α_s from scaling violations of hard parton-to-hadron fragmentation functions

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Abstract: We assess the prospects of a high-precision determination of the strong-coupling constant α_s from the scaling violations in fragmentation functions at the CERN Future Circular Collider operated in the e^+e^- annihilation mode (FCC-ee).

Introduction

The strong force acting between hadrons is one of the four fundamental forces of nature. It is now commonly believed that the strong interactions are correctly described by quantum chromodynamics (QCD), the SU(3) gauge field theory which contains colored quarks and gluons as elementary particles. The strong coupling constant $\alpha_s^{(n_f)}(\mu^2) = g_s^2/(4\pi)$, where g_s is the QCD gauge coupling, is a basic parameter of the standard model of elementary particle physics; its value $\alpha_s^{(5)}(M_Z^2)$ at the Z-boson mass scale is listed among the constants of nature in the Review of Particle Physics [1]. Here, μ is the renormalization scale, and n_f the number of active quark flavors, with mass $m_q \ll \mu$. In the modified minimal-subtraction ($\overline{\text{MS}}$) scheme, the evolution of $\alpha_s^{(n_f)}(\mu^2)$ is known through four loops [2] and its matching at the flavor thresholds through three [3] and even four loops [4].

There are a number of processes in which $\alpha_s^{(5)}(M_Z^2)$ can be measured (see Ref. [1] and these workshop proceedings, for recent reviews). A reliable method to determine $\alpha_s^{(5)}(M_Z^2)$ is through the extraction of the fragmentation functions (FFs) [5] in the e^+e^- annihilation process

$$e^+e^- \to (\gamma, Z) \to h + X, \tag{1}$$

which describes the inclusive production of a single hadron h. Here, h may either refer to a specific hadron species, such as $\pi^\pm$, π^0, η, $K^\pm$, K_S^0, $p/\overline{p}$, $\Lambda/\overline{\Lambda}$, or to the sum of all charged hadrons. In the parton model of QCD, the cross section of process (1), differential in the scaled hadron energy $x = 2E_h/\sqrt{s}$, may be evaluated, up to power corrections, as

$$\frac{d\sigma^h}{dx}(x) = \sum_a \int_x^1 \frac{dy}{y} \frac{d\sigma_a}{dy}(y, \mu^2, Q^2) D_a^h\left(\frac{x}{y}, Q^2\right), \tag{2}$$

where a runs over the gluon and the quarks and antiquarks active at energy scale $\sqrt{Q^2}$. The partonic cross sections $d\sigma_a(x, \mu^2, Q^2)/dx$ pertinent to process (1) can entirely be calculated in perturbative QCD with no additional input, except for $\alpha_s(\mu^2)$. They are known at next-to-leading order (NLO) [6] and even at next-to-next-to-leading order (NNLO) [7]. The subsequent transition of the partons into hadrons takes place at an energy scale of the order of 1 GeV and can, therefore, not be treated in perturbation theory. Instead, the hadronization of the partons is described by FFs $D_a^h(x, Q^2)$. Their values correspond to the probability that the parton a, produced at short distance of order $1/\sqrt{Q^2}$, fragments into the hadron h carrying the fraction x of the energy of a. For process (1), $\sqrt{Q^2}$ is typically of the order of the center-of-mass (CM) energy $\sqrt{s}$. Given their

x dependence at some scale Q_0, the evolution of the FFs with Q^2 may be computed perturbatively from the timelike Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [8],

$$\frac{dD_a^h(x,Q^2)}{d\ln Q^2} = \sum_b \int_x^1 \frac{dy}{y} P^T_{a\to b}(y,\alpha_s(Q^2)) D_b^h\left(\frac{x}{y},Q^2\right). \tag{3}$$

The timelike splitting functions $P^T_{a\to b}(x,\alpha_s(Q^2))$ are known at NLO [9] and NNLO [10].

This method to determine $\alpha_s^{(5)}(M_Z^2)$ is particularly clean in the sense that, unlike other methods, it is not plagued by uncertainties associated with hadronization corrections, jet algorithms, parton density functions (PDFs), *etc.* We recall that, similarly to the scaling violations in the PDFs, perturbative QCD only predicts the Q^2 dependence of the FFs. Therefore, measurements at different CM energies are needed in order to extract values of $\alpha_s^{(5)}(M_Z^2)$. Furthermore, since the Q^2 evolution mixes the quark and gluon FFs, it is essential to determine all FFs individually, which requires quark-flavor and gluon-jet tagging in the experimental data analysis.

This contribution is organized as follows. In Sec. 2, we discuss the α_s determination by fitting light-hadron FFs to experimental data of process (1) at NLO in the parton model of QCD. In Sec. 2, we present various ways how the standard NLO approach may be refined. In Sec. 2, we discuss the α_s determination by fitting the first Mellin moments of the FFs to experimental data of average hadron multiplicities in gluon and quark jets. Sec. 2, we assess the prospects of a high-precision determination of α_s at the FCC-ee.

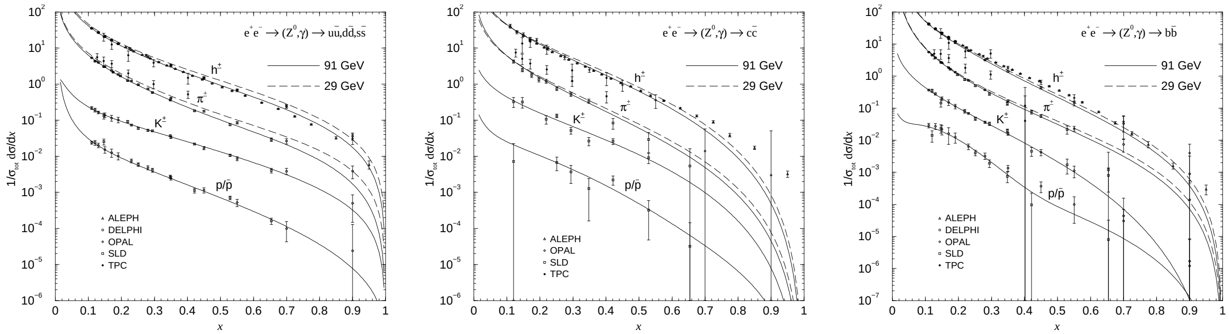
NLO fits of light-hadron FFs

NLO FFs for light hadrons were extracted via process (1) by several experimental [11,12] and theoretical [13,14,15,16,17] collaborations. Simultaneous determinations of α_s were performed in Refs. [11,14,15,16]. The latest one of the latter analyses [16] was focused on $\pi^\pm$, $K^\pm$, and $p/\overline{p}$ data. Collective charged-hadron data, including measurements of the gluon FF [12] and the longitudinal cross section, were excluded from the fit to avoid contaminations with charged particles other than the three lightest charged hadrons, but they were used for comparisons assuming the charged hadrons to be exhausted by $\pi^\pm$, $K^\pm$, and $p/\overline{p}$. The selected data were taken by TPC [18] at SLAC PEP, by ALEPH [19] and DELPHI [20] at CERN LEP, and by SLD [21] at SLAC SLC, and partially came as light-quark, charm, and bottom enriched samples. For the first time, the light-quark tagging probabilities obtained by OPAL [22] were included. The quality of the fit may be judged from Fig. 1. The fit result [16],

$$\alpha_s^{(5)}(M_Z^2) = 0.1176 \begin{matrix} +0.0053 & +0.0007 \\ -0.0067 & -0.0009 \end{matrix}, \tag{4}$$

is greatly dominated by the experimental error.

The fact that a high-precision determination of α_s from scaling violations in FFs is possible at all provides a stringent test of the DGLAP evolution. The universality of the FFs, as predicted by the factorization theorem of the QCD parton model, has been tested through extensive comparisons with experimental data of inclusive single hadron production in reactions other than process (1) [24,25].



(a) (b) (c)

Figure 1: Comparisons of the cross sections of process (1) for $h = h^\pm, \pi^\pm, K^\pm, p/\bar{p}$ with (a) $a = u, d, s$, (b) $a = c$, and (c) $a = b$ evaluated using the fitted FFs with the experimental data sets included in the fit [18,19,20,21] and one by OPAL [23] without hadron identification [16]. The theoretical results for $h = h^\pm$ are taken to be the sums of the respective ones for $h = \pi^\pm, K^\pm, p/\bar{p}$. The theoretical results for $a = g$ are distributed among the respective ones for $a = u, d, s, c, b$ according to the flavor of the $q\bar{q}$ pair produced along with the gluon.

Improvements

The fixed-order approach to FFs breaks down at small values of x because of the appearance of soft-gluon logarithms (SGLs). They manifest themselves at n loops in the timelike DGLAP splitting functions as terms which, in the limit $x \to 0$, behave as $(\alpha_s/2\pi)^n/x \ln^{2n-m-1} x$, where $m = 1, \ldots, 2n-1$ labels the SGL class. In Refs. [26,27], a general scheme for the evolution of FFs was introduced which resums both SGLs and mass singularities to any order in a consistent manner and requires no additional theoretical assumptions.

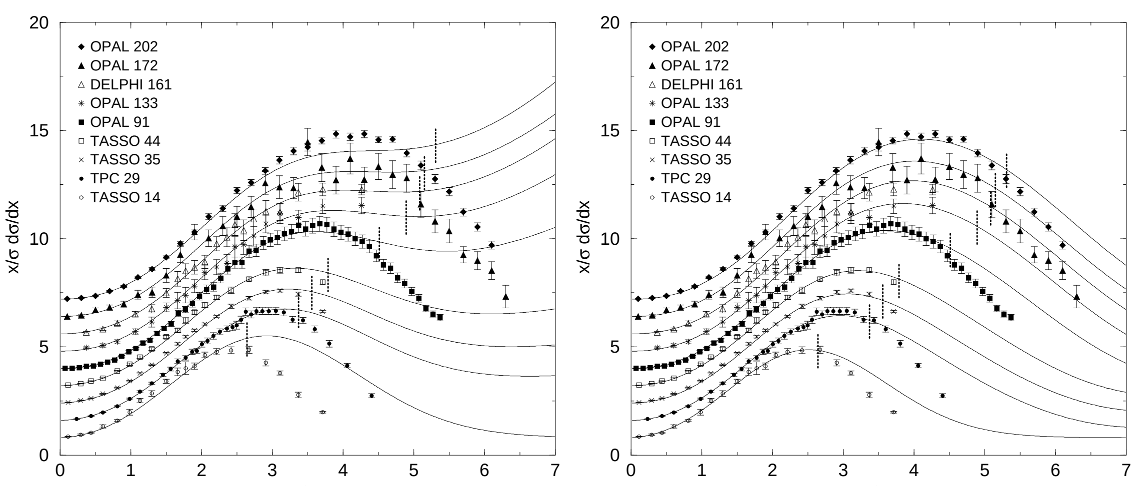


Figure 2: LO fits to small-x data of process (1) without (left) and with (right) resummation of double logarithms [27].

The double and single logarithms, with $m = 1$ and $m = 2$, respectively, in the timelike DGLAP splitting functions were resummed in Refs. [26,27,28]. The first approximation, with just double logarithms resummed and fixed-order contributions evaluated at LO, is already more complete than the modified leading logarithm approximation [29], which uses assumptions reaching beyond first principles of QCD. This dramatically improves the description of the *hump-backed plateau* as compared to the fixed-order treatment, as is illustrated in Fig. 2. The resummation of double logarithms in the coefficient functions of process (1) was explained in Ref. [30].

The fixed-order approach to FFs is spoiled by soft-gluon radiation also at large values of x. At n loops in the timelike DGLAP splitting functions, the corresponding divergences take the form $(\alpha_s/2\pi)^n[\ln^{n-r}(1-x)/(1-x)]_+$, where $r = 0,\ldots,n$ labels the class of divergence. Starting from the NLO results [9], the resummation was performed through next-to-leading-logarithmic (NLL) accuracy, for $r = 0, 1$, in Ref. [31]. The corresponding NLL expressions for the coefficient functions of process (1) may be found in Ref. [32]. The NLL resummation enhances the cross section of process (1) at large x values and reduces its theoretical uncertainty. In Ref. [17], this was found to improve the quality of the fit, as shown in Table 1 (left). In Ref. [31], the NLL effects were found to be comparable to the NNLO ones [10].

Particle	Unresummed	Resummed	Fitted mass (MeV)	PDG mass (MeV)
$\pi^\pm$	519.0	518.7	154.6	139.6
$K^\pm$	439.4	416.6	337.0	493.7
$p/\overline{p}$	538.0	525.2	948.8	938.3
K_S^0	318.7	317.2	343.0	497.6
$\Lambda/\overline{\Lambda}$	325.7	273.1	1127.0	1115.7

Table 1: Left: Minimized χ^2 values for the individual fits without charge-sign identification [17] without and with NLL resummation at large x values. Right: Fit results for light-hadron masses obtained in Ref. [17].

Furthermore, the quality of the fits may be significantly improved by including hadron mass effects [17,25,27]. In fact, including the light-hadron masses among the fit parameters, one obtains values that are amazingly close to the true values [17], as may be seen from Table 1 (right).

Multiple photon radiation from the initial state may have an appreciable effect on the line-shape of the differential cross section $d\sigma/dx$ of process (1). This effect is more pronounced for e^+e^- annihilation in the continuum than on resonances, such as the Z-boson one, because the photonic initial-state radiation shifts the CM energy of the hard e^+e^- collision to lower values, where the cross section is typically somewhat larger in the former case, but significantly smaller in the latter one. QED initial-state radiation may be conveniently incorporated via radiator functions, which allows for the resummation of leading logarithms of the form $(\alpha/2\pi)^n \ln^n(s/m_e^2)$ to all orders $n = 1, 2, \ldots$ [33]. The appropriate formalism was presented in Ref. [34].

Due to charge-conjugation invariance, the cross section of process (1) is the same for the positively and negatively charged hadrons of a given species. The latter are, therefore, routinely combined in experimental analyses. By the same token, it is impossible to discriminate FFs for h^+ and h^- hadrons by fitting to e^+e^- data. This is also true for $p\overline{p}$ data. However, the situation is different for ep and pp data. In Ref. [17], the charge-sign asymmetries of single hadrons inclusively produced in pp collisions as measured by BRAHMS, PHENIX, and STAR at BNL RHIC were used to constrain the valence-quark fragmentations. While this allows for a refined determination of α_s, it introduces additional theoretical uncertainties, associated with the PDFs. Furthermore, the fitting procedure is more complicated because the PDFs themselves depend on α_s.

Average hadron multiplicities in gluon and quark jets

Global fits to average hadron multiplicities in gluon and quark jets of energy $\sqrt{Q^2}$ produced in e^+e^- annihilation, $\langle n_h(Q^2)\rangle_a$ with $a = g, q$,* and their ratio $r(Q^2) = \langle n_h(Q^2)\rangle_g / \langle n_h(Q^2)\rangle_q$ also allow for high-precision determinations of α_s [35,36]. This requires the experimental ability to discriminate quark and gluon jets and the use of compatible jet algorithms in the experimental data analyses. By the definition of the FFs,

$$\langle n_h(Q^2)\rangle_a = \int_0^1 dx\, x^0 D_a^h(x, Q^2) = \tilde{D}_a^h(0, Q^2), \tag{5}$$

which is just the first Mellin moment of $D_a^h(x, Q^2)$. The respective expressions of the timelike DGLAP splitting functions, $\tilde{P}_a(0, \alpha_s(Q^2))$, are ill defined and require resummation. This was achieved with next-to-next-to-leading-logarithmic (NNLL) accuracy starting from the NNLO expressions [10] in Ref. [37]. The coupled system of the timelike DGLAP evolution equations in Mellin space is conveniently solved by diagonalization [35]. In this way, $\tilde{D}_a^h(0, Q^2) = \tilde{D}_a^{h+}(0, Q^2) + \tilde{D}_a^{h-}(0, Q^2)$ are decomposed into plus and minus components, which are numerically large and small, respectively. The ratios $r_\pm(Q^2) = \tilde{D}_g^{h\pm}(0, Q^2)/\tilde{D}_q^{h\pm}(0, Q^2)$ may be evaluated perturbatively in powers of $\sqrt{\alpha_s}$. At present, $r_+(Q^2)$ is known through $\mathcal{O}(\alpha_s^{3/2})$ [38] and $r_-(Q^2)$ through $\mathcal{O}(\alpha_s^{1/2})$ [36], i.e. through next-to-next-to-next-to-leading order ($\mathrm{N^3LO}$) and NLO, respectively. Global fits of $\langle n_h(Q^2)\rangle_g$ and $\langle n_h(Q^2)\rangle_q$ to experimental data coming from CLASSE CESR with $\sqrt{s} = 10$ GeV, SLAC PEP with 29 GeV, DESY PETRA with 12–47 GeV, KEK TRISTAN with 50–61 GeV, SLAC SLC with 91 GeV, CERN LEP1 with 91 GeV, and CERN LEP2 with 130–209 GeV yielded the results listed in Table 2. The $\mathrm{N^3LO_{approx}} + \mathrm{NLO} + \mathrm{NNLL}$ value of $\alpha_s^{(5)}(M_Z^2)$ therein corresponds to $\alpha_s^{(5)}(M_Z^2) = 0.1199 \pm 0.0026$ at the 68% CL. The quality of the fits may also be judged from Fig. 3.

	$\mathrm{N^3LO_{approx}} + \mathrm{NNLL}$	$\mathrm{N^3LO_{approx}} + \mathrm{NLO} + \mathrm{NNLL}$
$\langle n_h(Q_0^2)\rangle_g$	24.18 ± 0.32	24.22 ± 0.33
$\langle n_h(Q_0^2)\rangle_q$	15.86 ± 0.37	15.88 ± 0.35
$\alpha_s^{(5)}(M_Z^2)$	0.1242 ± 0.0046	0.1199 ± 0.0044
χ^2_{dof}	2.84	2.85

Table 2: Fit results for $\langle n_h(Q_0^2)\rangle_g$, $\langle n_h(Q_0^2)\rangle_q$, and $\alpha_s^{(5)}(M_Z^2)$ for $Q_0 = 50$ GeV obtained in the $\mathrm{N^3LO_{approx}} + \mathrm{NNLL}$ and $\mathrm{N^3LO_{approx}} + \mathrm{NLO} + \mathrm{NNLL}$ approaches [36]. The errors refer to the 90% confidence level (CL).

We observe from Fig. 4 that the experimental uncertainty overwhelms the theoretical one in the case of $\langle n_h(Q^2)\rangle_q$, while they are comparable in the case of $\langle n_h(Q^2)\rangle_g$. Since the theoretical predictions are unlikely to advance beyond the NNLL accuracy in the foreseeable future, the potential of high-precision measurements of $\langle n_h(Q^2)\rangle_g$ and $\langle n_h(Q^2)\rangle_q$ to reduce the error in $\alpha_s^{(5)}(M_Z^2)$ will be limited.

Conclusions

In this contribution, we explained how α_s may be determined from scaling violations in FFs by fitting the latter to experimental data of the inclusive production of single light hadrons in e^+e^-

*Here, q denotes the quark singlet component.

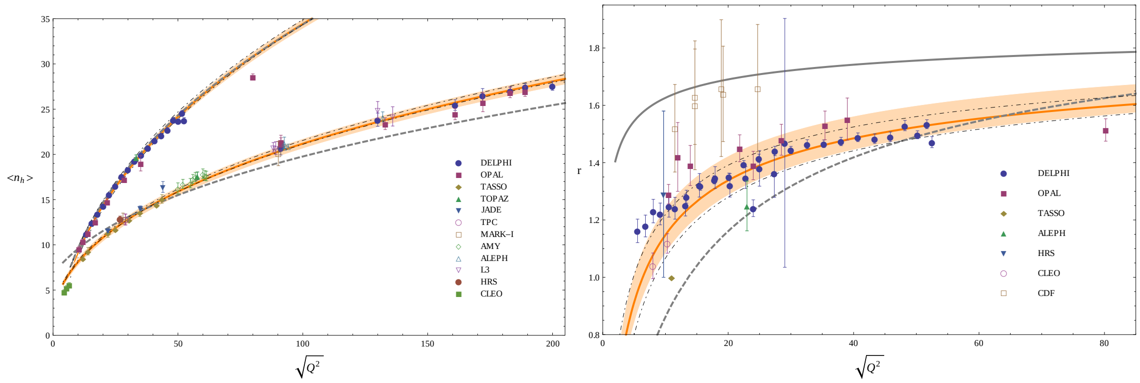


Figure 3: Comparisons of fit results for $\langle n_h(Q^2)\rangle_g$, $\langle n_h(Q^2)\rangle_q$ (left), and $r(Q^2)$ with experimental data (right) [36]. The experimental and theoretical uncertainties are indicated by the shaded/orange bands and the bands enclosed between the dot-dashed curves, respectively.

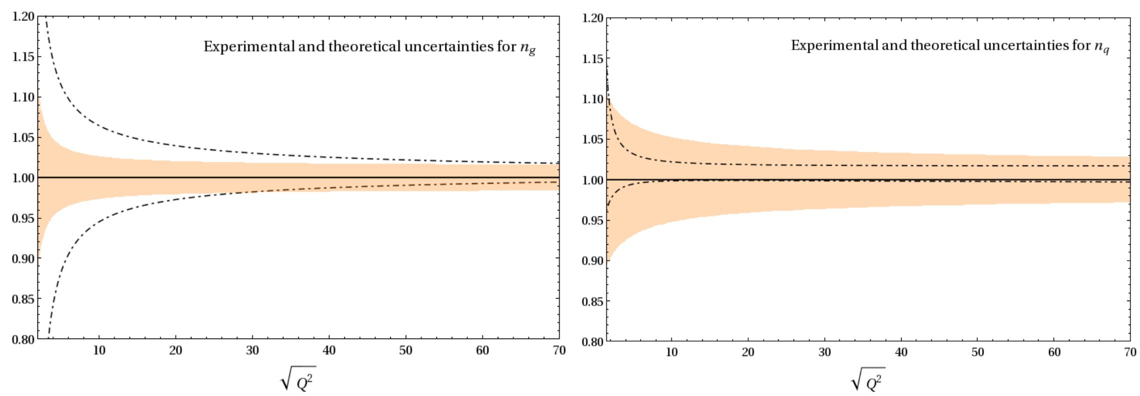


Figure 4: Experimental (shaded/orange bands) and theoretical (bands enclosed between dot-dashed curves) uncertainties in the $\mathrm{N^3LO_{approx}} + \mathrm{NLO} + \mathrm{NNLL}$ results for $\langle n_h(Q^2)\rangle_g$ (left), and $\langle n_h(Q^2)\rangle_q$ (right) normalized with respect to their default evaluations [36].

annihilation. We also discussed the analogous analysis based on the average hadron multiplicities in gluon and quark jets, which correspond to the first Mellin moments of the respective FFs. In both cases, we reviewed previous such α_s determinations in the literature and pointed out ways how they may be refined with regard to the theoretical description.

This allows us to usefully assess the prospects of a high-precision determination of α_s by fitting FFs or their first Mellin moments to experimental data to be collected at the FCC-ee. On the experimental side, it will be indispensable to measure the cross sections at widely separated values of $\sqrt{s}$ and with fine binnings in the x variable, to identify the hadron species, to discriminate gluon and quark jets, and to tag the quark flavors in the latter case.

As for fits of the x dependencies of the FFs, the experimental errors in α_s achieved so far are typically one order of magnitude larger than the theoretical ones, as may be seen from Eq. (4). By incorporating the theoretical improvements mentioned in Sec. 2, which was actually done in a determination of FFs for fixed value of α_s [17], and, to a lesser degree, by including the fixed-order corrections at the NNLO level, the theoretical error in α_s will be appreciably reduced, possibly by up to a factor of five or more. Moreover, the use of the generalized scheme of FF evolution [26,27] will allow one to also fully exploit the wealth of small-x data, which must be excluded from fixed-order analyses because of the lack of SGL resummation. The enormous luminosity envisaged for the FCC-ee and the high quality of the detectors to be installed there will allow for the experimental error in α_s to be pushed well below the theoretical one after a few years of running at different CM energies. In conclusion, a combined error of order ± 0.0001 in α_s appears to be reachable.

As for fits of the first Mellin moments of the gluon and quark FFs, the theoretical and experimental errors in α_s are presently comparable, as may be gleaned from Fig. 3. Since theoretical progress will be very hard to achieve in the foreseeable future, the theoretical error in α_s will soon limit the precision achievable via this method at the FCC-ee.

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