$L = \frac{f_\pi^{\,2}}{16}\ \mathrm{tr}\ \left(\partial_\mu U^+ \partial^\mu U\right) + \frac{1}{32a^2}\ \mathrm{tr}\ \left[U^+\partial_\mu U,\ U^+\partial^\nu U\right]^2 \tag{15}$$

where U(x) is a field taking on values in SU(2) which describes the Goldstone boson degrees of freedom, $f_\pi$ is the pion decay constant, and a is a dimensionless coupling constant. The first term is obligatory and when expanded in terms of the usual pion field according to

$$U = \exp i(\vec{\pi}\cdot\vec{\tau}/f_\mu) \tag{16}$$

gives all the standard low-energy theorems for multipion vertices. The second term is the simplest example of possible higher-dimension vertices which could be used to summarize corrections to the low-energy theorems.

This Lagrangian has an identically conserved current

$$B_\mu = \frac{1}{24\pi^2}\ \varepsilon_{\mu\nu\alpha\beta}\ \mathrm{tr}\ \left(U^{-1}\partial^\nu U\partial^\alpha U^{-1}\partial^\beta U\right) \tag{17}$$

whose conserved charge, B, is just the winding number of the mapping from three-dimensional physical space into the SU(2) group and which therefore takes on only integer values. The standard vacuum state, U(x)=1, is simply the lowest-energy configuration in the B=0 sector. The lowest energy configuration in the B=1 sector, on the other hand, turns out to be a localized, soliton-like configuration, referred to as a "Skyrmion" in honor of its inventor.[19] If the soliton is localized at $\bar{x}=0$, its explicit form is

$$U_{sk}(\vec{x}) = \exp i f(r)\hat{x}\cdot\bar{\tau} \tag{18}$$

where f(r) runs from $\pi$ to $\theta$ as r runs from zero to infinity, and $\hat{x}$ is the unit vector pointing away from the origin. The detailed form of f(r) is to be determined by minimizing the energy. This soliton has several collective coordinates: Its central position may of course be changed without changing its energy; its orientation within the group may also be changed according to

$$U_{sk}(\bar{x}) \rightarrow A\ U_{sk}(\bar{x})\ A^{-1} \tag{19}$$

without affecting the energy. A long-standing question was the physical meaning to attach to this soliton as well as to the excitations arising from rotations of its internal collective coordinate.

Skyrme suggested a long time ago that this soliton should be identified with the nucleon, although it was totally mysterious how a fermion could be built out of a bose field or why such a soliton should have non-zero baryon number. Recent developments show that he was exactly right.[20,21] The key point is that, although the preceding Lagrangian summarizes all standard soft-pion theorems which follow from spontaneous breaking of the flavor symmetry, it does not incorporate the effects of anomalous breaking of those symmetries.

For pions alone, anomalies show up in matrix elements involving vector currents (they determine the $\pi^0 \to 2\gamma$ decay rate), while if the theory is extended to include kaons, anomalies directly affect meson interactions. The baryon number current is a vector current and the anomaly in its matrix elements with pions turns out to mean that the topological current discussed above must be identified with the baryon number current. Thus, the soliton with B=1 does have the right baryon number to be identified with the nucleon. The anomalies in meson self-interactions have to be accounted for by an extra term in the Lagrangian known as the Wess-Zumino term.[22] The details of its construction are rather subtle and have been discussed in a previous talk, so I will not repeat them here. Suffice it to say that this term appends a phase, which may be either +1 or −1, to the path integral for a $2\pi$ rotation of the internal rotator collective coordinate, and that this phase can cause the rotator to be quantized with either integral or half-integral spin depending on the underlying gauge theory. For an underlying SU(3) color theory, the quantization of the rotator leads to a sequence of states with spin equal to isospin and spin taking on the values 1/2, 3/2, etc.[23] This is of course precisely the observed set of low-lying hadron states!

All the static properties of the nucleon and delta can be computed in this theory and a quantitatively reasonable one-parameter fit to observations is obtained.[24] With a modest degree of hyperbole one can say that this model allows one to obtain all the results of the quark and bag models without explicit quarks! A possible virtue of this approach (as yet unexploited) is its ability to attack time-dependent questions such as those concerning scattering cross-sections.

The second matter I wish to mention is the discovery of a new class of gravity anomalies which has implications for attempts to build supergravity or string theories.[25] The basic result is that theories of chiral fermions in 2, 6, 10, ... dimensions do not conserve energy-momentum! This is a potential disaster for higher-dimension theories with realistic fermion content. A remarkable corollary of this result is that in a very small subclass of such theories (favored by builders of superstring theories) this anomaly cancels between fields of different spin. It is possible to give a fairly physical argument for the occurrence of this anomaly by making use of the properties of axions and axion strings.[26] Due to lack of time, I will not give the argument here and refer the interested reader to the appropriate references.

## 6. Monopole Catalysis

The phenomenon of monopole catalysis of baryon decay ties together the foregoing topics in a remarkable way: On the one hand, it is a phenomenon raising new possibilities for experimental exploration of physics at the GUT scale; on the other, its theoretical basis lies in very non-trivial non-perturbative aspects of quantum field theory. It also provides a surprising example of a one-dimensional (and therefore nearly soluble) field theory playing a direct role in real physics.[27] The theoretical arguments for monopole catalysis have been hotly debated for the last two years and it now seems quite clear that the phenomenon happens as originally advertised: in a theory like SU(5) with baryon-number non-conservation built into the Lagrangian, the cross-section for

$$\text{proton} + \text{monopole} \rightarrow \text{monopole} + \text{positron} + \text{pion}$$

is determined by the size of the proton and not the size of the monopole core region, and is therefore of strong interaction magnitude. This effect would have a number of fascinating applications, some alrady mentioned, if only Nature would supply us with a few monopoles! Although the situation is not wildly promising, the experimentalists are busy looking, and we theorists can only sit and wait.

I would like to discuss two theoretical issues which have yet to be satisfactorily dealt with. The first is the question of how to obtain serious quantitative estimates of catalysis cross-sections and branching ratios. The second is the question whether, if baryon number is strictly conserved in the Lagrangian, catalysis can still occur at an interesting rate via the baryon number anomaly.

A fascinating application of the Skyrme soliton model of hadrons is to provide a quantitative framework for discussing monopole catalysis.[28] Consider the schematic description in Fig. 3 of what happens when a soliton collides with a monopole. In this figure, the blob is just the nucleon soliton described in the previous section, and the wiggly line attached to the point monopole stands for the monopole's string singularity: In spherical coordinates, in a standard gauge, the Dirac monopole's vector potential is

$$A_\phi = \frac{(1-\cos\theta)}{2r\sin\theta} \tag{20}$$

This is singular at $\theta=\pi$, i.e. along the negative z-axis. This "string" singularity is unphysical and may be moved about by a gauge transformation. In particular, the gauge transformation generated by $\Lambda=\phi$ changes the previous vector potential into

$$A_\phi = \frac{(1+\cos\theta)}{2r\sin\theta} \tag{21}$$

a form in which the singularity lies along the positive z-axis.

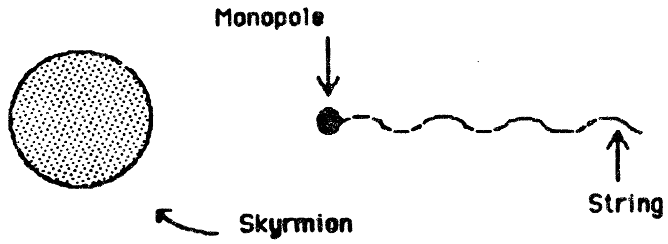


Figure 3: A collision between a monopole and a Skyrmion.

In the collision described in Fig. 3, the Skyrmion after the collision is speared by the string. In order to interpret this situation, it is best to carry out the gauge transformation which flips the string to the positive z-axis, thus removing it from the soliton (as in Fig. 4). But, since the soliton is made of electricallly charged fields, it is affected by this transformation, and it is by no means guaranteed that the new configuration still has baryon number one. A small calculation shows that it actually has baryon number zero and is therefore really an unbound collection of pions which will eventually disperse.

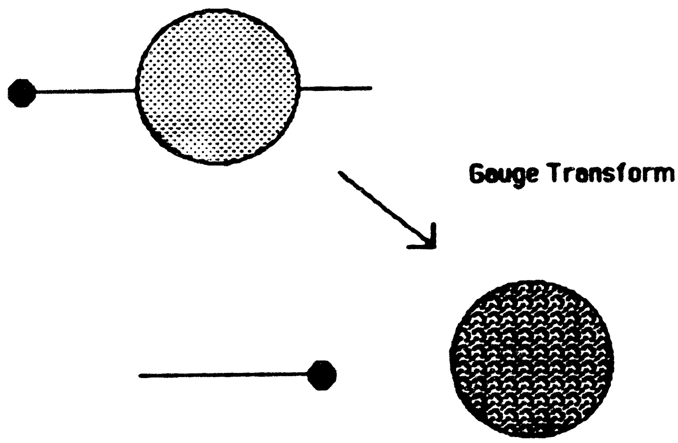


Figure 4: Removing the monopole string from the interior of the Skyrmion by a gauge transformation.

Non-head-on collisions are not so easy to analyse and it is reasonable to suppose that for small impact parameters the nucleon soliton will emerge intact, while for large ones it will be converted into a zero-baryon number collection of pions. It is apparent that a complete picture of monopole catalysed proton decay can be obtained by solving the Skyrme soliton equations of motion in the presence of a monopole vector potential for all possible impact parameters. This procedure will give a detailed, and probably quantitatively quite reasonable, picture of the final state in monopole catalysed proton decay. As far as I know, the chiral model is the only reasonable approximation to QCD capable of dealing with this problem.

Next I would like to discuss whether monopole catalysis at strong interaction rates can be caused by anomalous rather than explicit baryon number non-conservation. The point is that since explicit baryon number violation is an optional feature of grand unified theories, while the anomaly mechanism is always present, it is of interest to know how big catalysis cross-sections will be if they are due only to the anomaly. This is a subject about which there has been some confusion and controversy.

A study of the kinematics of the anomaly shows that the simultaneous presence of the ordinary magnetic field of the monopole and a weak electric field (i.e. the electric field carried by $Z^0$ boson) will lead to non-conservation of baryon number.[29] The explicit expression is

$$\dot{Q}_B = \frac{e^2}{16\pi^2} \int d^3x \; \vec{E}_{Z^0} \cdot \vec{B}_{\text{MONOPOLE}} \tag{22}$$

The magnetic field is the static background field of the monopole and is always present. The weak electric field, on the other hand, is present only if $Z^0$ electric charge sources are present. This charge can be carried either by external quarks and leptons scattering from the monopole or by the dyon degree of freedom associated with the monopole core.

Not all monopoles have a dyon degree of freedom capable of carrying $Z^0$ charge. In the examples I have been able to work through, the following rule holds: If the underlying GUT has explicit baryon number violation (as in standard SU(5)), then the dyon carries no $Z^0$ charge, while if the underlying GUT conserves baryon number (as in the $SU(4)\times SU(2)\times SU(2)$ model), the dyon does carry $Z^0$ charge. This is important because the baryon number anomaly effect is in general suppressed by the fact that the $Z^0$ electric field is screened with a screening length of order $1/\mu_{Z^0}$. Unless the $Z^0$ charge can be assembled on a scale smaller than that screening length, the net baryon number change in a typical scattering process will be suppressed by factors of $1/\mu_{Z^0}$. Roughly speaking,

$$\Delta B_{\text{SCATT}} \sim e^2 \left(\frac{E}{M_W}\right)^P \tag{23}$$

(where E is the scattering energy), indicating that the catalysis cross-section in this case is of weak interaction magnitude.[30] This analysis applies to the case where the $Z^0$ charge is carried only by external particles since their charge can only be concentrated on distance scales comparable to their deBroglie wavelength. If the dyon can carry $Z^0$

charge, the above argument does not apply because the dyon is an essentially point-like degree of freedom. In that case the typical baryon number change in a scattering event, due entirely to the anomaly, could be of order one (i.e. the catalysis cross-section would be strong).[31]

Putting this all together we get the following picture: in SU(5)-like theories, we get strong baryon number catalysis directly from the Lagrangian and the anomaly effect is swamped. In SU(4)×SU(2)×SU(2)-like theories there is no direct baryon number catalysis, but the indirect anomaly effect can be strong. Actually, if there are several generations of fermions, the fact that only the baryon number summed over generations is anomalous means that there must be many powers of intergeneration mixing angles in the anomaly-driven catalysis cross-section. In practice this will make the cross section weak again. In realistic theories, strong catalysis seems to occur only if baryon number is broken at the Lagrangian level. Otherwise catalysis will be an effect of weak interaction magnitude.

## 7. Higher Dimensions

The theoretical background of all the developments discussed in previous sections is "old-fashioned" renormalizable gauge field theory. Much gratifying progress has been made, but there are good reasons to believe that it will eventually be necessary to go beyond this framework. The closeness of the GUT scale to the Planck mass (along with several other things) hints that it may be necessary to include gravity in our schemes before we can achieve a complete understanding of all the mysteries of particle physics. This is disturbing because quantum gravity is a non-renormalizable theory, because we don't know how to compute with it and because we don't know that it is a consistent theory if taken by itself. The Planck scale is a long way away from any scale of which we have direct experimental experience, so it may well be premature to worry about such matters. Nonetheless, brave theorists have been speculating about ways to unify gravity with the other forces and some promising and beautiful ideas have been produced. Although their connection with experiment is as yet quite murky, I will briefly discuss two of the approaches I find attractive: Kaluza-Klein theory and superstring theory.

The Kaluza-Klein approach is an old idea adopted to modern circumstances.[32] One considers general relativity in D>4 dimensions and imagines that the higher-dimensional space decomposes into the product of four-dimensional Minkowski space and a compact (D-4)-dimensional space (perhaps a sphere) as in

$$M_D \to M_4 \times S_{D-4} \tag{24}$$

The components of the metric tensor referring to the collapsed space play the role of scalar and vector fields from the point of view of four-dimensional space:

$$g_{AB} = \left[\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\mu & \phi \end{array}\right] \tag{25}$$

The vector fields behave like non-abelian gauge fields and if the collapsed space is symmetric enough to have an isometry group G, the four dimensional theory looks like a gauge theory based on G. The net result is the unification of gauge theory and gravitation as well as the derivation of the gauge theory from the dynamics of geometry.

This approach to unification has many problems which require resolution. In the first place, general relativity is non-renormalizable in D=4 and is even less so in D>4! This is a serious embarrassment, to say the least. There is the question of the dynamics of the collapsed space: the classical equations by themselves do not give a stable solution. It has been suggested that the interplay of classical curvature and the Casimir effect (quantum corrections) due to matter fields would give a satisfacatory solution for the collapsed space.[33] It is not yet know whether this scheme works.

The most serious problem from the phenomenological standpoint has to do with the need to have chiral fermions. The Dirac operator has derivatives in the collapsed space as well as physical space:

$$[\gamma^\mu D_\mu + \gamma^a D_a]\,\psi = 0 \tag{26}$$

The typical eigenvalue of the collapsed space part of the Dirac operator will be the inverse of the collapsed space radius, a quantity typically on the order of the Planck mass. Since these eigenvalues play the role of effective four-dimensional masses for the fermions, the collapsed space Dirac operator must have zero eigenvalues in order to permit the existence of physical light fermions. The problem is that these zero eigenvalues always appear in groups corresponding to vector fermions (equal number of left- and right-handed fields) while chiral fermions are needed to describe the real world.[34] This problem can be solved if there exist extra fundamental gauge fields (not arising from geometry) which can take on monopole-like configurations in the collapsed space, but this to some extent contradicts the original motivation for the Kaluza-Klein approach. Kaluza-Klein theory almost certainly has a great deal still to teach us and is currently vigorously being explored.

Superstring theory is virtually impossible to summarize in a few paragraphs, but it has some remarkable features that I cannot resist mentioning. The basic dynamical element is not a point particle tracing out a world line but rather a "string" tracing out a world surface. In lowest approximation, one is attempting to describe the dynamics not of single species of particle but rather the dynamics of whole Regge trajectories of particles (see Fig. 5). The slope, $\sigma$, of the trajectories and Newton's constant, G, are the independent constants of the theory. Their product is a number of order one and consistent quantization may in fact require that product to have a definite value! Because of this relation, the spacing between particles on the trajectories is of order the Planck mass, an indication that the essential dynamics is occurring at the Planck scale. Low-energy physics will be dominated by the zero-mass particles on the various trajectories, and keeping only those degrees of freedom yields an effective field theory of the ordinary kind which should describe physics at energies below the Planck mass.

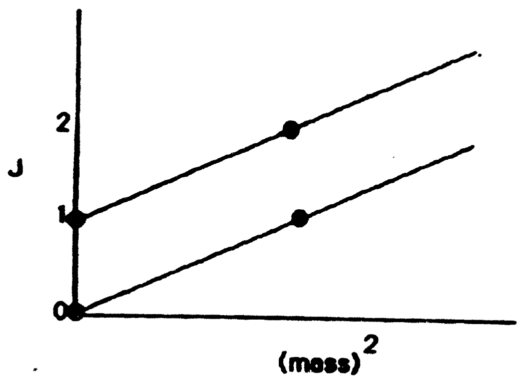


Figure 5: Typical mass vs. angular momentum spectrum of a string theory.

It has been difficult to find a string theory which is not obviously afflicted with unpleasant diseases. By insisting on the right kind of supersymmetry, a small set of string models has recently been found which does not have any obvious inconsistencies and which has a number of virtues.[35] Perhaps the greatest virtue is that at least one of these models, despite the fact that it incorporates gravity, may be not only renormalizable, but finite! The low-energy limit of this string theory turns out to be the special D=10 supergravity theory which manifests the miraculous anomaly cancellations mentioned in a previous section. From several points of view these string theories seem to be quite special and there is now some hope that they will provide us with the clues we need to make fundamental progress.

A few cautionary remarks are in order: No Lagrangian has yet been written down for these theories and there is no glimmer of an understanding of the non-perturbative effects which have proven so crucial to an understanding of gauge theories. Perhaps even more important, there is as yet no clear notion of how these theories relate to the known phenomenology of quarks and leptons. The believers feel that these theories are so beautiful that they must be true and that, if we are patient, all these mysteries will eventually be cleared up.

8. Conclusions

I stated at the beginning of this talk that I would not attempt a global review of particle theory, but would instead concentrate on important topics which had been "left out" of the plenary presentations. If we add to what I have said the subjects discussed earlier in these sessions, it seems to me that three conclusions can be drawn about the health of our subject: First, there has been steady progress in the development of tools to deal with conventional field theory. We have increasing command of the non-perturbative aspects of field theory and are making steady progress toward the goal of making reliable calculations. Second, there is currently under active discussion a healthy number of promising ideas for going beyond standard quantum field theory. Third, more than anything else, we theorists need some facts concerning the deeply hidden scales of particle physics in order to concentrate our attention and guide our thinking. We need to invent more unconventional, non-accelerator methods of digging out evidence of these scales. There are good new ideas emerging, and I urge the experimentalists to seriously consider them.

Needless to say, we hope that the exciting experimental hints of new structure beyond the standard model which have been presented at this conference will prove to be just the tip of the iceberg and that two years from now we will be able to present a convincing picture of the particle physics of the future.

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