Horizon, homogeneity and flatness problems – do their resolutions really depend upon inflation?

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In textbooks and review articles on modern cosmology [1, 2, 3, 4, 5, 6] one almost invariably comes across a section devoted to the subject of observed homogeneity and near-flatness of the universe, where it is argued that to explain these observations inflation is almost a must. In fact that was the prime motive of Guth [7] to propose inflation in the first place. We show that the arguments offered therein are not proper. The horizon problem, which leads to the causality arguments, arises only in the world models where homogeneity and isotropy (cosmological principle) is presumed to begin with. We do not know whether the horizon problem would still arise in non-homogeneous world models. Therefore as long as we are investigating consequences of the cosmological models based on Robertson-Walker line element, there is no homogeneity issue.

We also show that the flatness problem, as it is posed, is not even falsifiable. The usual argument used in literature is that the present density of the universe is very close (within an order of magnitude) to the critical density value. From this one infers that the universe must be flat since otherwise in past at 10^{-35} second (near the epoch of inflation) there will be extremely low departures of density from the critical density value (i.e., differing from unity by a fraction of order $\sim 10^{-53}$), requiring a sort of fine tuning. Actually we show that even if the present value of the density parameter (in terms of the critical density value) were very different, still at 10^{-35} second it would in any case differ from unity by a fraction of order $\sim 10^{-53}$. For instance, even if had an almost empty universe, with say, $\rho_o \sim 10^{-56}$ gm/cc or so (with density parameter $\Omega_o \sim 10^{-28}$, having a mass equivalent to that of Earth alone to fill the whole universe), we still get the same numbers for the density parameter at the epoch of inflation. So such a fine-tuning does not discriminate between various world models and a use of fine tuning argument amounts to *a priori* rejection of all models with $k \neq 0$, because inflation or no inflation, the density parameter in all Friedmann-Robertson-Walker (FRW) world models gets arbitrarily close to unity as we approach the epoch of the big bang. That way, without even bothering to measure the actual density, we could use any sufficiently early epoch and use "extreme fine-tuning" arguments to rule out all non-flat models. Thus without casting any whatsoever aspersions on the inflationary theories, we point out that one cannot use these type of arguments, viz. homogeneity and flatness, in support of inflation.

1. Horizon and homogeneity problem

Horizon in the cosmological context implies a maximum distance yonder which we as observers have not yet seen the universe due to a finite speed of light as well as a finite age of the universe. In other words these are the farthest regions of the universe (redshift $z \to \infty$) from which the light signals have just reached us. However when we look at the universe we find that distant regions in opposite directions seen by us have similar cosmic microwave background radiation (CMBR) temperatures. The object horizon problem in standard cosmological big bang model is that these different regions of the universe have not ever communicated with each other, but nevertheless they seem to have the same temperature, as shown by the CMBR which shows almost a uniform temperature (2.73° K) across the sky, irrespective of the direction. How can this be possible, considering that any exchange of information (say, through photons or any other means) can occur, at most, at the speed of light. How can such two causally disconnected regions have one and same temperature, unless one makes a somewhat "contrived" presumption that the universe was homogeneous and isotropic to begin with when it came into existence [1]?

One can illustrate the object horizon problem using a simple, though somewhat naive, argument in the following way. According to the big bang model the universe has only a finite age, say t_o. Then light (or information) from regions at a cosmological distance ct_o from us would have reached us just now, and could not have crossed over to similar distant regions on the other side of us. Then how two far-off regions on two opposite sides of us have managed to achieve the homogeneity so that we see them having same properties. Though the argument does contain an element of truth, but it could not be always true and its naive nature can be seen from the simplest of FRW models, namely empty universe of Milne ($\rho = 0, q_o = 0$), where even the most distant ($z \to \infty$) observable

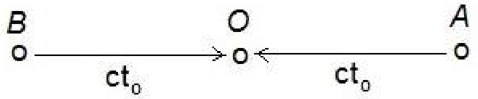

Figure 1. An observer at O (us) receiving signal from distant objects at A and B at time t_o, which is the time since big bang. Signal from A having just reached O could not have yet reached B and vice versa.

point in the infinite extent of the universe is within horizon for each and every point of the universe, let it be in any region in any direction from us in this infinite universe model. For instance, in this world-model B will receive signals from us (at O) and from A in same amount of time. In fact all regions in this universe at any time receive past signals from all other regions even though the universe is infinite. Thus horizon problem does not arise in this particular world-model. However, in more realistic cases of general relativistic cosmological models, say with finite density, almost invariably one comes across horizon problems.

From the observed CMBR, the universe appears to be very close to isotropic. At the same time Copernican principle states that earth does not have any eminent or privileged position in the universe and therefore an observer's choice of origin should have no bearing on the appearance of the distant universe. From this we infer that the cosmos should appear isotropic from any vantage point in the universe, which directly implies homogeneity. For this one uses Weyl's postulate of an infinite set of *equivalent fundamental observers* spread around the universe, who agree on a "global" time parameter, orthogonal to 3-d space-like hyper surfaces, and measured using some local observable like density, temperature, pressure etc. as a parameter [1, 3, 8]. Thus we are led to the cosmological principle that the universe on a sufficiently large scale should appear homogeneous and isotropic to all fundamental observers, and then one gets for such observers a metric for the universe known as Robertson-Walker metric.

Is there any other evidence in support of the cosmological principle? Optically the universe shows structures up to the scale of super clusters of galaxies and even beyond up hundreds of mega parsecs, but the conventional wisdom is that when observed on still larger scales the universe would appear homogeneous and isotropic. It is generally thought that radio galaxies and quasars, the most distant discrete objects (at distances of gig parsecs and farther) seen in the universe, should trace the distribution of matter in the universe at that large scale and should therefore appear isotropically distributed from any observing position in the universe.

But there is a caveat. In the 3CRR survey, the most reliable and most intensively studied complete sample of strong steep-spectrum radio sources, large anisotropies in the sky distributions of powerful extended quasars as well as some other sub-classes of radio galaxies are found [9]. If we include all the observed asymmetries in the sky distributions of quasars and radio galaxies in the 3CRR sample, the probability of their occurrence by a chance combination reduces to $\sim 2\times 10^{-5}$. Such large anisotropies present in the sky distribution of some of the strongest and most distant discrete sources imply inhomogeneities in the universe at very large scales (covering a fraction of the universe).

Also using a large sample of radio sources from the NRAO VLA Sky Survey, which contains 1.8 million sources, a dipole anisotropy is seen [10] which is about 4 times larger than the CMBR dipole, presumably of a kinetic origin due to the solar motion with respect to the otherwise isotropic CMBR. These unexpected findings have recently been corroborated by two independent groups [11, 12]. The large difference in the inferred motion (as much as a factor of ~ 4) cannot be easily explained. A genuine discrepancy in the dipoles inferred with respect to two different cosmic reference frames would imply a large ($\sim 10^3$ km/sec) relative motion between these frames, not in accordance with the cosmological principle.

If we ignore these and some other similar threats to the cosmological principle and trust the assumption of homogeneity and isotropy *for the whole universe for all times*, then the line element can be expressed in the Robertson-Walker metric form [2, 3, 4, 8],

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - R^2(t)\left[\frac{\mathrm{d}r^2}{(1-k\,r^2)^{1/2}} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \tag{1}$$

where the only time dependent function is the scale factor $R(t)$. The constant k is the curvature index that can take one of the three possible values $+1, 0$ or -1 and (r, θ, ϕ) are the time-independent comoving coordinates.

Using Einstein's field equations, one can express the curvature index k and the present values of the cosmic scale factor $R_{\rm o}$ in terms of the Hubble constant $H_{\rm o}$, the matter energy density Ω_m and the vacuum energy (dark energy) density Ω_Λ as [2, 3, 4],

$$\frac{k\, c^2}{H_{\rm o}^2 R_{\rm o}^2} = \Omega_o - 1, \tag{2}$$

where $\Omega_o = \Omega_m + \Omega_\Lambda$. The space is flat ($k = 0$) if $\Omega_o = 1$.

In general it is not possible to express the comoving distance r in terms of the cosmological redshift z of the source in a close-form analytical expression and one may have to evaluate it numerically. For example, in the $\Omega_m + \Omega_\Lambda = 1, \Omega_\Lambda \neq 0$ world-models, r is given by [2],

$$r = \frac{c}{H_{\rm o} R_{\rm o}} \int_1^{1+z} \frac{{\rm d}z}{(\Omega_\Lambda + \Omega_m z^3)^{1/2}}. \tag{3}$$

For a given Ω_Λ, one can evaluate r from equation (3) by a numerical integration.

However for $\Omega_\Lambda = 0$ cosmologies, where the deceleration parameter $q_{\rm o} = \Omega_m/2$, it is possible to express the comoving (coordinate) distance r as an analytical function of redshift [13],

$$r = \frac{c}{H_{\rm o} R_{\rm o}} \frac{z}{(1+z)} \frac{\left[1 + z + \sqrt{1 + 2q_{\rm o} z}\right]}{\left[1 + q_{\rm o} z + \sqrt{1 + 2q_{\rm o} z}\right]}, \tag{4}$$

which for the empty Milne universe ($q_{\rm o} = 0$) of negative curvature ($k = -1$) yields,

$$r = \frac{c}{2H_{\rm o} R_{\rm o}} \left[1 + z - \frac{1}{1+z}\right], \tag{5}$$

while for the Einstein - de Sitter world-model ($q_{\rm o} = 1/2$) with zero curvature ($k = 0$) we get,

$$r = \frac{2c}{H_{\rm o} R_{\rm o}} \left[1 - \frac{1}{\sqrt{1+z}}\right]. \tag{6}$$

From equation (5) we see that in the Milne universe, corresponding to redshift $z \to \infty$, the comoving coordinate $r \to \infty$ too, thus there is no finite horizon limit in this case and the whole universe is visible to any observer at any time, consistent with our discussion above. On the other hand from equation (6) we notice in the flat universe redshift $z \to \infty$ yields for r a finite value. However in this world model the range of coordinate r goes up to infinity. Thus there is a certain finite object horizon $r_{oh} = 2c/(H_{\rm o} R_{\rm o})$ beyond which we are unable to see because there is only a finite amount of time since the big bang singularity (corresponding to $z \to \infty$), and thus only a finite distance that photons could have travelled within the age of the universe. It turns out that *all* non-trivial (that is with finite density) FRW world-models starting with a big bang necessarily have a object horizon [3].

Appearance of object horizon in a world model is generally interpreted as that different parts of the universe in that model did not get sufficient time to interact with each other and thus may have yet no causal relations and therefore could not have achieved uniformity everywhere. Therefore inflation is invoked in which an exponential expansion of space takes place at time $t \sim 10^{-35}$ sec by a factor of $\sim 10^{28}$ or larger and the space-points now far apart (and thus apparently not in touch with each other so they appear to be causally unrelated) were actually much nearer before $t \sim 10^{-35}$ sec or so and could have had time to interact with each other before inflation.

A most crucial point that somehow seems to have been missed (or ignored) in these deliberations is that the question of horizon problem arises only when we to begin with assume that the Universe was "always" homogeneous and isotropic, because only then we can make use of Robertson-Walker element where we separate the time co-ordinate from the 3-d space which may or may not be flat and has the time-dependence only through a single scale parameter $R(t)$. It is only there that horizon makes an appearance which in turn has given rise to the oft-discussed question of the uniformity and homogeneity of the space. However, as long as we make use of the Robertson-Walker metric we are taking for guarantee that the universe was *ever* homogeneous and isotropic, and that a single parameter $R(t)$

can describe the past and determine the future of the universe. There is no doubt about the presence of a horizon, which follows from the Robertson-Walker line element. However that in itself does not imply a nonexistence or lack of homogeneity as horizon itself exists only in models where to begin with homogeneity is presumed. All we find from calculations is that in a universe which is which is isotropic and homogeneous on a large enough scale and where one can assign a single common parameter to all fundamental observers to use as time, the light signals in a finite amount of time are not able to cover the whole available range of space coordinate r in the universe. In fact some of the world models even in an infinite time all r may not get covered by light signals emitted from a point ("event horizon"). Cause and effect seem to reverse their roles. It is not that because horizon exists so uniformity is not possible, ironically it is where a uniformity is present to begin with that we seem to end up with a horizon problem. In these models we assume not only a single parameter t, but all other parameters describing the universe having common values of scale factor $R(t)$, density $\rho(t)$, Hubble parameter H_o, deceleration parameter q_o etc. to be the same everywhere, at any given time t (even beyond object or event horizons wherever we encounter such horizons)). It is yet to be seen whether such horizons would still arise in models where one does not begin with the cosmological principle and one has to deal with a genuine non-uniformity problem.

Actually if we follow the standard arguments in the literature then inflation in one sense makes the application of cosmological principle worse than ever. Though it may alleviate the problem of object horizon, yet it gives rise to much more acute event horizon problems. After all even just before inflation began, there were object horizons which because of a rapid expansion of the universe due to inflation will become even more "remote" from each other ending up in growth of large number of event horizons, with all such regions of the universe never able to interact with each other. Thus such a universe will comprise huge number of large patches still isolated from each others. Then how can one still apply the cosmological principle to such a disjointed universe which would conflict with our starting assumption (Weyl's postulate!), where we cannot even get a single parameter to act as cosmic time, orthogonal to 3-d space-like hyper surfaces, which is purely based on the condition of universal homogeneity and isotropicity. We cannot then even use Robertson-Walker line element to describe the geometry and then all our conclusions about the cosmological models would have to be abandoned and we will then be back to square one.

Once again, the only saviour here is that, inflation or no inflation, these horizons are encountered only in the models where we have already assumed cosmological principle. However, if we do want to really examine the question of homogeneity or its absence then we need to abandon the standard model based on the Robertson-Walker metric and then with some new model, where possibility of anisotropy or inhomogeneity is assumed to begin with, one has to examine if in such models also we come across horizons and if so, then we may have a genuine problem to explain.

2. Flatness problem

In the so-called flatness problem, the current density of the universe is observed to be very close to the critical value, needed for a zero curvature ($k = 0$). Since the density departs rapidly from the critical value with time, the early universe must have had a density even closer to the critical density, so much so that if we extrapolate the density parameter to the epoch of inflation ($t \sim 10^{-35}$ sec) we find it to be within unity within an extremely small fraction of order $\sim 10^{-53}$. This leads to the question how the initial density came to be so closely fine-tuned to the critical value. Cosmic inflation was proposed to resolve this issue along with the horizon problem [7]. However, as we will show, the flatness problem, as it is posed, is not falsifiable.

A general form of equation (2) valid at any epoch is,

$$H^2R^2(\Omega - 1) = kc^2. \tag{7}$$

Making use of equation (2) we get,

$$H^2R^2(\Omega - 1) = H_o^2R_o^2(\Omega_o - 1), \tag{8}$$

which for the epoch of inflation ($t \sim 10^{-35}$ sec) can be simplified [1] to,

$$(\Omega - 1) \approx 10^{-53}(\Omega_o - 1). \tag{9}$$

The usual argument prevalent in literature is that the present density of the universe is very close (within an order of magnitude) to the critical density value, i.e., $0.1 < \Omega_o < 1$. From this one infers that the universe must be flat since otherwise in past at 10^{-35} second (near the epoch of inflation) there will be extremely low departures of density from the critical density value (i.e., differing from unity by a fraction of order $\sim 10^{-53}$), requiring a sort of fine tuning. However this argument could be applicable to almost any present value of the observed density of the Universe. What is implied here is that even in a hypothetical almost empty universe where the density of universe is say, $\rho_o \sim 10^{-56}$ gm/cc) or so (with density parameter $\Omega_o \sim 10^{-28}$, having only a mass equivalent to that of Earth alone to fill the whole universe), from equation (9) the density parameter at the epoch of inflation would differ from unity by the same fraction, of order $\sim 10^{-53}$. Thus without casting any whatsoever doubts on the inflationary theories, we merely point out that one cannot use these type of arguments to support inflation.

Is there really any substance in this type of arguments as even a mass equal to that of earth alone spread over the universe will lead to the same low departures from unity of 10^{-53}? In fact even the presence of a mere single observer would imply the same departures from unity of 10^{-53}. So a use of fine tuning argument amounts to *a priori* rejection of all models with $k \neq 0$, because inflation or no inflation, the density parameter in all Friedmann-Robertson-Walker (FRW) world models gets arbitrarily close to unity as we approach the epoch of the big bang. That is the property of all these FRW models. That way, irrespective of the actual density, we could use any sufficiently early epoch and use the "extreme fine-tuning" arguments to reject all non-flat models. But that is not what one could call a falsifiable theory. Thus without casting any whatsoever aspersions on the inflationary theories, we point out that one cannot use these type of arguments to support inflation.

In fact flatness and homogeneity problems seem to contradict each other. If we say that the universe is flat ($k = 0$) then we are assuming that the density is exactly equal to the critical value and does not depart from it to even by a smallest fraction. Or in words, each and every particle is essential and is thus accounted for and even a single particle is not extra or less in the whole cosmos, as excess of even a single particle more than that needed for the critical density will ultimately turn the universe from a flat to a curved one. Which means this much information we have about the whole universe. Then how can we say that we may have no information about some other parts of the universe due to the so-called horizon problem? While on one hand we guarantee that in $k = 0$ world models (flat space), each and every particle in the universe is accounted for (as otherwise even a single extra or missing particle more than the critical density would cause the universe to deviate from the flat universe (a runaway case!), but on the other hand we are saying that we (one part of the universe) have no communication or information about distant parts of the universe and know nothing about them i.e., about the density, temperature, pressure etc. there, so that uniformity or homogeneity could not have been enforced since the "birth" of the universe. Are we not contradicting ourselves?

By opting for a flat universe, the least probable out of three possible curvature values, we seem to be following the example of Copernicus epicycles on philosophical grounds. Further, the argument of flatness perhaps has a catch. Inflation might make the universe flatter (by bringing density parameter closer to unity) but it can make it flat (by making the density parameter exactly equal to unity). What we mean is that if we think that inflation has brought about only a near-flatness then we are essentially assuming that $k \neq 0$, because otherwise if $k = 0$, then inflation does not have a role to play here as it cannot flatten it further. And if $k \neq 0$ then inflation cannot make it $k = 0$, even though it might bring the density parameter closer to unity. In fact by assuming a flat model we are assuming the ultimate finest-ever tuning imaginable where even the least amount of perturbation on this unstable equilibrium model (in the form of an excess or deficiency of the smallest amount of matter from the critical density - a single particle or atom extra or missing!) can ultimately take the universe away from the flat-space model to a curved one.

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