Polarization analysis of gravitational-wave backgrounds

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Abstract

The stochastic background of gravitational waves may be polarized by the mechanisms involving the parity violation in the early universe and/or the preferential alignment of unresolved sources. Here, we present the detection method of a polarized gravitational-wave background and discuss the sensitivity of space interferometers to the polarized gravitational waves.

1 Introduction

The stochastic background of gravitational wave contains valuable cosmological information to probe the dark side of the universe. In particular, primordial gravitational-wave background produced during inflation is one of the most fundamental prediction from the inflationary theory and detection of it provides a stringent constraint on the inflation model. Recently, several space missions to detect such a tiny signal have been proposed and feasibility of direct detection was intensively discussed. Although there still remain practical issues such as the subtraction of the overlapping signals coming from the neutron star-neutron star binaries and/or the unknown unresolved sources, aiming at the future direct detection, the infrastructure such as new characterization and/or data analysis technique of gravitational-wave signals should be developed furthermore.

Along the line of this discussion, one important aspect is the polarization character of the gravitational wave background (GWB). As it is well known, gravitational waves have polarization degree of freedom due to its spin-2 nature. While the standard prediction from inflation leads to an un-polarized GWB, there might exist some physical mechanisms to generate a polarized GWB [1]. Detection of polarized GWB may thus be important to identify the physical origin of each GWB.

In this article, based on the cross-correlation technique, we present a formalism to detect a polarized GWB. The observational characteristics for polarized GWB are discussed in a specific detector, LISA. Further, we discuss how the geometry of detectors affects the sensitivity to a polarized GWB.

2 Formalism

To begin with, let us write down the basic equation characterizing the gravitational waves. In the transverse-traceless gauge, the metric perturbation becomes

$$h_{ij}(\vec{x},t)=\sum_{A=+,\times}\int_{-\infty}^{\infty}df\int d\hat{\Omega}\,h_A(f,\hat{\Omega})\,e^{i\,2\pi\,f(t-\hat{\Omega}\cdot\vec{x})}\;e_{ij}^{A}(\hat{\Omega}), \tag{1}$$

where the unit vector $\hat{\Omega}$ is the propagation direction and the quantity $e_{ij}^{+,\times}$ is the polarization basis satisfying the transverse-traceless conditions. For our interest of the stochastic signals, the amplitude h_A has random nature, whose statistical properties including the polarization characters are described by the power spectra. Defining the right and the left-handed tensor amplitudes as, $h_R\equiv(h_+-ih_\times)/\sqrt{2}$ and $h_L\equiv(h_++ih_\times)/\sqrt{2}$, we have [2, 3]

$$\begin{pmatrix}\left\langle h_R(f,\hat{\Omega})\,h_R^*(f',\hat{\Omega}')\right\rangle & \left\langle h_L(f,\hat{\Omega})\,h_R^*(f',\hat{\Omega}')\right\rangle\\ \left\langle h_R(f,\hat{\Omega})\,h_L^*(f',\hat{\Omega}')\right\rangle & \left\langle h_L(f,\hat{\Omega})\,h_L^*(f',\hat{\Omega}')\right\rangle\end{pmatrix}=\frac{\delta_D(f-f')}{2}\,\frac{\delta_D(\hat{\Omega},\hat{\Omega}')}{4\pi}\begin{pmatrix}I+V & Q+i\,U\\ Q-i\,U & I-V\end{pmatrix}. \tag{2}$$

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The quantities I, V and $(Q \pm i\,U)$ are the stokes parameters for gravitational waves, which are the standard notations characterizing the polarization states, in analogy to characterize those of the electromagnetic waves. While the quantity I denotes the total intensity, V represents the asymmetry between left- and right-handed gravitational waves, leading to the circular polarization. On the other hand, $(Q \pm i\,U)$ characterize the linear polarization states and have spin-± 4 properties. These are all functions of frequency f and the direction $\hat{\Omega}$. For later convenience, we write down the power spectra in terms of the spherical harmonics. For the intensity I and V, the harmonic expansions become

$$I(f,\hat{\Omega}) = \sum_{\ell,m} I_{\ell m}(f)\, Y_{\ell m}(\hat{\Omega}), \qquad V(f,\hat{\Omega}) = \sum_{\ell,m} V_{\ell m}(f)\, Y_{\ell m}(\hat{\Omega}). \tag{3}$$

As for the linear polarization, due to its spin-4 nature, these are expanded by the spin-weighted spherical harmonics ${}_{\pm 4}Y_{\ell m}$ [3]:

$$(Q \pm\, i\,U)(f,\hat{\Omega}) = \sum_{\ell,m} P^{(\pm)}_{\ell m}\ {}_{\pm 4}Y_{\ell m}(\hat{\Omega}). \tag{4}$$

Notice that the combinations $(Q \pm\, i\,U)$ are not invariant under the rotation around a specific direction. For a better characterization, we introduce the electric- and the magnetic-mode decomposition of linear polarization:

$$\mathcal{E}_{\ell m}(f) = \frac{1}{2}\left\{P^{(+)}_{\ell m}(f) + P^{(-)}_{\ell m}(f)\right\}, \qquad \mathcal{B}_{\ell m}(f) = \frac{1}{2\,i}\left\{P^{(+)}_{\ell m}(f) - P^{(-)}_{\ell m}(f)\right\}. \tag{5}$$

These two combinations behave differently under parity transformation: while $\mathcal{E}$ remains unchanged, $\mathcal{B}$ changes its sign.

With the harmonic coefficients $I_{\ell m}$, $V_{\ell m}$, $\mathcal{E}_{\ell m}$ and $\mathcal{B}_{\ell m}$, anisotropies and frequency dependence of polarized GWBs are completely characterized. We then move to discuss how one can detect such polarized GWBs. First recall that the gravitational-wave signal received at the detector α, h_α, can be written as

$$h_\alpha(\vec{x},t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d\hat{\Omega}\, D^{ij}_\alpha(f,\hat{\Omega};t)\, e^A_{ij}(\hat{\Omega})\ h_A(f,\hat{\Omega})\, e^{i\,2\pi\,f\,(t-\hat{\Omega}\cdot vecx)}, \tag{6}$$

where the function D^{ij}_α represents the detector response tensor, which explicitly depends on time due to the (orbital) motion of detector. If we have another data set, h_β, obtained from the detector β which is located near the detector α, the cross-correlation analysis may be applied to detect a stochastic GWB and we have the non-vanishing amplitude of cross-correlation signal, $C_{\alpha\beta}(t) \equiv \langle h_\alpha(t)\, h_\beta(t)\rangle$. The Fourier counterpart of it, $\widetilde{C}_{\alpha\beta}$, which is related with $C_{\alpha\beta}$ by $C_{\alpha\beta}(t) = \int \frac{df}{2}\, \widetilde{C}_{\alpha\beta}(f;t)$, becomes [3]

$$\widetilde{C}_{\alpha\beta}(f;t) = \frac{1}{4\pi} \sum_{\ell,m,m'} \left\{\vec{\boldsymbol{S}}_{\ell m'}(f) \cdot \vec{\boldsymbol{\mathcal{F}}}_{\alpha\beta,\,\ell m}(f)\right\}\, \mathcal{D}^{(\ell)}_{m'm}\Big(\psi(t),\vartheta(t),\varphi(t)\Big), \tag{7}$$

where $\mathcal{D}^{(\ell)}_{m'm}$ is the rotation matrix. In deriving the expression (7), we have assumed that the time variation of detector's orientation is described by the Euler rotation with angles $(\psi(t),\vartheta(t),\varphi(t))$ in the co-moving frame of rigidly moving detectors α and β. Here, the vectors $\vec{\boldsymbol{S}}_{\ell m}$ and $\vec{\boldsymbol{\mathcal{F}}}_{\alpha\beta,\,\ell m}$ are

$$\begin{aligned} \vec{\boldsymbol{S}}_{\ell m}(f) &= \Big\{I_{\ell m}(f),\ V_{\ell m}(f),\ \mathcal{E}_{\ell m}(f),\ \mathcal{B}_{\ell m}(f)\Big\}, \\ \vec{\boldsymbol{\mathcal{F}}}_{\alpha\beta,\,\ell m}(f) &= \Big\{a^{(I)}_{\ell m}(f),\ a^{(V)}_{\ell m}(f),\ a^{(\mathcal{E})}_{\ell m}(f),\ a^{(\mathcal{B})}_{\ell m}(f)\Big\}, \end{aligned}$$

where quantities $a^{(X)}_{\ell m}$ $(X = I, V, \mathcal{E}, \mathcal{B})$ represent the multipole coefficients of antenna pattern function for each polarization state. Eq.(7) implies that polarized anisotropies of GWBs can be detected through the time variation of correlation signal $\widetilde{C}_{\alpha\beta}$ and the sensitivity to each polarization anisotropy depends on the amplitude of the multipole coefficient $a^{(X)}_{\ell m}$. The explicit expressions for each multipole coefficients are given as follows. Defining the quantity $F^{R,L}_\alpha(f,\hat{\Omega})$ by $F^{R,L}_\alpha = D^{ij}_\alpha(f,\hat{\Omega})e^{R,L}_{ij}(\hat{\Omega})$, we have [3]

$$a^{(I)}_{\ell m}(f) = \int d\hat{\Omega}\ e^{i\,2\pi\,f\,\hat{\Omega}\cdot(\vec{x}_\beta - \vec{x}_\alpha)} \left\{F^R_\alpha\, F^{R*}_\beta + F^L_\alpha\, F^{L*}_\beta\right\}\, Y_{\ell m}(\hat{\Omega}), \tag{8}$$

$$
\begin{aligned}
a_{\ell m}^{(V)}(f) &= \int d\hat{\Omega}\; e^{i\,2\pi\,f\,\hat{\Omega}\cdot(\vec{x}_\beta-\vec{x}_\alpha)}\left\{F_\alpha^R\,F_\beta^{R*}-F_\alpha^L\,F_\beta^{L*}\right\}\,Y_{\ell m}(\hat{\Omega}), && (9)\\
a_{\ell m}^{(\mathcal{E})}(f) &= \int d\hat{\Omega}\; e^{i\,2\pi\,f\,\hat{\Omega}\cdot(\vec{x}_\beta-\vec{x}_\alpha)}\left\{F_\alpha^R\,F_\beta^{L*}\;{}_{-4}Y_{\ell m}(\hat{\Omega})+F_\alpha^L\,F_\beta^{R*}\;{}_{4}Y_{\ell m}(\hat{\Omega})\right\}, && (10)\\
a_{\ell m}^{(\mathcal{B})}(f) &= i\int d\hat{\Omega}\; e^{i\,2\pi\,f\,\hat{\Omega}\cdot(\vec{x}_\beta-\vec{x}_\alpha)}\left\{-F_\alpha^R\,F_\beta^{L*}\;{}_{-4}Y_{\ell m}(\hat{\Omega})+F_\alpha^L\,F_\beta^{R*}\;{}_{4}Y_{\ell m}(\hat{\Omega})\right\}. && (11)
\end{aligned}
$$

Here, the vectors $\vec{x}_\alpha$ and $\vec{x}_\beta$ represent the position of detectors α and β, respectively. Note that the phase factor $e^{i\,2\pi\,f\,\hat{\Omega}\cdot(\vec{x}_\beta-\vec{x}_\alpha)}$ arises from the arrival-time difference of gravitational waves between detectors α and β.

3 Detector characteristic: low-frequency limit of LISA

The expressions (8)-(11) derived in previous section play a central role in detecting a polarization anisotropy of GWB. We then wish to understand the sensitivity of gravitational-wave detector to a polarized GWB in a specific detector configuration. In this section, as an illustrative example, we consider the low-frequency limit of LISA and investigate the characteristic properties of the polarization sensitivity. Here, the term, low-frequency, implies the frequency lower than the characteristic frequency $f_{\rm crit}$ given by $f_{\rm crit}=c/(2\pi L)$ with L being the arm-length of detector. With $L=5\times10^6$km, low-frequency limit of LISA indicates $f\ll 0.1$mHz. In this frequency range, LISA has effectively two output signals sensitive to the gravitational waves, called A and E variables, which are constituted by a time-delayed combination of six one-way data streams.

Adopting the coordinate system defined in Ref.[2], the projected detector responses $F_A^{R,L}$ and $F_E^{R,L}$ are explicitly written as

$$
\begin{aligned}
F_A^{R,L}(\hat{\Omega}) &= \frac{1}{2}(1+\cos^2\theta)\,\cos2\phi\mp i\,\cos\theta\sin2\phi \qquad (-:R,\ +:L), && (12)\\
F_E^{R,L}(\hat{\Omega}) &= \frac{1}{2}(1+\cos^2\theta)\,\sin2\phi\pm i\,\cos\theta\cos2\phi \qquad (+:R,\ -:L). && (13)
\end{aligned}
$$

Then, substituting the above equations into the expressions (8)-(11), we compute the multipole coefficients $a_{\ell m}^{(X)}$. Using the fact that $\vec{x}_\alpha=\vec{x}_\beta$, the resultant non-vanishing coefficients are summarized as follows:

$$
\begin{aligned}
&I\text{-mode}: && a_{00}^{(I)}=\frac{4\sqrt{\pi}}{5},\quad a_{20}^{(I)}=\frac{8}{7}\sqrt{\frac{\pi}{5}},\quad a_{40}^{(I)}=\frac{2\sqrt{\pi}}{105},\qquad a_{44}^{(I)}=\pm\frac{1}{3}\sqrt{\frac{\pi}{35}},\\
&\mathcal{E}\text{-mode}: && a_{40}^{(\mathcal{E})}=\frac{2}{3}\sqrt{\frac{2\pi}{35}},\quad a_{44}^{(\mathcal{E})}=\pm\frac{2\sqrt{\pi}}{3}\qquad (+:\mathrm{AA},-:\mathrm{EE})
\end{aligned}
$$

for the self-correlation signals, $(\alpha,\beta)=(A,A)$ or (E,E), and

$$
\begin{aligned}
&I\text{-mode}: && a_{44}^{(I)}=i\,\frac{1}{3}\sqrt{\frac{2\pi}{35}},\\
&\mathcal{E}\text{-mode}: && a_{44}^{(\mathcal{E})}=i\,\frac{2\sqrt{\pi}}{3},\\
&V\text{-mode}: && a_{10}^{(V)}=-i\,\frac{8}{5}\sqrt{\frac{\pi}{3}},\quad a_{30}^{(V)}=-i\,\frac{2}{5}\sqrt{\frac{\pi}{7}}
\end{aligned}
$$

for the cross-correlation signals, $(\alpha,\beta)=(A,E)$. From this, important properties of polarization sensitivity in the low-frequency limit can be found:

- Visible multipole components of anisotropic GWB are restricted to $\ell=0,\,2,\,4$ for I-mode, $\ell=1,\,3$ for V-mode and $\ell=4$ for $\mathcal{E}$-mode. As for the $\mathcal{B}$-mode, all the multipole moments vanish.
- There exists degeneracy between I- and $\mathcal{E}$-modes ($\ell=4$). To be precise, the relation $a_{4m}^{(I)}/a_{4m}^{(\mathcal{E})}=\sqrt{1/70}$ holds for the non-vanishing components $m=0$ and 4,

which are generic properties in the low-frequency limit of co-located detectors.

4 Improving the sensitivity to a polarized GWB: worked example

The polarization sensitivity of LISA shown in previous section seems a little bit problematic in a sense that no useful information about linear polarization modes ($\mathcal{E}$ and $\mathcal{B}$) can be obtained. While this may be a generic low-frequency property, we wish to remedy this by introducing some physical effects. There are two possible effects: **(i)** finite arm-length effect on detector's response ($f \gtrsim f_{\rm crit}$) and **(ii)** effect of finite separation between two detectors $\vec{x}_\alpha \neq \vec{x}_\beta$. Here, we examine the latter case and study how the sensitivity to the linear polarized GWB can be improved.

First note that the influence of finite separation is incorporated into the phase factor in Eqs.(8)-(11). The non-vanishing contribution of the phase factor $e^{i\,2\pi\,f\,\hat{\Omega}\cdot(\vec{x}_\beta-\vec{x}_\alpha)}$ leads to the frequency-dependent polarization sensitivity and depending on the propagation direction, the response to a polarized gravitational-wave signal can be different between two detectors. As a worked example, we consider the two set of LISA-type detector labeled by I and II, which are separated by a distance d. The two detectors are assumed to be co-aligned and the position vector $\vec{x}_{\rm I}-\vec{x}_{\rm II}$ is normal to the arms of each detector. In Fig.1, specifically focusing on the $\ell = 4$ component, the resultant multipole coefficients are plotted as ratio, $a_{4m}^{(X)}/a_{4m}^{(\mathcal{E})}$, which are given by a function of normalized frequency, f/f_* with $f_* \equiv c/(2\pi|\vec{d}|)$. The left panel shows that the degeneracy between I- and $\mathcal{E}$-modes can be broken as increasing the frequency f (or the separation d), while the right panel reveals that sensitivity to $\mathcal{B}$-mode polarization can be recovered.

Although the polarization sensitivity shown in Fig.1 indicates a monotonic dependence on the frequency, the actual detector response to each polarization mode is a complicated function of the frequency f/f_*. Relaxing the assumption of co-aligned detectors, there are six parameters to characterize the geometric configuration of two detectors: orientation, alignment and separation between two detectors. Optimizing these six parameters, one can obtain the most sensitive detector set to the polarized GWB at a given frequency band. Improvement of the sensitivity is important and helpful to extract useful cosmological information. This issue will be discussed in details in a separate publication.

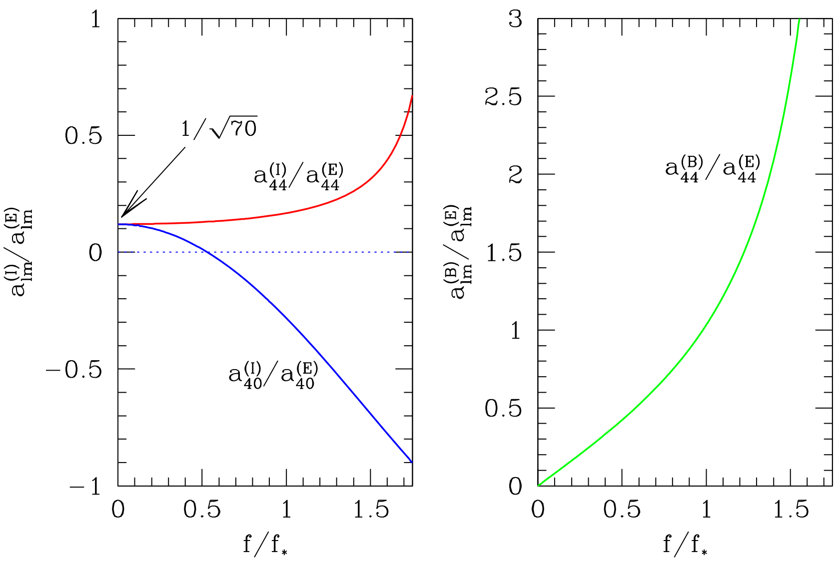

Figure 1: Ratio of the multipole coefficients for antenna pattern function, $a_{4m}^{(I)}/a_{4m}^{(\mathcal{E})}$ (*left*) and $a_{44}^{(\mathcal{B})}/a_{44}^{(\mathcal{E})}$ (*right*), as function of normalized frequency (see Eqs.(8)-(11) for definitions). In left panel, the blue and the red lines represent the ratios $a_{40}^{(I)}/a_{40}^{(\mathcal{E})}$ and $a_{44}^{(I)}/a_{44}^{(\mathcal{E})}$, respectively.

References

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