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A systematic study of mirror and triplet energy differences

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Abstract. Differences of excitation energy among analogue states in a broad mass range are calculated in the framework of the shell model by introducing isospin breaking interactions. Mirror and triplet energy differences along the N=Z line from mass $A \sim 20$ to $A \sim 60$ have been systematically studied using the same method. It is shown that in all cases an additional term not directly associated to the electromagnetic interaction is needed to reproduce the experimental data.

1. Introduction

The nuclear interaction may be considered with good approximation to be charge symmetric and charge independent. In the nucleus, these symmetries are of course broken by the Coulomb interaction. However, the experimental data show that the isospin symmetry is slightly broken even when Coulomb effects are taken into account. The degree of symmetry breaking by the nuclear interaction can be studied by comparing the binding energies or masses of isobaric nuclei, the so called Coulomb Displacement Energies (CDE) [1]. More recently, with the advent of high-resolution, high-efficient gamma-ray arrays, it has become possible to study differences in *excitation* energy between isobaric analogue states as a function of the angular momentum. These differences are of the order of tens of keV and constitute a very sensitive tool to probe some nuclear structure properties, such as the evolution of the nuclear structure, such as changes of the nuclear radius, the origin of the backbending in rotating nuclei, and other interesting properties [2].

These studies have been performed within the shell model framework, and in particular in nuclei in the $f_{7/2}$ shell, where energy differences between mirror nuclei and T=1 isobaric triplets can be well reproduced by large-scale calculations [2, 3]. From these studies, and after taking into account electromagnetic corrections, it was suggested that an additional isospin breaking term has to be considered to reproduce the data. Two schematic terms, consisting of only one matrix element, have been proposed by Zuker and collaborators [2]. In the case of the differences between excited states in mirror nuclei (MED), the isovector matrix element corresponds to two protons in the $f_{7/2}$ shell coupled to $J = 2$, while for the energy differences of an isobaric triplet (TED), the matrix element corresponds to two protons in the $f_{7/2}$ coupled to $J = 0$. In both cases the strength of the matrix elements is +100 keV and is deduced from the experimental data in mass $A = 42$.

In recent years the experimental studies have been extended from the nuclei in the $f_{7/2}$ shell to other mass regions due to the progress in the experimental techniques and the use of

radioactive beams. From the theoretical point of view, there have been studies by Ekman and collaborators [4] in the mass region $A = 57 - 61$ and more recently by Kaneko *et al.* on the excitation energy differences in $A = 67$ [5]. This latter study has been partly based on the work by Zuker *et al.* [2], but have not converged into a unified method for the description of the excitation energy differences in this mass region. In particular, the need of introducing an isospin breaking force does not emerge.

One would think that if a simple isospin breaking force arise so clear in the $f_{7/2}$ shell, it should be also present in the rest of the nuclear chart. Of course the $f_{7/2}$ shell is very particular as it is quite isolated from the rest of the shells and therefore some intrinsic properties can be easily found. In this paper we present some new results on the *sd* and upper *pf* shell where we have extended the same method applied to the energy differences in $f_{7/2}$ shell nuclei. This study puts in evidence the general character of the isospin symmetry breaking terms that have to be added to the nuclear interaction to reproduce the excitation energy differences measured so far.

2. Shell model description of MED and TED

The mirror (isovector) energy differences, which account for the charge symmetry breaking as a function of the nuclear spin J are defined as:

$$MED(J) = E_x(J, T, T_z) - E_x(J, T, -T_z). \tag{1}$$

The triplet (isotensor)energy differences, which refer to three isobaric nuclei with $T = 1$ are defined as:

$$TED(J) = E_x(J, T = 1, T_z = -1) + E_x(J, T = 1, T_z = +1) - 2E_x(J, T = 1, T_z = 0), \tag{2}$$

where E_x are the excitation energies of the states referred to the ground state or to the lowest state of the same T in each nucleus and $T_z = (N - Z)/2$.

To calculate these differences in the framework of the shell model, we follow the method described in refs. [2, 3]. The terms that contribute to the energy differences can be divided in a monopole Coulomb (V_{Cm}), a multipole Coulomb (V_{CM}), and an additional isospin breaking term V_B. MED and TED are therefore calculated as,

$$MED(J) = \Delta_M < V_{Cm}(J) > + \Delta_M < V_{CM}(J) > + \Delta_M < V_B(J) > \tag{3}$$

$$TED(J) = \Delta_T < V_{CM}(J) > + \Delta_T < V_B(J) > \tag{4}$$

where Δ_M and Δ_T indicate that the differences are obtained as in equation (1) and (2), respectively.

Monopole Coulomb contributions are responsible for single-particle shifts and bulk effects, while the multipole term takes into account the interaction between valence nucleons. The single-particle energy corrections have two different origins.

The energy of the proton orbits is affected proportionally to the square of the orbital momentum l in the harmonic oscillator representation. In particular, the single-particle energy of protons with principal quantum number N, above the closed shell Z_{CS} is modified by the following term [6]:

$$E_{ll} = \frac{-4.5 Z_{CS}^{13/12}[2l(l+1) - N(N+3)]}{A^{1/3}(N + 3/2)} \tag{5}$$

The relativistic electromagnetic spin-orbit interaction, that affects both the proton and neutron single-particle energies is [1, 7]:

$$V_{ls} = (g_s - g_l)\frac{1}{2m_N^2 c^2}(\frac{1}{r}\frac{dV_c}{dr})\vec{l}.\vec{s}, \tag{6}$$

where m_N is the nucleon mass, and g_s and g_l are the gyromagnetic factors of protons and neutrons.

There is an additional monopole effect due to the Coulomb interaction. This is due to changes of the nuclear radius as a function of the angular momentum. The radius of the nucleus depends on the orbitals that are occupied and the occupation number may change with J. The radial term is calculated in the shell model framework as discussed in Ref.[2] from the equation:

$$V_{Cr} = n\alpha_i\left(\frac{m_{i\pi}(g.s.) + m_{i\nu}(g.s.)}{2} - \frac{m_{i\pi}(J) + m_{i\nu}(J)}{2}\right) \tag{7}$$

where $n/2 = |T_z|, m_{i\pi}, m_{i\nu}$ are the proton and neutron occupation numbers of orbit i, respectively, and α_i is a parameter that depends on the particular orbit. It is important to note that due to the way TED are obtained, monopole effects cancel out.

In Ref. [2] it was shown that the effect of the isospin breaking interaction V_B in both MED and TED can be as large at the Coulomb terms. This was deduced from extracting the Coulomb contribution to the MED and TED in mass A=42. For the MED, after subtracting to the experimental value the Coulomb contribution, a large difference (100 keV) was observed for the J=2 state, while for the TED, the maximum difference was found for the J=0 state. The extrapolation to other shells is not straightforward. In mass $A = 42$, the wavefunctions can be approximately considered of pure $f_{7/2}$ nature while this is not the case for other mass regions since all the orbits within the model space can contribute to the V_B term. We generalize the prescription of the $f_{7/2}$, adding the relevant matrix elements of all the orbits within the model space with the same strength that allows a good description of all the available data. For more details see Ref. [2, 3].

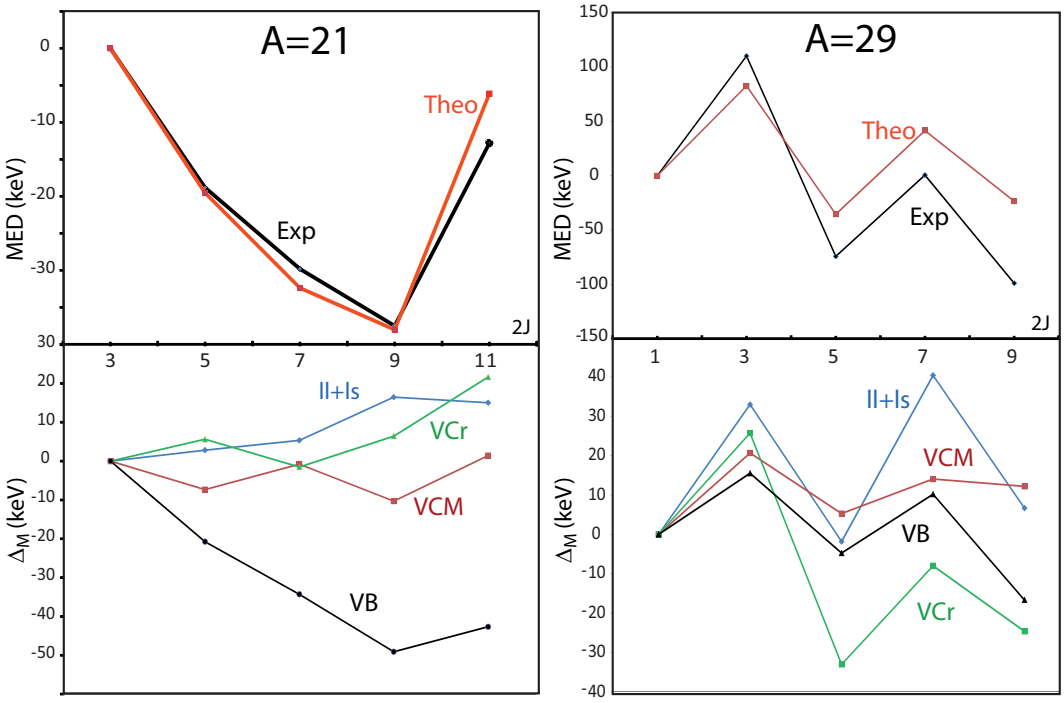

Figure 1. MED (left panels) for the $T = 1/2$ yrast states in the mirror pair ^{21}Na-^{21}Ne. The lower panels show the contribution of the different components to the MED.

3. Results

We calculate the mirror energy differences and the triple energy differences of nuclei in the mass range $A \sim 20 - 60$ where experimental data are available. The calculations were done by using the shell-model code ANTOINE [8, 9] with different effective nuclear interactions. For the *sd* shell, the USD interaction [10, 11] has been used while the KB3G [12] interaction was used for nuclei in the $f_{7/2}$ shell. For $A = 57 - 61$, the GXPF1A [13] interaction has been adopted.

In this section, we show some few illustrative examples of the results obtained for MED and TED of nuclei in the *sd* shell and upper *fp* shell. The result of MED and TED of the $f_{7/2}$ shell can be found in [3].

3.1. MED and TED in the sd shell

The calculations of the MED and TED in the *sd* shell are in very good overall agreement with the data. We report in figure 1 the MED results for mass $A = 21$ and $A = 29$, together with the single contributions. The radial term in this case is due to the changes in occupation number of the $s_{1/2}$ orbit with strength $\alpha = 100$ keV. For the V_B term we consider two matrix elements with two protons in the $d_{3/2}$ and two protons in the $d_{5/2}$ coupled to $J = 2$ with a strength of 100 keV, as that used in the $f_{7/2}$ shell. As can be deduced from figure 1, the contribution of the V_B term is essential to allow a good description of the data, in particular, in $A = 21$.

Following the prescription of Ref. [2, 3], for the TED, the V_B term corresponds to matrix elements with two protons in the $d_{3/2}$, $d_{5/2}$, $s_{1/2}$ coupled to $J = 0$ with a strength of 100 keV for all the orbits. We report in figure 2 the TED for the $T = 1$ triplets in the *sd* shell. The calculated curves are in very good agreement with the experimental data. Interestingly, the contributions from the multipole Coulomb term and the isospin breaking V_B term are very similar.

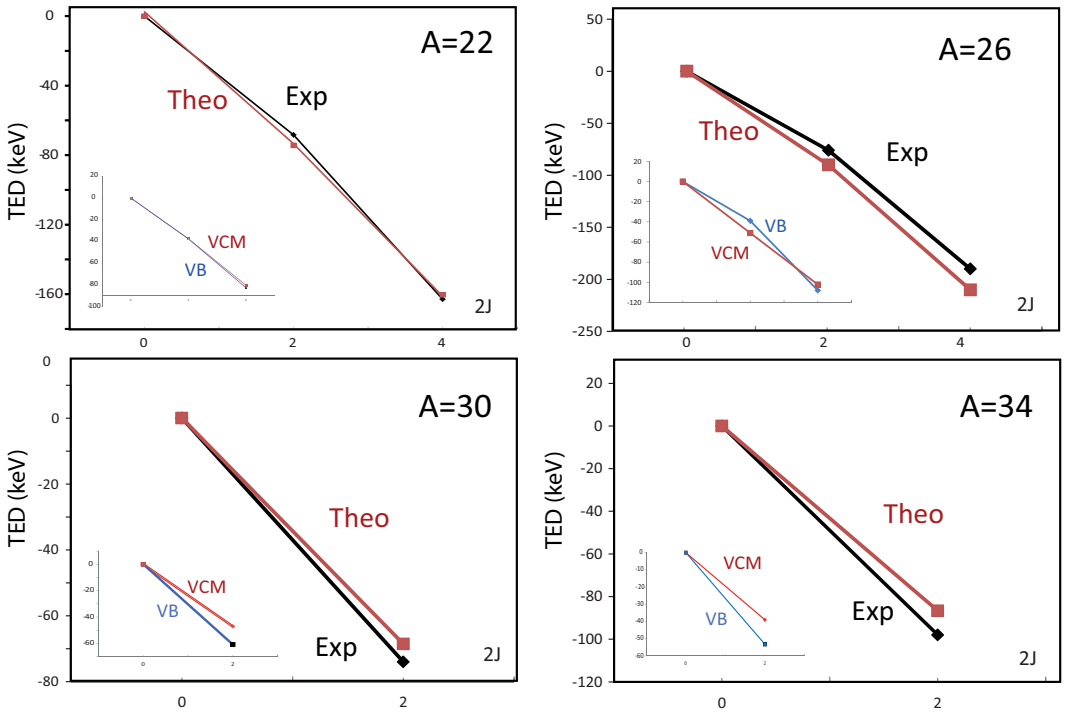

Figure 2. TED for the $T = 1$ yrast states in the isobaric multiplets of mass A= 22, 26, 30 and 34. In the left bottom of each figure the two contributions to the TED in equation (4) are displayed.

3.2. MED and TED of masses in the mass range A=57 to A=61

The calculations of the MED and TED in the upper fp shell are performed following an extension of the prescription of Ref. [2, 3], using the GXPF1A interaction. For the MED, the radial term is due to changes in the occupation number of the $p_{3/2}$ and the $p_{1/2}$ orbits with equal strength α = 200 keV. A different prescription is used for the INC term V_B: instead of increasing the contribution of the $J = 2$ matrix elements by 100 keV, we decrease by the same amount those coupled to J =0, i.e. we take into account matrix elements with two protons in the $f_{7/2}, p_{3/2}, f_{5/2}$ and $p_{1/2}$ coupled to $J = 0$ with a strength of -100 keV. This choice is justified by a parallel theoretical work by Bentley and collaborators [14] where a best fit of the available MED data in the $f_{7/2}$, allowing all the matrix elements for two protons in the $f_{7/2}$ to vary, show that it is the relative difference between the J =0 and the J =2 matrix elements which counts. So, an increase of the J =2 matrix element by 100 keV, may be equivalent to a decrease of the J =0 matrix element by 100 keV.

The results are in overall agreement with the available data, with the exception of $A = 59$. We report in figure 3 the results for mass A=58 together with the single contributions. For the TED, we follow the same prescription as for the other mass regions: the V_B term corresponds to matrix elements with two protons in all the orbits coupled to $J = 0$ with a strength of 100 keV. The TED for $A = 58$ are reported in figure 3. As for the *sd* shell, the contribution of the V_B term is clearly relevant and very similar to that of the Coulomb contribution.

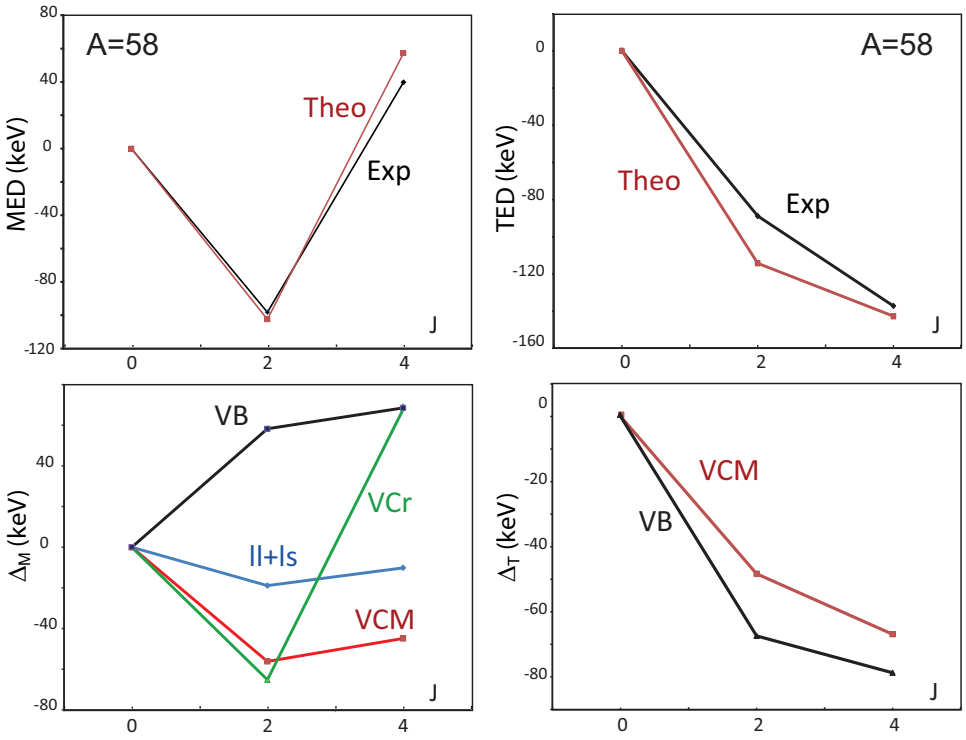

Figure 3. MED and TED for the $T = 1$ yrast states in the isobaric multiplets of mass A = 58. The lower panels show the contribution of the different components to the MED and TED.

4. Conclusions

We have performed a systematic study of the MED and the TED in different mass regions that cover the *sd* shell up to the upper fp shell. The theoretical calculations reproduce with the same prescription the TED in the different mass regions. In all cases so far investigated the

theoretical calculations are in excellent agreement with the experiment. It is shown that an additional isospin breaking term $V_B = 100$ keV for two protons in the different shells coupled to $J = 0$ is needed to reproduce the data. This term results very similar to that due to the Multipole Coulomb contribution. Here we present some significant examples but it is important to note that the same parameterization reproduces all cases in which data are available. Recently, Kaneko *et al.* have used a similar V_B term for the description of the TED in $A = 66, 70$ within the fp shell [15].

For the MED, the extension and generalization to the *sd* shell of the method introduced in refs. [2, 3], describes very well the data up to mass $A \sim 30$. The need of a generalized isospin symmetry breaking term V_B corresponding to matrix elements with two protons in the same shell coupled to $J = 2$ and strength +100 keV emerges clearly. At higher masses, excitations to the upper fp shell become important. On the other hand, in the upper fp shell, a better agreement with the data is obtained using a V_B term where the protons are coupled to $J = 0$ and with negative strength of -100 keV. This alternative prescription gives also rather good results in the *sd* and the $f_{7/2}$ shells.

The results show a consistent description of the MED and TED in the shell model framework. Moreover, they demonstrate that the evidence of an isospin symmetry breaking term is not constrained to the $f_{7/2}$ shell, but has to be systematically included to account for the experimental results. The microscopic origin of this term is still not understood.

It is important to note that in this analysis, the strengths of the different terms contributing to the MED and TED have not been fitted in order to have the best reproduction of the data. Very schematic values have been used aiming at putting in evidence the general character of the method. A more detailed presentation of the results, that includes new data is in preparation.

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