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Dark Matters

A UV Complete Picture of Asymmetric Dark Matter and Coannihilations

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Abstract

Asymmetric dark matter models simultaneously explain the predominance of matter over antimatter in our universe (baryogenesis) and why $\Omega_{DM} \simeq 5\Omega_{VM}$. We look at asymmetric dark matter models that have a conserved global $U(1)$ symmetry, which creates equal and opposite asymmetries in the dark and visible sectors. Recently the idea of an asymmetry forming through coannihilation has been introduced, but has not been widely studied, with most asymmetric dark matter models relying on other means to generate an asymmetry. This thesis looks at high energy completions of a toy model of coannihilation, and explores a new model, the lepton portal. We study the CP violation and asymmetry formation of these theories. We find that the lepton portal is capable of correctly producing the relic abundance of dark matter, and explaining the matter-antimatter asymmetry.

Statement of contributions

Chapter 1 is an original review, following in the same vein as the cited papers. Chapters 2-5, as well as Appendices A and B, are original research conducted by the author unless otherwise stated. In addition to the work appearing in this thesis, the author contributed a section on UV completions to [40].

Acknowledgements

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Contents

Chapter 1

Introduction and Review

Cosmology has come a long way since Ptolemy's celestial spheres, but we still lack a clear understanding of the material that makes up our universe and how this material came to exist. The Standard Model (SM) does an excellent job of describing the matter we observe on earth and in terrestrial experiments (baryonic matter), but astrophysical observations show us that the SM only describes 15% of the matter in the universe[1]. This "dark matter", though it is invisible, has a profound impact on the formation of structure in our universe; without it we would not have the rich array of galaxies we see today. Further, the SM cannot explain how the luminous matter came to exist. In fact the SM predicts an almost empty universe. Asymmetric dark matter (ADM) models explain the formation of this baryonic matter (baryogenesis) via interactions between luminous matter and dark matter. This thesis explores a recent avenue for achieving ADM, where coannihilations of particles, rather than the typical decay, cause baryogenesis. In particular, we are looking for a renormalisable theory that can accomplish this.

1.1 Dark Matter

Dark matter is one of the oldest cosmological problems. As far back as 1933 [1] there has been evidence that a significant fraction (85%) of the universe's matter was non-luminous, or "dark". In the 80 years since then, it has grown increasingly certain that five-sixths of the matter in the universe does not strongly interact with SM particles (via the strong or electromagnetic forces). Unfortunately, despite the significant indirect evidence that dark matter exists, we are yet to have a clear and uncontroversial discovery of dark matter via more direct means. This is due mainly to the weak interactions between dark and visible matter - we only have evidence of gravitational interactions. Unfortunately because of this it is extremely difficult to narrow down the large number of dark matter candidates which, while giving theorists a large amount of freedom, deprives us of a clear direction for explaining this excess matter. To remedy this, theorists attempt to incorporate dark matter into a larger theoretical framework in order to provide new ways to understand it.

Though initial evidence for dark matter came from rotation curves of galaxies, more compelling evidence for non-baryonic matter exists. Gravitational lensing indicates that galaxies and clusters of galaxies possess significant amounts of non-luminous matter, while at the same time demonstrating that massive astrophysical bodies could not provide this extra mass [2, 3]. Data from the Bullet Cluster (two colliding clusters of galaxies) indicates that, at least at low temperature, dark matter does not have significant self interactions [4]. Moreover, non-baryonic matter is required to explain the formation of large scale structure in the universe [5], as well as the acoustic peaks of the CMB [6].

[1]Only 30% of the universe is any kind of matter; the lion's share of the universe consists of the mysterious "dark energy".

Dark Matter Candidates

Due to the large amount of freedom theorists have in constructing dark matter models, it is not surprising that there are many candidates for the elusive dark matter. Asymmetric dark matter theories will be discussed later in section 1.4.

WIMPs

Historically, the most popular candidate for dark matter is that of a weakly interacting massive particle (or WIMP). As the name would suggest, WIMPs interact with normal matter with a strength similar to that of the weak force. WIMPs do not couple at tree level to photons or gluons, though they often have tree level interactions with the W and Z bosons. WIMPs naturally arise in many beyond the SM theories (such as supersymmetry) and with the so-called "WIMP miracle" can naturally produce the correct relic abundance of dark matter [7]. To see this, note that if the relic density is determined by thermal freeze out then

$$\Omega_{DM} \sim \frac{x_f T_0^3}{\rho_c M_{pl}} \langle \sigma_A v \rangle^{-1}, \tag{1.1}$$

where $x_f = m_{DM}/T_{\text{freeze out}}$ (typically ~ 20), T_0 is the present day temperature, ρ_c is the critical density, $\langle \sigma_A v \rangle$ is the velocity averaged cross section of the annihilations, Ω_{DM} (Ω_{VM}) is the ratio of dark matter (visible matter) density to the critical density and $M_{Pl} \simeq 1.22 \times 10^{19}$ GeV is the Planck Mass. For a weak scale cross section,

$$\sigma_A v \simeq \frac{g_{weak}^4}{16\pi^2 m_{DM}^2} \times (1 \text{ or } v^2), \tag{1.2}$$

where the factor of 1 (v^2) is for S-wave (P-wave) interactions and $g_{weak} \simeq 0.65$ is the weak gauge coupling. Plugging in the numbers, a dark matter mass of $100 - 1000$ GeV reproduces the correct relic density. As this is a very natural mass scale for the weak force (the W, Z and Higgs bosons are all ~ 100 GeV), we have the WIMP miracle.

Unfortunately there is a lot of conflicting evidence regarding the potential discovery of WIMP particles, and minimal supersymmetry is disfavoured by the LHC. More complex theories of supersymmetry and unrelated theories (such as Kaluza-Klein dark matter from extra dimensions) can still produce WIMPs [8].

Exotics

Unlike WIMPs, which are produced thermally, exotics are (usually) produced by non-thermal effects. These less familiar particles, such as wimpzillas and axions, are often produced through phase transitions or inflationary effects [9, 10]. For example, axions were introduced to solve the strong CP problem (why there is no charge-parity (CP) violation in the strong force) as a pseudo-Nambu-Goldstone boson due to the breaking of the proposed Peccei-Quinn symmetry. They can be produced non-thermally by the phase transition associated with this symmetry breaking. Unlike WIMPs, axions do not naturally produce the correct abundance of dark matter, which must be put in by hand.

While this is clearly an incomplete list of the various dark matter theories, it gives some idea of the cornucopia of theories at a physicist's disposal when trying to describe dark matter. See [7] for a more in depth discussion of dark matter theories.

1.2 Baryogenesis

A puzzling feature of the universe is that it consists almost entirely of matter. In colliders, whenever we create a particle of matter, a corresponding antimatter particle is made. Indeed, in perturbative SM interactions, the total baryon number, B, is conserved. So how then can we explain this difference in abundance between matter and antimatter? We might consider different initial conditions for matter and antimatter, but inflation dilutes any initial nonzero B to be negligible [11]. So it is necessary to have a dynamically generated net baryon number. The forming of this asymmetry is called "baryogenesis". We define the baryon number of a proton to be 1, so quarks have a baryon number of $\frac{1}{3}$ and antiquarks have a baryon number of $-\frac{1}{3}$. We can similarly define a lepton number, L, where electrons and neutrinos have a lepton number of 1.

Evidence for Baryogenesis

On Earth clearly there is effectively no antimatter (any sizeable amount of antimatter would soon destroy those searching for it). But this does not tell us that there are no stores of antimatter elsewhere in the universe. By looking for the tell-tale gamma rays from hydrogen annihilating with anti-hydrogen (such as at the borders between a matter and an antimatter region), we can search the observable universe for antimatter. This rules out antimatter regions on scales smaller than 10^{20} solar masses [12], so if the universe was baryon symmetric we would need to segregate matter and antimatter on scales of at least 10^{20} solar masses.

We can also employ the freeze out condition to determine the relic abundance of matter and antimatter. Freeze out occurs when the rate of an interaction maintaining equilibrium becomes less than the rate of expansion of the universe. If B is conserved, annihilations freeze out at a temperature of around 22 MeV. This corresponds to $\frac{n_b}{s} = \frac{n_{\bar{b}}}{s} = 7 \times 10^{-20}$, where $\frac{n_b}{s}$ is the number density of baryons weighted by the entropy. However we know that the average density of the universe is nine orders of magnitude larger [13]. Further, to avoid this annihilation through segregation of matter and antimatter, the mechanism must occur before the abundances drop below $\frac{n_b}{s} = \frac{n_{\bar{b}}}{s} = 8 \times 10^{-11}$, their present value. This corresponds to a temperature of 38 MeV. At 38 MeV the horizon of the universe contains only 10^{-7} solar masses, so we could not create a segregation of the order 10^{20} solar masses without violating locality [14].

The need for baryogenesis seems inescapable.

Sakharov Conditions

Now that we know baryogenesis is necessary, we might wonder what conditions are required for the formation of a baryon asymmetry. We can use these to identify theories that might effectively lead to baryogenesis. It turns out that there are three conditions we must meet, known as the Sakharov conditions [15] .

1. *B* violation. This first condition seems self evident. No matter what else is happening, if there are no interactions that violate baryon number then no net baryon number may form. We might ask ourselves if the SM can provide baryon number violating terms; while baryon number is an accidental global U(1) symmetry of the SM, it is in fact anomalous [16]. Sphalerons, the product of tunneling between different non-trivial

vacuum gauge configurations, violate B, while conserving $B - L$.

2. C and CP violation. If matter and antimatter are not treated differently, then any B violating interactions will be cancelled by corresponding charge reversed B violating interactions. For example, consider

$$ff \to \bar{f}f, \tag{1.3}$$

with f being a fermion charged under B. This interaction violates baryon number by $-2B(f)$. If C, charge conjugation, is not violated then this reaction rate is exactly the same as the rate of

$$\overline{ff} \to \bar{f}f, \tag{1.4}$$

which violates baryon number by $+2B(f)$. So C must be violated to get a net baryon number. When we average over spins we find that we also need CP violation. The weak interaction has C maximally violated but little CP violation. Exactly how CP violation generates an asymmetry is discussed in section 1.3.

3. Thermal Non-equilibrium. So say we have a theory with B, C and CP violation. If the particles are in chemical equilibrium, then the chemical potentials of any non conserved quantum number are zero. In addition, if the particles are also in kinetic equilibrium then they will share a temperature with their CP conjugates. CPT invariance guarantees that a particle and its antiparticle have the same mass [17]. So as the phase space densities are given by $[1 + \exp{(p^2 + m^2)/T}]^{-1}$, they are necessarily equal at thermal equilibrium. This is essentially due to requiring unitarity and CPT invariance [18]. Fortunately, due to the expansion of the universe, it is not too hard to find non-equilibrium conditions. For example, in the SM a small baryon asymmetry could be caused by the electroweak phase transition (EWPT). Unfortunately this transition is second order, and consequently does not lead to a significant abundance of matter [19]. Roughly speaking, a first order phase transition is discontinuous in the first derivative of the free energy, like water boiling, and a second order phase transition is discontinuous in the second derivative. Viable electroweak baryogenesis theories add interactions that make the electroweak phase transition first order.

While the SM satisfies all three Sakharov conditions, the baryon asymmetry created within the SM is several orders of magnitude too small [20] to explain the observed matter abundance. The presence of B and L violating (but $B - L$ conserving) interactions in the SM cannot be ignored, as they are often very useful in theories of baryogenesis. In particular, leptogenic theories create a nonzero L, and then use sphalerons to transfer the asymmetry to the baryons [21]. In Chapter 4, we will follow a similar path with the lepton portal.

Boltzmann Equations

Once we invent some interactions satisfying the Sakharov conditions, we need to know the magnitude and nature of the asymmetry they generate. The equations that govern the evolution of the number density of a particular particle are known as the Boltzmann equations. For the process $\psi + a + b + \ldots \longrightarrow i + j + \ldots$, evolving in the Friedmann-Robertson-Walker metric, the evolution of n_ψ is given by [17]:

$$\frac{dY}{dx} = -\frac{x}{H(m)s} \times C(\psi), \tag{1.5}$$

where $d\Pi \equiv g\frac{1}{(2\pi)^3}\frac{d^3p}{2E}$, $x = \frac{m}{T}$, $Y \equiv \frac{n_\psi}{s}$, $H(m) = 1.67 g_*^{1/2} m^2/m_{Pl}$ is the expansion rate of the universe, $|\mathcal{M}|^2$ is the matrix element squared, f_i is the phase space density of particle species i, s is the entropy per comoving volume and

$$\begin{aligned} C(\psi) \equiv & \int d\Pi_\psi \, d\Pi_a \, d\Pi_b \ldots d\Pi_i \, d\Pi_j \ \ldots (2\pi^4) \, \delta^4(p_i + p_j \ldots - p_\psi - p_a - p_b \ldots) \\ & \times \Big[|\mathcal{M}|^2_{\psi+a+b+\ldots\to i+j+\ldots} f_a f_{b\psi} - |\mathcal{M}|^2_{i+j+\ldots\to\psi+a+b+\ldots} f_i f_j \ldots \Big]. \end{aligned} \tag{1.6}$$

$C(\psi)$ is often referred to as the collision term. In this case, g does not denote a coupling but rather the number of internal degrees of freedom and g_* is effective degrees of freedom of the radiation bath. If the process conserves time reversal (T) symmetry (which CP violating processes do not), then

$$|\mathcal{M}|^2_{i+j+\ldots\to\psi+a+b+\ldots} = |\mathcal{M}|^2_{\psi+a+b+\ldots\to i+j+\ldots}. \tag{1.7}$$

To solve the Boltzmann equations, it is necessary to write them in a more malleable form. As we will be using Maxwell-Boltzmann statistics as a (good) approximation throughout this thesis, we can factor out the chemical potential from the phase space

$$f_\psi = e^{(\mu_\psi - E_\psi)/T} = e^{\mu_\psi/T} f_\psi^{eq}, \tag{1.8}$$

where f^{eq} and the soon to be used n^{eq} refer to the equilibrium values when the chemical potential is zero. We can then use conservation of energy[2] to make the replacement

$$f_\psi^{eq} f_a^{eq} \ldots \to f_i^{eq} f_j^{eq} \ldots, \tag{1.9}$$

and define:

$$r_\psi \equiv \frac{n_\psi}{n_\psi^{eq}} = e^{\mu_\psi/T}. \tag{1.10}$$

If time reversal (T) symmetry is respected then the collision term can be written as:

$$\begin{aligned} C(\psi) =& (r_i r_j \ldots - r_\psi r_a \ldots) \\ & \times \int d\Pi_\psi d\Pi_a d\Pi_b \Pi_i d\Pi_j \ldots (2\pi^4) \delta^4(p_i + p_j \ldots - p_\psi - p_a - p_b \ldots) f_1^{eq} \ldots |\mathcal{M}(\psi, a \ldots \to i, j \ldots)|^2 \\ & \equiv (r_i r_j \ldots - r_\psi r_a \ldots) W(\psi, a \ldots \to i, j \ldots), \end{aligned} \tag{1.11}$$

where $W(\beta \to \alpha)$ is the equilibrium reaction rate density. To satisfy the Sakharov conditions, there must be CP (and by the CPT theorem, T) violation. We parametrize the CP violation as,

$$\epsilon_{\psi,a\ldots\to i,j\ldots} = \frac{W(\psi, a \ldots \to i, j \ldots) - W(i, j \ldots \to \psi, a \ldots)}{W(\psi, a \ldots \to i, j \ldots) + W(i, j \ldots \to \psi, a \ldots)}, \tag{1.12}$$

and define the CP symmetric reaction rate density as:

$$W_{sym} = \frac{1}{2}\Big[W(i, j \ldots \to \psi, a \ldots) + W(\psi, a \ldots \to i, j \ldots)\Big]. \tag{1.13}$$

In general both $\epsilon_{\psi,a\ldots\to i,j\ldots}$ and W_{sym} will be temperature dependent. Putting this all together we have

$$\begin{aligned} C(\psi) &= r_i r_j (i, j \ldots \to \psi, a \ldots) - r_\psi r_a (\psi, a \ldots \to i, j \ldots) \\ &= W_{sym}\Big[r_i r_j \ldots (1 - \epsilon_{\psi,a\ldots\to i,j\ldots}) - r_\psi r_a \ldots (1 + \epsilon_{\psi,a\ldots\to i,j\ldots})\Big]. \end{aligned} \tag{1.14}$$

[2]Technically in an expanding universe there is no timelike Killing vector that would allow us to define energy as a conserved quantity. Fortunately in practice the relatively slow expansion of the universe allows us to treat energy as conserved. Similarly, particles are never quite in equilibrium.

The evidence of a baryon asymmetric universe tells us that the SM fundamentally cannot explain the universe we live in. This provides some of the strongest evidence for physics beyond the SM, both scientifically and psychologically. The abundance of matter is almost entirely dependent on the asymmetry between matter and antimatter; any theory related to the abundance of matter must explain this discrepancy.

1.3 CP Violation

Charge, Parity and Time Reversal symmetries have played a pivotal role in the development of modern physics. In QCD, QED and classical mechanics these are all exact symmetries. Due to this many physicists were very surprised to see C and P violated by the weak interaction [22]. They were even more surprised when CP violation was discovered in 1964 [23]; CP violation implies that there is a preferred direction of time in the laws of physics (by the CPT theorem [24]). As CP violation is one of the Sakharov conditions, it is fortunate for us that this is the case.

While C, P and T are all broken symmetries, as long as Lorentz invariance holds the CPT theorem guarantees that CPT is conserved. In theories of spontaneous baryogenesis, CPT and Lorentz invariance are spontaneously broken, so the Sakharov conditions do not hold [17]. CP violation and thermal non-equilibrium are not required as the mass of a particle and its antiparticle are now different. We do not consider models of this type.

It is also worth mentioning that if we want a process, such as a decay, to violate CP we must have more then one final state allowed. This is due to a combination of CPT and unitarity; it is easy enough to show [18] that

$$\sum_j |\mathcal{M}(i \to j)|^2 = \sum_j |\mathcal{M}(\bar{i} \to j)|^2, \tag{1.15}$$

where $\bar{i}$ is the CP transform of i.

How do we get CP violation?

While charge conjugation is relatively straightforward, CP violation has several subtle features. It is possible to show that for CP violation to occur we must go beyond tree level [25]. To give a clearer picture of how the complexity of the couplings and higher order diagrams come into play in the generation of CP violation, consider a toy model where we have complex couplings [17],

$$\mathcal{L} = g_1 X \overline{f}_1 f_2 + g_2 X \overline{f}_3 f_4 + g_3 Y \overline{f}_1 f_3 + g_4 Y \overline{f}_2 f_4 + H.c, \tag{1.16}$$

where the g are dimensionless couplings, X and Y are scalar fields and the f are fermionic fields. When looking for CP violation, we desire theories with complex couplings. Compare the CP and Hermitian conjugate transforms[3]

$$\begin{aligned} gX\overline{f}_1 f_2 &\xrightarrow{CP} gX\overline{f_2} f_1 \\ gX\overline{f}_1 f_2 &\xrightarrow{H.c} g^* X\overline{f_2} f_1. \end{aligned} \tag{1.17}$$

[3]See [26] for a complete list of C, P and T transforms

We can see from this that the Hermitian conjugate of a term is structurally the same as a CP conjugate, but has the complex conjugate of the coupling constant, g. So the degree to which a term violates CP is measured by the complexity of the coupling. To ensure the couplings are indeed complex, it is important to consider rephases of the fields when looking at CP violation. A charged particle f can always be redefined, $f \to e^{i\theta} f$, which can be used to remove a complex phase. We will refer to this as "rephasing" a particle. As this is just a field redefinition, none of the physics is altered: no change will be made to any observable. But there is a clearer picture of how many physical phases there actually are. For example, in the toy Lagrangian above, if X and Y are complex scalars, the fs, as well as X and Y can be rephased. However, the couplings are constructed so there are not four linearly independent equations. Because of this, while any of the g can be chosen to be real, $g_1 g_2^* g_3 g_4^*$ cannot be made real, so there is one physical CP violating phase. Further, CP violating effects must involve all four of the g. If some different, less prescient definition of our fields were made the observables would not change but it would be unclear why only one combination of phases kept appearing in the cross sections.

If we compare, say, the process $X \to \overline{f}_1 f_2$ with its conjugate, we get

$$\begin{aligned} \Gamma(X \to \overline{f}_1 f_2) &= |g_1|^2 I_{\text{tree}} + g_1 g_2^* g_3 g_4^* I_{\text{int}} + (g_1 g_2^* g_3 g_4^* I_{\text{int}})^* + |g_2|^2 |g_3|^2 |g_4|^2 I_{\text{loop}} + \dots, \\ \Gamma(\overline{X} \to \overline{f}_2 f_1) &= |g_1|^2 I_{\text{tree}} + g_1^* g_2 g_3^* g_4 I_{\text{int}} + (g_1^* g_2 g_3^* g_4 I_{\text{int}})^* + |g_2|^2 |g_3|^2 |g_4|^2 I_{\text{loop}} + \dots, \end{aligned} \tag{1.18}$$

where we have simply used the usual matrix element perturbation series and pulled the coupling constants out from the functions of momentum. I_{tree} denotes the momentum part of the tree level decay width and I_{int} is the interference between tree level and the one loop graph, as shown in fig. 1.1.

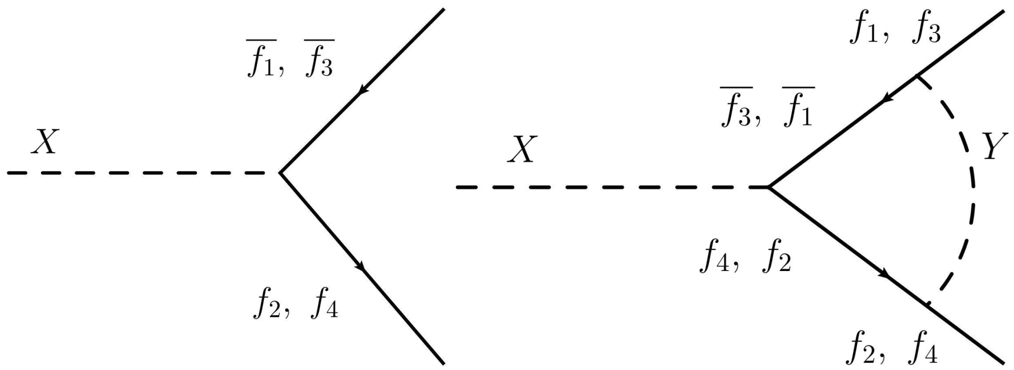

Figure 1.1: Graphs contributing to CP violation.

The difference between the two decay rates is given by $4\text{Im}(g_1 g_2^* g_3 g_4^*)\text{Im}(I_{\text{int}})$ at lowest order. From this it is easy to see that to get CP violation we must get a net complex phase, and I_{int} must similarly be complex. With the exception of particle antiparticle mixing (such as in kaons), the requirement that I_{int} is complex is equivalent to the requirement that the loop particles must be on shell [18]. The imaginary part of I_{int} is often referred to as the "absorptive phase".

To evaluate the imaginary parts of the phase space function I_{int}, we will use Cutkosky rules [27]. It is possible to show that, ignoring complex couplings, the only way to have a complex matrix element is to have a branch cut in the centre of mass energy [28]. To find the discontinuity of the matrix element associated with a Feynman diagram (Disc(M) = $2i$Im(M)), we simply follow three steps:

1. Cut through the diagram in all ways that allow one to simultaneously put cut propagators on shell, splitting the graph into two halves. We must make sure to keep the first half of the graph containing all the initial particles, and the second half containing all the final particles.

2. Replace the cut propagators ($\frac{1}{p^2-m^2+i\epsilon}$; for fermions we do not replace the numerator of the propagator) with $2\pi i\delta(p^2-m^2)$ and evaluate the loop integrals.

3. Sum over all possible cuts.

This is computationally much more efficient than calculating the full matrix element and taking the imaginary part, and usually allows one to bypass divergences and renormalisation concerns. By calculating Im(I_{int}), and knowing the complex phases of a theory, we know exactly how CP violation enters our interactions. An example calculation can be found in Appendix A. It should be noted that these are the cutting rules for zero temperature quantum field theory; when thermal effects dominate it is necessary to use the rules contained in [29, 30]. As we will only be interested in the dominant contributions to CP violation near freeze out we will use the zero temperature cutting rules.

1.4 Asymmetric Dark Matter

Previously we noted that $\Omega_{DM} \simeq 5\Omega_{VM}$. In scenarios where the abundances of dark and visible matter are unrelated (such as the standard WIMP scenario), this must be chalked up to a massive coincidence. The core concept of asymmetric dark matter (ADM) is that dark and visible matter share a common origin, with their abundance determined by the same process. As we noted in the preceding section, the abundance of visible matter is determined by baryogenesis - by the asymmetry between matter and antimatter. In the purest incarnation of this idea, baryogenesis is caused by interactions between dark and visible matter. While this idea has been around for a while [31], it is only recently that the abundances of dark and visible matter were confirmed to be so similar. Since then, there has been a large increase in interest in ADM models.

Types of Asymmetric Dark Matter

As we desire to relate the asymmetry in normal matter to the abundance of dark matter, it makes sense to define a dark analogue of baryon number for dark matter. For visible and dark matter to be stable, both symmetries must be preserved at low energies. For baryogenesis to occur, some combination of the two must be broken. We classify three basic types of models based on the preservation or non preservation of the global symmetries. Denoting the visible baryon number B, and the dark baryon number D, the possibilities are:

1. A (nontrivial) linear combination of B and D is conserved, but a linearly independent combination is not. This is the most common option in ADM, as it naturally gives $\Omega_{DM} \simeq \Omega_{VM}$. It is easy enough to show that we can always consider the conserved combination to be $B-D$ and the broken combination to be $B+D$ [32]. To avoid anomalies breaking the conserved linear combination, the visible baryon number is often actually a combination, $B-L$, of baryon and lepton number. The models studied fall into this category.

2. One of B or D is broken, and the other is conserved. In these models some extra mechanism is required to reprocess the asymmetries until they are roughly equal (see [33] for an example).

3. No combination of B and D is conserved. To regain the relation $\Omega_{DM} \simeq \Omega_{VM}$, some process must force the asymmetry in dark and visible matter to be similar (e.g [34]).

We will focus on the first type of ADM, where there is a conserved global symmetry. The most obvious value for the DM mass is ~5 GeV. This assumes that the the dark matter has $Q(DM)_D = \pm 1$.[4] As long as the symmetric component of dark matter annihilates away we regain the relationship $\Omega_{DM} \simeq 5\Omega_{VM}$. It is possible to have dark matter carry a different baryonic number, and consequently gain freedom in the dark matter mass. After the electroweak phase transition is taken into account, we find that [32]

$$m_{DM} \simeq Q(DM)_D \times (1.7 - 5)\ \text{GeV}. \tag{1.19}$$

where $Q(DM)_D$ is the charge of dark matter under D. The value of 1.7 GeV occurs for totally asymmetric dark matter with a Q_D of 1 where baryogenesis occurs entirely before the EWPT. This is a wonderful constraint on the mass of dark matter for a given model. Alas most ADM theories do not contain an explanation as to why the mass of dark matter is so similar to the proton mass. The exceptions to this are Mirror Matter and models involving the breaking larger gauge groups [31, 35]

As in normal baryogenesis, there are a number of mechanisms to create the asymmetry. Decays of heavy particles [34], first order phase transitions [36] and the Affleck-Dine mechanism [37] have all been used to create a baryon asymmetry, but we will focus on a more recent mechanism.

Asymmetric freeze out was first used in an ADM context in 2006, through the neutron portal model [38]. Consider an interaction of the type

$$\overline{d_i} + X \rightarrow d_j + u_k, \tag{1.20}$$

where X, the dark matter, is a Dirac spin half fermion. This conserves a global $U_{B-L-D}(1)$ symmetry while violating D and B. These coannihilation interactions can violate CP, and so freeze out occurrs at different times for dark matter and its antiparticle. If X freezes out after $\overline{X}$, then there will be extra quarks created through the above processes. These same processes leads to proton decay [39], as well as the dark matter particle decaying. In the original proposal, a strongly temperature dependent coupling was introduced to resolve this, suppressing baryon number violating interactions at low temperature. Unfortunately the original proposal does not lead to baryogenesis due to unitarity constraints; it is possible to extend this model so that baryogenesis occurs and protons do not decay [39]. We will discuss this extension in Chapter 3. This method has been generalised recently in [40] with the introduction of a toy model, and has been used in [41] to create a version of the neutron portal capable of achieving baryogenesis (albeit with no dark matter candidate). While coannihilations have been considered in a leptogenesis context [42, 43], the CP violation (in particular its temperature dependence) is different for these models. Notably, while in leptogenesis these scatterings are usually negligible, we will show that it is possible to generate the full baryon asymmetry of the universe with just scatterings when dealing with ADM.

[4] This also assumes that baryogenesis occurs after the electroweak phase transition.

1.5 UV completions

In all these cases, and many others in particle cosmology, effective field theories (EFTs) were used to reduce model dependence and study the broad behaviour without needing too many specific details. In an EFT heavy particles are integrated out, giving couplings with non-renormalizable mass dimensions (mass dimensions less than zero). Unfortunately, this comes at the cost of removing some of the information which a renormalisable (or UV complete) theory possesses. Renormalisable theories function at high energies where an EFT would break down, revealing high energy phenomena.

The move from an EFT to a UV complete theory has a strong precedent in the SM. Originally, through the Fermi "current-current" model the weak force was thought of as a contact interaction between four fermions; $u,\ d,\ e$ and ν, given by the operator[5] [44]

$$A\bar{d}\gamma^{\mu}u\bar{e}\gamma_{\mu}\nu. \tag{1.21}$$

While, with some modification of the Dirac structure, this models the low energy behaviour of the weak interaction well, problems arise when considering the coupling A. The mass dimension of A is -2, so the cross sections at tree level of the $2 \rightarrow 2$ scatterings at some energy E are given by

$$\sigma(u\bar{d} \rightarrow e^{+}\nu) \simeq \frac{A^2E^2}{\pi}, \tag{1.22}$$

which is unbounded. In fact, the perturbation series expansion, which consists of an infinite series of terms $\propto A^{2n}E^{4n-2}$, blows up at $E \sim 1/\sqrt{A} \sim 100$ GeV. This violates unitarity. While this may seem like a fatal blow to the theory, it is more proper to think of it as an indication of new physics. If a new massive particle is included to mediate the interaction then (1.2) is broken up into two renormalisable operators. This is often referred to as "opening up" the operator, shown in fig. 1.2. In the case of the weak interaction, a ~ 100 GeV vector boson, the W boson, can accomplish this, though massive vector bosons themselves violate unitarity. As this is the story of baryogenesis, and not the Higgs mechanism, we will not be concerned with this. Now the operators are

$$\frac{g}{\sqrt{2}}\bar{e}_L\gamma^{\mu}\nu_L W^{-}_{\mu} + \frac{g}{\sqrt{2}}\bar{d}_L\gamma^{\mu}u_L W^{-}_{\mu} + H.c, \tag{1.23}$$

where g is the dimensionless weak coupling constant, and $_L$ refers to the left chiral component of the respective fermion.[6] When the Higgs mechanism is included, this theory now produces sensible answers at any energy scale (or at least until gravitational effects become important). To see some of the benefits in UV completing this EFT, note that in the EFT we should have also included the operators

$$B\bar{d}\gamma^{\mu}u\bar{u}\gamma_{\mu}d, \;\; C\bar{\nu}\gamma^{\mu}e\bar{e}\gamma_{\mu}\nu, \tag{1.24}$$

as they are allowed by the same symmetries as (1.2). But there is no obvious connection between A, B and C, as well as the respective couplings for different families of the fermions. When looking at the UV completion, however, it is clear that they all must be the same: Fermi's constant $G_F = g^2/(4\sqrt{2}m_W^2)$. This inability to predict relationships between the different parameters is one of the main limitations of EFTs. Many physicists live in hope

[5] This operator is often written in terms of the proton and neutron for historical reasons, but we have neglected to do so here for clarity.

[6] This is the modification of the Dirac structure mentioned before.

that we will one day find a theory to which the SM itself is the low energy limit and so explain the many mysteries of the SM. Relevant to our work, EFTs are incapable of properly calculating CP violation. While EFTs can often predict at what order CP violation should occur, they do not give the full dependence on the couplings of the underlying theory; as above an EFT often overestimates the number of independent couplings. We shall show that the CP violation that was assumed to occur in the EFTs of [40, 41] is in fact non-minimal, requiring multiple heavy intermediaries. Further, EFT's cannot give a full measure of how complex or natural a theory is; it is possible to write down two EFT's with the same number of parameters that have minimal UV completions of differing complexity.

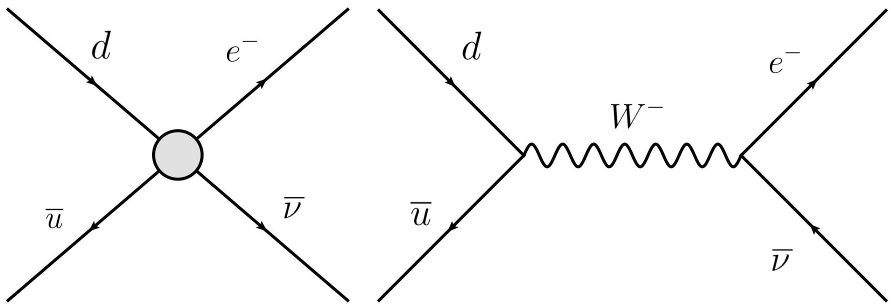

Figure 1.2: Effective operator (left) compared to a UV completion (right). The non-renormalisable vertex (denoted by the circle) has been replaced with an intermediate particle. It is clear that the UV completion reduces to the EFT when the energies are small compared to the mass of the W; explicitly the propagator $1/(p^2 - m_W^2)$ goes to $-1/m_W^2$ in this limit (neglecting couplings and Lorentz structure).

1.6 Thesis Outline

In this research we explore UV completions of coannihilation models of ADM. In Chapter 2 we use the toy model introduced in [40] as a testing ground for these techniques, and to study coannihilation in a clean environment. Having found UV completions of the toy model, we calculate the CP violation of these models, which come from one loop corrections. Once we know the CP violation, we solve the Boltzmann equations for the evolution of the number densities and asymmetries of the particles. We then extend these methods in order to study the more realistic scenarios, in particular those which lead to stable dark matter. In 3, we explore extensions of the neutron portal, and introduce a new model which we will refer to as the lepton portal. We analyse a case study of the lepton portal in Chapter 4, and show that it can lead to viable ADM.

Chapter 2

Toy Model

To gain an insight into more realistic models of asymmetric dark matter, we must first cut our teeth on the toy model introduced in [40]. This is the simplest case of asymmetric dark matter via coannihilations and, while not physically realistic, it contains all the essential ingredients for baryogenesis. Originally the only method to generate a baryon asymmetry was thought to involve a process that violates both baryon number and CP. However, we will show that it is possible to divide the task of satisfying Sakharov's conditions between different processes. In this way, baryogenesis can be caused indirectly, by first forming a flavour asymmetry. The flavour asymmetry is generated by processes that violate CP while conserving $B-L$, and then is transferred to a baryon asymmetry by $B-L$ violating (but CP conserving) processes. This shows that there are two ways for baryogenesis to occur with $2 \to 2$ scatterings. In order for the toy model to work as originally intended, we will show that it is necessary to introduce two copies of the mediating particle. While after this extension flavour effects can still be important, the original source terms are far more effective at generating an asymmetry.

2.1 Minimal Completion

We consider the toy model

$$\mathcal{L}_{int} = \kappa^{ij}\overline{\psi_i^c}\psi_j\overline{f^c}f + \epsilon^{ijkl}\overline{\psi_i^c}\psi_j\overline{\psi_k}\psi_l^c + G\overline{f^c}f\overline{f}f^c, \tag{2.1}$$

where the ψs and f are Dirac fermions [40]. In this model the fs will form the dark matter, and carry a dark baryon number equivalent that we will call D, with, for simplicity, $Q_D(f) = 1$. The ψ sare gauge singlets under the SM, carrying a nonzero SM $B-L$ of 1. Again we have chosen $B-L$ of 1 for simplicity. $B-L$ and D are broken, but there is a global $U_{B-L-D}(1)$ which is conserved. This global U(1) symmetry protects the model from Majorana mass terms. If the ψs were Majorana we would not be able store an asymmetry in the ψs.[1]

The simplest way to break up the effective operators into mass dimension 4 operators is by introducing a scalar. It is also possible to introduce further Dirac structure (γ_μ and possibly γ_5), allowing us to break this operator with a massive vector boson. If this vector boson comes from a gauge boson, with a broken symmetry, then all the couplings will be the same within families. Because this is non minimal and gives us less freedom to choose the couplings (asymmetric couplings enhance the asymmetry formed) we will not consider this version. The Lagrangian terms are

$$\mathcal{L}_{int} = -\lambda_{ij}\overline{\psi_i^c}X\psi_j - g\overline{f^c}X^*f + H.c, \tag{2.2}$$

where X is a complex scalar carrying a Q_{B-L-D} of -2, and is significantly more massive than the other particles. We have imposed a $\mathbb{Z}_2$ symmetry to stop ψs decaying to f via terms involving $\overline{f}X\psi_i$, which would be allowed if we only had a U(1) symmetry. The minimum

[1] In our case the ψs are not ultra relativistic, so we cannot use helicity as a substitute for baryon number as in [45].

number of generations of ψ for a CP violating phase to arise is two, though written in this form it is difficult to see what, if any, symmetries the matrix λ_{ij} has. As there are charge conjugate terms in the couplings, the diagonal terms are not necessarily real. Writing out the full Lagrangian we have

$$\begin{aligned} &\lambda_{11}\overline{\psi_1^c}X\psi_1 + \lambda_{11}^*\overline{\psi_1}X^*\psi_1^c \\ &+\lambda_{12}\overline{\psi_1^c}X\psi_2 + \lambda_{12}^*\overline{\psi_2}X^*\psi_1^c \\ &+\lambda_{21}\overline{\psi_2^c}X\psi_1 + \lambda_{21}^*\overline{\psi_1}X^*\psi_2^c \\ &+\lambda_{22}\overline{\psi_2^c}X\psi_2 + \lambda_{22}^*\overline{\psi_2}X^*\psi_2^c \\ &+g\overline{f^c}X^*f + g^*\overline{f}Xf^c. \end{aligned} \tag{2.3}$$

While λ_{ij} is not symmetric, it has the same degrees of freedom as a symmetric matrix. To see this, we make use of the identity $\overline{\phi}\chi^c = \overline{\chi}\phi^c$. Hence, we now define $\lambda_3 = \lambda_{12} + \lambda_{21}$ and rename λ_{11} and λ_{22} as λ_1 and λ_2 respectively. So there are four phases which are complex (before any field rephasing); g, λ_1, λ_2 and λ_3.

It appears that, with four fields to rephase and four complex phases, all the couplings can be made real. Fortunately, there are not four linearly independent equations; the three λ cannot be made real simultaneously. We end up with a net complex phase from the combination $\lambda_3^2\lambda_1^*\lambda_2^*$. For our example solutions, we have assumed maximal CP violation, with $\lambda_3^2\lambda_1^*\lambda_2^*$ being purely imaginary. In general the asymmetry formed is roughly proportional to the imaginary part of $\lambda_3^2\lambda_1^*\lambda_2^*$.

The Lorentz structure of the operators could be changed, say to $\kappa^{ij}\overline{\psi_i^c}f\overline{f^c}\psi_j$. Unfortunately this makes it impossible to introduce a symmetry (such as $\mathbb{Z}_2$) to protect the model from $\psi \to \overline{fff}$ as the λ couplings cannot be forbidden. We would be forced to kinematically disallow this decay by requiring $M_{\psi_2} \simeq M_f$. This is overly restrictive for exploring the effects of coannihilation across a large parameter space, so we do not consider this version.

The toy model was designed so that the ψs go out of equilibrium as a result of the annihilations freezing out. As there is CP violation, the annihilations for ψ and $\overline{\psi}$ occur at different times, creating an asymmetry between ψ and $\overline{\psi}$. If ψ experiences freeze out before $\overline{\psi}$, $\overline{\psi}$ will be Boltzmann suppressed from the continued annihilations. This leaves us with a net $B-L$. We will show that it is not necessary to have CP violation in the $B-L$ violating terms. We have, for simplicity, $M_{\psi_1} > M_{\psi_2} \gg M_f$.

It should be noted that in this model the symmetric components of ψ_2 and f are not annihilated; while an asymmetry is generated, both ψ_2 and $\overline{\psi_2}$ populations remain high. To fulfil the ADM philosophy, the final abundance of dark matter must be determined by the baryon asymmetry. A further interaction must be introduced to accomplish this. For example, the ψ could decay into SM particles, and a new interaction could annihilate the f so that they form asymmetric dark matter. In our solutions we assume that the f are close to equilibrium for their entire thermal history. As the interactions responsible for asymmetry generation by coannihilation must freeze out, this will be a generic problem for these ADM models.

2.2 CP Violation

With two generations of ψ, there is exactly one CP violating phase and all the λ couplings must be included for the phase to present itself. At low temperatures (below M_X), graphs requiring that Xs in the loop are on shell become massively Boltzmann suppressed. For this reason the only graphs[2] (fig. 2.1) which contribute to the ϵ_a are negligible.

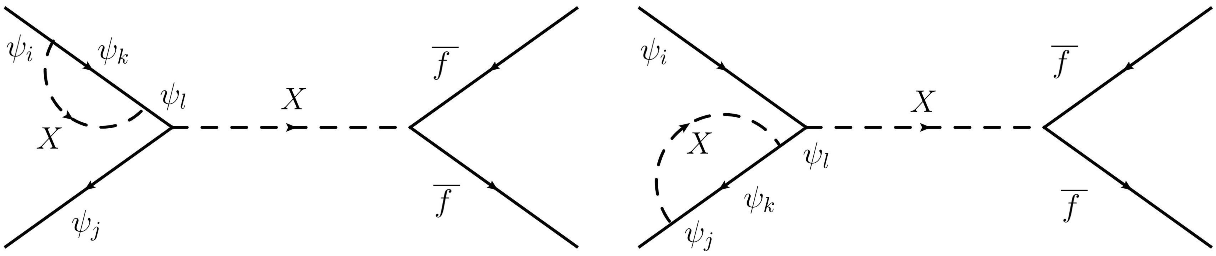

Figure 2.1: Graphs contributing to ϵ_a. At temperatures around m_ψ, these are negligible.

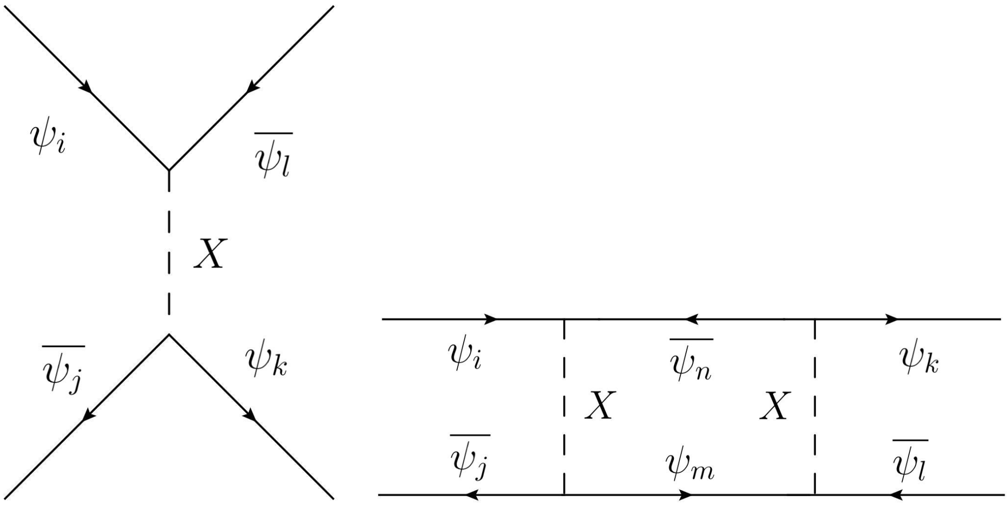

Figure 2.2: Tree level (left) and one loop (right) graphs contributing to CP violation in ϵ_{ijkl}.

CP violation for the $\psi_i\psi_j \to \psi_k\psi_l$ and $\psi_i f \to \overline{\psi_j f}$ processes are similarly negligible, with the same graphs contributing. Significant CP violation does occur in $\overline{\psi_i}\psi_j \to \overline{\psi_k}\psi_l$ processes. This comes from interference between the tree graph and the box graph (fig. 2.2), which can be cut at energies above the ψ masses. As these processes are baryon number conserving, it may seem irrelevant, but this CP violation allows us to generate a flavour asymmetry.

We do not write down, or gain much insight from, the general calculation of this graph as it would extend this thesis by nine pages and give the author a repetitive strain injury. Instead we will proceed directly to evaluations of the Boltzmann equations. For the curious, Appendix A has an example calculation following the Cutkosky rules that is the same in all respects save the trace structure. The traces were evaluated with Feyncalc [47], and the phase space integrals performed in Mathematica [48]. The results have been checked to satisfy the unitarity conditions, though small errors due to the numerical integration of the velocity averaged cross sections must be removed.

[2]This Feynman diagram, and all others throughout this work, were drawn with [46].

2.3 Boltzmann Equations

To work out precisely the asymmetry and calculate the number densities of our particles, we must solve the Boltzmann equations. The final baryon asymmetry is caused by the ψ freezing out, so we consider a senario where all the Xs have decayed. As $M_X \gg M_\psi$ (so we reclaim the EFT at low energies) this is a perfectly valid assumption. We also neglect thermal field theoretic effects in this analysis; as the purpose of the work is exploratory we only require accuracy up to an order of magnitude. Similarly, we will use Maxwell-Boltzmann statistics, neglecting Pauli blocking and stimulated emission. In order to focus on how coannihilations lead to baryogenesis we will kinematically disallow the decay $\psi_1 \to \overline{\psi_2}\psi_2\psi_2$. This means that the only decay is $\psi_1 \to \overline{\psi_2 f f}$, and by the CPT theorem it is CP conserving [18]. We use the notation $W(a+b \to c+d) \equiv n_a^{eq} n_b^{eq} \langle v\sigma(a+b \to c+d)\rangle$, and parameterize the CP violation as

$$\epsilon \equiv \frac{\langle v\sigma(a+b \to c+d)\rangle - \langle v\sigma(c+d \to a+b)\rangle}{\langle v\sigma(a+b \to c+d)\rangle + \langle v\sigma(c+d \to a+b)\rangle}.$$

We compute $\langle v\sigma(a+b \to c+d)\rangle$ from the cross sections by using the result from [49],

$$\langle v\sigma(a+b \to c+d)\rangle = \frac{g_a g_b T}{8\pi^4 n_a^{eq} n_b^{eq}} \int_{(m_a+m_b)^2}^{\infty} p_{ab} E_a E_b v_{\rm rel} \sigma K_1(\frac{\sqrt{s}}{T}) ds, \tag{2.4}$$

where s is the square of the centre-of-mass energy, p_{ab} is the centre-of-mass momentum of a and b and $K_i(x)$ is a modified Bessel function of the second kind of order i. This must be solved numerically, which can allow small artificial violations of unitarity to arise. To deal with these small errors, we enforce the unitarity relations (1.15), which will be discussed further below. We can now catalogue the interactions relevant to asymmetry formation. As a note of caution, we will not use the Einstein summation convention when discussing Boltzmann equations.

Annihilations

For annihilations, we have

$$W(\psi_i\psi_j \to \overline{ff}) \overset{\rm CPT}{=} W(ff \to \overline{\psi_i\psi_j}) = (1+\epsilon_a)W_a, \tag{2.5}$$

$$W(\overline{\psi_i\psi_j} \to ff) \overset{\rm CPT}{=} W(\overline{ff} \to \psi_i\psi_j) = (1-\epsilon_a)W_a, \tag{2.6}$$

where $a = 1,2,3$; with 1 corresponding to two ψ_1, 2 corresponding to two ψ_2 and 3 corresponding to the off-diagonal coannihilation. This numbering is introduced to avoid double counting. We write the CP conserving part of the interaction as W_a and use $\epsilon_a W_a$ for the CP violating component.

Flavour changing scatterings

We note that for flavour changing scatterings, there are

$$W(\psi_i\psi_j \to \psi_k\psi_l) \overset{\rm CPT}{=} W(\overline{\psi_k\psi_l} \to \overline{\psi_i\psi_j}) = (1+\epsilon_{ab})Z_{ab}, \tag{2.7}$$

$$W(\overline{\psi_i\psi_j} \to \overline{\psi_k\psi_l}) \overset{\rm CPT}{=} W(\psi_k\psi_l \to \psi_i\psi_j) = (1-\epsilon_{ab})Z_{ab}, \tag{2.8}$$

with $Z_{ab} = Z_{ba}$ and $\epsilon_{ab} = -\epsilon_{ba}$.

$$W(\overline{\psi_i}\psi_j \to \overline{\psi_k}\psi_l) \overset{\text{CPT}}{=} W(\overline{\psi_l}\psi_k \to \overline{\psi_j}\psi_i) = (1+\epsilon_{ijkl})T_{ijkl}, \tag{2.9}$$

$$W(\overline{\psi_j}\psi_i \to \overline{\psi_l}\psi_k) \overset{\text{CPT}}{=} W(\overline{\psi_k}\psi_l \to \overline{\psi_i}\psi_j) = (1-\epsilon_{ijkl})T_{jilk}, \tag{2.10}$$

with $T_{ijkl} = T_{klij} = T_{lkji} = T_{jilk}$ and $\epsilon_{ijkl} = \epsilon_{lkji} = -\epsilon_{jilk} = -\epsilon_{klij}$.

The flavour changing terms do not appear in the Boltzmann equation for the overall baryon asymmetries as they do not violate $B-L$ or D. But they are (indirectly) important for the formation of a baryon asymmetry.

B − L and D violating scatterings

Finally, there is

$$W(\psi_i f \to \overline{\psi_j}\overline{f}) \overset{\text{CPT}}{=} W(\psi_j f \to \overline{\psi_i}\overline{f}) = S_{ij}, \tag{2.11}$$

$$W(\overline{\psi_i}\overline{f} \to \psi_j f) \overset{\text{CPT}}{=} W(\overline{\psi_j} \to \psi_i f) = S_{ij}. \tag{2.12}$$

At low energy there is no CP violation in the $B-L$ and D violating scatterings, and $S_{ij} = S_{ji}$.

From these we write down the Boltzmann equation for the number densities of the ψ:

$$\begin{aligned}\frac{dn_{\psi_1}}{dt} + 3Hn_{\psi_1} = {} & r_{\bar{f}}^2[2(1-\epsilon_1)W_1 + (1-\epsilon_3)W_3] - r_1^2 2(1+\epsilon_1)W_1 - r_3(1+\epsilon_3)W_3 \\ & + 2\sum_a (r_{\bar{a}}r_{\bar{f}} - r_1 r_f)S_{1a} + 2\sum_a r_{\bar{a}}^2[2(1-\epsilon_{1a})Z_{1a} + (1-\epsilon_{3a})Z_{3a}] \\ & - 2\sum_a [2r_1^2(1+\epsilon_{1a})Z_{1a} - r_3^2(1+\epsilon_{3a})Z_{3a}] + \sum_{jkl}[r_k r_{\bar{l}}(1-\epsilon_{1jkl})T_{1jkl} - r_i r_{\bar{j}}(1+\epsilon_{1jkl})T_{1jkl}] \\ & + 2n_1^{eq}\Gamma_1(r_{\bar{2}}r_{\bar{f}}^2 - r_1)\end{aligned} \tag{2.13}$$

and

$$\begin{aligned}\frac{dn_{\psi_2}}{dt} + 3Hn_{\psi_2} = {} & r_{\bar{f}}^2[2(1-\epsilon_2)W_2 + (1-\epsilon_3)W_3] - r_2^2 2(1+\epsilon_2)W_2 - r_3(1+\epsilon_3)W_3 \\ & + 2\sum_a (r_{\bar{a}}r_{\bar{f}} - r_2 r_f)S_{2a} + 2\sum_a r_{\bar{a}}^2[2(1-\epsilon_{2a})Z_{2a} + (1-\epsilon_{3a})Z_{3a}] \\ & - 2\sum_a [2r_2^2(1+\epsilon_{2a})Z_{2a} - r_3^2(1+\epsilon_{3a})Z_{3a}] + \sum_{jkl}[r_k r_{\bar{l}}(1-\epsilon_{2jkl})T_{2jkl} - r_2 r_{\bar{j}}(1+\epsilon_{2jkl})T_{2jkl}] \\ & + 2n_1^{eq}\Gamma_1(r_{\bar{1}} - r_2 r_{\bar{f}}^2),\end{aligned} \tag{2.14}$$

with r_3^2 interpreted as $r_1 r_2$. The ϵ_{ijkl} (and ϵ_{ab}, if they were non-zero) act as source terms for a flavour asymmetry. These can be combined with the Boltzmann equations for $\overline{\psi}$ to obtain the Boltzmann equation for baryon number,

$$\frac{dn_{B-L}}{dt} + 3Hn_{B-L} = 2\sum_a (r_{\overline{a}}^2 - r_a^2 + r_{\overline{f}}^2 - r_f^2)W_a - \; 2\sum_a (r_{\overline{a}}^2 + r_a^2 + r_{\overline{f}}^2 + r_f^2)\epsilon_a W_a$$
$$+ 2\sum_{ij}(r_{\overline{j}} r_{\overline{f}} - r_i r_f)S_{ij} + \; 2n_1^{eq}\Gamma_1(r_{\overline{1}} - r_1 + r_{\overline{2}} r_{\overline{f}}^2 - r_2 r_f^2) \tag{2.15}$$
$$= \frac{dn_D}{dt} + 3Hn_D.$$

Looking at this it is clear that the ϵ_a would act as a source term for a baryon asymmetry. For clarity, we will only refer to baryon number violating source terms in our Boltzmann equations as source terms, and simply speak of a flavour asymmetry generation when referring to non baryon number violating source terms. While it may appear that if the ϵ_a are zero, then no net $B-L$ may form, this is not true if there is a flavour asymmetry. This does not violate Sakharov's conditions; a flavour asymmetry still requires CP violation to form. We simply divide the task of satisfying Sakharov's conditions between two processes. Consider a flavour asymmetry with $r_1 = c\overline{r_1}$, $cr_2 = \overline{r_2}$ and $\overline{r_1} = r_2 = r$. Focusing on the first term of (2.15),

$$\begin{aligned} \frac{n_{B-L}}{dt} + 3Hn_{B-L} &= 2\sum_a (r_{\overline{a}}^2 - r_a^2 + r_{\overline{f}}^2 - r_f^2)W_a \\ &= 2r^2(1-c^2)(W_1 - W_2), \end{aligned} \tag{2.16}$$

it is clear that this gives us a nonzero baryon number when $W_1 \neq W_2$. This occurs when the couplings or masses of the ψ are different. Even if there were a source term, this can still be a significant effect if there is an efficient way to transfer the asymmetry, such as B violating decays.

The Boltzmann equations are similar to those in [40], as we would expect for a UV completion. In this work we have not neglected CP violation in the T_{ijkl} terms. Additionally all CP violation is calculable from the single underlying CP violating phase. To evaluate these, we use a change of variables to write the Boltzmann equations in terms of temperature, given by [17]

$$H = \left(\frac{4\pi^3}{45}\right)^{1/2} g_*^{1/2} \frac{T^2}{M_{Pl}} = \frac{1}{2t}\,. \tag{2.17}$$

As the total entropy in the universe is conserved, we will normalise the number densities by the entropy densities. This will hold in the absence of a first order phase transition. This defines $Y \equiv n/s$, where

$$n_a^{eq} = \frac{g_a m_a^2 T}{2\pi^2} K_2\left(\frac{m_a}{T}\right) \tag{2.18}$$

$$s = \frac{2\pi^2}{45} g_* T^3. \tag{2.19}$$

This holds true for a radiation dominated universe. When recast in this way, the Boltzmann equations become

$$\frac{dY_a}{dT} = -\left(\frac{\pi}{45}\right)^{1/2} \frac{M_{Pl} g_*}{s^2 g_{eff}^{1/2}} \left(1 + \frac{T}{4g_{eff}} \frac{dg_{eff}}{dT}\right) C_a, \tag{2.20}$$

where C_a is the collision term for a particle species a.

Unitarity Constraints

The requirement that CPT and unitarity must hold imposes an important restriction on our CP violating terms. While, if all calculations are performed correctly, these should automatically hold, these relations are very important for error checking and correction. They are also quite illustrative as to how CP violation behaves. The relevant unitarity condition is obtained from (1.15). This can be transferred into our language [50–52],

$$\sum_j W(i \to j) = \sum_j W(j \to i) = \sum_j W(i \to \bar{j}) = \sum_j W(\bar{i} \to j). \tag{2.21}$$

As the CP symmetric components of these equations cancel, we are left with a restriction on the ϵ. To obtain the unitarity condition relevant for our case, consider the initial (or final) state $\overline{\psi_1}\psi_2$. After the symmetric parts cancel, we have

$$\sum_{ij} \epsilon_{12ij} T_{12ij} = 0. \tag{2.22}$$

It is easy to see that ϵ_{1212} is zero, as the initial state is the same as the final state (since the phase space is symmetric, ϵ_{1212} is both symmetric and antisymmetric). This gives us

$$\epsilon_{1221} T_{1221} + \epsilon_{1222} T_{1222} + \epsilon_{1211} T_{1211} = 0. \tag{2.23}$$

When combined with (2.13) and (2.14), this constraint ensures that if all particles are in equilibrium then no flavour asymmetry may form, enforcing the third Sakharov condition. The actual structure of the epsilon is non-trivial. Each epsilon is the result of two graphs, with different phase spaces. By calculating these graphs we get the temperature dependence of the epsilon. From dimensional analysis, we naively expect $\epsilon_{ijkl} T_{ijkl} \propto T^4/m_X^6$. This is true at high temperatures for ϵ_{1222} and ϵ_{1211}, but may not be true for $\epsilon_{1221} T_{1221}$ (fig. 2.3). If the couplings are all equal, then cancelations between the two graphs that compose $\epsilon_{1221} T_{1221}$ lead to it being constant. It is straightforward to check that the unitarity conditions are satisfied; as the absorptive phase is symmetric under exchanging the initial, final and loop particles we get the necessary cancelation.[3]

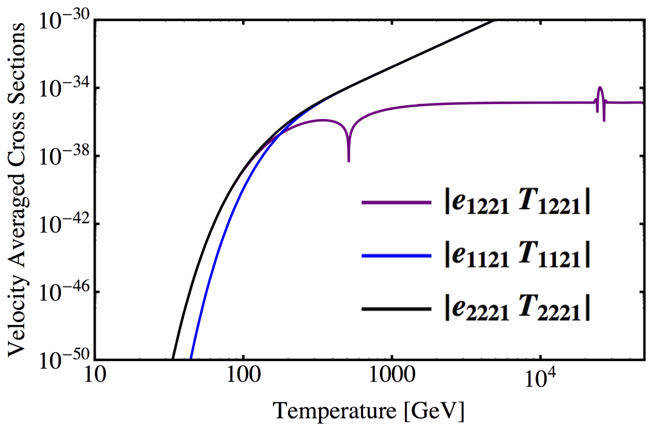

Figure 2.3: CP violating components of the annihilation cross sections. At high temperature ϵ_{1222} and ϵ_{1211} are proportional to T^4, but $\epsilon_{1221} T_{1221}$ is constant.

[3]Technically, it is the integrand of the velocity averaged cross section that is invariant. For just the cross section, the initial particles are treated in a privileged way as they are not averaged over.

2.4 Numerical Solutions

Now we have an understanding of the CP violation and Boltzmann equations, we use Mathematica to numerically solve the four coupled differential equations. We treat the number densities of $\psi_1, \overline{\psi_1}, \psi_2$ and $\overline{\psi_2}$ as independent, and assume that f stays close to equilibrium. These solutions have been found to be numerically stable under changes to the precision and the initial conditions.

In order to form significant asymmetry, it is necessary to have $T_{\text{Freeze out}} \simeq M_{\psi_1}$ (see fig. 2.4). If the freeze out temperature is any lower than this, there is a massive Boltzmann suppression of the asymmetry, as shown in fig. 2.4. Raising the freeze out temperature similarly will reduce the asymmetry, as all particles are closer to equilibrium and decays cease to be efficient. The decay of ψ_1 is crucial for an efficient transfer of asymmetry. If we kinematically disallow all decays, such as in fig. 2.5, the total asymmetry drops by an order of magnitude.

While we do not achieve the full asymmetry required for baryogenesis, $Y_B \simeq 10^{-10}$, this is a simple toy model and does not have the full range of interactions we expect for a more complete model. Further, the range of parameters in which a reasonable asymmetry can be formed is quite narrow. It is interesting to see whether including source terms for baryon number will lead to a significant increase in the asymmetry. As the original EFT was capable of reaching an asymmetry of order 10^{-11}, we suspect that source terms will dominate.

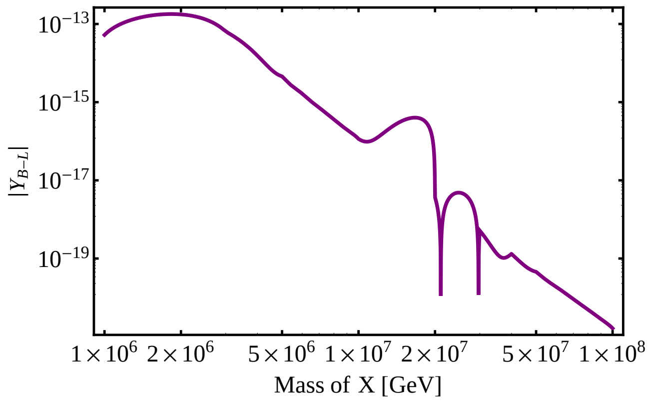

Figure 2.4: Asymmetry formed as a function of M_X.

2.5 Extended Toy Model

To extend this model so that the source terms are not zero we must introduce a second copy of the intermediate scalar X. We assume that this new X is similarly much heavier than the ψ and f, both to generate maximal asymmetry and to restore the EFT in the low energy limit. This gives us the Lagrangian

$$\mathcal{L}_{int} = -\lambda_{ijp}\overline{\psi_i^c}X_p\psi_j - g_p\overline{f^c}X_p^*f + H.c, \tag{2.24}$$

where p runs from 1 to 2. Using the same arguments as before, we relabel $\lambda_{11p} \to \lambda_{1p}$, $\lambda_{22p} \to \lambda_{2p}$ and $\lambda_{12p} \to \lambda_{3p}$. After considering field rephasings, we now have four complex phases, which we will write as

$$\begin{aligned} CP_1 &= \text{Phase}(\lambda_{22}\lambda_{21}^*\lambda_{11}\lambda_{12}^*),\ CP_2 = \text{Phase}(\lambda_{31}^2\lambda_{21}^*\lambda_{11}^*), \\ CP_3 &= \text{Phase}(\lambda_{32}^2\lambda_{22}^*\lambda_{12}^*),\ CP_4 = \text{Phase}(g_2g_1^*\lambda_{11}\lambda_{12}^*). \end{aligned} \tag{2.25}$$

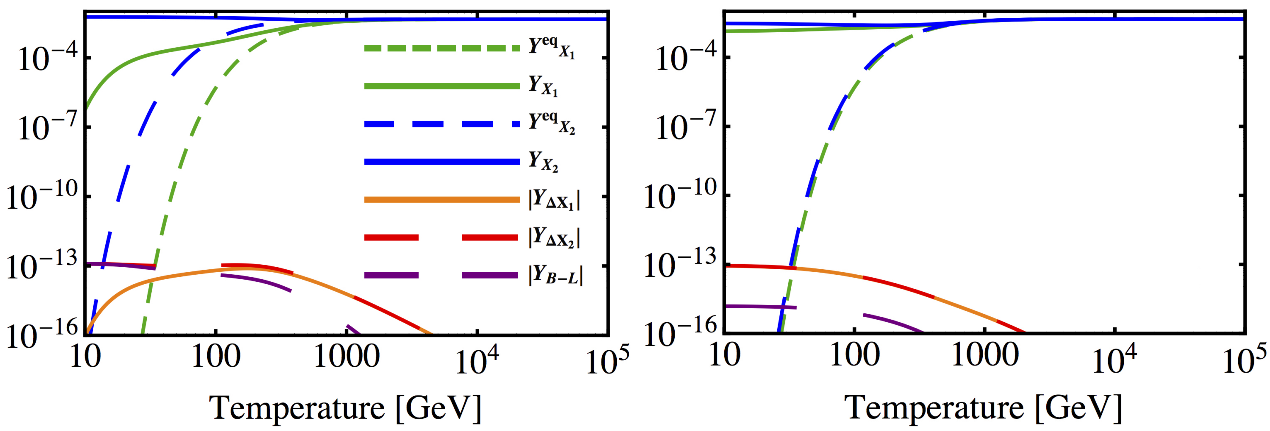

Figure 2.5: Example solutions to the Boltzmann equations with $m_X = 10^6$ GeV, $m_1 = 1000$ GeV, $m_2 = 400$ (950) GeV and $m_f = 50$ GeV for the left (right) hand side. Without decays a significantly smaller $B-L$ is formed.

Looking at this we can see that CP_1 and CP_4 depend on mixings between different scalars whereas CP_2 and CP_3 only require couplings from one scalar. This model is completely symmetric under interchange of the two intermediate scalars.

Simply increasing the number of complex couplings will not necessarily lead to nontrivial source terms - we must have new graphs that can be cut at low energy. When the masses, or couplings, of the X are different, there is a contribution to the ϵ_a from the bubble graph in fig. 2.6. The calculation of this graph can be found in Appendix A. If we send the mass of the second scalar to infinity (or couplings to zero), all the CP violating phases cease to matter save CP_2, our original complex phase. This can be seen from the calculation in Appendix A, where the CP violating terms vanish as the mass of the second scalar goes to infinity. This restores the minimal toy model.

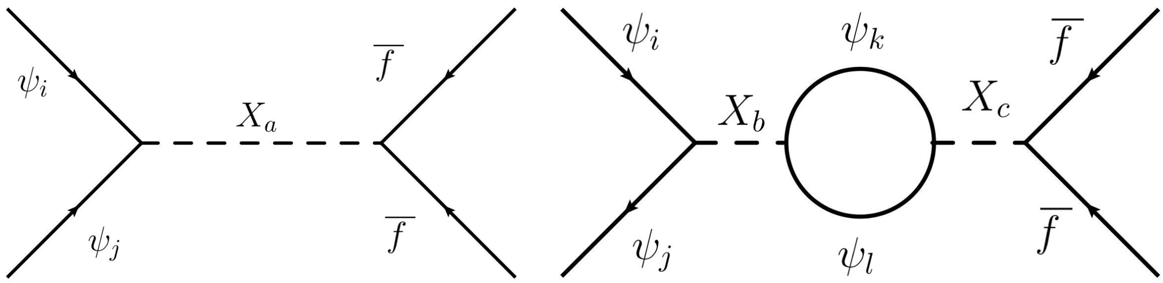

Figure 2.6: Tree level and one loop graph contributing to ϵ_a.

As before, we can use (2.21) to get constraints on the CP violation from unitarity. By considering ff and $\psi_i\psi_j$ as initial states, we derive

$$\sum_a \epsilon_a W_a = 0, \tag{2.26}$$

$$\sum_{ab} \epsilon_{ab} Z_{ab} = 0, \tag{2.27}$$

in addition to (2.23). It is simple enough to check that the results from 2.6 satisfy this for any values of the couplings. We also get new contributions to ϵ_{ijkl}, where we now allow the

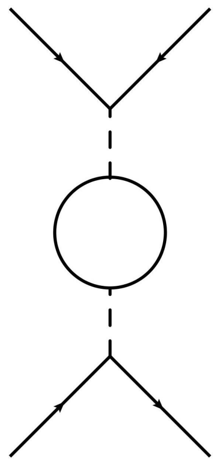

Figure 2.7: A non-example of a one loop correction to the transverse graphs. All cuttings of the loop particles are kinematically forbidden.

scalars to be X_1 or X_2 in fig. 2.2. There is no contribution to the T channel processes S_{ij} and T_{ijkl} from graphs as in fig. 2.7, since all cuttings are kinematically forbidden.

We now have two competing generation mechanisms for the baryon asymmetry; to see which of these dominate we must turn to the Boltzmann equations. The Boltzmann equations are the same as before but now ϵ_a and ϵ_{ab} are non-zero.

Numerical Solutions

The extended toy model has significantly more independent degrees of freedom: five masses, eight couplings and four phases. These degrees of freedom extend the range of parameter space, allowing a significant asymmetry to form with higher m_{X_a}. Fig. 2.8 shows the dependence of the source terms on the mass difference of the intermediate scalars. Source terms dominate over flavour violation when the decays are not efficient, or the two Xs are close in mass. Through these source terms, we are now capable of reaching the same asymmetry as [40], $\Delta B - L \sim 10^{-11}$ (see fig. 2.8). However, to maximise the asymmetry formed it is necessary to minimise the cancelation from the various contributions to the CP violation. This requires an asymmetric treatment of the couplings; the ψ should not couple equally to X_1 and X_2. As the source terms do not rely on efficient decays to form an asymmetry, there is a larger range of masses where an asymmetry can be formed, as shown in fig. 2.8. Unfortunately, if the two scalars are more than an order of magnitude apart, asymmetry formation from source terms is suppressed, as shown in fig. 2.8.

Introducing the source terms leads to an increase in the asymmetry of approximately two orders of magnitude for peak values. Generically, source terms tend to be more forgiving than relying on flavour violating processes. Further, in many models of interest, such as the lepton portal, the heavy particles which would store the flavour asymmetry are in fact Majorana.

2.6 Summary

In this chapter we have explored minimal UV completions of the toy model introduced in [40], and shown that there are two methods to generate an asymmetry with $2 \to 2$ annihilations. Flavour effects can play an important role in asymmetry generation, as long as there is an effective way to transfer the flavour asymmetry to a baryon asymmetry, typically decay.

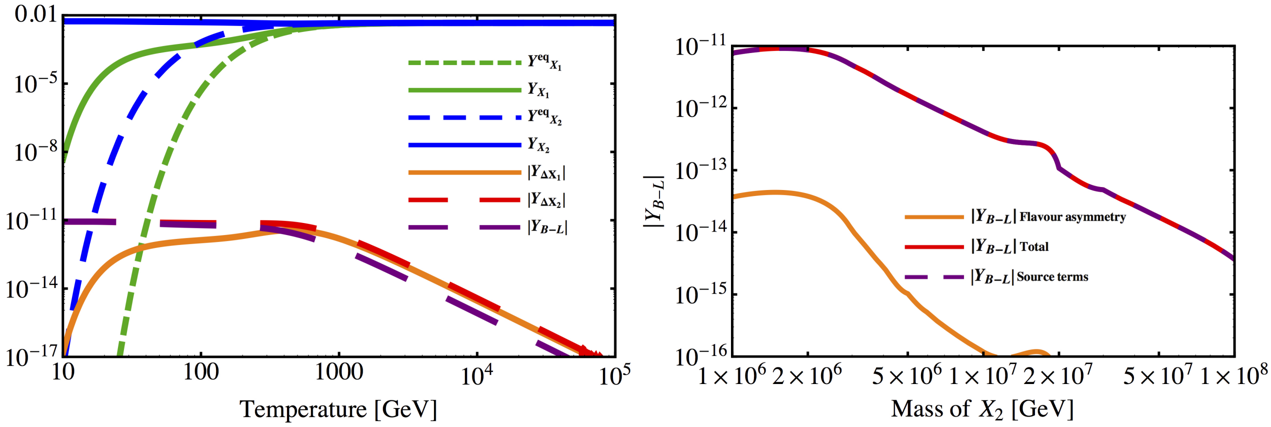

Figure 2.8: (Left) Example solution of the toy model with source terms. The final asymmetry is of order 10^{-11}. (Right) Variations of the mass of the second scalar, keeping the first fixed at 1.4×10^6. We can see that while there is an order of magnitude in which a reasonable asymmetry is formed, asymmetry is maximised for a factor of two difference. Source terms dominate over flavour effects.

However, for generating a large asymmetry, direct baryon number violating source terms are much more effective and can generate an asymmetry over a larger range of masses. Generating these source terms required two copies of the intermediate scalar, X, with similar masses, though they can be as much as an order of magnitude different. As we soon show, our realistic model of ADM, the lepton portal, also requires two intermediate scalars, so many of the same conclusions about mass ranges will hold. With the methods shown here, we are now equipped to deal with a full theory.

Chapter 3

Towards a Realistic Model of Asymmetric Dark Matter

While the toy model is an excellent testing ground for the concepts of $2 \rightarrow 2$ scatterings, to explain baryogenesis a physically realistic model is required, involving SM particles. There are two main avenues of interest - extending the neutron portal and a new model we will refer to as the lepton portal. Extending the neutron portal we mentioned in section 1.4 requires additional particles so that the asymmetry is in fact generated by $2 \rightarrow 3$ scatterings. Ultimately the neutron portal is not a generic method for achieving ADM via coannihilations. The lepton portal will prove promising and is studied further in Chapter 4.

3.1 Extensions of the Neutron Portal

The simple neutron portal of [38] cannot form an asymmetry via coannihilations. Recently [41] has considered a version of the neutron portal capable of baryogenesis, but with no dark matter candidate. To turn this into a fully functional asymmetric dark matter theory, it is necessary to move beyond $2 \rightarrow 2$ scatterings. The essential neutron portal operator is

$$\overline{X} u_R \overline{d^c_R} d_R, \tag{3.1}$$

where X is Dirac. Without a temperature dependent coupling (which would make the model very difficult to test), X decays into neutrons. Consequently new particles must be introduced to store the asymmetry in and to ensure the stability of X, if X is Dirac. If instead X is Majorana there must be a $\mathbb{Z}_2$ symmetry between X and a new particle to form ADM, though it is not necessary for X to be stable. Both cases are essentially the same, so we will only explicitly consider the Dirac case.

That X must be stable in the Dirac case is clear from considering an asymmetry being generated by the neutron portal and stored in another particle, Y, such as in hylogenesis [53]. These models generically have a conserved U(1) symmetry that makes the lightest dark particle stable, leading to the constraint

$$\Delta X + \Delta Y = 0. \tag{3.2}$$

If X were to decay, the conserved charge requires that the asymmetry stored in Y would then vanish. Any asymmetry generated by the coannihilations would therefore be wiped clean.

Because of this, X must be kept stable by kinematically disallowing the decay. Dark matter will then be an admixture of X and the lightest other particle charged under the global symmetry. The simplest way to make a stable version of the neutron portal is to introduce the effective operator $\overline{X} u_R \overline{d^c_R} d_R \sigma$, where X is a SM singlet and σ is a complex SM singlet scalar (similar to a model proposed by [39]). There is a global U(1) symmetry giving X and σ a conserved charge. As in [41], to form an asymmetry there must be multiple generations of either X or σ. We chose to introduce another generation of σ as it turned out to have a simpler

UV completion, though similar results hold for a second generation of X.

There are multiple ways to open up this effective operator, shown in fig. 3.1. The only UV completion that is capable of generating an asymmetry is the first. In all others, there are rapid $2 \to 2$ flavour changing scatterings (fig. 3.2), which are problematic. As the $2 \to 2$ scatterings are mediated by only one heavy intermediate scalar, they dominate over the $2 \to 3$ scatterings. This delays departure from thermal equilibrium; as we desire CP violating effects involving the $2 \to 3$ scatterings to be as large as possible during freeze out this is a serious suppression. The first UV completion gives us the Lagrangian

$$\begin{aligned}\Delta\mathcal{L} = &-\lambda_i \phi \overline{X} u_{Ri} - \lambda_{ij} \theta \overline{d^c_{Ri}} d_{Rj} - \kappa_l \phi \theta \sigma_l^* + H.c \\ &+ \text{quartic terms},\end{aligned} \tag{3.3}$$

where l runs from 1-2.

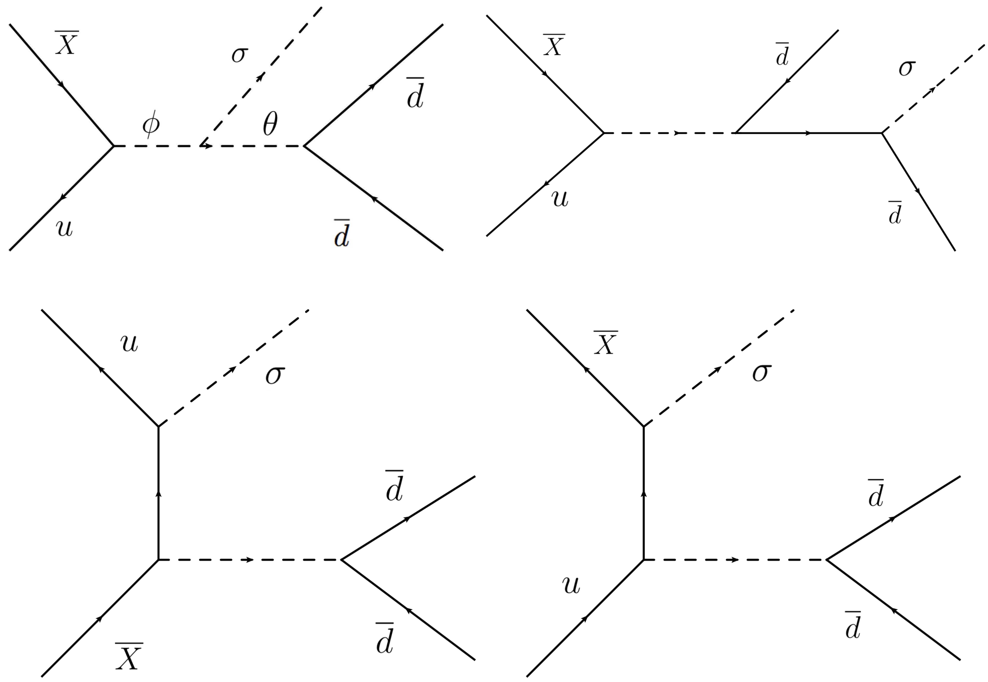

Figure 3.1: UV completions of the stable neutron portal operator.

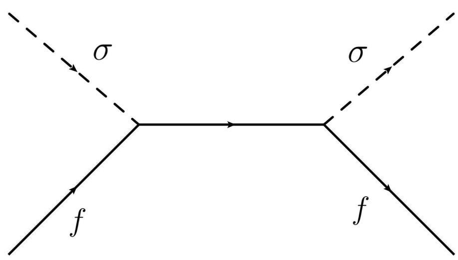

Figure 3.2: Flavour changing scatterings - f can be u, d or X depending on the model.

The mediating scalars ϕ and θ have quantum numbers $(\overline{3}, 1, -4/3, D)$ and $(\overline{3}, 1, -4/3, 0)$ respectively, with D being the dark conserved charge. For the sake of simplicitly we take $M_\phi = M_\theta$. Unfortunately, flavour changing scatterings are necessary to generate an asymmetry [39]. Without these, the unitarity constraints derived from (2.21) cancel all baryon number violating source terms. In this case it is the lack of $2 \to 2$ scatterings, rather than their rapidity, that reduces the asymmetry formed. It is possible to introduce a new mechanism to relate σ_1 to σ_2, but it must be tuned to be roughly equal in rate to the $2 \to 3$ scatterings at freeze out. But there is no obvious reason why two unrelated processes should be so similar at freeze out. To give a concrete example, consider introducing a SM singlet real scalar η. The new Lagrangian is

$$\Delta\mathcal{L} = -\lambda_{ia}\phi_a\overline{X}u_{Ri} - \lambda_{ij}\theta\overline{d^c_{Ri}}d_{Rj} - g_l\phi\theta\sigma^*_l - \kappa_{lk}\sigma^*_l\sigma_k\eta - \gamma\overline{X}X\eta \ + H.c \\ + \text{quartic terms.} \tag{3.4}$$

With a sufficient hierarchy between the couplings of σ_1 and σ_2 to η it is possible to use the η terms to simultaneously provide an efficient means to generate asymmetry in σ_2 and annihilate the symmetric components of X and σ_2. To allow $\sigma\sigma^* \to \eta\eta^*$, η must be a light scalar, and its couplings to σ_1 must be heavily suppressed so that departure from thermal equilibrium is unaffected. Requiring that the $2 \to 2$ scatterings are less rapid than the $2 \to 3$ scatterings at freeze out yields the suppression

$$\gamma \lesssim (T_{\text{freeze out}}/m_\phi)^{3/2}. \tag{3.5}$$

In addition, there must be a large branching fraction for $\sigma_1 \to \sigma_2\eta$ so that the asymmetry stored in σ_1 can be transferred to σ_2 and the unitarity constraints do not stop asymmetry formation. From this it is easy to show that

$$\gamma \gtrsim (m_{\sigma_1}/m_\phi)^{3/2}. \tag{3.6}$$

If the freeze out temperature is approximately the mass of σ_1 it is possible to satisfy both of these bounds, but only for a very narrow range of parameter space. Unfortunately in coannihilation models peak asymmetry production tends to occur when the freeze out temperature is a bit below the mass of particle going out of equilibrium, squeezing this parameter space even more. Further, as these are completely separate processes, that they should be connected in this way requires explanation. A similar argument holds if η is a very heavy scalar.

It seems that the only way to obtain a working ADM neutron portal is to have unnatural couplings, which is not obviously an improvement over temperature dependent couplings. Any full theory of $2 \to 3$ scatterings must explain why two seemingly unrelated processes conspire to allow an asymmetry to be formed. If instead X is Majorana, a similar conclusion is reached. We must look elsewhere for a generic method for ADM via coannihilations.

3.2 Alternative to the Neutron Portal: the Lepton Portal

To produce a realistic model of ADM via coannihilations using the neutron portal, we were forced to use $2 \to 3$ scatterings, which had significant problems. This raises the question of whether it is possible to use another operator to obtain a viable $2 \to 2$ scattering model. This new operator must have only two SM particles in order to avoid the pitfalls of the neutron portal. Fortunately for us, one combination of two SM particles is a gauge singlet, $\overline{L}H$.

From this we construct an effective operator, $\overline{L}YH\phi$, with ϕ a complex gauge singlet and Y a Majorana fermion. Our ADM candidate will be ϕ, carrying a non-zero $B - L$. To satisfy unitarity constraints we will have two copies of the Ys and one ϕ. We choose the Ys to be Majorana in this model, as kinematically disallowing decays would require the rather implausible mass difference between ϕ and Y of $\sim m_\nu$. This can be avoided if the Ys decay

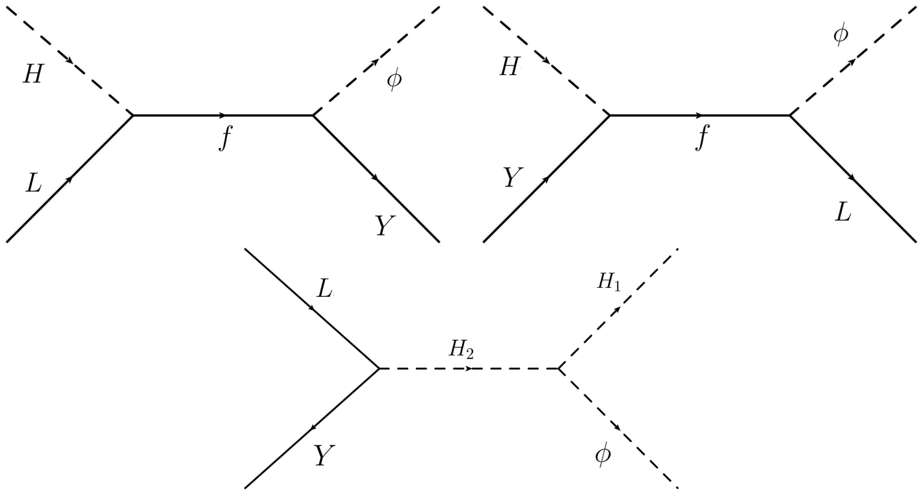

Figure 3.3: UV completions of $\overline{L}YH\phi$. We label them as case 1 (top left), case 2 (top right) and case 3 (bottom).

after the EWPT, but only as a special case. We envision the Ys as the most massive particles, going out of equilibrium first. There are three possible UV completions (fig. 3.3).

The first completion has a Dirac SM singlet, f as intermediary. We impose a global lepton number conservation (or $B-L$ conservation), to preclude the Y from acting as an intermediate particle and coupling to the leptons. For CP violation to arise at one loop it is necessary to include two copies of f.

The second case may be discarded as the intermediate fermion has the same quantum numbers as L, and must gain its mass from the Higgs mechanism, never leading to $\overline{L}YH\phi$ as an EFT. The case study in Chapter 4 and [41] show that a very heavy intermediary is optimal for an asymmetry to form from coannihilations. Further, f is essentially a copy of the left handed leptons and so forbidden by the limits on a fourth generation of leptons.

The third completion is an extension of inert two Higgs doublet models (IDM) [54]. The intermediate particle, H_2, has the same quantum numbers as the SM Higgs but cannot play the same role. As the SM is consistent with LHC data [55, 56], this new particle cannot significantly influence the EWPT and fermion mass generation. IDM can resolve these issues by introducing a new parity. By having the H_2, ϕ and the right handed neutrino be negative parity states the stability of the lightest negative parity state is ensured. A global U(1) symmetry can be imposed, with H_2 and ϕ being charged under $B-L$ and D, respectively, so $B-L-D$ is conserved. While CP violation is possible with just one inert Higgs doublet, we will show that to get a sufficient asymmetry to form, two inert Higgs doublets are necessary.

As the IDM completion is particularly educational, we will study this case in more detail in Chapter 4. The thermal histories of both realistic UV completions (cases 1 and 3) are essentially the same. At high temperature, the Ys are kept in equilibrium by the rapid $2 \to 2$ interactions from fig. 3.3. Around the mass of the heavier copy of Ys, these annihilations freeze out. As these annihilations violate CP, $B-L$ is stored in the ϕ, creating a baryon asymmetry (as long as this occurs above the EWPT). Subsequently, the Ys decay into leptons and ϕ, a process that also violates CP, potentially leading to further asymmetry formation. To see which process dominates, as well as the size of the asymmetry formed, it is necessary to solve the Boltzmann equations.

Chapter 4

Case study: Lepton Portal

To see a realistic model in action, we will study the inert Higgs doublet completion of the lepton portal. As in IDM, we add a massive scalar SU(2) doublet which will function as our heavy intermediary. We also introduce a new parity, with the Lagrangian symmetric under $H_2 \to -H_2$, $Y \to -Y$, and $\phi \to -\phi$. All SM particles are even under this parity. We call H_2 inert as it does not acquire a vacuum expectation value (vev), and does not play a role in the fermion mass generation.[1] However, H_2 still interacts with the SM Higgs and the electroweak gauge bosons. Where this work differs from traditional IDM is that in our case H_2 carries a non-zero $B-L$. The relevant additions to the SM Lagrangian are

$$\Delta\mathcal{L} = -m_{H_2}^2|H_2|^2 - \lambda_{ia}H_2\overline{L}_i Y_a - \kappa H_1 H_2^\dagger \phi - \lambda_1|H_1|^2|H_2|^2 - \lambda_2|H_1^\dagger H_2|^2 - \lambda_3|H_1|^2|\phi|^2 - \lambda_4|\phi|^4 + H.c. \quad (4.1)$$

We consider the Y to have a mass higher than the EWPT, and H_2 to be heavy enough that it can always be considered off shell, typically several orders of magnitude heavier than the next lightest particle, Y_1.There is a conserved global symmetry $U(1)_{B-L-D}$, where $D = \Delta\phi$. As the lepton portal does not violate baryon number directly, sphalerons are required to reprocess the lepton asymmetry into a baryon asymmetry. Thus, for a maximal baryon asymmetry all processes, including decays, should finish before the EWPT. When decays are unimportant it is possible to relax this condition; it is only necessary to have $T_{\text{freeze out}} > T_{EWPT}$. In this work we only consider the former case. In addition we assume that there is an additional process that keeps the ϕ in equilibrium, but remain agnostic as to which process mediates this. For ϕ to be ADM this is necessary to ensure the symmetric component is annihilated. For an example, ϕ quartic coupling to a light real scalar field could be used for this purpose. It is also possible for ϕ to decay into lighter particles, and for them to have new interactions that annihilate their symmetric component, but we do not consider that scenario in this work.

After considering field rephasings, there are two CP violating phases left, that we write as

$$\begin{aligned} CP_1 &= \text{Phase}(\lambda_{22}\lambda_{11}\lambda_{21}^*\lambda_{12}^*), \\ CP_2 &= \text{Phase}(\lambda_{32}\lambda_{11}\lambda_{31}^*\lambda_{12}^*). \end{aligned} \quad (4.2)$$

For our example solutions we choose $CP_1 = \pi/2$ and $CP_2 = \pi/4$.

While normally in IDM neutrinos gain Majorana masses through radiative corrections [57], this is prohibited in our model as $B-L$ is conserved. In this model neutrinos can gain a Dirac mass through the Higgs mechanism (with the addition of Dirac SM singlets). The model proposed is the simplest extension of the IDM that gives asymmetric dark matter; the mass ranges required for H_2 to be ADM instead of ϕ are excluded by a combination of collider searches and electroweak precision tests [58].

[1] As H_2 does not acquire a vev, it is slightly misleading to label it a Higgs boson. We have chosen to do so here as it conforms to the literature.

4.1 Phenomenological Constraints

As we are introducing a second copy of the Higgs, as well as a light singlet scalar, there may be concerns about unwanted phenomenological effects, such as the electroweak phase transition becoming first order. Fortunately, due to the mass range of H_2 we are considering, most of the couplings in our theory are unconstrained.

Electroweak Phase Transition

While the population of H_2 is negligible during the EWPT, it is worth considering the effects of the EWPT on H_2. In addition, while ϕ is a SM singlet it can still influence the EWPT. During the electroweak symmetry breaking (EWSB), H_1 acquires a vev v and acts as the SM Higgs. On the other hand H_2 only experiences mass splitting. It is possible to parameterize H_2 as

$$H_2 = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \tag{4.3}$$

where H^+ and H^0 are complex scalars. After EWSB we have

$$\begin{aligned} m_{H^+}^2 &= m_{H_2}^2 + \lambda_1 v^2 \\ m_{H^0}^2 &= m_{H_2}^2 + (\lambda_1 + \lambda_2) v^2. \end{aligned} \tag{4.4}$$

There is also a contribution to the mass of ϕ when H_1 acquires a vev, as well as mass mixing between H^0 and ϕ. This gives us a mass matrix

$$\begin{pmatrix} m_\phi^2 + \lambda_3 v^2 & \kappa v \\ \kappa v & m_{H_0}^2 \end{pmatrix}. \tag{4.5}$$

We expect this to diagonalise to a heavy state and a light state, which we will label ϕ_{Heavy} and ϕ_{Light} respectively with a mixing angle of $\kappa v / m_{H^0}^2$. This occurs for small mixing angles, $v\kappa/m_{H^0} \lesssim (m_\phi^2 + \lambda_3 v^2)^{1/2}$. This requirement for small mixing is our only real constraint relevant to asymmetry formation. For $\kappa \simeq m_Y$ and $m_\phi \simeq 0$, we have $m_{\text{Light}} \simeq \lambda_3 v^2$. As we envision ϕ_{Light} to be ADM, we must have $\lambda_3 \sim 5 \times 10^{-5}$ to get the correct relic density ($m_{\text{Light}} \simeq 1.7$ GeV as in (1.19)). Intriguingly it is possible to have the entire mass of ϕ_{Light} to be generated by EWSB.

Introducing ϕ can potentially alter the EWPT, but only via its quartic coupling to H_1. One might worry that if the EWPT is made first order, there will be two competing methods for baryogenesis - electroweak baryogenesis and coannihilations. From the analysis in [59] it can be shown that the EWPT is first order if

$$\frac{\lambda_3^4}{128\pi^2} - \frac{1}{3}\lambda_{H_1}^2(\lambda_3 + \lambda_4) > \left[\lambda_{H_1}\left(\frac{\lambda_3}{6} + \frac{y_t}{2}\right) - \frac{\lambda_3^3}{32\pi^2}\right]\left(\frac{m_\phi}{v}\right)^2, \tag{4.6}$$

where λ_{H_1} is the quartic self coupling of H_1 and y_t is the Yukawa coupling of H_1 to the top quark. As λ_3 is small, the EWPT remains second order. We make the approximation that the electroweak phase transition occurs the same way as in the SM.

Electroweak Precision Tests

As H_2 still couples to the electroweak gauge bosons, electroweak precision tests are in principal sensitive to H_2. In particular the electroweak precision tests are only sensitive to the mass splitting of H_2 [54]. Standard electroweak precisions tests can be cast in terms of the "oblique parameters" S, T and U, which parametrize the radiative corrections due to new physics. A full explanation of the S, T and U terms can be found in [60]. From [54] the main contributions to electroweak precision tests (at one loop order) can be written as

$$\begin{aligned}\Delta T &\simeq \frac{\lambda_2}{32\pi^2\alpha},\\ \Delta S &\simeq \frac{\lambda_2 v^2}{8\pi^2 m_{H^+}^2}.\end{aligned} \tag{4.7}$$

This gives a ΔT of ~ 1 for $\lambda_2 \sim 1$. The experimental values are $\Delta T_{U=0} = .08 \pm .07$ and $\Delta S_{U=0} = .05 \pm .09$, so $\lambda_2 \lesssim .1$ is required to satisfy electroweak precision tests [61]. Fortunately, λ_2 does not affect asymmetry production, so this constraint does not seriously affect the model. ΔS is negligible for all sensible parameter choices.

Self Interactions

While the self couplings of ϕ do not affect asymmetry production directly, they will contribute to the thermal mass of ϕ. The most stringent limits on dark matter self interactions come from the Bullet Cluster. From this there is the constraint, [62, 63]

$$\frac{\sigma_{self}}{m_{\text{Light}}} < 2 \times 10^{-24} \frac{cm^2}{\text{GeV}}. \tag{4.8}$$

The self interaction is simple enough to calculate, yielding

$$\frac{\lambda_4^2}{m_{\text{Light}}^3} \lesssim 10^6 \ \text{GeV}^{-3}. \tag{4.9}$$

This leaves λ_4 essentially unconstrained. As the thermal mass of ϕ will be determined by λ_4, this freedom is useful for the kinematics.

Fortuitously only one of these constraints affect parameters required for asymmetry formation, so we are free to choose parameters to maximise the asymmetry formed. The only limitation is that $v\kappa/m_{H^0} \lesssim (m_\phi^2 + \lambda_3 v^2)^{1/2}$.

4.2 Thermal Masses

While in the toy model all thermal field theoretic effects were neglected, there is one effect that must be included in our realistic model. At high temperatures particles can gain an effective mass through interactions with the plasma. This mass is determined by which particle species are abundant and the interactions of the individual particle species. For H_1, L and ϕ, these thermal masses are kinematically significant. For a particle to gain a thermal mass, there must be particles in the plasma which could appear in a loop correction to the propagator of that particle. H_2 is too massive to appear in the plasma and so does not acquire a thermal mass, nor does it contribute to thermal masses. Similarly, as the Y only interact via H_2 they also do not acquire a thermal mass. The main effect of thermal masses is to change the kinematics; it has been shown that effects such as the apparent breaking of chiral symmetry can be neglected, to good enough approximation for our purposes [64–66]. The prescription is simply to replace the mass of all the particles (save Y_1, Y_2 and H_2) with the relevant thermal masses, both in the phase space and propagators. From [65, 67], the thermal masses are given by:

$$m_{H_1}^2 = \left(\frac{3}{16}g_2^2 + \frac{1}{16}g_Y^2 + \frac{1}{4}y_t^2 + \frac{1}{2}\lambda_{H_1}\right)T^2, \tag{4.10}$$

$$m_L^2 = \left(\frac{3}{32}g_2^2 + \frac{1}{32}g_Y^2\right)T^2, \tag{4.11}$$

$$m_\phi^2 = \frac{\lambda_4 T^2}{2}, \tag{4.12}$$

where g_2 is the coupling constant of $SU(2)_L$ and g_Y is the coupling constant of $U(1)_Y$ in the SM. The electric charge and the Weinberg angle are related to g_2 and g_Y by $\sin\theta_W g_2 = e$ and $\cos\theta_W g_Y = e$. We have neglected all other Yukawa couplings and contributions from λ_3. In our solutions we use $m_{H_1} = .71T$, $m_\phi = .59T$ and $m_L = .19T$. In general, the asymmetry formed increases when the thermal masses are increased. We can safely neglect the Lagrangian mass of ϕ at the temperatures we consider.

Interestingly, as the Y do not acquire a thermal mass, at temperatures above $1.4m_{Y_a}$ the SM Higgs can decay into Y rather than the other way round (depending on the thermal mass of ϕ). While there could be concerns about the CP violation of these decays, as H_1 is in equilibrium these are ineffective. This is in agreement both with ansatz calculations of the CP violation and the results from [66].

4.3 Interactions and CP violation

We now catalogue the relevant interactions.

Annihilations

Our CP violating annihilations are given by:

$$W(Y_a L \to H_1\phi) \overset{\text{CPT}}{=} W(\phi^* H_1^* \to \overline{L} Y_a) = (1+\epsilon_a)W_a, \tag{4.13}$$

$$W(\overline{L} Y_a \to \phi^* H_1^*) \overset{\text{CPT}}{=} W(H_1\phi \to Y_a L) = (1-\epsilon_a)W_a, \tag{4.14}$$

where we have included an implicit sum over the three families of leptons. Because all the leptons have essentially the same mass and chemical potential, it is not really necessary to consider them as separate species except when summing the couplings for a process. These annihilations will be the main generators of the asymmetry. The relevant unitarity constraint, derived from (2.21), is

$$\epsilon_1 W_1 = -\epsilon_2 W_2. \tag{4.15}$$

As before, we calculate the CP violation due to interference between one loop and tree graphs. The only CP violation for the process $Y_a L \to H_1\phi$ comes from graphs like fig. 4.1, involving a Majorana mass insertion. Following the Cutkosky rules, we find that at high temperatures

$$\epsilon_1 W_1 \sim -\frac{\kappa^2}{256\pi^2}\frac{m_{Y_1} m_{Y_2}}{m_{H_2}^6}, \tag{4.16}$$

where we have assumed order one couplings for the λ. As we were forced to use the Majorana mass of Y to get a CP violating graph, from dimensional analysis it is clear that there is no temperature dependence (at high temperature) in the CP violation[2]. As we will soon show, this will suppress the asymmetry by about an order of magnitude.

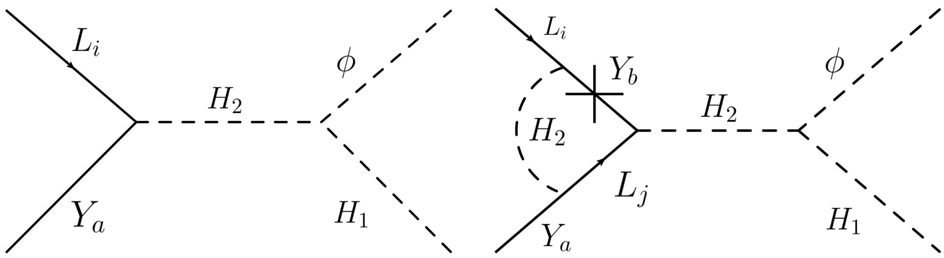

Figure 4.1: Graphs contributing to ϵ_a. The one loop graph (right) has a Majorana mass insertion.

We also have CP conserving scatterings, which we label

$$W(Y_aH_1 \to L\phi^*) = T_a, \tag{4.17}$$
$$W(Y_a\phi \to H_1^*L_i) = U_a, \tag{4.18}$$
$$W(Y_aL \to Y_bL) = S_{ab}, \tag{4.19}$$
$$W(\overline{L}L \to Y_aY_b) = P_{ab}. \tag{4.20}$$

While $\overline{L}L \to Y_aY_b$ is not technically CP conserving, CP violation in this term only leads to a flavour asymmetry. As Y is Majorana, there is no method to store this asymmetry or transfer it to a non-zero $B - L$ so these terms can be safely neglected. Even if the Y were Dirac, from the toy model in Chapter 2 it is clear that they would be subdominant. The rest are similar to the T channel processes in the toy model - all Cutkosky cuts that could lead to CP violation in these processes are kinematically forbidden.

Decays

At different times, Y_1, Y_2 and H_1 can all decay, though we are only interested in the CP violation of the Y decays.We will label these decays by:

$$\Gamma(H_1 \to YL\phi^*) = \Gamma_{H_1a} \tag{4.21}$$
$$\Gamma(Y_1 \to Y_2\overline{L}L) = \Gamma_{1A} \tag{4.22}$$
$$\Gamma(Y_1 \to \overline{L}H_1\phi) = \frac{1}{2}(1+\epsilon_D)\Gamma_{1B} \tag{4.23}$$
$$\Gamma(Y_1 \to LH_1^*\phi^*) = \frac{1}{2}(1-\epsilon_D)\Gamma_{1B} \tag{4.24}$$
$$\Gamma(Y_2 \to \overline{L}H_1\phi) = \Gamma(Y_2 \to LH_1^*\phi^*) = \frac{1}{2}\Gamma_2, \tag{4.25}$$

where we have parameterized the CP violation in decays by ϵ_D. Contributions to ϵ_D come from the graphs in fig. 4.2. There is a further contribution to the CP violation from the $3 \to 3$ scattering $\overline{L}H_1\phi \to LH_1^*\phi^*$. While the CP symmetric component of this scattering is negligible, CP violation in this process serves to cancel the CP violation in the decays at thermal equilibrium (as in [41]). This can be seen by applying the unitarity relation (2.21) to the state $\overline{L}H_1\phi$, to derive

$$\epsilon_{\overline{L}H_1\phi\to LH_1^*\phi^*}W_{\overline{L}H_1\phi\to LH_1^*\phi^*} = \frac{1}{2}\epsilon_D n_1^{eq}\Gamma_{1A}, \tag{4.26}$$

[2]This is, of course, ignoring the implicit temperature dependence of the thermal masses, but the overall point still stands.

which we include in our Boltzmann equations.

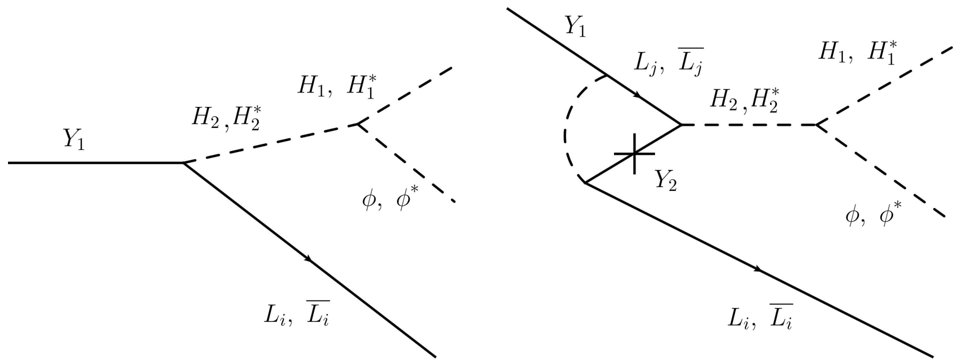

Figure 4.2: Graphs contributing to ϵ_D. The one loop graph (right) has a Majorana mass insertion.

4.4 Chemical Potentials

Since ϕ, as well as the leptons, will be kept close to equilibrium above the EWPT it is unnecessary to have a Boltzmann equation for each particle species. Rather, we can solve our Boltzmann equations with the chemical potentials of these species, using (1.8) to obtain their number densities. This is where the advantage of using Maxwell-Boltzmann statistics lies: we can separate out the chemical potentials. Here we are not considering processes (or the relevant chemical potentials) below the EWPT. Our task is made simpler by the fact that all our chemical potentials can be written in terms of μ_ϕ. As our model is similar to standard treatments of B and L violation, such as leptogenesis, we can borrow the chemical potentials from [68], and just note that the segregation of $B-L$ into ϕ is the only source of $B-L$ violation. By making the replacement

$$B - L = \mu_\phi, \tag{4.27}$$

we obtain the chemical potentials:

$$\mu_L = \frac{-7}{79}\mu_\phi \tag{4.28}$$

$$\mu_{H_1} = \frac{-4}{79}\mu_\phi. \tag{4.29}$$

These can be related to the asymmetry generated in ϕ by using

$$n_\phi - n_{\overline{\phi}} = \frac{T^2}{3}\mu_\phi. \tag{4.30}$$

To write this all in terms of the baryon asymmetry, we use

$$B = \frac{28}{79}\mu_\phi. \tag{4.31}$$

Thus armed, we are ready to tackle the Boltzmann equations.

4.5 Boltzmann Equations

Because of the simplifications we gain from using the chemical potentials, we have only three coupled differential equations to solve, even less than in the toy model. These Boltzmann equations are similar to those in [41]; in both cases we have two heavy Majorana particles, which experience CP violation through decays and $2 \to 2$ scatterings. For Y_1 and Y_2, we have:

$$\begin{aligned}
\frac{dn_{Y_1}}{dt} + 3Hn_{Y_1} =& \Gamma_{1a} n^{eq}_{Y_1}[(r_l r_{\overline{H_1}} r_{\overline{\phi}} + r_{\bar{l}} r_{H_1} r_\phi)/2 - r_{Y_1}] + \Gamma_{1b} n^{eq}_{Y_1}[r_l r_{\bar{l}} r_{Y_2} - r_{Y_1}] \\
&+ \Gamma_{H_1 a} n^{eq}_{H_1}[r_{H_1} + r_{\overline{H_1}} - r_l r_{\overline{\phi}} r_{Y_1} - r_{\bar{l}} r_\phi r_{Y_1}] - \frac{1}{2}\epsilon_D \Gamma_{1a} n^{eq}_{Y_1}[r_l r_{\overline{H_1}} r_{\overline{\phi}} - r_{\bar{l}} r_{H_1} r_\phi] \\
&+ W_1[r_{H_1} r_\phi + r_{\overline{H_1}} r_{\overline{\phi}} - r_{Y_1}(r_{\bar{l}} + r_l)] - \epsilon W_1[r_{H_1} r_\phi - r_{\overline{H_1}} r_{\overline{\phi}} + r_{Y_1}(r_{\bar{l}} - r_l) \\
&+ T_1[r_\phi r_{\bar{l}} + r_{\overline{\phi}} r_l - r_{Y_1}(r_{H_1} + r_{\overline{H_1}})] + U_1[r_{H_1} r_{\bar{l}} + r_{\overline{H_1}} r_l - r_{Y_1}(r_\phi + r_{\overline{\phi}})] \\
&+ S_{12}[(r_{\bar{l}} + r_l)(r_{Y_2} - r_{Y_1})] + 2P_{11}[r_{\bar{l}} r_l - r^2_{Y_1}] + P_{12}[r_{\bar{l}} r_l - r_{Y_1} r_{Y_2}]
\end{aligned} \tag{4.32}$$

and

$$\begin{aligned}
\frac{dn_{Y_2}}{dt} + 3Hn_{Y_2} =& \Gamma_2 n^{eq}_{Y_2}[(r_l r_{\overline{H_1}} r_{\overline{\phi}} + r_{\bar{l}} r_{H_1} r_\phi)/2 - r_{Y_2}] - \Gamma_{1b} n^{eq}_{Y_1}[r_l r_{\bar{l}} r_{Y_2} - r_{Y_1}] \\
&+ \Gamma_{H_1 b} n^{eq}_{H_1}[r_{H_1} + r_{\overline{H_1}} - r_l r_{\overline{\phi}} r_{Y_2} - r_{\bar{l}} r_\phi r_{Y_2}] + W_2[r_{H_1} r_\phi + r_{\overline{H_1}} r_{\overline{\phi}} - r_{Y_2}(r_{\bar{l}} + r_l)] \\
&+ \epsilon W_1[r_{H_1} r_\phi - r_{\overline{H_1}} r_{\overline{\phi}} + r_{Y_2}(r_{\bar{l}} - r_l) + T_2[r_\phi r_{\bar{l}} + r_{\overline{\phi}} r_l - r_{Y_2}(r_{H_1} + r_{\overline{H_1}})] \\
&+ U_2[r_{H_1} r_{\bar{l}} + r_{\overline{H_1}} r_l - r_{Y_2}(r_\phi + r_{\overline{\phi}})] - S_{12}[(r_{\bar{l}} + r_l)(r_{Y_2} - r_{Y_1})] \\
&+ 2P_{22}[r_{\bar{l}} r_l - r^2_{Y_2}] + P_{12}[r_{\bar{l}} r_l - r_{Y_1} r_{Y_2}].
\end{aligned} \tag{4.33}$$

The Boltzmann equation for $B - L$ is:

$$\begin{aligned}
\frac{dn_{B-L}}{dt} + 3Hn_{B-L} =& \Gamma_{1a} n^{eq}_{Y_1}[r_l r_{\overline{H_1}} r_{\overline{\phi}} - r_{\bar{l}} r_{H_1} r_\phi] - \frac{1}{2}\epsilon_D \Gamma_{1a} n^{eq}_{Y_1}[2r_{Y_1} - (r_l r_{\overline{H_1}} r_{\overline{\phi}} + r_{\bar{l}} r_{H_1} r_\phi)] \\
&+ \Gamma_{2a} n^{eq}_{Y_2}[r_l r_{\overline{H_1}} r_{\overline{\phi}} - r_{\bar{l}} r_{H_1} r_\phi] + \Gamma_{H_1 a} n^{eq}_{H_1}[r_{H_1} - r_{\overline{H_1}} + r_{\bar{l}} r_\phi r_{Y_1} - r_l r_{\overline{\phi}} r_{Y_1} + r_{\bar{l}} r_\phi r_{Y_1}] \\
&+ \Gamma_{H_1 b} n^{eq}_{H_1}[r_{H_1} - r_{\overline{H_1}} + r_{\bar{l}} r_\phi r_{Y_2} - r_l r_{\overline{\phi}} r_{Y_2} + r_{\bar{l}} r_\phi r_{Y_1}] + \sum_{a=1,2} W_a[r_{\overline{H_1}} r_{\overline{\phi}} - r_{H_1} r_\phi + r_l r_{Y_a} - r_{\bar{l}} r_{Y_a}] \\
&+ \sum_{a=1,2} T_a[r_{H_1} r_{Y_a} - r_{\overline{H_1}} r_{Y_a} + r_{\overline{\phi}} r_{\bar{l}} - r_\phi r_l] + \sum_{a=1,2} U_a[r_{\overline{H_1}} r_l - r_{H_1} r_{\bar{l}} + r_{\overline{\phi}} r_{Y_a} - r_\phi r_{Y_a}] \\
&+ \epsilon W_1[(r_l + r_{\bar{l}})(r_{Y_1} - r_{Y_2})] \\
=& \frac{dn_D}{dt} + 3Hn_D.
\end{aligned} \tag{4.34}$$

As in the toy model, the terms with ϵ_1 and ϵ_D are source terms for the asymmetry. We have used (4.15) to write the Boltzmann equations solely in terms of these two CP violating terms. We can now numerically solve these to see the evolution of an asymmetry.

4.6 Numerical Solutions

To solve this set of coupled differential equations, we will again call on Mathematica. As before, these solutions are stable under changes to the precision, starting temperature (for sensible values of the temperature) and initial conditions. We start our solutions at high temperature, where $2 \to 2$ scatterings can wash out any pre-existing asymmetry, and then track the evolution of $B - L$ down to the EWPT. To good approximation, at the EWPT sphalerons simply switch off, freezing the value of B. In general, Y_1 or Y_2 may come to dominate the energy density at some time, causing the universe to become matter dominated (instead of radiation dominated, as we assume). This would lead to a dilution factor, approximated in [69] as

$$d = \mathrm{Max}\left[1.8 g_*^{1/4} \frac{Y_2|_{Tfo} m_{Y_2}}{(\Gamma_{Y_2} M_{Pl})^{1/2}},\ 1\right], \tag{4.35}$$

where $Y_2|_{Tfo}$ is the density to entropy ratio of Y_2 (the slowest decaying particle) at freeze out. For the interesting regions of parameter space (those which lead to large asymmetries) this condition is not satisfied, but for completeness we do include the dilution factor.

For example solutions see fig. 4.3. As in [41], for regimes where the two Y are relatively close in mass coannihilations dominate the asymmetry. As there are two mass scales in this problem (once we choose a mass for Y_1), we scan over κ and m_{H_2}, keeping in mind that $v\kappa/m_{H^0} \lesssim (m_\phi^2 + \lambda_3 v^2)^{1/2}$. The asymmetry is maximised when $\kappa \sim m_{Y_1}$ and $m_{H^0} \sim 10^2 \times m_{Y_1}$. Unfortunately, this model does not generate the full asymmetry required for baryogenesis; from fig. 4.4 it can be seen that the maximum asymmetry is of order $Y_{B-L} = 10^{-11}$. This is quite puzzling: with such similar Boltzmann equations and asymmetry production methods, how can this model fail where [41] succeeded? The answer lies in the CP violation. Whereas in the neutron portal case there were graphs that gave the CP violation in coannihilations the property $\epsilon(X\overline{u} \to dd) \propto T^2 W(X\overline{u} \to dd)$, there is no temperature dependence in our CP violating terms. As was suggested in [41], which we will show explicitly, in the neutron portal case this temperature dependence made the coannihilations relevant at a higher temperature, enhancing the asymmetry production. To obtain similar temperature dependence, we must have CP violating graphs without Majorana mass insertions. To get this crucial temperature dependence we must, as in the toy model, add a second intermediate scalar. In fact, this conclusion also holds for [41]; to get the full asymmetry found in the EFT studied it is necessary to have two heavy intermediate scalars regardless of the number of CP violating phases.

4.7 Extended Lepton Portal

Three copies of the Higgs boson is not a far-fetched notion - three Higgs models have been explored in both an inert and general context [70, 71]. We will not write down the full potential, as most of the terms are irrelevant for asymmetry formation, but simply note that to satisfy electroweak precision tests it is necessary to avoid significant mixing between the two inert scalars [72]. The Lagrangian now looks like

$$\Delta\mathcal{L} = -m_{H_p}^2 |H_p|^2 - \lambda_{iap} H_p \overline{L}_i Y_a - \kappa_p H_1 H_p^\dagger \phi + H.c, \tag{4.36}$$

where $p = 2, 3$. With 14 relevant couplings there are now 8 CP violating phases, which for the sake of the example we will choose to be

$$\begin{aligned}\mathrm{Phase}(\lambda_{221}) =& \frac{\pi}{5},\ \mathrm{Phase}(\lambda_{321}) = \frac{\pi}{7},\ \mathrm{Phase}(\lambda_{122}) = \frac{\pi}{5},\ \mathrm{Phase}(\lambda_{212}) = \frac{\pi}{20}\\ \mathrm{Phase}(\lambda_{222}) =& \frac{7\pi}{20},\ \mathrm{Phase}(\lambda_{312}) = \frac{\pi}{28},\ \mathrm{Phase}(\lambda_{322}) = \frac{7\pi}{20},\ \mathrm{Phase}(\kappa_2) = \frac{\pi}{3},\end{aligned} \tag{4.37}$$

where we have chosen phases that avoid cancellations between the various CP violating graphs. The first two phases correspond to those in the unextended lepton portal. Unlike the previous models, we have chosen to specifically assign phases to the various couplings because of the sheer number of phases we are now dealing with. We choose all other couplings to be real, without loss of generality. As in the toy model, there are additional

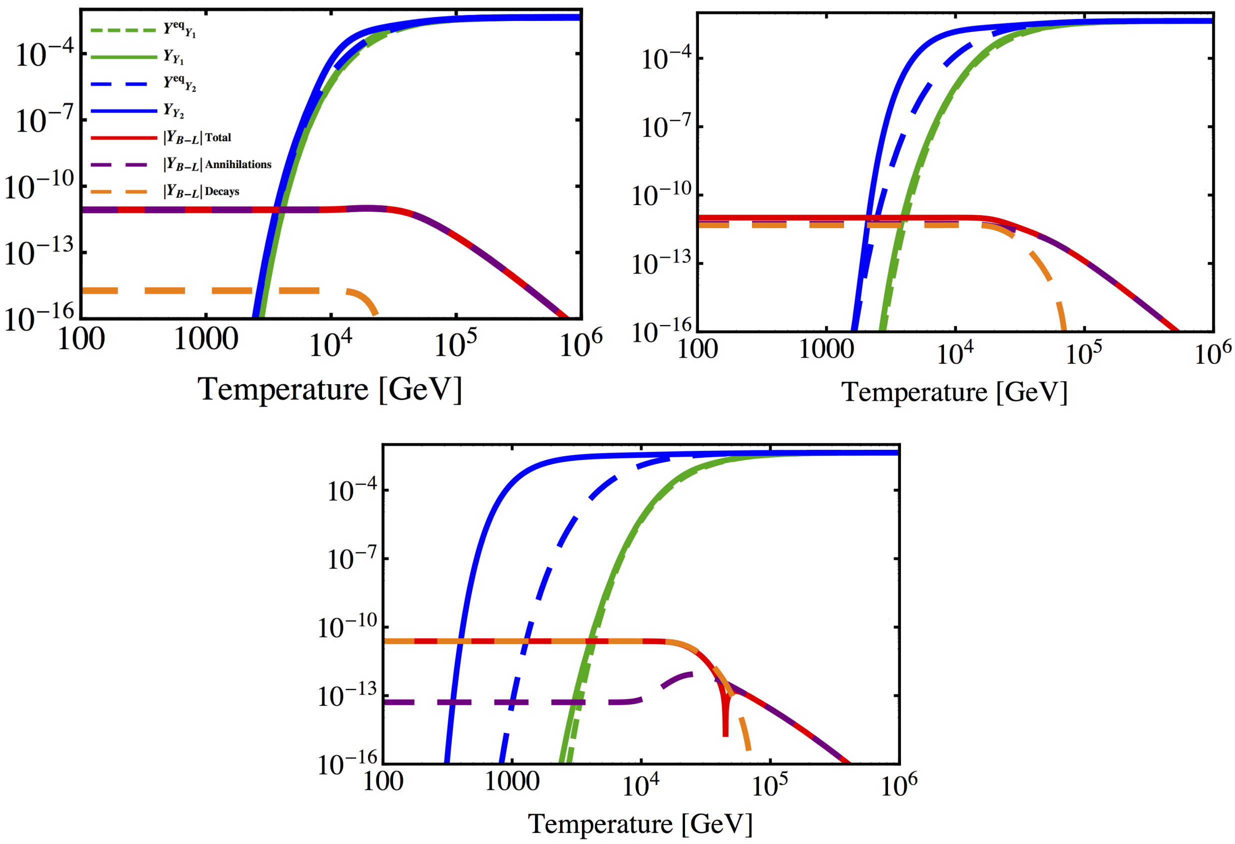

Figure 4.3: Example solutions with $m_{Y_1} = 100,000$, $m_{Y_2} = 90,000$ (top right); $m_{Y_1} = 100,000$, $m_{Y_2} = 60,000$ (top left) and $m_{Y_1} = 100,000$, $m_{Y_2} = 30,000$ (bottom). Coannihilations dominate over decays in asymmetry formation when $\frac{m_{Y_2}}{m_{Y_1}} \gtrsim \frac{3}{5}$.

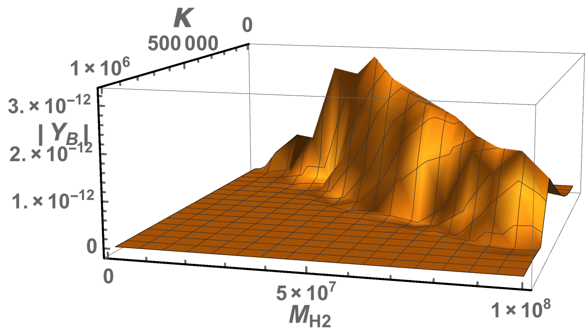

Figure 4.4: Asymmetry formed as a function of κ and m_{H_2}. There is a ridge of values where the asymmetry formed is significant, corresponding to a freeze out temperature of order M_{Y_1}. Maximal asymmetry corresponds to $\kappa \sim m_{Y_1}$ and $m_{H^0} \sim 10^2 \times m_{Y_1}$.

graphs involving a closed fermion loop (fig. 4.5). As there are no Majorana mass insertions, this graph exhibits the desired temperature dependence, $\epsilon_1 W_1 \propto T^2 W_1$. There are similar contributions to the CP violation in decays, which we also include. Armed with this new CP violation, we can again solve the Boltzmann equations for the evolution of $B - L$.

Numerical Solutions

With the temperature dependent CP violation, we see a significant increase in the asymmetry. As long as there are no significant cancellations between the various contributions to the ϵ_a the full baryon asymmetry of the universe is generated (see fig. 4.6). Fittingly for ADM,

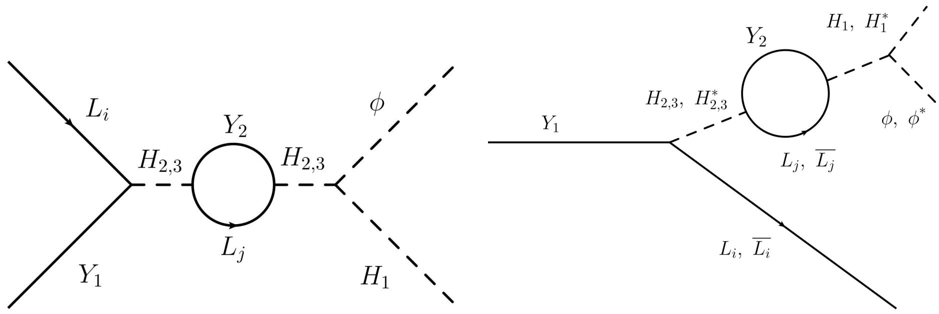

Figure 4.5: New one loop graphs contributing to CP violation. The graph on the left (right) contributes to ϵ_a (ϵ_D). Neither graph has a Majorana mass insertion.

cancellations are minimised when the λ_{iap} are not symmetric under changing families (of the leptons, as well as the Y and inert Higgs). This asymmetry in the couplings does not need to be more than an order of magnitude for significant enhancement, as in (fig. 4.7). As in the toy model, we can scan across a single κ_p or m_{H_p} (fig. 4.8). It is preferable for κ_2 and κ_3 to be approximately an order of magnitude different, and m_{H_2} and m_{H_3} to be within an order of magnitude of each other. Comparing this to the asymmetry formed when only the mass insertion graphs are included (fig. 4.6) we see that the asymmetry starts forming much earlier, culminating in a significantly higher asymmetry.

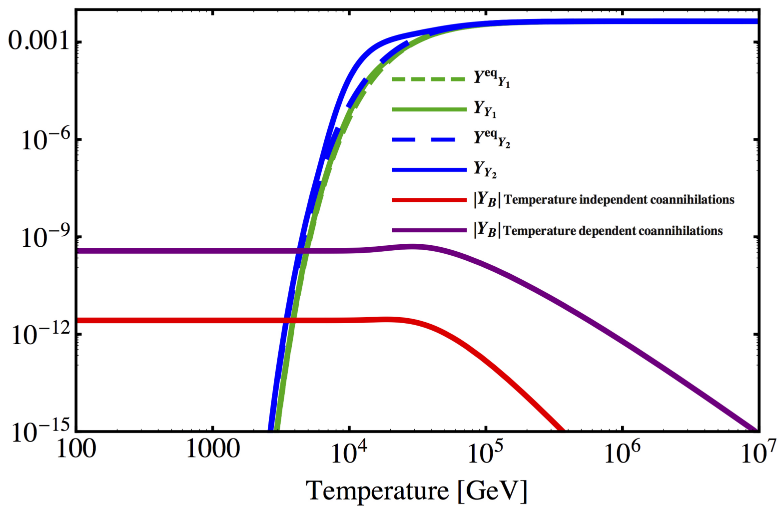

Figure 4.6: Example solution with $m_{Y_1} = 100,000$, $m_{Y_2} = 90,000$. The asymmetry generated is $Y_B = 1.1 \times 10^{-10}$.

Due to the similarity of the asymmetry production, Boltzmann equations and CP violation between this model and [41], it is clear that main asymmetry production in the neutron portal EFT studied in [41] was also due to this temperature dependent CP violation (rather than the mass insertion diagrams that are also there in the neutron portal case). One can also see this by comparing the slope of asymmetry formation in [41] with our own model: when temperature dependence is introduced the slopes become similar. This provides compelling evidence that for coannihilations to dominate a heavy intermediate particle is necessary (to provide the dimensionality for temperature dependence). As many cosmological models are EFTs, this will often be satisfied. Further, multiple intermediate particles (leading to bubble graphs) seem to be a generic feature of coannihilation models.

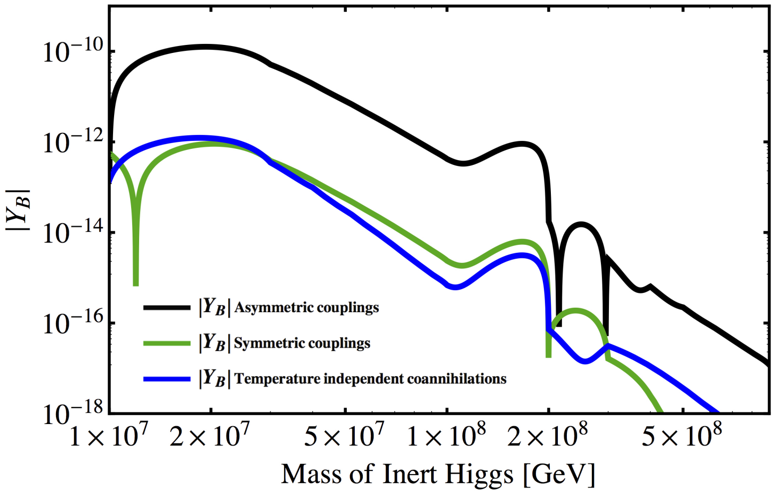

Figure 4.7: Asymmetry formed vs mass of Y. For asymmetric couplings the asymmetry formed is as much as two orders of magnitude higher than when all the couplings are the same. When there is little or no discrimination between the couplings, the asymmetry formed is essentially the same as the unextended lepton portal.

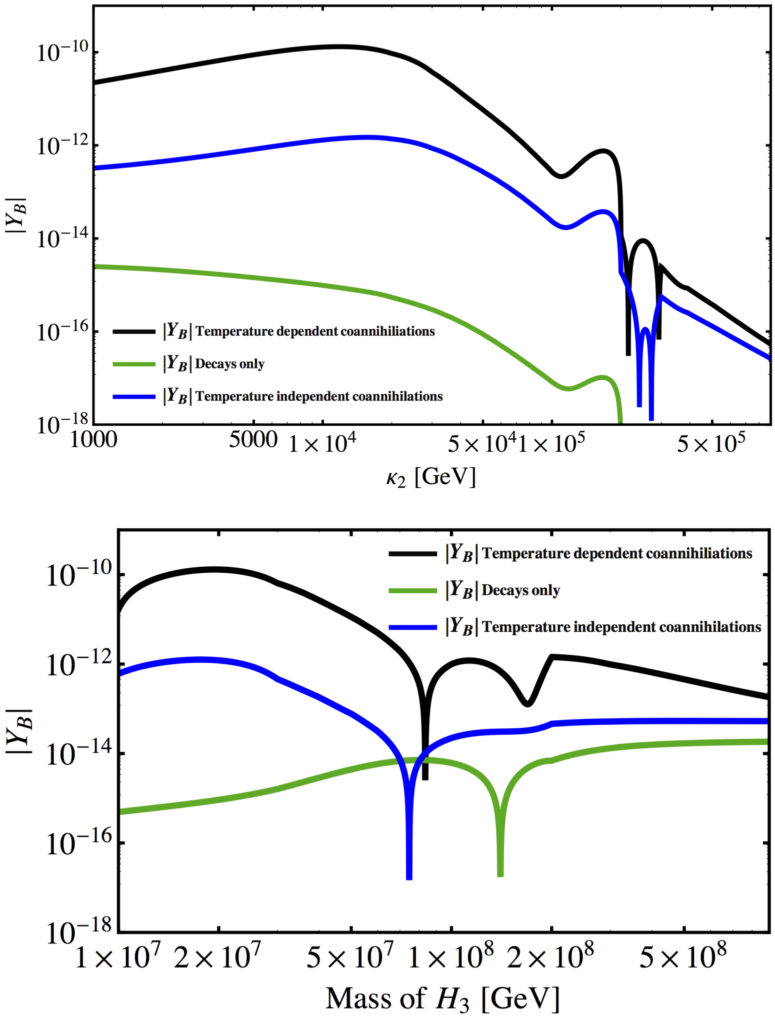

Figure 4.8: (Top) Asymmetry formed vs κ_2, with κ_1 held at 2×10^5 GeV. (Bottom) Asymmetry formed vs the mass of the second intermediate scalar, with M_{H_2} held at 4.5×10^7 GeV.

We have shown that it is possible to create a realistic model of ADM, with the primary mechanism of asymmetry formation being coannihilations. Further, we have demonstrated explicitly the importance of temperature dependence in the CP violating terms.

Chapter 5

Conclusion

The predominance of matter over antimatter in our universe, as well as the presence of dark matter, are two of the most confounding features of the world we live in. That the SM cannot explain these essential facts is one of the main driving factors towards new physics. This thesis explored models that combine these problems, positing a common origin for dark and visible matter. As the effects of coannihilations in ADM have not been explored at detail, we looked at both a toy model of coannihilations and the new lepton portal model in a UV complete manner.

In the toy model we showed that there is a second mechanism to produce ADM with $2 \to 2$ scatterings. Instead of relying on scatterings which both violate CP and baryon number, a flavour asymmetry can be formed and then transferred to a baryon asymmetry. In the simplest UV completion of the toy model, this is the only source of a baryon asymmetry. Even when the model is extended to allow CP violation in the baryon number violating scatterings these flavour effects can still be important. However, for the creation of a large asymmetry the original method proves to be far more effective.

To understand the behaviour of these models in the wild we applied the techniques of the toy model to more realistic scenarios. In particular, we examined the feasibility of the neutron portal providing a viable ADM candidate. While it is possible to construct such a model, this requires serendipitous connections between unrelated processes and moving to $2 \to 3$ scatterings. As the aim was to explore simple and general models of coannihilations we introduced a new model using the lepton portal. In the lepton portal case we showed that it is possible to create a fully functional model of ADM, with the asymmetry produced through thermal freeze out. By studying the UV completions of this model, we explicitly demonstrated the importance of temperature dependence in the CP violation of the coannihilations. For future model builders to see a significant asymmetry caused by $2 \to 2$ scatterings the quadratic temperature dependence will be a key feature. An interesting avenue for future research is the annihilation of the symmetric part; it would be minimalistic to have the same process create an asymmetry and annihilate the symmetric component of ADM.

Exploring these kinds of avenues towards new physics is crucial for understanding both the matter in the universe and how this matter came to exist. As there is no confirmed experimental detection of dark matter, for the time being it is these theoretical considerations that may help reveal the dark side of matter.

Appendix A

Cutkosky Rules: an Example

Cutkosky rules allow us to evaluate the imaginary part of a matrix element without performing the full calculation; in particular we wish to calculate the imaginary part of the interference between a one loop and tree level graph. For the phase space to be complex, there must be on shell loop particles. As an example, we calculate the CP violation in $\psi_i\psi_j \to ff$ due to the graphs in fig. A.1. Though this is the simplest example of CP violation used in this thesis, the principles and techniques are used throughout. The two main sources of complication are the trace structure and kinematics.[1] Here and in all other Feynman diagram calculations we use the Feynman rules of [73]. These rules are more general than those in most QFT textbooks, allowing us to handle charge conjugate and Majorana fields. The calculation is carried out in the centre of momentum frame.

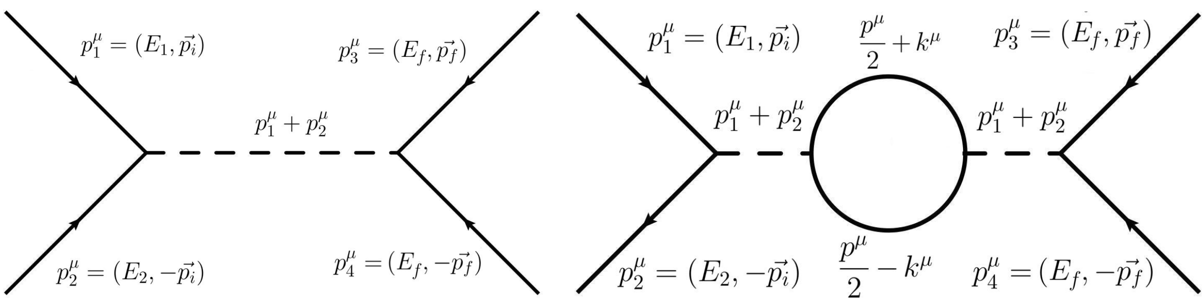

Figure A.1: Kinematics of the process $\psi_i\psi_j \to ff$. Note that if all the scalars are the same, or the scalars are degenerate in mass and couplings, the complex couplings will cancel.

We define $\mathcal{X}_{T^*L}$ to be $\frac{1}{4}\sum_{a,b,c,d} \mathcal{M}_T^*\mathcal{M}_L$, where $\mathcal{M}_T$ and $\mathcal{M}_L$ are the matrix elements of the tree and loop graphs, respectively. Reading off from the graph, we have

$$\begin{aligned}\mathcal{X}_{T^*L} =& \frac{-i}{4m_{X_p}^2 m_{X_q}^2 m_{X_r}^2} \sum_{a,b,c,d} \overline{u_{\psi_i a}} v_{\psi_j b} \overline{v_{fc}} u_{fd} \overline{u_{fd}} v_{fc} \overline{v_{\psi_j b}} u_{\psi_i a} \\ &\times \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left(\frac{-(\not{p}/2 - \not{q}) + m_l}{(p/2-q)^2 - m_l^2 + i\epsilon} \frac{\not{p}/2 + \not{q} + m_k}{(p/2+q)^2 - m_k^2 + i\epsilon} \right),\end{aligned} \tag{A.1}$$

where $p = p_1 + p_2$. We have used here that the center of mass energies are much lower than m_X and, for clarity during the calculation, we will neglect the couplings. This is equivalent to calculating I_{int}. In writing this, we use the identity $v = C\overline{u}^T$, which is easy enough to prove just by writing the spinors explicitly. As in [28], we use a symmetrical distribution of the momenta in the loop – there is no loss of generality. For the curious reader, appendix D of [74] has a useful section on handling charge conjugate fields. For completeness the field operators are given here (only writing out the creation and annihilation structure):

$$\psi = c_r u_r + d_r^\dagger v_r, \quad \overline{\psi} = d_r \overline{v_r} + c_r^\dagger \overline{u_r}, \quad \psi^c = d_r u_r + c_r^\dagger v_r, \quad \overline{\psi^c} = c_r \overline{v_r} + d_r^\dagger \overline{u_r}. \tag{A.2}$$

[1]For example, the box diagram in fig. 2.2 has the same kinematics, but a horrendous trace structure involving six γ matrices and the decay shown in fig. 4.2 has a three body on shell intermediate state and final state.

Evaluating the traces, we get

$$\mathcal{X}_{T^*L} = \frac{32i}{m_{X_p}^2 m_{X_q}^2 m_{X_r}^2}(E_iE_j + p_{\text{int}}^2 - m_im_j)p_{\text{fin}}^2 \int \frac{d^4q}{(2\pi)^4} \frac{(E_kE_l + p_{\text{loop}}^2 - m_km_l)}{(p/2-q)^2 - m_l^2 + i\epsilon} \frac{1}{(p/2+q)^2 - m_k^2 + i\epsilon}, \tag{A.3}$$

It is now time to implement the Cutkosky rules stated in section 1.3: we replace the propagators of the loop particles $\frac{1}{p^2-m^2+i\epsilon} \to -2\pi i\delta(p^2 - m^2)$. In this case there is only one way to cut the loop propagators, straight down the middle. This gives us 2i times the imaginary part of $\mathcal{X}_{T^*L}$. Unfortunately the integral over k^μ makes it difficult to evaluate despite the Dirac deltas. To fix this, we will make the change:

$$\int d^4q \to \int d^4p_1^{'} d^4p_2^{'} \delta^4(p_1^{'} + p_2^{'} - p), \tag{A.4}$$

where $p_1^{'} = p - k$, $p_2^{'} = p + k$. Altogether, we now have:

$$\begin{aligned} \text{Im}\mathcal{X}_{T^*L} =& \frac{-16}{m_{X_p}^2 m_{X_q}^2 m_{X_r}^2}(E_iE_j + p_{\text{int}}^2 - m_im_j)p_{\text{fin}}^2 \\ &\times \int \frac{d^4p_1^{'}d^4p_2^{'}}{(2\pi)^2}(E_kE_l + p_{\text{loop}}^2 - m_lm_k)\delta(p_1^{'2} - m_l^2)\delta(p_2^{'2} - m_k^2)\delta^4(p_1^{'} + p_2^{'} - p). \end{aligned} \tag{A.5}$$

The Cutkosky Dirac deltas remove the time integral, putting the loop particles on shell:

$$\text{Im}\mathcal{X}_{T^*L} = \frac{-4}{m_{X_p}^2 m_{X_q}^2 m_{X_r}^2}(E_iE_j + p_{\text{int}}^2 - m_im_j)p_{\text{fin}}^2 \int \frac{d^3p_1^{'}d^3p_2^{'}}{(2\pi)^2} \frac{E_kE_l + p_{\text{loop}}^2 - m_lm_k}{E_kE_l} \delta^4(p_1^{'} + p_2^{'} - p_1 - p_2). \tag{A.6}$$

The $4E_kE_l$ on the denominator is due to the Dirac delta being written in terms of the square of the energy. Using the 3-momentum part of the remaining delta functional, one of the integrals over 3-momentum can be removed,

$$\text{Im}\,\mathcal{X}_{T^*L} = \frac{-4}{m_{X_p}^2 m_{X_q}^2 m_{X_r}^2}(E_iE_j + p_{\text{int}}^2 - m_im_j)p_{\text{fin}}^2 \int \frac{dp_1^{'}d\Omega}{(2\pi)^2} \frac{p_{\text{loop}}^2(E_kE_l + p_{\text{loop}}^2 - m_lm_k)}{E_kE_l} \delta(E_l + E_k - \sqrt{s}), \tag{A.7}$$

where $\sqrt{s}$ is the centre of mass energy and $d\Omega$ is the usual differential solid angle. As we still have a Dirac delta left to handle the momentum integral a simple change of variables is all that is required to finish the calculation. Using the on shell condition, $E^2 = p^2 + m^2$, to write p in terms of E we get

$$\text{Im}\,\mathcal{X}_{T^*L} = \frac{-4}{\pi\sqrt{s}m_{X_p}^2 m_{X_q}^2 m_{X_r}^2} p_{\text{fin}}^2 p_{\text{loop}}(E_iE_j + p_{\text{int}}^2 - m_im_j)(E_kE_l + p_{\text{loop}}^2 - m_lm_k). \tag{A.8}$$

To relate this to the overall cross section, the usual two body kinematic factors are added,

$$\text{Im}\,I_{\text{int}} = \frac{-1}{16s^{3/2}\pi^2 m_{X_p}^2 m_{X_q}^2 m_{X_r}^2} \frac{p_{\text{fin}}^3 p_{\text{loop}}}{p_{\text{int}}}(E_iE_j + p_{\text{int}}^2 - m_im_j)(E_kE_l + p_{\text{loop}}^2 - m_lm_k). \tag{A.9}$$

While at first this does not look symmetric under interchange of the initial and loop particles, that is because we have not velocity averaged the cross section yet. The offending factor which ruins the symmetry is $\frac{p_{\text{loop}}}{p_{\text{int}}}$; when the velocity average is taken, there will be an extra factor of p_{int}^2.[2]

[2]One factor of p_{int} is hidden in $E_1E_2v_{\text{rel}}$.

Appendix B

Cross sections of the Lepton Portal

For the sake of completeness, here we catalogue the cross sections of the case study in Chapter 4. We will not include the cross sections of the toy model as space does not permit. E_1, E_2, E_3 and E_4 refer to the energies of the particles in the order listed. For the $2 \to 2$ scatterings, we will denote the initial momentum p_i and the final momentum p_f. All cross sections are written in the centre of mass frame, except when otherwise stated. It is important to remember that the masses of the L, ϕ, and H_1 are thermal and so have an implicit temperature dependence.

B.1 Minimal Lepton Portal

Cross sections (CP conserving component)

We include some of the factors from (2.4) to highlight the symmetry under interchange of particles.

$\underline{Y_a L_i \to H_1 \phi}$

$$p_i E_1 E_2 \sigma v = \frac{|\kappa^2 \lambda_{ia}^2|}{16\pi \sqrt{s} m_{H_2}^4} p_i p_f (E_1 E_2 + p_i^2). \tag{B.1}$$

$\underline{L_i \phi \to Y_a H_1}$

$$p_i E_1 E_2 \sigma v = \frac{|\kappa^2 \lambda_{ia}^2|}{8\pi \sqrt{s} m_{H_2}^4} p_i p_f E_1 E_3. \tag{B.2}$$

$\underline{L_i H_1 \to Y_a \phi}$

$$p_i E_1 E_2 \sigma v = \frac{|\kappa^2 \lambda_{ia}^2|}{8\pi \sqrt{s} m_{H_2}^4} p_i p_f E_1 E_3. \tag{B.3}$$

$\underline{Y_a L_i \to Y_b L_j}$

$$p_i E_1 E_2 \sigma v = \frac{|\lambda_{jb}^2 \lambda_{ia}^2|}{4\pi \sqrt{s} m_{H_2}^4} p_i p_f \left[(E_1 E_2 + p_i^2)(E_3 E_4 + p_f^2) + E_1 E_2 E_3 E_4 + 1/2 p_i^2 p_f^2 \right]. \tag{B.4}$$

$\underline{\overline{L}_i L_j \to Y_a Y_b}$

$$p_i E_1 E_2 \sigma v = \frac{|\lambda_{jb}^2 \lambda_{ia}^2| + |\lambda_{ja}^2 \lambda_{ib}^2|}{4\pi \sqrt{s} m_{H_2}^4} p_i p_f \left[E_1 E_2 E_3 E_4 + 1/2 p_i^2 p_f^2 \right]. \tag{B.5}$$

Cross sections (CP violating component)

$\underline{Y_1 L_i \to H_1 \phi}$

$$p_i E_1 E_2 (\sigma - \overline{\sigma}) v = -\sum_j \frac{m_{Y_1} m_{Y_2} \,\mathrm{Im}(\lambda_{i1}^* \lambda_{j1}^* \lambda_{i2} \lambda_{j2}) \kappa^2}{16\pi^2 s m_{H_2}^6} p_i p_f p_{\mathrm{loop}} E_2 E_{L_j}, \tag{B.6}$$

where E_j is the energy of L_j in the loop and p_{loop} is the momentum of the particles in the loop. The CP violation in $Y_1 L_i \to H_1 \phi$ is simply the negative of this. CP violation in $Y_a L_i \to Y_b L_j$ is similar but only leads to flavour violation and is so neglected.

Decays (CP conserving component)

To calculate the three body decays, we used the usual trick of decomposing N-body phase space into a series of 2-body phase spaces. While there are other ways to handle three body decays, this decomposition is particularly useful when dealing with the CP violation, which has a three body intermediate state. All integrals are numerically integrated. The decay rates appearing in section 4.5 are in fact thermally averaged [42],

$$\Gamma^{\text{thermal}} = \frac{K_1(m/T)}{K_2(m/T)}\Gamma. \tag{B.7}$$

Since the relative masses of the particles changes with temperature, at different times the Y, H_1 and ϕ can all decay depending on the thermal masses (in particular the choice of λ_4). We will only write down the Y decays, as the others are similar and not particularly important. In our example solutions we chose λ_4 so that at high temperatures $H_1 \to L_i Y_a \phi$ occurs. Keeping this in mind we now catalogue the decays.

$\underline{Y_a \to L_i H_1^* \phi^*}$

$$\Gamma = \int_{(m_\phi + m_{H_1})^2}^{(m_{Y_a} - m_L)^2} ds \frac{4|\kappa^2 \lambda_{ia}^2|}{(2\pi)^3 \sqrt{s} m_{H_2}^4 m_{Y_a}^2} E_L(s) \left(\sqrt{s + p_L^2(s)} + \sqrt{m_L^2 + p_L^2(s)} \right) p_L(s) p_\phi(s), \tag{B.8}$$

where $E_L(s)$ is the energy of L_i, $p_L(s)$ $[p_\phi(s)]$ is the momentum of L_i $[\phi]$ in the centre-of-momentum frame [rest frame of the mediating particle H_2] and s is $p_{phi}^\mu p_{H_1\mu}$. When we say the rest frame of H_2, we mean a fictitious on-shell particle with mass $\sqrt{s}$ in place of H_2. By integrating over the mass squared of this fictitious particle, we take into account varying kinetic energies of the final particles.

$\underline{Y_1 \to \overline{L_i} Y_2 L_j}$

$$\Gamma = \int_{(m_{L_j} + m_{Y_2})^2}^{(m_{Y_1} - m_{L_i})^2} ds \frac{4(|\lambda_{jb}^2 \lambda_{ia}^2| + |\lambda_{ja}^2 \lambda_{ib}^2|)}{(2\pi)^3 \sqrt{s} m_{H_2}^4 m_{Y_a}^2} E_{L_i}(s) \left(\sqrt{s + p_{L_i}^2(s)} + \sqrt{m_{L_i}^2 + p_{L_i}^2(s)} \right) p_{L_i}(s) p_{L_j}(s) \left(E_{L_j}(s) E_{Y_2}(s) + p_{L_j}^2(s) \right), \tag{B.9}$$

where $E_{L_{i\ (j)}}(s)$ and $p_{L_{i\ (j)}}(s)$ are the energy and momentum of $L_{i\ (j)}$, respectively, and s is $p_{Y_2}^\mu p_{L_j \mu}$. Note that $p_{L_j}(s)$ is in the rest frame of H_2 and all others are in the rest frame of H_2.

Decays (CP violating component)

To evaluate the CP violation in the decays can require some deft changing of Lorentz frames. Fortunately the loop particles have the same energy running through them as the pair ϕ and H_1 and so can be dealt with naturally in the same reference frame as ϕ and H_1.

$\underline{Y_1 \to L_i H_1^* \phi^*}$

$$\Gamma - \overline{\Gamma} = \int_{Low}^{(m_{Y_1} - m_L)^2} ds \sum_j \frac{m_{Y_2} \operatorname{Im}(\lambda_{i1}^* \lambda_{j1}^* \lambda_{i2} \lambda_{j2}) \kappa^2}{(2\pi)^4 s^{3/2} m_{H_2}^6 m_{Y_1}} E_{L_j}(s) \left(m_{Y_1}^2 - s - m_{L_i}^2 \right) p_{L_i}(s) p_{L_j}(s) p_\phi(s), \tag{B.10}$$

where s is $p^{\mu}_{L_i} p_{H_1\mu}$ and Low $\equiv$ Max$[(m_\phi + m_{H_1})^2, (m_{Y_2} + m_{L_j}])^2$. The kinematics of L_i were written in the centre-of-momentum frame, and the other particles kinematics are in the rest frame of H_2.

B.2 Extended Lepton Portal

When a second inert Higgs doublet is added, H_3, we get many more diagrams contributing to the various processes. For most processes, these diagrams are simply allowing the intermediate particle to be H_2 or H_3, with some interference terms. Due to the lack of space, and triviality of extension, we do not write these down. But there are important new contributions to the CP violation in both the decays and coannihilation. The new cross section is:

$\underline{Y_1 L_i \to H_1 \phi}$

$$\begin{aligned} p_i E_1 E_2 (\sigma - \overline{\sigma}) v = &- \sum_{jprs} \frac{m_{Y_1} m_{Y_2} \operatorname{Im}(\lambda^*_{i1p} \lambda^*_{j1r} \lambda_{i2r} \lambda_{j2s} \kappa^*_p \kappa_s)}{16\pi^2 s m^2_{H_p} m^2_{H_r} m^2_{H_s}} p_i p_f p_{\text{loop}} E_2 E_j \\ &+ \sum_{jprs} \frac{\operatorname{Im}(\lambda^*_{i1p} \lambda_{i1r} \lambda^*_{j2r} \lambda_{j2s} \kappa^*_p \kappa_s)}{8\pi^2 s m^2_{H_p} m^2_{H_r} m^2_{H_s}} p_i p_f p_{\text{loop}} \left(E_1 E_2 + p_i^2 \right) \left(E_{L_j} E_{Y_2} + p^2_{loop} \right), \end{aligned} \tag{B.11}$$

where $p,\ r,\ s \in 2,\ 3$ and E_{Y_2} is the energy of Y_2 in the loop. We note that if all the masses and couplings are the same for H_2 and H_3 the second term cancels, and CP violation is reduced to the minimal lepton portal. The contributions to the decays are given by:

$\underline{Y_1 \to L_i H^*_1 \phi^*}$

$$\begin{aligned} \Gamma - \overline{\Gamma} = &- \int_{Low}^{(m_{Y_1} - m_L)^2} ds \sum_{jprs} \frac{m_{Y_2} \operatorname{Im}(\lambda^*_{i1p} \lambda^*_{j1r} \lambda_{i2r} \lambda_{j2s} \kappa^*_p \kappa_s)}{(2\pi)^4 s^{3/2} m^2_{H_p} m^2_{H_r} m^2_{H_s} m_{Y_1}} E_{L_J}(s) \left(m^2_{Y_1} - s - m^2_{L_i} \right) p_{L_i}(s) p_{L_j}(s) p_\phi(s) \\ &+ \int_{Low}^{(m_{Y_1} - m_L)^2} ds \sum_{jprs} \frac{2 \operatorname{Im}(\lambda^*_{i1p} \lambda_{i1r} \lambda^*_{j2r} \lambda_{j2s} \kappa^*_p \kappa_s)}{(2\pi)^4 s^{3/2} m^2_{H_p} m^2_{H_r} m^2_{H_s} m^2_{Y_1}} E_{L_J}(s) \left(\sqrt{s + p^2_{L_i}(s)} + \sqrt{m^2_{L_i} + p^2_{L_i}(s)} \right) \\ &\times \left(E_{L_j} E_{Y_2} + p^2_{loop} \right) p_{L_i}(s) p_{L_j}(s) p_\phi(s). \end{aligned} \tag{B.12}$$

Similar to the CP violation in the scatterings, in the limit of degenerate couplings and masses the second term disappears. This completes the cataloguing of the cross sections used to solve the Boltzmann equations in section 4.5.

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