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PHASE MATCHING APPLICATION IN HARD X-RAY REGION OF HePS*

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Abstract

For the 6 meters long straight-section of HEPS, a double collinear double-cryogenic permanent magnet undulator (CPMU) structure is designed for high energy photon users to achieve higher brightness. Angular profiles of radiation produced by the double undulator configuration has been derived analytically. The efficiency of phase shifter on improving the brightness of double-CPMU is therefore evaluated with the beam energy spread is taken into account.

INTRODUCTION

In the first phase of HEPS construction, a total of 14 ID-based beamlines are required for constructed, of which 7 are based on in-vacuum undulators.[1] In order to satisfy the requirement of high-energy users, the phase error of these Ids should be reduced to below 2-3 ° especially for the application of harmonics higher than 9th. Therefore, the maximum length of in-vacuum undulator has to be less than 3 meters due to the limitations of the current manufacturing process. This leads to the necessity of installing two undulators in series on one 6 meters long straight section. In this case, if is it necessary to install an additional phase shifter between the two undulators and its effects on the radiation performance when considering the real beam parameters become a significant problem should be investigated.

To specify the performance of a synchrotron radiation (SR) source, photon flux density in the 4D phase space i.e. brilliance is the most common figure of merit. In general case, brilliance should be first calculated by the method of Wigner function [2] and then convoluted with the electron beam distribution in phase space to include the effects of emittance and energy spread. A widely used model to calculate the radiation brilliance from a single undulator is Gaussian approximation in the case of Gaussian electron beam distribution [3] which could help to simplify this calculation process. The only difference should be considered is that energy deviation of electrons will change the phase slip between the two undulators. Moreover, in most practical cases, it is sufficient to use on-axis brilliance to evaluated the SR performance. Therefore, we only calculate the on-axis brilliance in this paper.

RADIATION MODEL OF DOUBLE UNDULATOR CONFIGURATION

Radiation Model of Double-Undulator

To start with the calculation of the spectra of the combination of two undulators with the phase shifter between them analytically, we illustrate the whole structure in Fig. 1 [4].

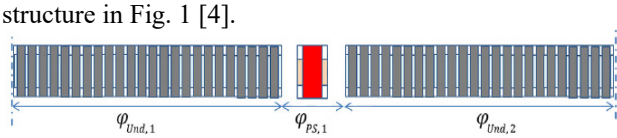

Figure 1: structure of the double undulator configuration.

Where φund,1 and φund,2 represent to the phase slip in each undulator. φps is the phase slip between the two undulator. We ignore the front-ends of the two undulators for it only cause an additional phase slip which contain in φps. The radiation field then is expressed by the sum of the two complex field emitted from both undulators as

$$E_{Double}(\omega,\theta,t)=\left[1+e^{i(\varphi_{Und,1}+\varphi_{PS})}\right]E_{Single}(\omega,\theta,t),$$

Where Esingle denote the field emitted from a single undulator. And the on-axis radiation intensity is written as

$$I_{\mathrm{Double}}(\delta)=\left|e^{i\left[(1+\delta)n\varphi+\frac{2Nn\pi}{1+\delta}\right]}+1\right|^2 I_{Single}(\delta)$$

$$\propto 2\{1+\cos[(1+\delta)n\varphi-2Nn\pi\delta]\}Sinc^2(Nn\pi\delta).$$

Where δ= Δ ω/ωn is regarded as the detune factor. It is important to note that Δ ω is the offset of the nth harmonic energy due to the electron energy deviation. It is differ from an arbitrary energy offset compare with the reference harmonic energy. N is the undulator periods number and φ represent the phase slippage between the two undulator without electron energy deviation.

On-axis Angular Flux Density

If we assumed the distribution of the energy spread is Gaussian with the RMS σε, it caused the δ obeys the Gaussian distribution with the RMS 2σε. The on-axis angluar flux density is then a convolution shown as below.

$$I_{total}=\frac{1}{2\sqrt{2\pi}\sigma_\varepsilon}\int_{-\infty}^{\infty}e^{-\frac{\delta^2}{8\sigma_\varepsilon}}I_{Double}(\delta)d\delta$$

$$=\frac{1}{\sqrt{2\pi}\sigma_\varepsilon}(I_1+I_2)$$

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$$I_1 = \int_{-\infty}^{\infty} e^{-\frac{\delta^2}{8\sigma_\varepsilon}} Sinc^2(Nn\pi\delta)d\delta$$

$$I_2 = \int_{-\infty}^{\infty} e^{-\frac{\delta^2}{8\sigma_\varepsilon}} \cos[(1+\delta)n\varphi - 2Nn\pi\delta]Sinc^2(Nn\pi\delta)d\delta$$

It is indicated that Itotal can be divided into two parts. The first part is the intrinsic angluar flux density emitted from each undulator and the second part is the contribution of coherence relate to the phase slippage φ.

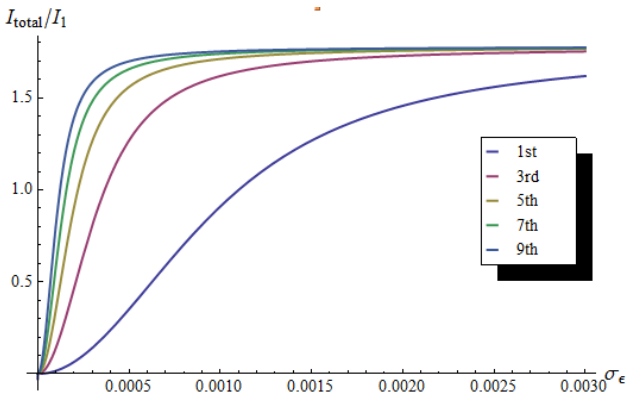

Figure 2: gain of on-axis angular flux density of different harmonics vary with energy spread on the condition of phase mismatched.(N=100 in both case)

We introduce a gain factor defined by I_{total}/I_1 in order to tell if the phase shifter is necessary more intuitively. If the radiation from the two undulators was completely incoherence, the gain factor would be equal to 2. That means the total flux density is the sum of that from two undulators simply i.e. the phase shifter has little effects on the gain. It is clearly shown in Fig. 2 that even at the range of large energy spread the gain factor is not tend to 2 which indicated the significance of phase matching by the phase shifter.

Angular Distribution of Photon Flux Density

We next investigate the angular flux distribution of double undulator configuration. We only interesting about the photon energy equals to the resonance energy without any deviation. In this case, the detune factor δ can be rewrite as a function of electron energy γ and observe angle θ given by

$$\delta(\theta,\gamma) = \frac{\omega_n(\gamma_0,0) - \omega_n(\gamma,\theta)}{\omega_n(\gamma,\theta)}$$

$$\omega_n(\gamma,\theta) = \frac{2\gamma^2}{\left(1+k^2/2+\gamma^2\theta^2\right)}\frac{2\pi c}{\lambda_u},$$

where λ_u is the period length of undulator and γ_0 represent to the energy without any offset. If we assume the offset of the electron energy $\Delta\gamma \ll \gamma_0$ it could be expressed the detune factor $\delta(\Delta\gamma,\theta)$ by

$$\delta(\Delta\gamma,\theta) = \frac{\gamma_0^2\theta^2}{1+k^2/2} - 2\frac{\Delta\gamma}{\gamma_0} \quad .[5]$$

Substituted this expression into I_{double} derived above, angular flux distribution of nth harmonic is able to obtain analytically.

Figure 3 shows the comparison of the analytic results with the SPECTRA result.

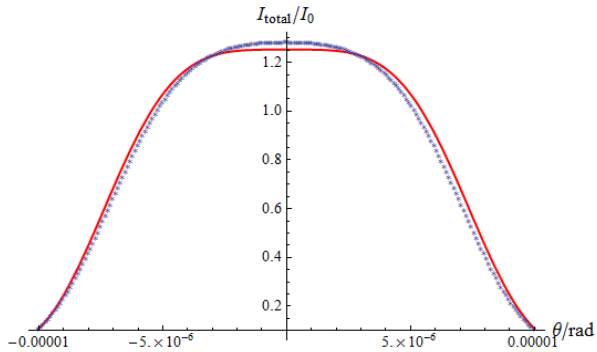

Figure 3: comparison of analytical results of angular flux density distribution for the 3rd order harmonic with the SPECTRA result. The dash line represent to the numerical results calculated by SPECTRA and the red line represent to the analytic result.

It is also to see that energy spread extend the angular distribution range of radiation central cone according to Fig. 4. The RMS angular divergence can be derived from the angular flux distribution analytical as

$$\sigma_{r'}^2 = \frac{\int_{-\infty}^{\infty}\theta^2 I_{total}(\theta,\sigma)d\theta}{\int_{-\infty}^{\infty} I_{total}(\theta,\sigma)d\theta} = \frac{DI_{total}}{TI_{total}}$$

Substitutes the expression of I_{total} into the form above, the RMS angular divergence is obtained. Note that in the case of phase mismatched, the angular distribution of angular flux density is like a ring, only in the case of phase matched it makes the expressions above meaningful. The result of RMS angular divergence is shown in the Fig. 5.

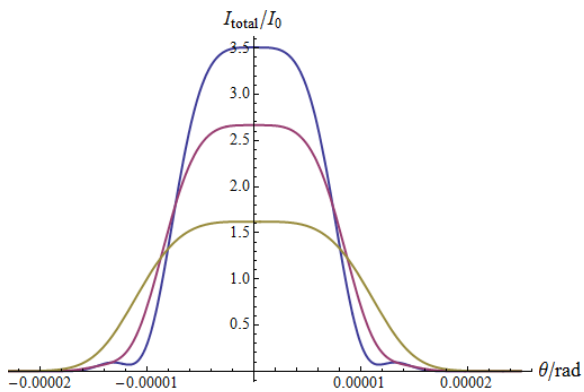

Figure 4: angular flux distributions with different energy spreads of first harmonic. Where Blue line, red line and

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yellow line represent to σε=0.05%, σε=0.1%, σε=0.2% respectively.

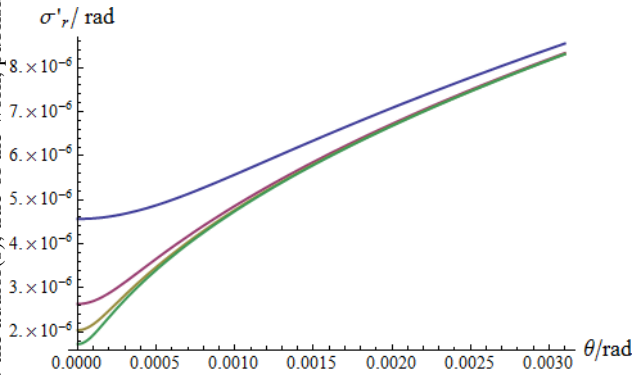

Figure 5: RMS angular divergences of different harmonics vary with energy spread. The condition of calculation is same as before.

Brilliance and Optimized Beta-Functions

Phase shifter works on the condition of phase matched in most cases for improving the brilliance. We also calculate the brilliance on this condition. For the angular distribution of photon density is near Gaussian, it is appropriate to use the well-know expression of brilliance in the Gaussian approximation as [3]

$$B = \frac{F}{4\pi^2 \Sigma_x \Sigma'_x \Sigma_y \Sigma'_y}$$

$$= \frac{F}{4\pi^2 \left[\sigma_x^2 + \sigma_r^2(\sigma_\varepsilon)\right]\left[\sigma'^2_x + \sigma'^2_r(\sigma_\varepsilon)\right]\left[\sigma_y^2 + \sigma_r^2(\sigma_\varepsilon)\right]\left[\sigma'^2_y + \sigma'^2_r(\sigma_\varepsilon)\right]}$$

where

$$F = 2\pi\sigma'^2_r \, I_{total}.$$

In order to calculate the brilliance, it is necessary to obtain the expression of photo source size which should obtain by the Fourier transform at the source points in general. We make a simplification treatment in this paper as shown below[6]:

$$\sigma_r(\sigma_\varepsilon) = Qs(\sigma_\varepsilon)\sigma_r(0)$$

$$\sigma_r(0) = \frac{\lambda_n}{4\sigma'_r(0)}$$

$$Qs(x) = 4\left[\frac{1}{2I_1(x/4) + 2I_2(x/4)}\right]^{2/3}$$

Figure 6 shows the brilliance tune-curve of CPMU18. The beam parameters are chosen according to the latest HEPS lattice scheme which εx=34.2pm, βx=1.9m, εy=4pm, βy=2.2m, σε=0.106% and current I=0.2A. We compare the brilliance of double undulator configuration with the single undulator which the length is doubled. It is clearly to see the beam parameters effects on double undulator configuration is more significant than the latter. In the Fig. 7 brilliance of CPMU18 calculated under two different lattice schemes of HEPS are compared. The brilliance of double undulator configuration is more sensitive to the energy spread than to the emittance especially for the high harmonics.

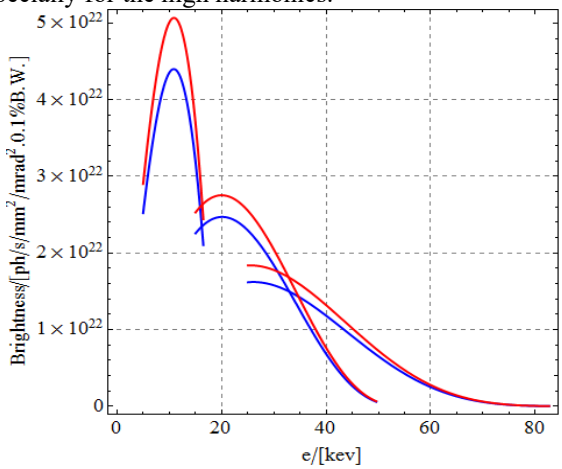

Figure 6: brilliance tune-curve of CPMU18. Where εx=34.2pm, βx=1.9m, εy=4pm, βy=2.2m and current I=0.2A. the remain parameters are the same as before. The red line represent to the brilliance of single undulator with the length doubled. The blue line is the brilliance of double undulator configuration.

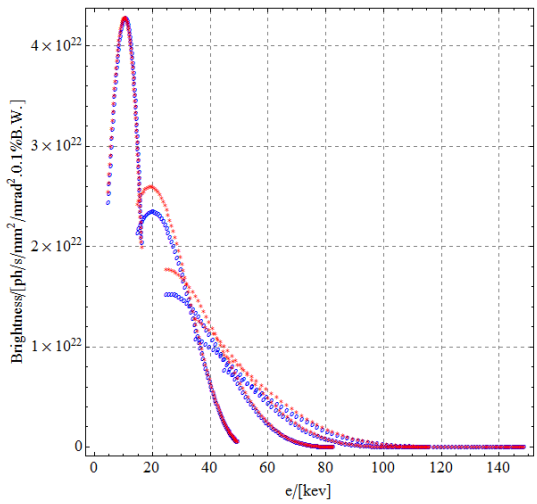

Figure 7: brilliance comparison of CPMU18 between the two different parameter schemes. The blue points refer to the scheme with emittance εx=34.2pm and energy spread σε=0.106% while the red points refer to the scheme with emittance εx=40pm and energy spread σε=0.081%. Any other parameters are the same as that in Fig. 6.

SUMMARY

An analytic expression of the on-axis brilliance and angular flux density distribution are derived. The analysis above indicates that the double undulator configuration with a phase shifter in the middle appears more sensitive to the energy spread than a single undulator which makes us to pay more concern about the energy spread during the lattice design of the storage ring. However, energy spread can not undermine the coherence thoroughly yet i.e. phase shifter is necessary in any cases.

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