Study of the Sagitta Resolution of MDT-Chambers

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Abstract

The cosmic ray measurement facility in Garching, Germany, provides a good environment to validate the Geant4 simulation of the MDT-chambers with real data. The sagitta resolution of the muon tracks, reconstructed with the three MDT-chambers of this setup, allows a meaningful test of this comparison. The sagitta resolution of the measurement facility was measured with cosmic muons recorded between 2004 and 2005. This measured resolution was reproduced to a very high accuracy within a detailed Geant4 simulation of the measurement facility and the MDT-chambers.

Given this validated simulation, other effects of the sagitta and momentum resolution of the MDT-chambers in the cosmic ray measurement facility, in the ATLAS detector and the H8-testbeam construction are studied. In particular effects of multiple scattering, single tube resolution and alignment are discussed.

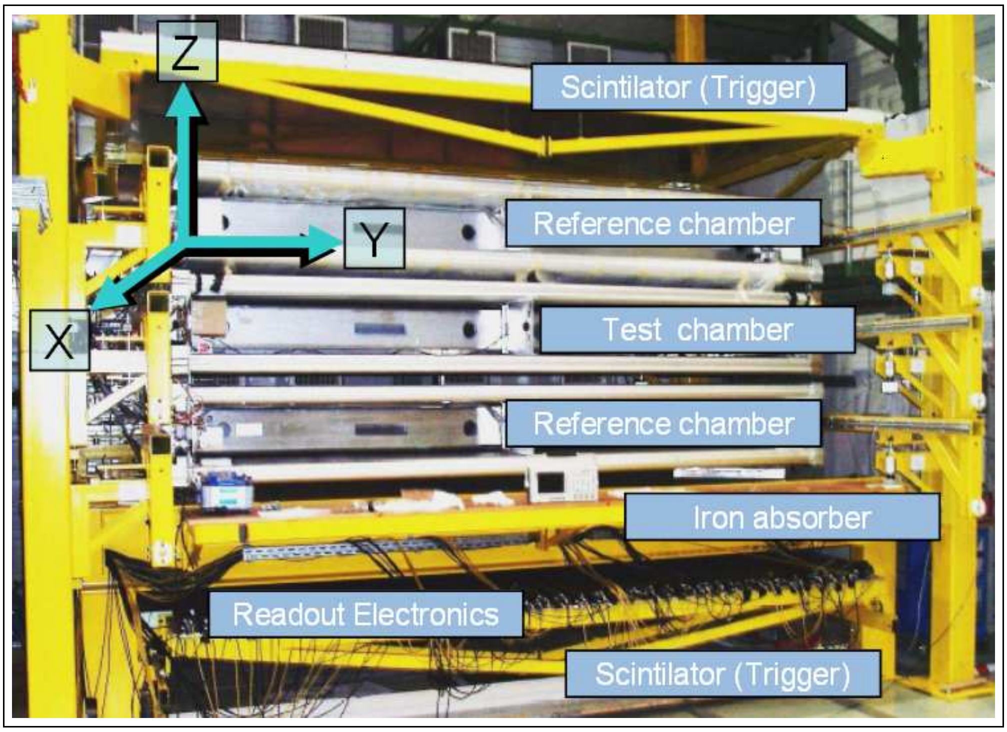

Figure 1: The cosmic ray measurement facility in Garching and the appropriate coordinate system

1 Introduction

The cosmic ray measurement facility in Garching, Germany, is designed for measurement and calibration of ATLAS MDT-chambers (See Figure 1). The measurement facility consists of three MDT-chambers, which are positioned one on top on the other. The upper and the lower chamber are used as reference chambers. The properties of these two chambers are known to a very high precision from tomography measurements made at CERN [1]. The so-called *test chamber* is sandwiched between the two reference chambers. Furthermore, an iron absorber is placed below the lower reference chamber to cut on low energy muons. Since no magnetic field is applied at the cosmic ray measurement facility, the track of an incident muon is expected to be - on average - a straight line. With this assumption the rt-relation, the wire positions and the geometry of the test-chamber can be determined. Detailed information is given in [1] and [2].

The coordinate system shown in Figure 1 was used in this study. The horizontal xy-plane corresponds to the wire chamber plane with the x-axis along the MDT tubes. The z-axis is perpendicular to the other two axis and describes the vertical direction. The origin of the coordinate system was chosen to be the geometric center of the test-chamber.

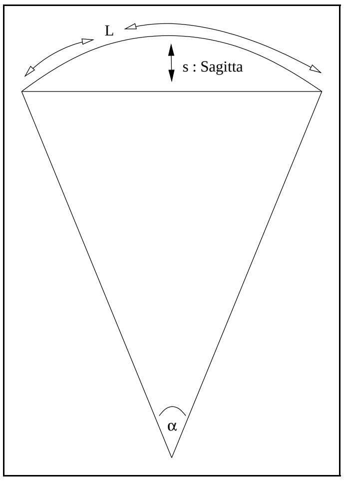

Figure 2: Definition of sagitta

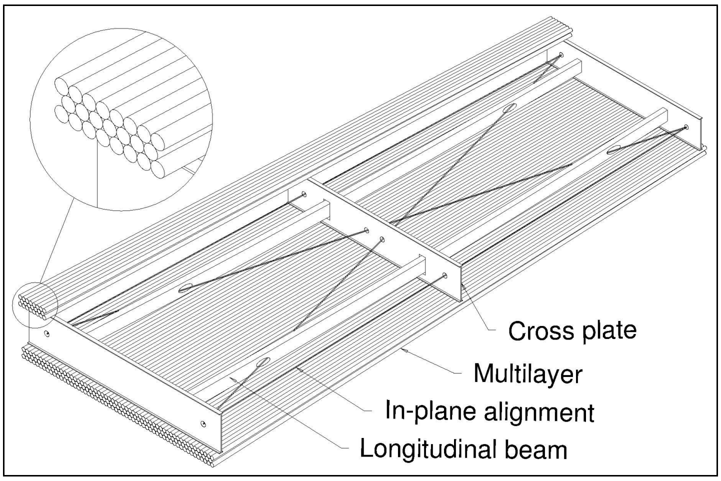

Figure 3: Schematic drawing of a MDT chamber from [3] with its two multilayers each consisting of 3 tube-layers

The measured track positions may not be perfect fit to a straight line, because of the finite resolution of the tubes. Moreover the muon track may not be perfectly straight because of multiple scattering on the material of the MDT-chambers or δ-electron emissions which may mask or displace the signal of the muon. Hence, the muon trajectory is in general not a straight line even without the presence of a magnetic field, but can be described by a parabola which is the next order approximation. This muon tracjectory uncertainty results in a limited sagitta resolution. A theorical estimation of the expected sagitta resolution can be found in the appendix A.

The sagitta is defined through a segment of a circle as shown in Figure 2.

$$s = r(1 - \cos\frac{\alpha}{2}) \approx r\frac{\alpha^2}{8} \tag{1}$$

The dependence of the sagitta s on the transverse momentum p_T of a particle in a magnetic field is given by

$$s \approx \frac{1}{8}\frac{L^2 B}{p_T} \tag{2}$$

From this equation follows that the measurement of sagitta determines the transversal momentum of the particle. The errors of s and B lead to an uncertainty on p_T via

$$\Delta p_T = \left(-\frac{1}{8}\frac{L^2 B}{s^2}\right)\Delta s + \left(\frac{1}{8}\frac{L^2}{s}\right)\Delta B. \tag{3}$$

2 Sagitta study for the Cosmic Ray Measurement Facility

2.1 Definitions and algorithms

An incident muon passes six multilayers - two multilayers per chamber (Figure 3) - on its way through the measurement facility, which corresponds in the ideal case to 18 measured drift circles. The aim is now to fit a parabola tangential to these 18 drift circles which could be done by the χ^2-method. The χ^2-algorithm finds parameters for a given function that minimizes the distances of the function to the drift circles. In this case the parameters a,b and c of a parabola $f(x) = ax^2 + bx + c$ have to be found.

The reason for using a parabola and not an arc segment as fitting function is due to mathematical requirements of the χ^2-method. Since there is no magnetic field and the measured cosmic muons have high momentum ($\geq 600\,MeV$), small sagittae in comparison with the arc length are expected. In this limit the circle can be well approximated by a parabola. In order to achieve an optimal approximation by a parabola, a transformation of the global coordinate system into a new coordinate system, whose x-axis is defined to be parallel to the slope of the incoming muon as shown in Figure 4, is performed. This ensures that the vertex of the parabola is placed in the center of the measured muon track section. The slope of the incident muon is determined by a simple staight line fit to the measured drift circles.

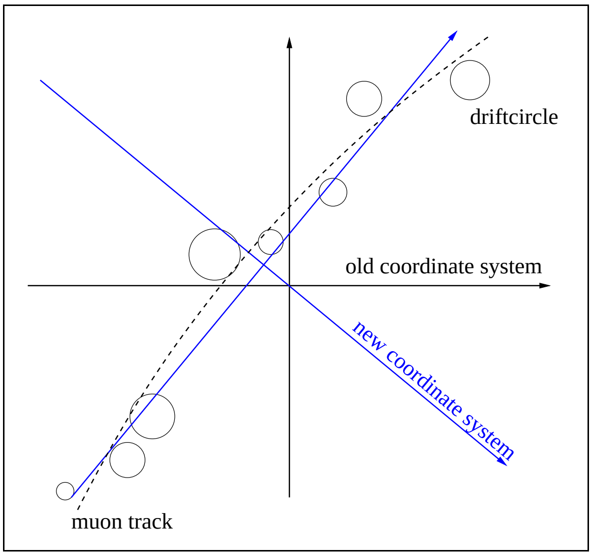

Figure 4: Approximation of a track with a parabola

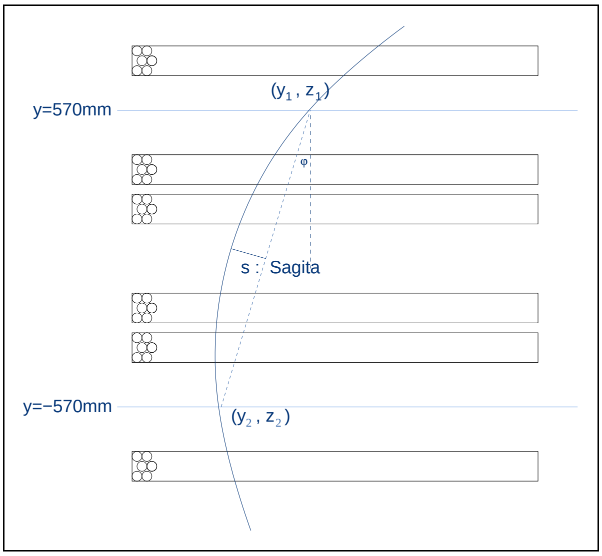

Figure 5: Definition of Sagitta at the cosmic ray measurement facility

Figure 5 shows schematically a parabola which has been fitted to the measured drift circles. The intersections of the parabola and the center planes of the reference chambers at $z_1 = 570\,mm$ and $z_2 = -570\,mm$ define an arc segment which is the basis for the definition of the sagitta.

The ambiguities of fitting a function to the drift circles lead to local minima, since the χ^2 of the fitted function gets minimal on both sides for each drift circle. Clearly only one side corresponds to the real muon track. One possible solution to that problem is to ignore the drift-radius-information in a first step and fit the parabola only to the centres of the drift-circles. The aim of this procedure is to find suitable starting parameters of the fitting function that should lie on the correct side of the drift-circles. These parameters are used as starting-values for a second fit which makes use of the drift-radius information. A Monte-Carlo study of this method shows that the chance of finding only a local minimum could be reduced from 2.8% to 1.1%.

In the following the overall procedure from data to the measured sagitta is summarized:

1. Apply pattern recognition to identify the drift circles which correspond to an incident muon and generate a group of drift circles.

2. Fit a straight line to the centers of the drift circles in order to measure the slope of the incident muon. Rotate the coordinates of the drift circles so that their x-axis is parallel to the measured slope.

3. Fit a parabola to the centers of the drift circles.

4. Delete the drift circles out of the group which have a minimal distance of more than 18 mm from the fitted parabola. These drift circles were wrongly identified by the pattern recognition.

5. Optimize the parabola fit of step 3 by using the drift radius information to reduce the problem of finding only a local minimum solution

6. Calculate the relative residuum R_σ of each drift circle

$$R_\sigma = \frac{r_{dc} - r_p}{\sigma}$$

   where r_{dc} is the radius of the drift circle, r_p is the minimal distance of the drift circle to the fitted parabola and σ is the error on the measured drift circle.

7. Delete the drift circle with the largest R_σ from the group and repeat step 3.

8. Calculate the corresponding sagitta, if there are at least 16 drift circles left in the group

Step 7 is optional since no significant change in the measured sagitta resolution was observed. This step was introduced to minimize δ-electron effects. It was also tested to neglect hits which were close to the wire, since the measured drift radius has a large error, but also here no siginificant change on the sagitta distribution could be observed. It turned out that some tubes have significanlty more hits with $R_\sigma > 10$ than all others. This is a hint to a systematic error e.g. due to noise and therefore signals from these tubes were neglected during this study.

2.2 Geant4 simulation of the cosmic ray measurement facility

The simulation of the cosmic ray measurement facility (CMF-Simulation) is based on Geant4 and was fully implemented within the Athena-Framework. One should note that we developed an implementation of the GeoModel description [4] of the measurement facility and the digitization part of simulation, which is independent of the meanwhile existing implementation in the official ATLAS muon software release.

The Athena-Package *Cosmic Generator* was used to generate cosmic muons with the correct energy and momentum spectrum. This Athena-package interfaces existing *F77* source code [1].

Furthermore, we checked that all aspects of the simulation were properly implemented: It was reviewed that the simulation uses the correct rt-relation and single tube resolution and also takes into account the flight time of the muons and the signal propagation along the wire. In addition, further details such as wire-sagging were implemented.

2.3 Results and comparison between real and simulated data

2.3.1 Sagitta study with three chambers

The distribution in Figure 6 shows the measured sagitta for about 100.000 events that have been recorded at the cosmic ray measurement facility. The simulated sagitta distribution for about 30.000 events can be seen in Figure 7.

Two effects influence the sagitta resolution, as already mentioned in section 1: Multiple scattering, which is energy dependent and the single tube resolution which is energy independent. Figure 8 shows the energy dependence of the reconstructed sagitta for simulated events, where the energy is known from the Monte Carlo truth.

As expected, Figure 8 exhibits a strong energy dependence.Figure 9 and Figure 10 confirm that this observation is not due to the lower statistics at higher energies. There is a wide sagitta distribution for low muon energies since multiple scattering effects are dominating in this regime. It is useful to study the different energy regions separately in order to analyse the overall distribution (summed over all energies), which is the only accessible quantity at the cosmic ray measurement facility since no magnetic field

[1] Using this interface we found a subtle semantic bug, which leads to a shifted and distorted energy spectrum if a minimal muon energy is defined via *joboptions*. This error was corrected for this study and reported.

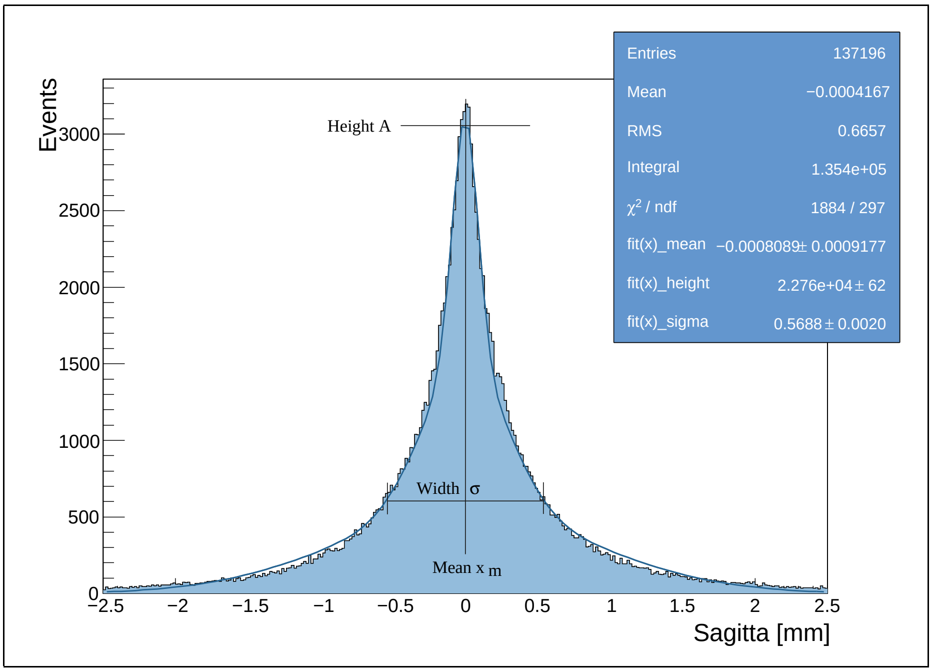

Figure 6: Measured sagitta distribution for 100,000 events. The blue curved corresponds to Equation 7

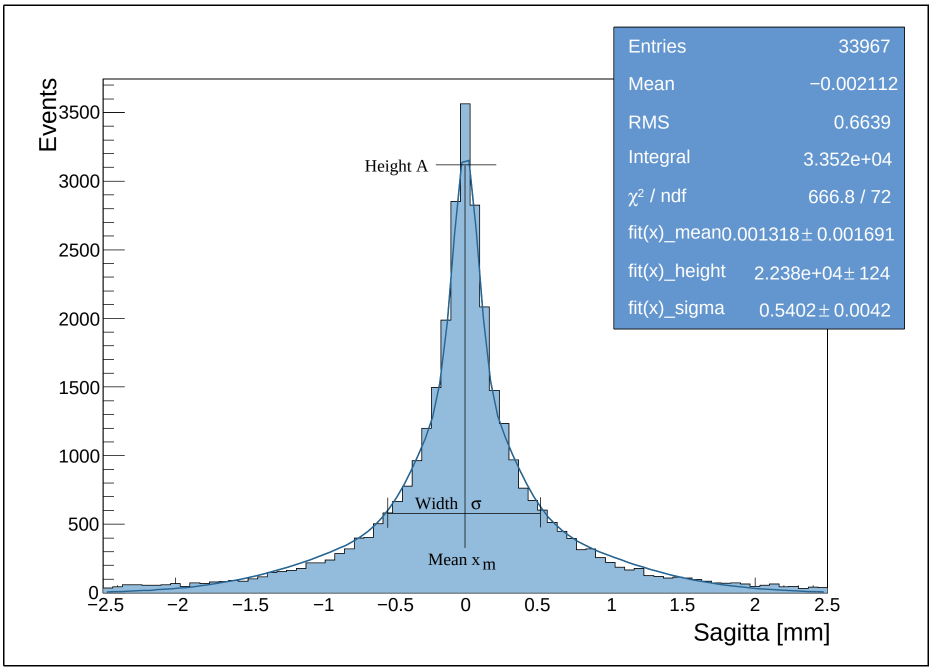

Figure 7: Simulated sagitta distribution of the cosmic ray measurement facility

is applied. A gaussian function can be used to describe the sagitta distribution of the simulation in each energy range separately as shown in Figure 9 for the energy region 2.0 GeV to 2.2 GeV and in Figure 10 for the energy region 5.0 GeV to 6.0 GeV. In Figure 11 the width of the fitted Gauss functions vs. the muon energies is shown. The single tube resolution begins to dominate the sagitta resolution at a muon energy of about 10 GeV, where the error from multiple scattering falls below $\sim 60\mu m$.

In order to describe the overall sagitta distribution we tried an Ansatz based on a sum of three gaussian functions (Function 4) to account for the wide range of muon energies. Each of the three gaussian should therefore describe one energy region while the sum of the three gaussians describes the overall sagitta distribution.

$$y(x, x_m, A_1, A_2, A_3, \sigma_1, \sigma_2, \sigma_3) = \frac{A_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-x_m)^2}{\sigma_1^2}} + \frac{A_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-x_m)^2}{\sigma_2^2}} + \frac{A_3}{\sqrt{2\pi}\sigma_3} e^{-\frac{(x-x_m)^2}{\sigma_3^2}} \tag{4}$$

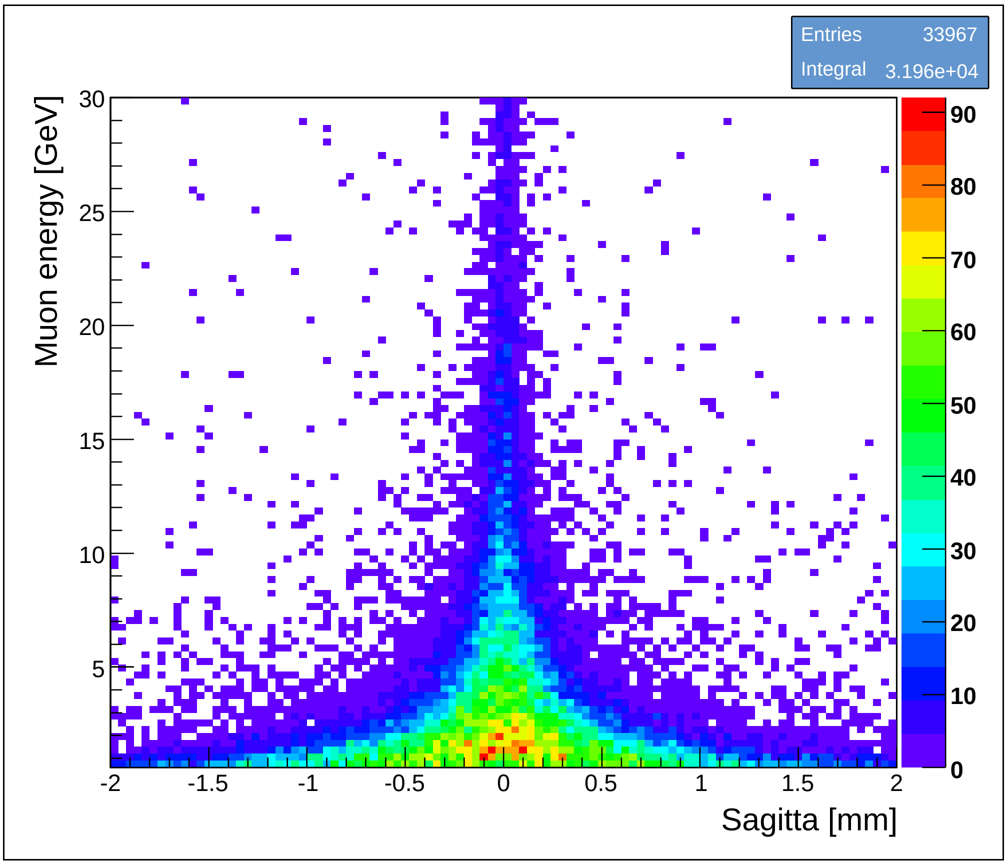

Figure 8: Simulation of the sagitta vs. corresponding muon energy. The color scale indicates the number of events in a given bin.

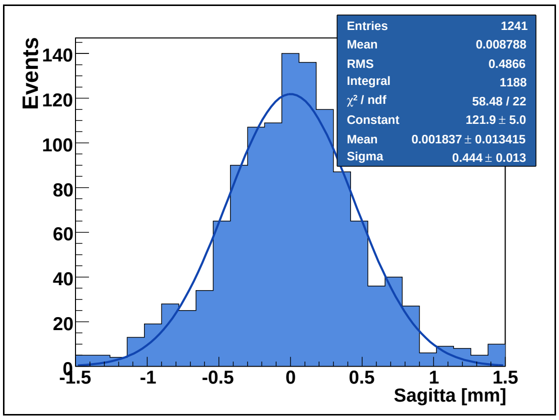

Figure 9: Description of the simulated sagitta distribution with a Gaussian function between muon energies of 2.0 GeV to 2.2 GeV

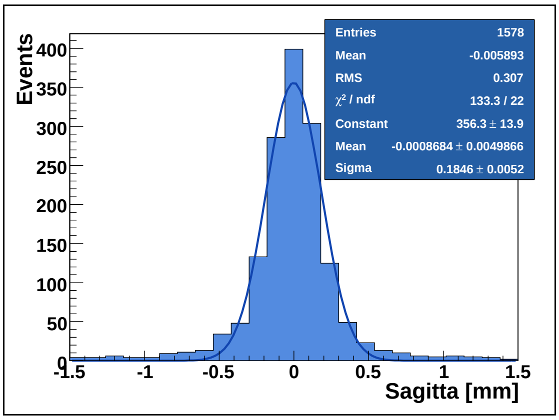

Figure 10: Description of the simulated sagitta distribution with a Gaussian function between muon energies of 5.0 GeV to 6.0 GeV

In a first step function 4 was fitted to the measured sagitta distribution of chamber BOS-4C-16. It is seen in Figure 12 that the choice of three Gaussian is sufficient to describe the measured sagitta distribution. The resulting values of the fitting parameters are shown in Table 1. This function has seven free parameters, and therefore it is difficult to directly compare the real and simulated sagitta distribution. Hence it is desirable to have a fitting function depending only on one width and one normalization parameter which describes the overall sagitta distribution. We assume that the ratios

$$a_1 = \frac{A_1}{A_2} = 0.4, \qquad a_3 = \frac{A_3}{A_2} = 0.9, \qquad s_1 = \frac{\sigma_1}{\sigma_2} = 0.28, \qquad s_3 = \frac{\sigma_3}{\sigma_2} = 2.84 \tag{5}$$

are constant for all studied sagitta distributions. The definition of

$$\sigma = n_\sigma \sigma_2, \quad A = A_2 \tag{6}$$

leads to a fitting function depending only on a single width parameter σ and a single normalisation parameter A. The parameter n_σ is arbitrary and can be choosen in such a way that an interval $[x_m \pm \sigma]$ contains 68% of all events. This definition allows a direct comparison between real and simulated data. The fitting function for the measured and simulated data is hence given by

$$y(x, x_m, A, \sigma) = \frac{1.57A}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_m)^2}{0.065\sigma^2}} + \frac{1.14A}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_m)^2}{0.77\sigma^2}} + \frac{0.35A}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_m)^2}{6.51\sigma^2}} \tag{7}$$

where x_m is the mean value, A the normalization and, σ the width of the distribution. These three parameters are fit parameters of the overall distribution. The parameter

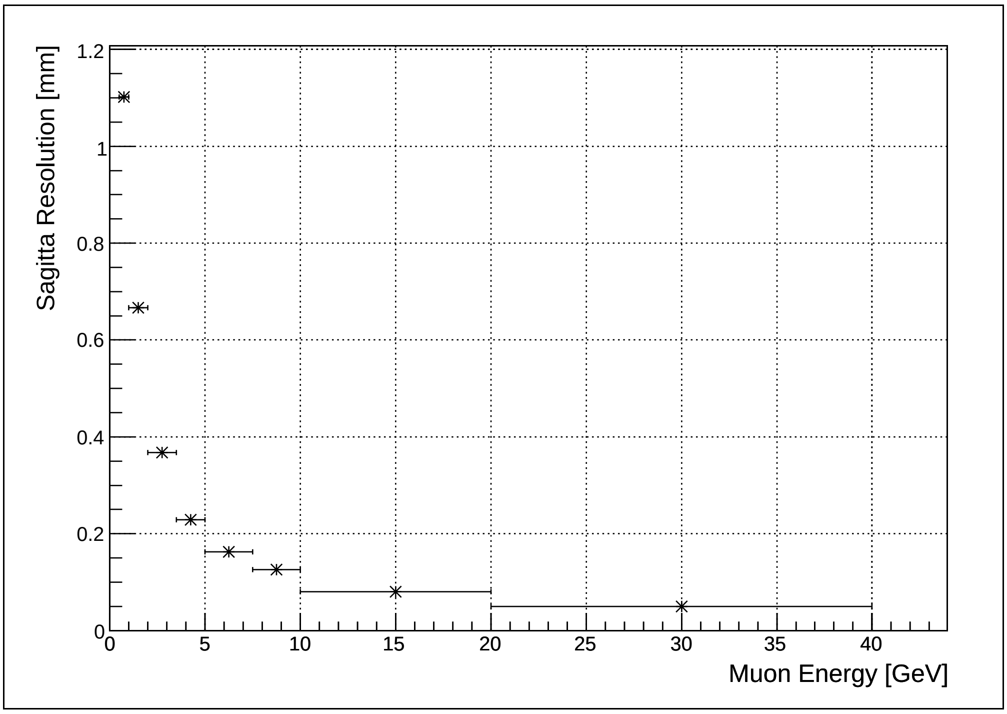

Figure 11: Reconstructed sagitta resolution of the CMF simulation in dependence of muon energy.

x_m	σ_1	σ_2	σ_3	A_1	A_2	A_3
0.0008	$0.14mm$	$0.50mm$	$1.42mm$	3563	8899	7877

Table 1: Fitted parameters of function 4 to the measured sagitta resolution of chamber BOS-4C-16

n_σ was set to 0.88. With these choises, the interval $[x_m \pm \sigma]$ contains 68% and the interval $[x_m \pm 2\sigma]$ 91% of all events.

The function of Equation 7 is drawn as blue line in Figure 6 and Figure 7. The width σ of the distribution was found to be $\sigma = 0.569\,mm$ on this grounds. The width σ_{data} describing the real data agrees within 5% to the width σ_{sim} for the simulated data, i.e.

$$\frac{\sigma_{data} - \sigma_{sim}}{\sigma_{data}} \approx 5\% \tag{8}$$

This result seems to confirm that the Geant4 simulation of multiple scattering, the material description and the energy spectrum of cosmic muons are implemented with reasonable accuracy in the CMF-simulation. In fact, our energy spetrum is quite soft, which leads to a high sensitivity to the simulation of multiple scattering and to material

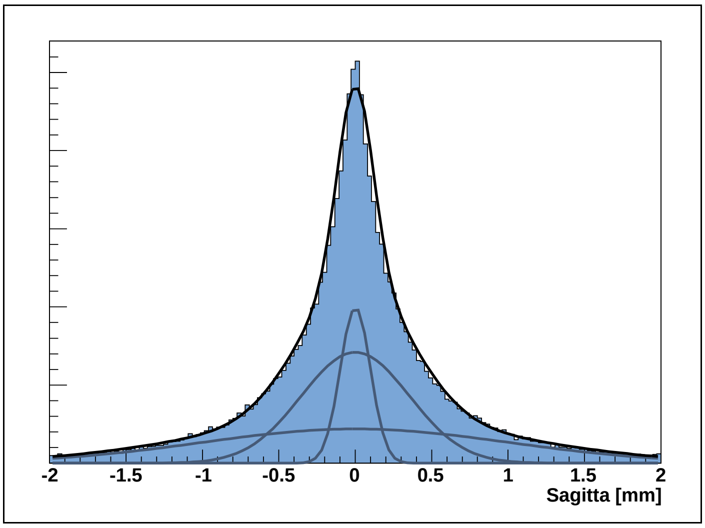

Figure 12: Fitting the overall sagitta distribution with the sum of three Gaussians from Equation 7.

description [2].

2.3.2 Sagitta studies with a single MDT-chamber

As discussed in the previous subsection the sagitta resolution of the whole measurement facility is dominated by multiple scattering effects and not by the single tube resolution[3]. A single MDT chamber, however, provides a good opportunity to study effects of the single tube resolution, because there is comparatively much less scattering material between the first and the last measured point. (Figure 13). Furthermore, the sagitta resolution of a single MDT chamber could be an important tool for alignment and B-field studies.

The same procedure as in the previous section is used: In a first step, the simulated sagitta distribution is studied in order to find a suitable function which describes the measured overall sagitta distribution. Gaussian functions were fitted to the simulated sagitta distributions for several energy regions. The energy dependence is much smaller compared to the study of the whole measurement facility as shown in Figure 14. It can be concluded that multiple scattering effects are not dominating for the sagitta measurement within a single MDT-chamber above 1 GeV.

The sagitta distributions of the real and simulated measurements can be discussed as shown in Figure 15 and Figure 16, respectively. Because of the small energy dependence

[2] Also other functions (single gaussian function, Breit-Wigner function) where used to describe the overall sagitta distribution in real data and simulation which led to no significant improvement

[3] A theorical estimation of the expected sagitta resolution of a single MDT chamber is given in the appendix B.

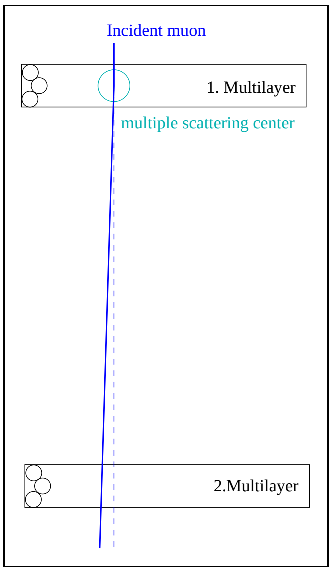

Figure 13: Illustration of multiple scattering in a single MDT chamber

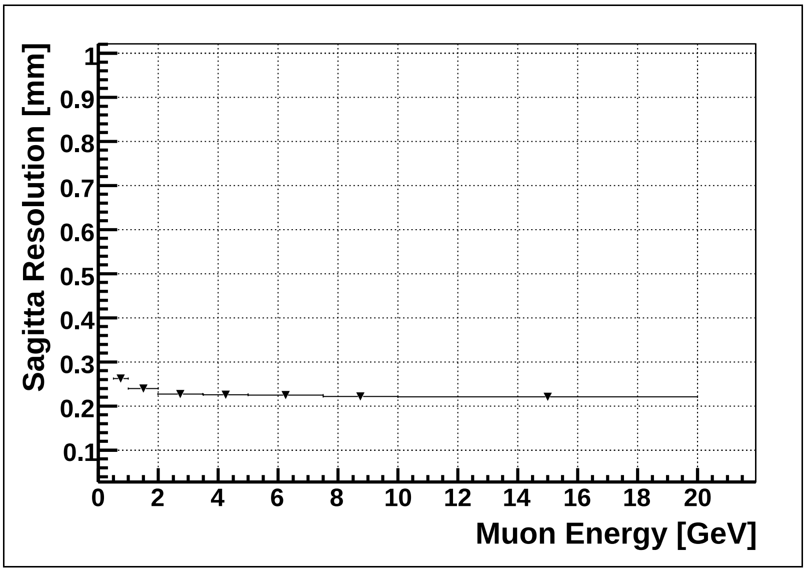

Figure 14: CMF-simulation: Sagitta resolution of one MDT-chamber vs. muon energy

it is sufficient in this case to use a single Gaussian function with constant background to describe the overall sagitta distribution. The sagitta resolution of a single MDT-chamber was found to be $\sigma = 0.237\,mm$. As shown in Figure 16 and Figure 15 the width parameters of the fitting Gaussian function match within the statistical errors, which is a convincing indication that the single tube resolution and other effects like the rt-relation, the flight time of the muons, the signal propagation along the wire or wire-sagging on this level of simulation are well understood.

2.4 Alignment

In general the nominal position of the test chamber in the measurement facility differs from its real position. This leads to systematic errors during reconstruction since wrong wire positions and therefore wrong centers of the drift circles are assumed. The mean value of the sagitta distribution in z- and y-direction is in principle sensitive to displacements of the test chamber. A displacement in y-direction will affect the mean value of the sagitta distribution strongly since most of the incident muons are nearly perpendicular to the y-axis. A displacement in the z-direction becomes only detectable with muons that have a large angle of incidence.

The test chamber has been displaced independently in z- and y- direction within the simulation, in order to get a quantitative relation between the displacement in both directions and the mean value of the sagitta distribution. Subsequently the simulated data have been analysed and the mean value of the sagitta distribution was calculated.

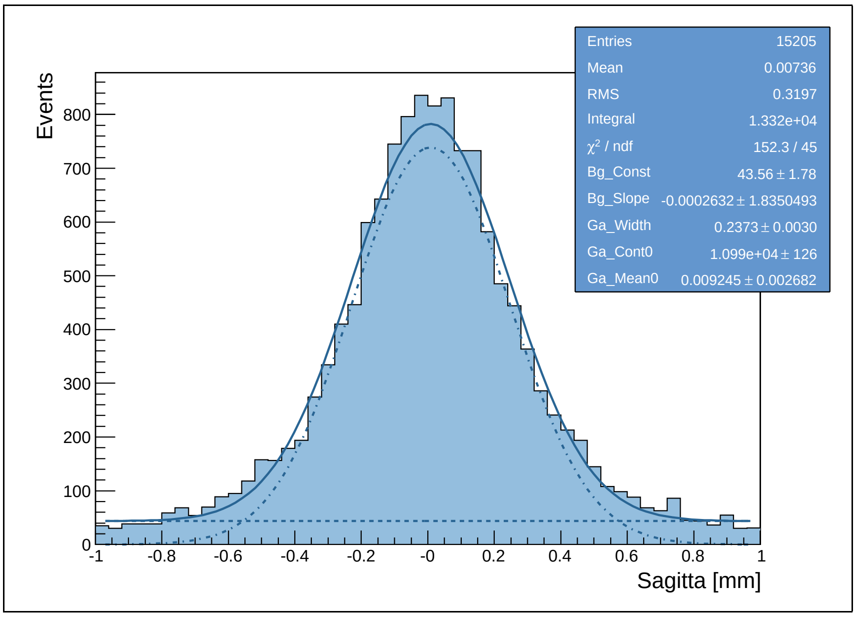

Figure 15: Measured sagitta distribution of one MDT-chamber. The dotted lines describe the constant background and the pure gaussian part. The full line corresponds to the sum of the two dotted lines.

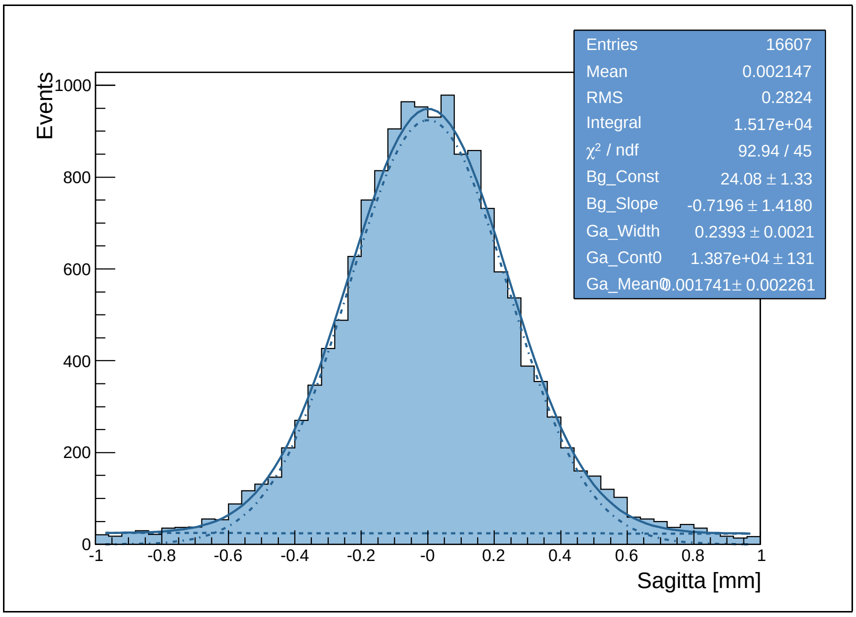

Figure 16: Simulated sagitta distribution of one MDT-chamber. The dotted lines describe the constant background and the pure gaussian part. The full line corresponds to the sum of the two dotted lines.

The results are shown in Figure 17 for the displacement in y-direction and in Figure 18 for the displacement in z-direction.

The relation for both directions can be described by a linear function between the mean value and the displacement which is valid at least for small displacements. The linear functions are given by

$$x_{m,y} = (0.393 \pm 0.001) y_{dis} + (0.005 \pm 0.0019) \, [\mathrm{mm}] \tag{9}$$

$$x_{m,z} = (-0.184 \pm 0.001) z_{dis} + (0.004 \pm 0.0024) \, [\mathrm{mm}] \tag{10}$$

where y_{dis} is the displacement of the chamber in y-direction and z_{dis} the displacement of the chamber in z-direction. Naively, it might be expected that the displacement of the chamber translates fully into a shift of the mean value of the sagitta distribution by the same amount. However, the defining distance for the definition of sagitta starts and ends in the center of the upper and lower reference chamber as illustrated in Figure 5 and therefore the misaligned chamber is part of the overall fitting procedure. Furthermore, the definition of sagitta in this study is more complex because there are not only 3 measurement points for the definition of sagitta but on average 18. This explains the deviation of the slope in Equation 9 from 1.

A full and powerful set of alignment algorithms was developed for the cosmic ray measurement facility and have been applied before reconstructing the muon tracks [2]. The precision of these alignment algorithms could be tested with Equation 9 and 10, since a misaligned test-chamber results in a displacement of the mean of the sagitta distribution. The constant offset terms of $4\,\mu m$ resp. $5\,\mu m$ in both equations can be neglected for displacements above $O(20\ \mu m)$ in y-direction and above $O(50\ \mu m)$ in z-direction.

The mean of the fitting function 7 of the sagitta-distribution for the chamber BOS-4C-16 was found to be $-0.81\ \mu m \pm 0.9\ \mu m$ (Figure 6). Using this value as $x_{m,y}$ and $x_{m,z}$ in Equation 9 and Equation 10, respectively, the maximum displacements y_{dis} and z_{dis} in both directions can be calculated.

This leads to the conclusion that the alignment algorithms work within a precision of $O(10\ \mu m)$ in y- and $O(30\ \mu m)$ in z-direction. This is a conservative estimation, since these values are dominated by the constant offset terms in Equation 9 and 10 which are expected to be zero for larger statistics. The width of the sagitta distribution is not significantly altered by a small displacement of the chamber.

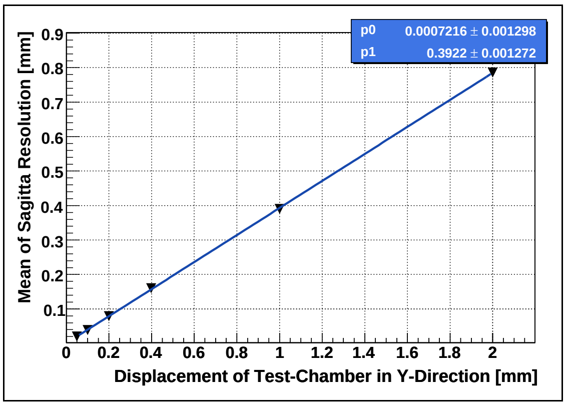

Figure 17: Displacement of the test-chamber in y-direction vs. mean value of sagitta distribution

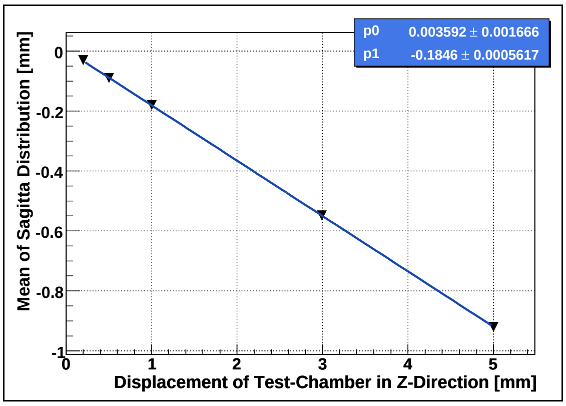

Figure 18: Displacement of the test-chamber in z-direction vs. mean value of sagitta distribution

3 H8/ATLAS Studies

3.1 Setup of the simulation

The CMF-Simulation describes well the response of MDT chambers to cosmic muons, as shown in the previous section. It is a useful exercise to use the simulation validated by the cosmic ray measurement facility and extend it for the setup at H8 or ATLAS.

In order to create a H8-like simulation three similar MDT-chambers were placed at distances corresponding to the ones at H8 [4]. The incident muons are assumed to come from a point-like source with a small opening angle. The muons were generated with energies between 30 GeV and 300 GeV as in the H8-testbeam.

The trigger signals in the real H8-assembly were measured by Resistive Plate Chambers (RPC). Multiple scattering effects due to the RPCs could not be neglected. Therefore blocks with the dimensions of the RPCs which consists out of the respective materials were introduced in the simulation.

Figure 19 shows the measured and simulated sagitta resolutions in the Geant4 simulation. The sagitta distributions within a small energy region can be described with a standard Gaussian function (Figure 20). The measured sagitta resolutions for different energies in the H8-testbeam are also shown in Figure 19 [5]. The comparison of the results shows that our simulated sagitta resolution is slighty worse than the measurements of the H8-testbeam. Keeping in mind the rough approach of this simulation it is nevertheless a quite good agreement.

The setup of the H8-testbeam differs only by the distances between the MDT-chambers

[4]The distances between the centers of the chambers are 2199 mm and 2514 mm and correspond to the center of the chambers

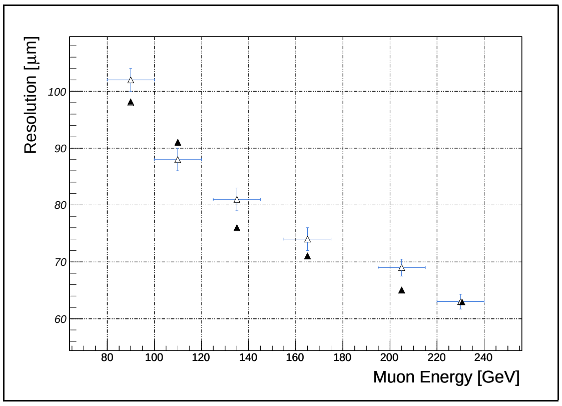

Figure 19: Simulated sagitta resolution (open triangle) and measured sagitta resolution (full triangle) in the H8 setup as a function of the muon energy [4]

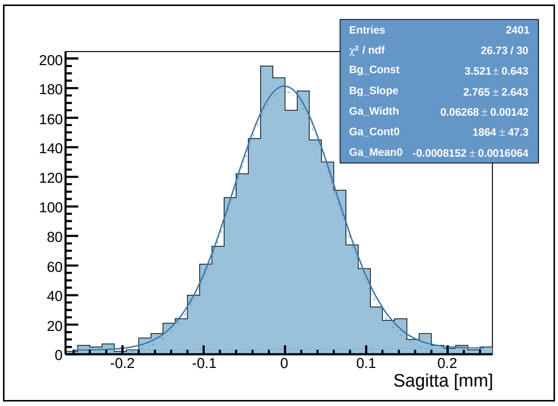

Figure 20: Sagitta resolution of the simulated H8 setup for muons with an energy between 220 GeV and 240 GeV.

from the final ATLAS layout. To modify the CMF-Simulation according to the ATLAS geometry the distances of the chambers where chosen to be 2580 mm and 3550 mm. The RPCs were included in the same manner as it has been done in the H8-simulation. The energy spectrum of the generated muons was chosen to be equally distributed between 5 GeV and 1 TeV.

The reconstructed sagitta resolution in the modified CMF-Simulation versus muon energy is shown in Figure 21. The energy dependence of the sagitta resolution in the case of a perfect single tube resolution is presented in Figure 22. It is clearly visible that the single tube resolution starts to dominate multiple scattering effects at a muon energy of roughly 80 GeV in agreement with the TDR[5] [3].

Using Equation 3 the sagitta resolution can be translated into the momentum resolution. For simplicity we assume a constant magnetic field with a field-strength of 0.5 T. Using this value the energy dependent momentum resolution of the modified CMF-Simulation can be calculated, which is shown in Figure 23. One should note that this study does not include energy loss fluctuations, which dominate the resolution at small energies at the ATLAS Muon Spectrometer. Furthermore, a perfect alignment of the detector was assumed.

Nevertheless, the resolution of the ATLAS Muon Spectrometer due to the single tube resolution and due to multiple scattering effects can be compared with the values given in the TDR [3]. The high energy regime provides a possibility to compare the single tube resolution since this is the dominating effect at these energies. For 1 TeV a momentum resolution of 8.8% is expected in the ATLAS detector. The simulation in this study predicts a momentum resolution of 8.7% for muon energies of 1 TeV, which

[5] Chapter 5.1.1

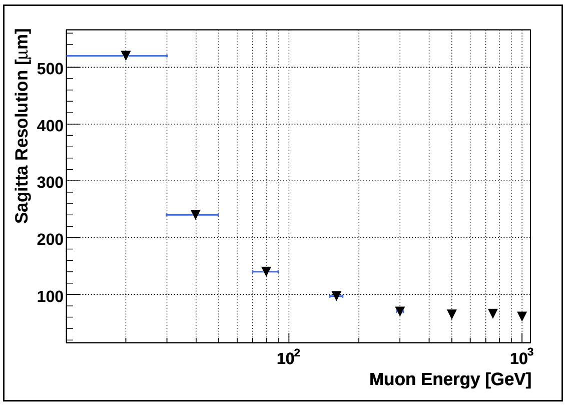

Figure 21: Sagitta resolution of simulated ATLAS setup in dependence of the muon energy with an average single tube resolution of about 100 μm

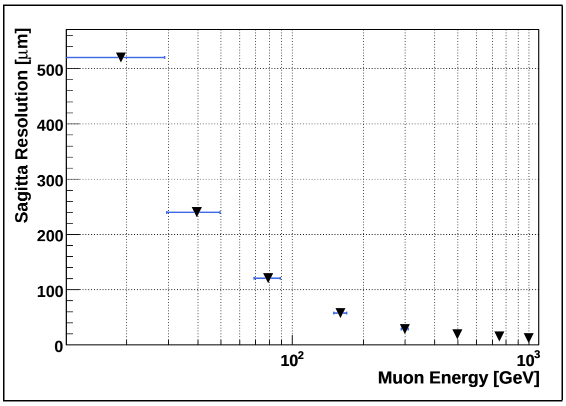

Figure 22: Sagitta resolution of simulated ATLAS setup in dependence of the muon energy with infinite single tube resolution

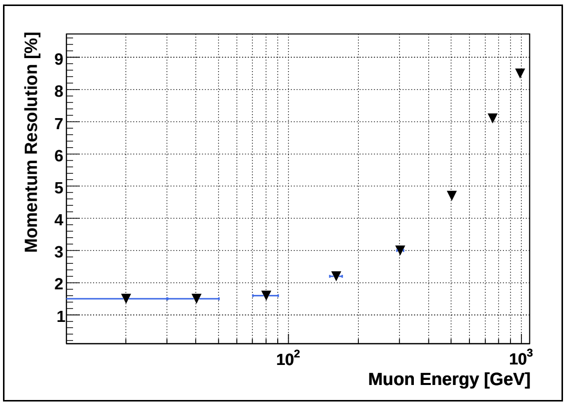

Figure 23: Momentum resolution of simulated ATLAS setup in dependence of the muon energy

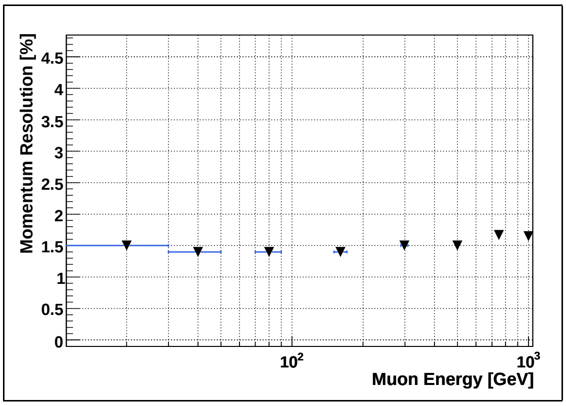

Figure 24: Momentum resolution of simulated ATLAS setup in dependence of the muon energy with infinite single tube resolution

is a good agreement but should not be over interpreted, given the differences in the details of simulation, digitization and magnetic field between the CMF-simulation and the TDR studies [3]. Nevertheless this is a good cross-check between these two studies.

At lower energies multiple scattering becomes the dominating effect for the momentum resolution. In order to investigate these in detail the simulation was run with the assumption of a perfect single tube resolution. This leads to the momentum resolution shown in Figure 24, which only includes multiple scattering effects. We obtain a resolution due to multiple scattering effects of about 1.5%. The results of the TDR[6] [3] range from $1.6 - 2.2\%$.

[6] Chapter 5.1.1

3.2 Alignment at the ATLAS Muon Spectrometer

Effects of misalignment on sagitta can also be studied using the CMF-Simulation, which was modified corresponding to the ATLAS geometry. Within this setup a direct proportionality between a misalignment of the middle chamber in y-direction and the mean value is expected. The results of this study are shown in Figure 25 and the dependence can be described by the phenomenological function

$$x_{m,y} = 1.128\, y_{dis} - 0.002\,[\mathrm{mm}] \tag{11}$$

which agrees with the expectation of a proportionality factor close to unity for the MDT geometry in the ATLAS detector. Hence, we see a strong effect on the mean of the sagitta distribution.

3.3 Effects of wire-displacement

Another important aspect is the precision of the wire positions. The effect on the sagitta resolution has been studied both for a single MDT-chamber and for the ATLAS-geometry. In order to study the effect of the precision of wire position on the sagitta resolution each wire-position was shifted by δy in y-direction and δz in z-direction in

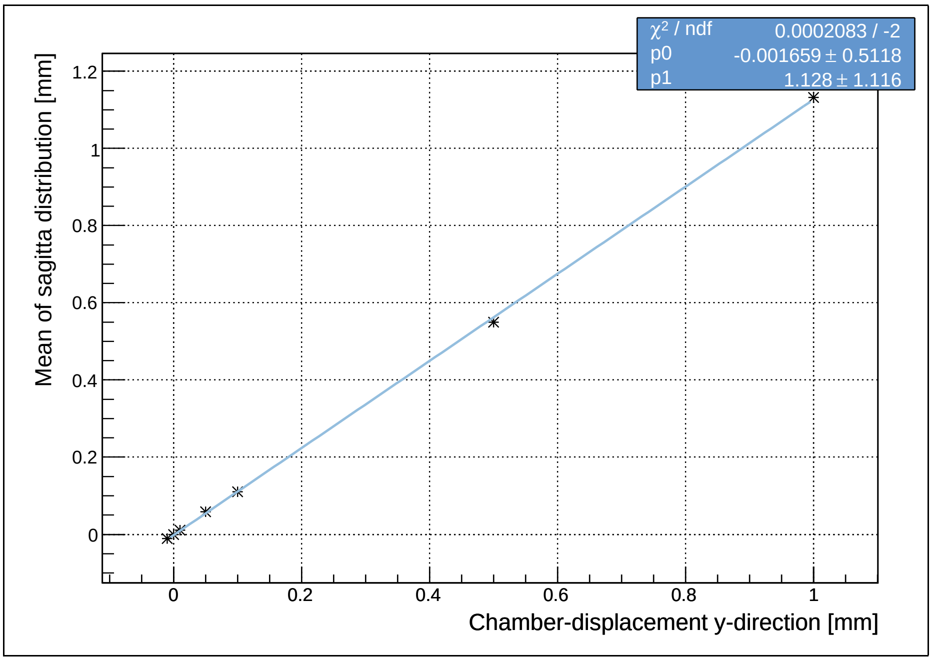

Figure 25: Impact of the displacement of the middle MDT-chamber on the mean value of the sagitta distribution

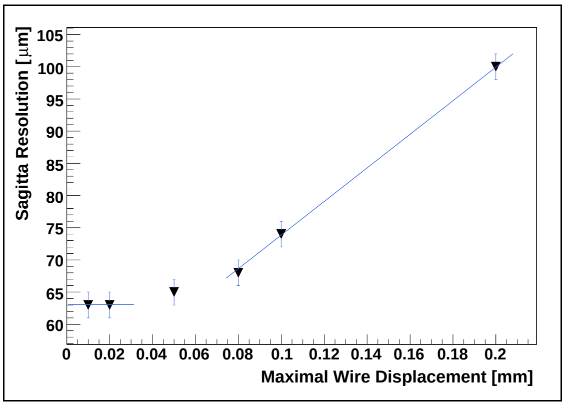

Figure 26: Sagitta resolution of the simulated ATLAS setup in dependence of the maximal displacement of the wire positions

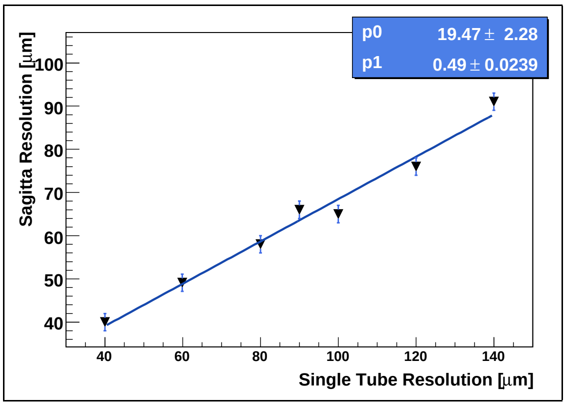

Figure 27: Sagitta resolution of the simulated ATLAS setup in dependence of the single tube resolution

the simulation. The displacement parameters δy and δz were set randomly to values between $[-d, d]$, simulating the deviations between the nominal and the true wire positions. 10.000 events were generated and reconstructed for several value of d.

The dependence of the sagitta resolution on the maximal displacement d is shown in Figure 26 for the modified CMF-Simulation, where muons with an energy of 1 TeV have been used to study the effect.

As expected, the sagitta resolution is dominated by the single tube resolution for small displacements of the nominal wire position. For values of d around 0.06 mm the wire displacements start to dominate the sagitta resolution, for larger values of d the dependence becomes linear. This behaviour can be phenomenologically described by

$$\sigma_{sagitta} = \sqrt{a^2 + b^2 d^2} + c\,[\mathrm{mm}]\,. \qquad (12)$$

The single tube resolution is described by $a + c$, while b is the proportional factor of the linear dependence. This functions is fitted to measured data. In case of the ATLAS-setup the fitted values are

$$a_{ATLAS} = 0.02\,\mathrm{mm}\,, b_{ATLAS} = 0.25\,\mathrm{mm}\,, c_{ATLAS} = 0.043\,\mathrm{mm} \qquad (13)$$

and for a single MDT-chamber

$$a_{MDT} = 0.13\,\mathrm{mm}\,, b_{MDT} = 1.01\,\mathrm{mm}\,, c_{MDT} = 0.12\,\mathrm{mm} \qquad (14)$$

3.4 Effects of the single tube resolution

Last but not least we studied the effect of the single tube resolution on the sagitta resolution. Figure 27 shows the sagitta resolution for the modified CMF-Simulation in dependence of the single tube resolution. Muons with energies between $0.95 - 1.0\,\text{TeV}$ have been used to study this dependence in the adapted CMF-Simulation to avoid large multiple scattering effects. The simulated data in Figure 27 can be fitted by the linear function

$$\sigma_{sagitta}\,[\text{mm}] = 0.5\sigma_{st}\,[\text{mm}] + 0.02\,[\text{mm}]\,. \tag{15}$$

where σ_{st} describes the single tube resultion.

4 Conclusion

The cosmic ray measurement facility offers an excellent opportunity to compare real data of ATLAS-components with a Geant4 simulation. The sagitta resolution predicted by a Geant4 simulation of the entire measurement facility agrees with the real data to within 5%. This measurement, which uses a setup of three MDT-chambers, is dominated by multiple scattering effects. In contrast, the sagitta measurement based on a single MDT chamber is dominated by single tube resolution. We find good agreement between the simulated sagitta resolution of a single MDT-chamber (BOS-Type) and the real data within the statistical uncertainties. Hence the two main input parameters on the sagitta resolution (single tube resolution and multiple scattering) could be verified to be simulated correctly.

Several aspects of the sagitta resolution were studied with this validated simulation of the MDT-chamber: The alignment algorithms for the MDT chambers in the cosmic ray measurement facility could be confirmed with a precision of $4\mu m$ in y-direction and $8\mu m$ in z-direction.

Furthermore, this simulation was extrapolated both to the setup of the MDT chambers for the H8 testbeam measurements and to the final MDT setup in ATLAS. We found good agreement between our adapted simulation on the one hand and the sagitta resolution measured in H8 and the momentum resolution as presented in the ATLAS TDR, respectively. Finally, the impact of single wire displacements and the single tube resolution on the sagitta resolution in the ATLAS setup was studied.

A Theoretical Estimation of Sagitta Resolution at the Cosmic Ray Measurement Facility

The expected sagitta resolution of the CMF δs_{CMF} is determined by the single tube resolution and multiple scattering effects. The overall resolution is therefore given by

$$\delta s_{CMF} = \sqrt{\delta s^2_{Drifttube} + \delta s^2_{MultipleScattering}} \tag{16}$$

The magnitude of s_{DT} can be estimated with the *Glückstern* formulas [6]. A parabola

$$y = \frac{1}{2}ax^2 + bx + c \tag{17}$$

can be fitted to N equidistant measurement points x_i, where uncorrelated errors ϵ on each single measurement are assumed. The errors on the parameters a, b and c are then given by

$$< a^2 >= \frac{\epsilon^2}{L^4} A_N \tag{18}$$

$$< ba >= -\frac{1}{2}\frac{\epsilon^2}{L^3} A_N \tag{19}$$

$$< b^2 >= \frac{\epsilon^2}{L^2} B_N \tag{20}$$

where L is the projected trajectory length $L(x_0 = 0, x_n = 1)$. For $N > 10$ we find for the parameters A_N und B_N:

$$A_N = \frac{720}{N+5} \tag{21}$$

$$B_N = \frac{192}{N+4} \tag{22}$$

This procedure can be applied to the CMF. The six multilayers are equidistant to a good approximation and the single tube resolution is about $100\,\mu m$ (averaged over all radii) which results in a estimated multilayer resolution of $100\,\mu m \sqrt{3} \approx 60\,\mu m$. The projected muon trajectory has an estimated length of $L = 1080\,mm$ for vertical incident muons since we expect small sagittae. Equations 18 and 21 lead then to the estimated error on the opening parameter α of the parabola

$$< \alpha^2 >= 1.7 \times 10^{-7} \frac{1}{m^2} \rightarrow \alpha = 4.1 \times 10^{-4} \frac{1}{m} \tag{23}$$

which corresponds to a sagitta resolution of

$$\Delta s = 120 \mu m. \tag{24}$$

This value is only due to the single tube resolution. It is expected that the overall sagitta resolution of the Cosmic Ray Measurement Facility (CMF) is dominated by multiple scattering effects since the muons are low energetic and transverse a relative large amount of aluminum. Each drift tube has a wall thickness of $0.4\,mm$.

To estimate the contribution of δs_{MS}, we use the formula for calculating the multiple scattering angle

$$\theta_0 = \frac{13.6 MeV}{p \beta c} z_c \sqrt{\frac{s}{X_L}} [1 + 0.038 \ln \left(\frac{s}{X_L} \right)] \tag{25}$$

where p is the momentum of the scattered particle, βc its velocity, z_c its charge and s the thickness of the transversed material. The parameter X_L is the radiation length of the scattering medium. This formula is based on the assumption that the muons are only scattered once inside the CMF. The middle part of the CMF is therefore approximated by an aluminum block with an effective thickness of one MDT-chamber which leads to the situation described in Fig.28. For this case we choose the parameters $X_L = 0.089m$, $s = 0.4\,mm \times 24$ and $p \approx 4GeV$ which leads to a scattering angle of $\theta_0 = 1.03 \times 10^{-3}$. The overall estimated resolution is

$$\Delta s = 0.565 \frac{\theta_0}{2} \approx 290 \mu m. \tag{26}$$

The overall sagitta resolution of the CMF can be calculated with Equation 16 and has a numerical value of $310 \mu m$. This estimation is based on several approximations and should be considered with care. Nevertheless it can be concluded that the overall sagitta resolution is clearly dominated by the multiple scattering contribution.

B Theoretical Estimation of Sagitta Resolution of a single MDT chamber

In this section we estimate the sagitta resolution of a single MDT chamber and ignore the information of the other two chambers. We expect an average angle variation of $\theta_0 \approx 5.4 \times 10^{-4}$ of an incident muon in the upper multilayer due to multiple scattering effects. A single tube resolution of $40 \mu m$ would be needed to measure this effect, which

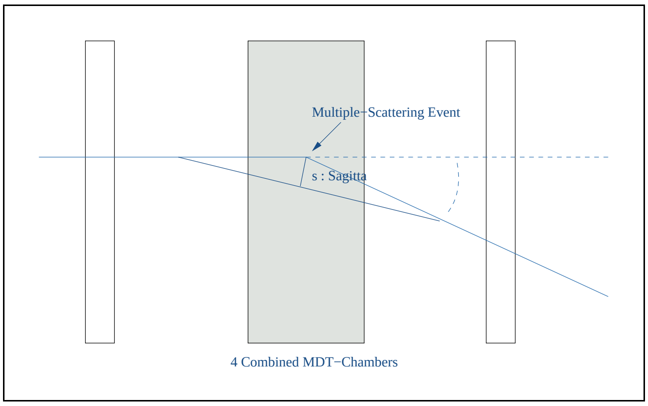

Figure 28: Approximation of the CMF for the estimation of multiple scattering effect

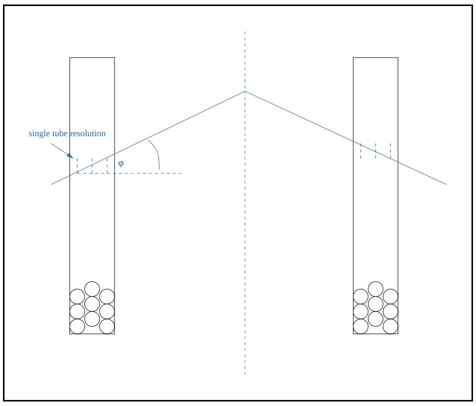

Figure 29: Approximation of the sagitta calculation with a single MDT chamber

is better than the actual resolution. Therefore the sagitta measurement within a single MDT chamber is not sensitive to multiple scattering effects.

To estimate the sagitta resolution in this case, one has to study the parabolas which can be fitted within the errors of the six measurement points. A parabola can be described by two straight lines in a first approximation as sketched in Figure 29.

This leads to an estimated sagitta resolution of

$$\Delta s = \frac{200\,mm \times \sigma_e}{90\,mm} \tag{27}$$

where σ_e is the single tube resolution, $90\,mm$ the thickness of one multilayer and $200\,mm$ the distance to the middle of the MDT chamber. A single tube resolution of $100\mu m$ leads to an expected sagitta resolution of $200\mu m$ which agrees well with the measured value.

References

[1] *A Cosmic Ray Measurement Facility for ATLAS Muon Chambers*, Biebel, O et al, ATLAS Note ATL-MUON-2004-004, 30. Jul. 2003

[2] *Integration of calibration and alignment procedures for the reconstruction of muons in the ATLAS*, M.Obermaier, Diploma thesis, LMU München, Germany, (2005).

[3] *Technical Design Report of the ATLAS Muon Spectrometer, Geneva, (1997)* .

[4] *Introduction to GeoModel*, A.Salzburger, https://uimon.cern.ch/twiki/bin/view/Atlas/GeoModel, (2005).

[5] *System Test of the ATLAS Muon Spectrometer in the H8 Beam at the Cern SPS*, E.Ezion,S.Rosati, IEEE Nuclear Science Symposium, Rome, Italy, (2004).

[6] *Uncertainties in track momentum and direction, due to multiple scattering and measurement errors*, R.L. Gluckstern, Nucl.Instrum.Methods.24,381-389,(1963).