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Towards a SM prediction of $N_{\rm eff}$, the effective number of neutrinos

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Abstract. In this work, we demonstrate how $\mathcal{O}(e^{(3)})$, where e is the elementary electric charge, corrections to the QED partition function, previously deemed to be too small to be considered, will affect the effective number of neutrinos in the universe $N_{\rm eff}$. Working in the instantaneous decoupling approximation, we find that their inclusion results in $\delta N_{\rm eff} \simeq -0.001$. We use this result to argue that $\mathcal{O}(e^{(3)})$ corrections should be included in a full calculation of $N_{\rm eff}$ that would include a complete treatment of neutrino decoupling, and accounting for neutrino oscillations. We predict that relative to $N_{\rm eff} = 3.044$ computed in the most recent such calculation, this should give $N_{\rm eff} = 3.043$.

1. Introduction

The ΛCDM paradigm of cosmology has been incredibly successful at explaining the large scale structure of the universe. Indeed, the parameters that describe it have been measured to a better than 1% precision [1–3].

Underlying these parameters, though, is the input of a theoretical quantity conventionally parameterised as the effective number of neutrinos, $N_{\rm eff}$ — the ratio of the Standard Model (SM) neutrinos' energy density relative to the energy density of the photons at some time well after the epoch of $e^{\pm}$ annihilation at $T \sim m_e/3$, with m_e the mass of the electron. This suggests, then, that the theoretical precision of $N_{\rm eff}$ limits how well the ΛCDM parameters can themselves be inferred from observations.

In the SM, $N_{\rm eff}$ is defined to be 3 — reflecting the number of SM neutrinos — plus some percent level corrections. These corrections stem from three main sources. Firstly, the annihilation epoch is not temporally localised (instantaneous), and as long as the neutrinos decouple at some temperature $T_d < \infty$, they will receive a share of the entropy transferred during annihilations [4–7]. The amount of entropy transferred unsurprisingly depends on T_d. Secondly, the QED partition function (and therefore energy density, by way of standard thermodynamic relations) is altered by higher order effects, such as charge screening and both the photon and electron gaining thermally generated masses [8–10]. These can be calculated by way of finite temperature QED (FTQED) [11, 12]. Previous works have considered $\mathcal{O}(e^{(2)})$ corrections [4, 6, 7, 13–18], but we include $\mathcal{O}(e^{(3)})$ and beyond [19]. Finally, not only do neutrinos also not decouple instantaneously — as a result, T_d cannot be clearly defined — but they oscillate between the three flavour states, altering the energy density of each neutrino flavour. We simplify the analysis by not considering the final effect in this work, and refer the reader to e.g. [17, 20].

While the canonical value of $N_{\rm eff} = 3.044$ [20] — a 2019 update of the oft quoted $N_{\rm eff} = 3.046$ [14] — a recent calculation gave a discrepant result of $N_{\rm eff} = 3.052$ [16]. One aim in our work is to therefore investigate this discrepancy. The current best observational constraint is $N_{\rm eff} = 2.99^{+0.34}_{-0.33}$ (95% C.I.) [2], derived from the Planck TT+TE+EE+lowE+lensing+BAO data combination for a 7-parameter vanilla ΛCDM+$N_{\rm eff}$ model, meaning that the discrepancy is too small to impact the parameter inference of current observations, but theoretical uncertainties of this size will before long use up much of the observational error budget. Indeed, the CMB-S4 experiment is expected to reach $\sigma(N_{\rm eff}) \sim 0.02 - 0.03$ [21].

This manuscript is meant as a short summary of [19], in which we investigate the percent level corrections to $N_{\rm eff}$, and is organised as follows. In sections 2, we discuss how entropy conservation can be used to give an estimate of the change in $N_{\rm eff}$ brought about by the finite temperature QED corrections to the partition function that we discuss in section 3. Finally, we summarise our results and give our conclusions in section 4. For further details, the reader should refer to [19].

2. Entropy conservation

In a realistic scenario, the neutrino decoupling process is not temporally localised (instantaneous) at some temperature T_d. Instead, decoupling is a drawn-out process that overlaps with the annihilation epoch. Higher momentum neutrinos decouple later, gaining a greater share of the entropy transfer from annihilations than their lower-momentum counterparts as a result. This creates momentum dependent distortions to the distribution, and thus generates entropy. Typically, in this scenario, the Boltzmann equation and the continuity equation are used to determine the evolution of the neutrino and overall energy densities respectively [13, 14, 17, 20].

However, under the instantaneous decoupling approximation, the scenario is simplified. Since the discrepancy between 0.044 [17] and 0.052 [16] stems from the finite temperature part of the calculation, this is a well-justified approximation for our purposes. After decoupling, which under this approximation can be defined clearly using the temperature parameter T_d, we can consider the comoving entropy of the photon and neutrino fluids to be individually conserved, since entropy is not generated. By defining $N_{\rm eff} = 3 + \delta N^x_{\rm eff}$, and as is detailed in [19], we can write

$$\delta N^x_{\rm eff} = 3 \left[\left(1 + \frac{\delta s^x}{s^{(0)}\big|_{T_d/m_e \to \infty}} \right)^{-4/3} - 1 \right] \tag{1}$$

where $s^{(0)}\big|_{T_d/m_e\to\infty}$ is the QED entropy density under the approximations $T_d/m_e \to \infty$ and an ideal gas (represented by superscript "(0)"). Meanwhile, δs^x is the scenario-dependent change in energy density due to: (a) relaxing the assumption that neutrinos are never coupled with the plasma (which we call $\delta s^{\not{\rm N}\not{\rm C}}$); and (b) the inclusion of thermal corrections $\delta s^{(n)}$ at order $\mathcal{O}(e^{(n)})$. Since the contributions to δs^x sum linearly, we can consider each scenario individually, and then the thermal corrections order by order. For example, setting $\delta s^x = \delta s^{\not{\rm N}\not{\rm C}+(2)+(3)+(4)} = \delta s^{\not{\rm N}\not{\rm C}} + \delta s^{(2)} + \delta s^{(3)} + \delta s^{(4)}$ is to consider simultaneously all effects discussed in this work. For reference, the $(n) = (2)$ term can itself be split into a term that is regularly used in calculations $(2)\not{\ln}$ and a logarithmic term that is usually considered to be too small to be worth calculating $(2)\ln$.

3. Finite temperature corrections

At non-zero temperatures, interacting quantum fields exhibit behaviours not encountered at zero temperature[11, 12]. In the context of this paper, the dominant contribution is expected to stem from corrections to the QED partition function. While $2 \leftrightarrow 2$ interactions between the

Table 1. Summary of the contributions to $N_{\rm eff}$ considered in this work. $T_d^{\rm eff} = 1.46$ MeV.

x	$\delta N_{\rm eff}^x(T_d^{\rm eff})$	Included in [17, 20]
NNC	0.033903	Yes
(2)l.h.	0.010173	Yes
(2) ln	−0.000043	No
(3)	−0.000951	No
(4)	$\simeq 3.5 \times 10^{-6}$	No
Total	**0.043086**	

$e^{\pm}$ and the photon fluids allow them to be treated at zero order as an ideal gas with a shared temperature T, there will be higher order contributions that must be considered too.

The QED partition function Z can be expanded in powers of the QED coupling constant e, $\ln Z = \ln Z^{(0)} + \ln Z^{(2)} + \ln Z^{(3)} + \cdots$, which gives through standard thermodynamic relations the entropy density $s^{(n)} = V^{-1}\partial_T \left[T \ln Z^{(n)}\right]$, i.e. the correction to the entropy density at $\mathcal{O}(e^{(n)})$, with each order having a clear diagrammatic origin [11].

Previous works have included the $\mathcal{O}(e^{(2)})$ diagram into calculations of $N_{\rm eff}$ in the form of a "quasiparticle" picture [13, 16, 17, 20]. In doing so, the corrections are often interpreted as an ideal gas with a temperature-dependent shift to the masses of both the electron and photon, $m_e^2 \to m_e^2 + \delta m_e^2(T)$ and $m_\gamma^2 \to \delta m_\gamma^2(T)$ respectively. However, this is a somewhat misleading interpretation because to change the mass of every particle in the plasma is to double-count the interaction energy between pairs [19]. As a result, when taking this viewpoint, one must take care to expand any relevant quasiparticle thermodynamic quantity about the thermal mass and insert a factor 1/2 by hand (as first detailed in [8]). This approach of *ad-hoc* insertion of numerical factors is certainly not self-consistent and may lead to mistakes.

Not only this, but it is not obvious that a similar treatment of any higher-than-$\mathcal{O}(e^{(2)})$ term can be made. The use of FTQED to calculate such corrections to the partition function, then, eliminates the possibility of these three issues.

4. Results

We begin by noticing that our $\mathcal{O}(e^{(2)})$ thermal corrections give $\delta N_{\rm eff}^{(2)\rm l.h.} = 0.0106$ in the limit $T_d/m_e \to \infty$, in agreement with [20] and which disagrees with [16] by a factor 1/2. This disagreement is likely due to a misunderstanding of [8] by [16], as described previously. Once established, we match $\delta N_{\rm eff}^{\rm NNC+(2)l.h.}$ to 0.044 [20] and invert for an effective decoupling temperature $T_d^{\rm eff}$, in order to subsequently include the $\mathcal{O}(e^{(3)})$ correction. In doing so, it is possible to determine the relative size of each contribution considered, as well as to predict what one may get when including the $\mathcal{O}(e^{(3)})$ term in a full calculation of $N_{\rm eff}$ inclusive of neutrino oscillations and non-instantaneous decoupling. These are presented in table 1. We therefore predict that the shift in $N_{\rm eff}$ due to the inclusion of the $\mathcal{O}(e^{(3)})$ corrections is $\delta N_{\rm eff} \simeq -0.001$ and argue that this is large enough that it should be considered in future $N_{\rm eff}$ calculations.

In figure 1, we vary T_d in order to consider how sensitive each of the FTQED corrections are to the decoupling temperature. As long as the decoupling temperature does not fall sufficiently low (see [19] for a new estimation of T_d in the instantaneous decoupling approximation), it is clear that $\delta N_{\rm eff}^{(3)}$ is large enough to affect the final digit at which $N_{\rm eff}$ is conventionally given, though any corrections higher than $\mathcal{O}(e^{(3)})$ may be ignored until we require a theoretical precision of $N_{\rm eff}$ of 5 s.f or better.

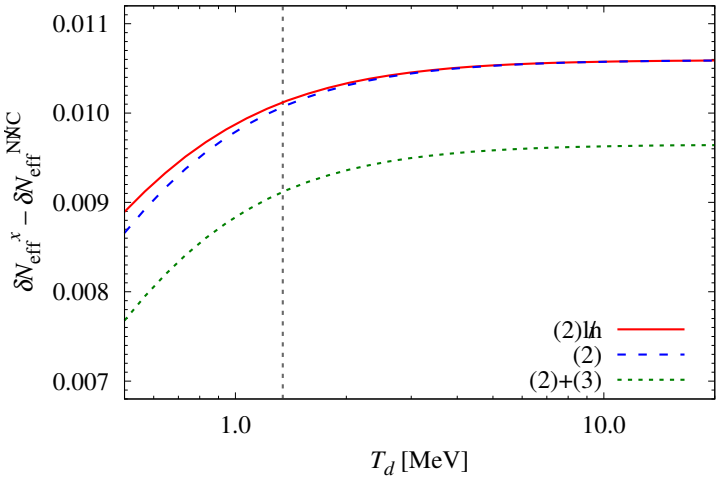

Figure 1. Corrections to $\delta N_{\rm eff}^{x}$ with the NNC contribution subtracted for different contributions considered.

Naturally, the results of this work cannot give a new definitive $N_{\rm eff}$. However, we make the prediction that the new SM value of $N_{\rm eff}$ including a full treatment of neutrino decoupling and our new $\mathcal{O}(e^{(3)})$ corrections should be lowered to $N_{\rm eff} = 3.043$.

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