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DESIGN OF THE EXPANSION BELLOWS FOR THE 40-INCH HYDROGEN BUBBLE CHAMBER

INTRODUCTION

The cross section of the toroidal expansion bellows for the bubble chamber is shown in Fig. 1. Such bellows have the property that alternating stresses are relatively easily absorbed, which in view of the required fatigue life virtually precludes other shapes. Furthermore, the pressure stresses in "omega" type bellows are quasi-independent of the major diameter. A right-circular toroidal convolution is similar in behaviour to a closed coil of thin tubing, in which the stresses at any pressure are a function of the minor radius and of the wall thickness of the tubing. As the torus is hydraulically formed, it is free from circumferential welds, which could act as local stress concentrators. The toroidal convolution is formed about heavy, integral reinforcing bands in such a manner that only the concave part of the torus is subject to flexure. As the inversely shaped parts of the bellows are adequately supported against pressure at all times and as their flexing is minimal, it is convenient to examine the stresses in the circular part only. This leads to considerable simplification in the calculations at the cost of an error estimated at ~7% in the final figures.

The treatment is based on the solution of the inhomogeneous differential equations determining rotationally-symmetric stress distributions in thin elastic shells of revolution. The method of dealing with the coupled, second-order equations most suited for our purposes has been given by Clark and Reissner[1] and by Clark[2,3] in the asymptotic approximation of $\mu \gg 1$ where $\mu = \sqrt{12(1 - \nu^2)}\, \frac{b^2}{ah}$. Here ν is Poisson's ratio for the material in question, a is the major radius of the torus taken between centers, b is the minor radius of the cross section, and h is the thickness of the shell. In our case where $a = 20$ inches, $b = 1.5$ inches, $h = 0.030$ inch, and $\nu = 0.305$, $\mu = 12.38$, so that the approximation is sufficiently good. Experience shows that an accuracy to within 5 to 10% of the measured values[4] can be achieved with this method, without recourse to elaborate computer calculations. If λ defines the parameter b/a (≈ 0.075 in our case) then, in general, the relative accuracy of the formulae used is determined by the order at which the expansion is stopped, usually $\sim \lambda/\mu^{1/3} \approx 3.2 \times 10^{-2}$.

It is certainly possible to make a more refined analysis; it seems doubtful, however, if more meaningful results could be obtained.

STRESSES DUE TO MOTION OF BELLOWS

Consider a geometrical arrangement such as shown in Fig. 2. Introduce a parameter ξ such that in the $r - z$ plane, $r = a + b \sin \xi$; $z = - b \cos \xi$ where r, z, and Θ form a right-handed set of coordinates describing any point on the middle surface. Let P denote the axial force applied to the joint, without specifying the origin of the force P, and ξ_1 and ξ_2 correspond to the inner and outer points respectively, where the meridian meets the plane of symmetry. ξ is a convenient computational parameter, without any special meaning.

It is assumed that the behaviour of the shell due to a vertical deflection of the edge of the slit may be approximated to by considering separately the action of the top and bottom halves of the torus. Thus, in the top half of the omega joint the tangents to the meridian remain vertical at the outer and inner edges. These edges also exhibit zero radial shear. Within the approximation of $\mu \gg 1$, it is apparent that the most significant stresses are then the circumferential direct stress $\sigma_{\theta D}$ and the meridianal bending stress $\sigma_{\xi B}$, defined in terms of the normal stress resultants N_ξ and N_θ and the stress couple M_ξ as

$$\sigma_{\theta D} = \frac{N_\theta}{h} ; \quad \sigma_{\xi B} = \frac{6M_\xi}{h^2} \tag{1}$$

According to the solution of reference (2), the maximum hoop stress occurs very near the crown line $\xi = \xi_0$, while the maximum meridianal bending stress occurs at points on either side of the crown line.

Then, the method of solution given in reference (3), leads to

$$\sigma_{\theta D,max} = \sigma_{\theta D}(\xi = 0) = 0.342\ (1-\nu^2)^{1/3} \left(\frac{bh}{a^2}\right)^{1/3} \frac{P}{h^2} \times \left[1 - \frac{0.342}{\mu^{2/3}} + O\left[\frac{1}{\mu}\right]\right] \quad (2)$$

$$\sigma_{\xi B,max} = \sigma_{\xi B}(\xi_M) = \pm \frac{0.475}{(1-\nu^2)^{1/6}} \left(\frac{bh}{a^2}\right)^{1/3} \frac{P}{h^2} \times \left[1 - \frac{0.054}{\mu^{2/3}} + O\left[\frac{1}{\mu}\right]\right] \quad (3)$$

$$\xi_M = \pm \left[\frac{1.225}{\mu^{1/3}} - \frac{0.017}{\mu} + O\left[\frac{1}{\mu^{4/3}}\right]\right] \quad (4)$$

Also, the total axial deflection of the bellows due to bending only is given by twice the difference in the deflections at $\xi = \xi_o$ and $\xi = \xi_M$, or

$$\delta = \left[12\ (1 - \nu^2)\right]^{\frac{1}{2}} \frac{bP}{Eh^2} \left[1 - O\left[\frac{1}{\mu}\right]\right] \quad (5)$$

In our application δ is determined by the expansion ratio, and is known. Hence P can be eliminated from Eqs. (2), (3), and (5), viz.,

$$\sigma_{\theta D,max} = \frac{0.0987\ E\delta}{(1 - \nu^2)^{1/6}} \left(\frac{h}{a^2 b^2}\right)^{1/3} \left[1 - \frac{0.342}{\mu^{2/3}}\right] \quad (6)$$

and

$$\sigma_{\xi B,max} = \pm \frac{0.1371\ E\delta}{(1 - \nu^2)^{2/3}} \left(\frac{h}{a^2 b^2}\right)^{1/3} \left[1 - \frac{0.054}{\mu^{2/3}}\right] \quad (7)$$

STRESSES DUE TO INTERNAL PRESSURE

Since the edges of the torus are effectively restrained from moving radially, the stresses due to internal pressure are assumed to be the same as those existing in a complete toroidal shell subjected to a pressure in a like manner. The stresses are obtained from an elementary consideration of equilibrium in thin shells:

$$\sigma_{\theta D} = \frac{1}{2}\frac{bp}{h} \; ; \; \sigma_{\xi B} = \frac{bp}{h} \times \frac{1 + \frac{1}{2}\lambda \sin \xi}{1 + \lambda \sin \xi} \tag{8}$$

where p is now the internal pressure.

DESIGN OF THE BELLOWS

Assuming that the bellows will be made of ASTM A240 type 316L stainless steel, the following parameters are to be regarded as fixed:

$E = 28.0 \times 10^6$ psi (ambient temperature)

$\nu = 0.305$

$p = 150$ psig (dynamic design and test pressure)

[$p = 225$ psig static test pressure]

$a = 20$ inches - dictated by the bubble chamber geometry

$\delta = \pm\, 0.125$ inch - dictated by the maximum expansion ratio.

The parameters h and b are to be optimized. Equations (6) and (7) can be written as:

$$\sigma_{\theta D,max} = \alpha \left[\frac{h}{b^2}\right]^{1/3} - \beta \left[\frac{h}{b^2}\right] \tag{9}$$

$$\sigma_{\xi B,max} = \pm\, \gamma \left[\frac{h}{b^2}\right]^{1/3} \mp \eta \left[\frac{h}{b^2}\right] \tag{10}$$

where

$$\alpha = \frac{0.0987\ E\delta}{(1 - \nu^2)^{1/6}\ a^{2/3}} = 4.765 \times 10^4$$

$$\beta = \frac{0.0987\ E\delta}{(1 - \nu^2)^{1/2}} \times \frac{0.342}{12^{1/3}} = 5.42 \times 10^4$$

$$\gamma = \frac{0.1371\ E\delta}{(1 - \nu^2)^{2/3}\ a^{2/3}} = 6.952 \times 10^4$$

$$\eta = \frac{0.1371\ E\delta \times 0.054}{(1 - \nu^2) \times 12^{1/3}} = 1.248 \times 10^4$$

Equation (8) becomes

$$\sigma_{\theta D} = 75 \left[\frac{b}{h}\right]; \quad \sigma_{\xi B,max} = 75 \left[\frac{b}{h}\right] \left(\frac{2a + b}{a + b}\right) \tag{11}$$

The maximum stresses are obtained by algebraically summing Eqs. (9), (10), and (11):

$$\sigma_{\theta D,max} = \alpha \left[\frac{h}{b^2}\right]^{1/3} - \beta \left[\frac{h}{b^2}\right] + 75 \left[\frac{b}{h}\right] \tag{12}$$

and

$$\sigma_{\xi B,max} = \gamma \left[\frac{h}{b^2}\right]^{1/3} - \eta \left[\frac{h}{b^2}\right] + 75 \left[\frac{b}{h}\right] \left(\frac{2a + b}{a + b}\right) \tag{13}$$

Both Eqs. (12) and (13) have a stationary point (minimum) where the parameters b and h reach their optimum value. An exact solution cannot be obtained however. For orientation purposes the line of minima is given approximately by

$$h^4 = 10.58 \times 10^{-8}\ b^5 \tag{14}$$

and

$$h^4 = 3.40 \times 10^{-8}\, b^5 \left(\frac{2a + b}{a + b}\right)^3 \tag{15}$$

With $b = 1.5$ inches, relation (14) gives $h \approx 0.030$ inch, and relation (15) gives $h \approx 0.037$ inch. For a value of b determined by the design of the chamber, these relations facilitate the choice of suitable values of h. Note that it is not possible to minimize $\sigma_{\theta D,max}$ and $\sigma_{\xi B,max}$ simultaneously at the same values of b and h. The respective minimum values of the stresses at these points are

$$\sigma_{\theta D,max}\ (b = 1.5,\ h = 0.030) = 14{,}335 \text{ psi}$$

and

$$\sigma_{\xi B,max}\ (b = 1.5,\ h = 0.037) = 23{,}321 \text{ psi}$$

The variation of the principal stresses as a function of b and h over a selected range of values is shown in Figs. 3 and 4.

Within the limitations imposed by the geometry, a toroidal expansion bellows with $a = 20$ inches, $b = 1.5$ inches and $h = 0.030$ inch satisfies the design criteria best.

FATIGUE ANALYSIS

The fatigue life of the actual bellows cannot be calculated with any degree of accuracy as the process of fabrication necessarily introduces departures from the theoretical shape. Nevertheless, as a long fatigue life is the most important prerequisite for the bellows, an analysis of the ideal configuration yields sufficient insight into the behaviour of the material to be of some usefulness.

The analysis is performed for the set of parameters chosen above. The usual sign convention is followed: positive forces denote tension.

The maximum, mean and amplitude variation values of the principal stresses are tabulated below, and the result of a conventional Sodeberg diagram-type of analysis shown in Fig. 5. A more elaborate stress cycle analysis has also been performed without, however, significantly departing from the more approximate values. A distortion energy calculation leads essentially to the same results.

		Maximum Stress		Mean Stress	Ampl. of Variable Stress	
		Expansion S_e	Compression S_c	S_m	S_{alt}	$S_{equiv.}$
Hoop Stress		+14335	- 6835	3750	10585	11184
Meridianal Stress	Outer	- 9049	+23525	7238	16287	18165
	Inner	+23525	- 9049	7238	16287	18165
Shear Stress	Outer	-11692	+15180	1744	13436	13780
	Inner	+ 4595	- 1107	1744	2851	2924

Note: $S_{equiv.} = \dfrac{S_{alt}}{1 - \dfrac{S_m}{S_u}}$

CONCLUSION

Hydroformed stainless steel omega bellows will be used for the bubble chamber expansion piston seal, having the following characteristics:

Material: ASTM-#A240 - 316L stainless steel

Pressure: 150 psig under dynamic test conditions

Major radius of torus: 20 inches nominal

Minor radius of torus: 1.5 inches nominal

Thickness of material: 0.031 inch nominal

Maximum deflection: ±0.125 inch.

The analysis as performed suggests that at the calculated stress levels a fatigue life in excess of 5×10^7 cycles can be expected at the temperature of liquid hydrogen.

REFERENCES

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2. R. A. Clark, J. Math. and Phys. 29, 146 (1950).
3. R. A. Clark, Quart. of Appl. Math. 16, 47 (1958).
4. N. C. Dahl, A.S.M.E. Trans. 75, 497 (1953).

ADDENDUM

While this note was being prepared, it became apparent from discussions with manufacturers of expansion bellows, that omega bellows with a circular cross section in the $r - z$ plane are very difficult to produce. It is therefore of considerable interest to examine the magnitude of the stresses existing in a torodial shell of elliptical and of arbitrary cross section, insofar as this is possible by approximate methods.

In terms of the parameter ξ introduced in the preceding, the parametric equation of a shell of arbitrary cross section is given by

$$\begin{aligned} r &= r(\xi) \\ z &= z(\xi) \end{aligned} \qquad \text{(A1)}$$

from which the radius of curvature at any point, R_ξ can be obtained, viz.,

$$R_\xi = \frac{\rho^3}{r'z'' - z'r''} \tag{A2}$$

where

$$\rho = \left[(r'^2) + (z')^2 \right]^{1/2}$$

For an elliptical section with major radius b and minor radius c

$$\rho = \left[b^2 \cos^2 \xi + c^2 \sin^2 \xi \right]^{1/2}$$

and

$$R_\xi = \frac{\rho^3}{bc} \tag{A3}$$

Now, assuming that the asymptotic method of solution holds for the parameter

$$\mu = \sqrt{12 (1-\nu^2)} \; \frac{\rho_o^3}{ahR_o} \tag{A4}$$

where now ρ_o and R_o are the values of $\rho(\xi)$ and $R(\xi)$ on the crown line of the torus.

As $\rho_o = b$ and $R_o = \frac{b^2}{c}$

$$\mu = \sqrt{12 (1-\nu^2)} \; \frac{bc}{ah} \tag{A5}$$

Suppose that bellows can be made which approach the roundness of the specified 3 inch nominal diameter to within 1/16 inch. Then $b = 1.53$ and $c = 1.47$ inches so that $\mu = 12.4$. The approximation should still be good.

By pursuing the method outlined in Ref. (3) the maximum bending stress for an elliptical section becomes approximately

$$\sigma_{\xi B,max} = \pm \frac{0.247\ E\ \delta}{(1 - \nu^2)^{2/3}} \left(\frac{hc^2}{a^2b^4}\right)^{1/3} \tag{A6}$$

which yields $\sigma_{\xi B,max} = 13{,}270$ psi .

The membrane stresses due to internal pressure acting on a non-circular cross section will clearly be very large. Consider the membrane stress at the points $R_2 = a + b$, that is on the outermost circumference of the torus, in the plane of symmetry $z = 0$. It follows then that

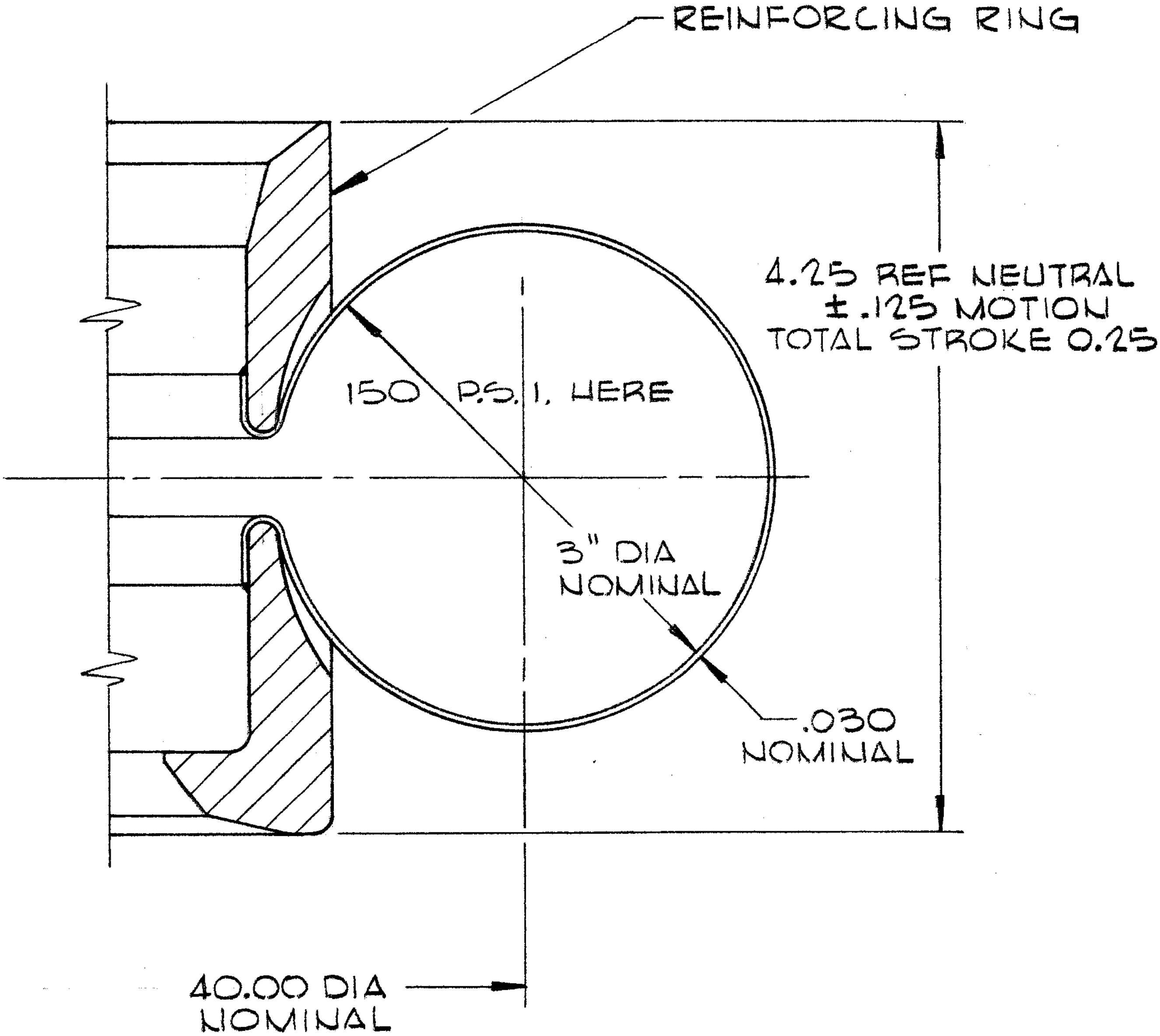
$$\sigma_{\theta D} = \frac{pa}{h} \left\{ \left(1 + \frac{b}{a}\right) - \frac{b^2}{c^2} \left(1 + \frac{b}{2a}\right) \right\} \tag{A7}$$

With $p = 150$ psig , $\sigma_{\theta D} = 48{,}290$ psi .

If for manufacturing reasons $b - c = 0.0625$ or more, $\sigma_{\theta D} \sim 83{,}700$ psi .

These numbers clearly indicate that the maximum effort should be made to obtain round bellows, and that a quasi-elliptical cross section is wholly inappropriate.

A torus with a distorted circular section presents a similar problem, except that no simple approximate method of calculation exists, which would yield the stresses as a function of the distortion.



OMEGA BELLOWS
CROSS SECTION

FIG. 1

STANFORD LINEAR ACCELERATOR U. S. ATOMIC ENERGY COMMISSION	TITLE	
ENGR. ______ CHK'D ______	DATE	
DFTS. ______ APPV'D ______	SCALE	

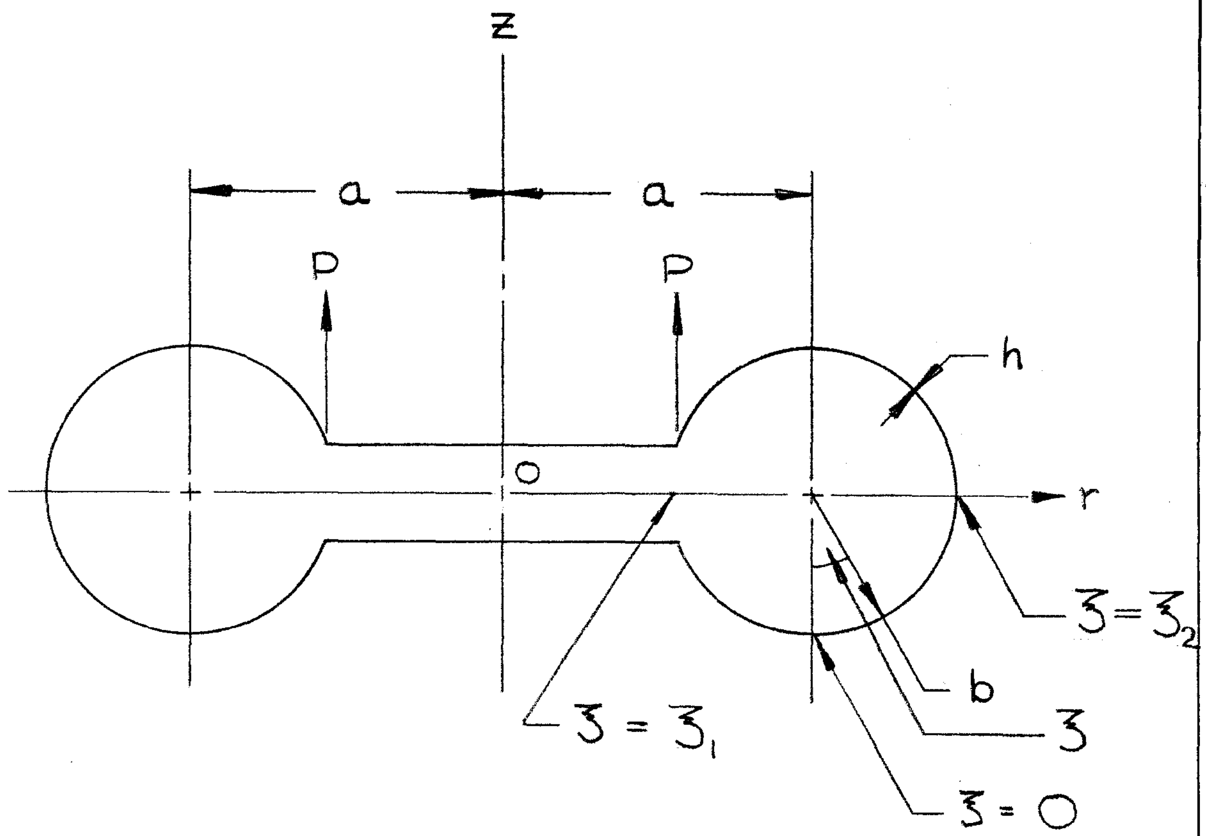


FIG. 2

STANFORD LINEAR ACCELERATOR U.S. ATOMIC ENERGY COMMISSION	TITLE
ENGR. ____ CHK'D ____	DATE
DFTS ____ APPV'D ____	SCALE

FIG. 3

HOOP STRESS VS. SHELL THICKNESS AS A FUNCTION OF SHELL RADIUS b

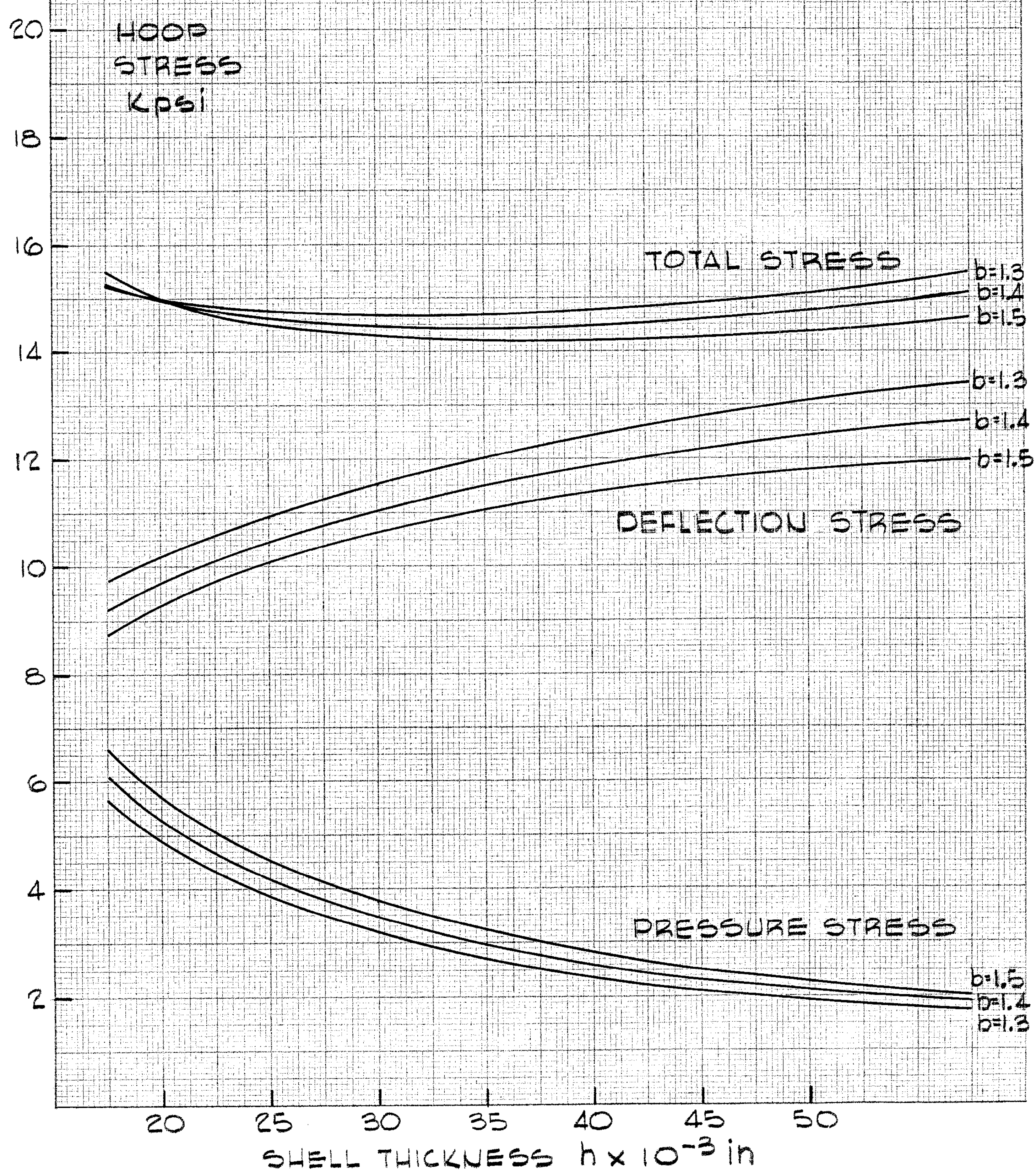
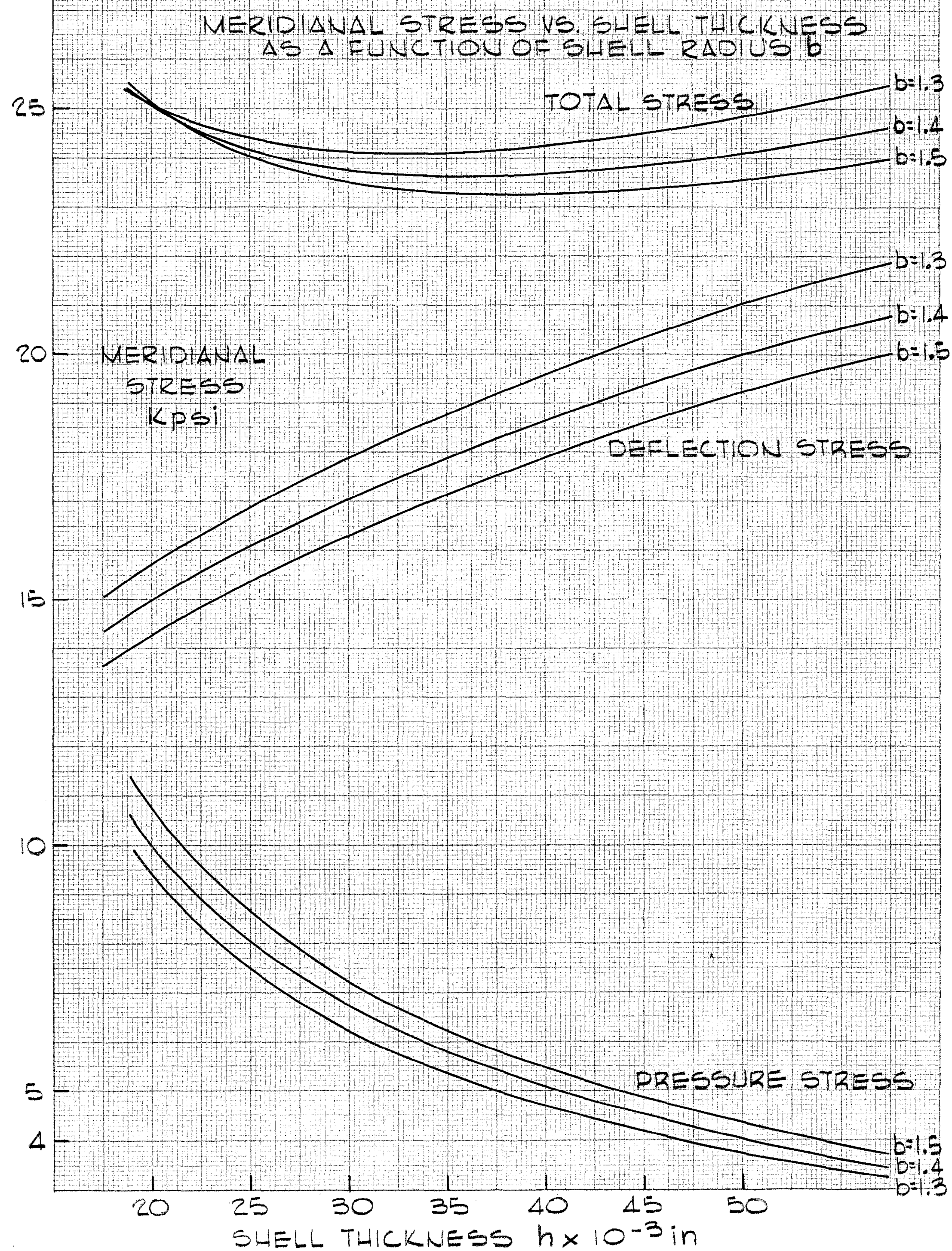


FIG. 4



10 X 10 TO THE CENTIMETER 46 1510
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FIG. 5

SODEBERG FATIGUE DIAGRAM