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Cosmic Axions from Cosmic Strings*

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ABSTRACT

The possibility of a new constraint on the Peccei-Quinn symmetry breaking scale, arising from the decay of cosmic axion strings, is discussed.

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1. Foreword

That a spontaneously broken, anomalous, global $U_{PQ}(1)$ symmetry can be introduced into the standard model to solve the strong CP problem is a well known fact.[1] This model has a pseudo-goldstone boson, the axion.[2,3] The axion is phenomenologically acceptable if the symmetry breaking scale is not too low($f_a > \times 10^9$Gev)[♮1] , for stars would burn up too fast;[5–7] and if it is not too high($f_a < 4 \times 10^{12}$Gev), for the axions would dominate the universe.[8–11] For details on these matters the interested reader is referred to the literature.

Here I am concerned with the upper bound on the PQ symmetry breaking scale. This bound was derived by looking at the zero momentum mode of the axion field as the universe passes through the quark-hadron phase transition. Above the phase transition the axion is massless and the zero momentum mode contributes nothing to the energy density, so the value of the field can be any constant from 0 to f_a without affecting the physics. Causality arguments imply that it is $o(f_a)$. At the phase transition instanton phenomena give mass to the axion field and it begins to undergo spatially homogeneous coherent oscillations about zero, with frequency m_a. Consideration of the damping needed to prevent the associated energy from dominating the universe gives the cosmological bound.

In this letter I would like to discuss other sources of axions which might contribute to this bound, the non-zero momentum modes. The first source is a thermal distribution: the axions were in thermal equilibrium at the PQ symmetry breaking, but then were decoupled from the subsequent thermal history of the

♮1 This bound may be lowered by a factor $\sim .01$ in models where the axion does not couple to electrons. Also, since the physics of the interiors of stars is imperfectly understood, one must take this bound with some skepticism. Present observational limits on the solar axion flux imply that $f_a > 10^7$Gev.[4]

universe. Though decoupled, these axions maintained the "thermal" spectrum as the universe cooled, with "temperature" simply related to the temperature of the surrounding thermal bath all the way down to the quark-hadron phase transition. In the analysis described above it was assumed that these axions get redshifted away and are therefore not cosmologically important. In section 2 I will show explicitly that this assumption is correct.

But mainly I am interested in another, hitherto unrecognized source of axions which could have cosmological import. These arise from the inevitable(barring inflationary models) appearance of axion strings after the PQ phase transition. The strings lose energy by emission of axions into a characteristic energy spectrum. Of no significance above the quark-hadron phase transition, the energy density is enhanced when passing through, and this contributes to the upper bound on the PQ scale. This material is treated in section 3, and comments on the relevance of this bound are in the conclusion. An important technical note is reserved for the appendix.

2. Energy Density of Decoupled Thermal Axions

The energy density just below the PQ symmetry breaking scale has a thermal spectrum

$$\rho_a = \frac{1}{2\pi^2}\int \frac{k^3 dk}{e^{\frac{k}{T}} - 1}.$$

Sometime after the symmetry breaking(long before the quark-hadron phase transition) the axions decouple from all other matter and radiation, and the axion field redshifts as the universe expands and cools. At lower temperatures this "thermal" distribution is maintained, only the effective temperature of the axion field may be different from the actual temperature of the universe. The energy density is

$$\rho_a = \frac{1}{2\pi^2}\int \frac{k^3 dk}{e^{\frac{k}{T^\star}} - 1},$$

where $T^\star$ can be derived in an adiabatically expanding universe,

$$T^\star = \left[\mathcal{N}(T)/\mathcal{N}(T_{dec})\right]^{1/3} T.$$

$\mathcal{N}(T)$ is the number of interacting particle degrees of freedom with masses less than T. At the quark-hadron phase transition the axion mass starts to turn on. How does this effect ρ_a? In the appendix I have outlined a WKB calculation for the proper integral insertion; the energy density is

$$\rho_a = \frac{\tilde{t}^2}{2\pi^2 t^2}\int \sqrt{\frac{(k^2 + \frac{t}{\tilde{t}} m_a^2)}{(k^2 + \tilde{m}^2)}}\, \frac{k^3 dk}{e^{k/\tilde{T}^\star} - 1}, \qquad (2.1)$$

where $\tilde{t}$ is roughly the time when the Compton wavelength of the axion comes within the horizon. I call the axion mass at that time $\tilde{m} = 1/\tilde{t}$. The value of $\tilde{t}$ can be estimated from instanton physics: the corresponding temperature is $\tilde{T} \sim 800\text{Mev} - 1\text{Gev}$,[8] and $\tilde{m} \sim 10^{-9} - 10^{-8}$ev.

Without performing the integral we can see that the phase transition does not significantly alter ρ_a. The insertion in eq.(2.1) is different from 1 only in the region $k \lesssim m_a \sim 10^{-5}$ev. But since the quark-hadron phase transition occurrs at $T \sim 100$Mev, and since for any plausible value of $\left[\mathcal{N}(T)/\mathcal{N}(T_{dec})\right]^{1/3}$ we must have $\tilde{T}^\star \gg m_a$, the other term in the integrand is essentially zero in that region. In other words, the effective temperature of the axion bath is high enough that the axions remain relativistic through the phase transition. Since they become non-relativistic at nearly the same time as other matter, and because their mass is so small, these thermal axions pose no threat of dominating the universe.

3. Energy Density of Cosmic String Axions

At the PQ phase transition a random network of cosmic axion strings materializes. Densely knotted, the strings will start to oscillate under their own tension after axion decoupling, when frictional effects cease to matter. In a previous article[12] I have demonstrated that strings from broken global symmetries lose their energy very efficiently by radiation of Goldstone bosons. Here I will assume that string excitations will be damped out so rapidly that loop formation and decay is not an important effect. Since oscillations can only occur when a kink enters the horizon, the string system will tend to straighten out on scales less than the horizon, maintaining its Brownian knottiness on larger scales. The system remains in a tangle, but the step length grows with the horizon, while the radiation of axions is continuous and accumulating. To begin I will derive the energy density of these massless axions.

It is quite general that a static, straight axion string has mass per unit length

$$\mu = \int [(\partial\rho)^2 + \frac{\rho^2}{r^2} + V(\rho)] r dr d\phi,$$

where V is a symmetry breaking potential and the field ρ behaves like $\sim r$ inside the string and goes exponentially to f_a outside. V and the functional form of ρ are model dependent, and therefore so is μ, but because of the middle term μ is dominated by a long range $1/r$ potential which is unambiguous. Thus we can write

$$\mu > 2\pi f_a^2 \ln(\Lambda/\delta),$$

where δ is the thickness of the string core and Λ is a cutoff provided by the inter-string spacing.

Let us divide space up into expanding cubic cells of size t, the age of the radiation dominated universe. Also, let us assume that at any given t there is one straight string of length t per cell. Then the string mass in a cell at any time is

$$E(t) = 2\pi f_a^2 t \ln(t/\delta). \tag{3.1}$$

In order to maintain this form of $E(t)$, axions must be radiated because kinks appear in a cell as it expands. We can calculate the amount of radiation if we were to suppose that the energy were to have the correct form at some time t_0, but afterwards the strings did not oscillate and radiate at all. At later times the energy per cell would be

$$\bar{E}(t) = 2\pi f_a^2 \sqrt{tt_0}(\frac{t}{\sqrt{tt_0}})^3 \ln(\sqrt{tt_0}/\delta) = 2\pi f_a^2 t^2 \ln(\sqrt{tt_0}/\delta),$$

in which I have included both the of stretching of the original cell, $t_0 \to t_0\sqrt{\frac{t}{t_0}}$, and the fact that a cell of size t contains $(t/\sqrt{tt_0})^3$ of the original cells. The rate at which energy must be radiated to maintain the form of eq.(3.1) is

$$\frac{d}{dt}E_{rad}(t)\mid_{t=t_0} = \frac{d}{dt}\bar{E}(t)\mid_{t=t_0} - \frac{d}{dt}E(t)\mid_{t=t_0} = 2\pi f_a^2[\ln(t_0/\delta) - 1/2].$$

The increment to the energy density at time t is

$$\Delta\rho(t) = \frac{\Delta E_{rad}(t)}{t^3} = 2\pi f_a^2[\ln(t/\delta) - 1/2]\frac{\Delta t}{t^3},$$

and the total energy density that is accumulated between some very early time $t^\star$, when axions decoupled, and a later time t is

$$\rho(t) = 2\pi f_a^2 \int_{t^\star}^{t} (\frac{\tau}{t})^2[\ln(\tau/\delta) - 1/2]\frac{d\tau}{\tau^3} = \frac{\pi f_a^2}{t^2}[\ln(t/\delta)\ln(t/t^\star) - \ln(t/t^\star)], \tag{3.2}$$

where the $(\tau/t)^2$ factor in the integrand accounts for the cosmological redshift in the radiation dominated epoch. All that remains is to write eq.(3.1) as a spectral distribution.

Radiation emitted at time τ will originally have a wavelength of $\sim \tau$ and momentum $\sim 2\pi/\tau$. To be careful, let us suppose that radiation emitted at τ has an effective momentum $k = \Omega/\tau$, where we expect $\Omega \gtrsim 2\pi$. At a later time t that momentum is redshifted by $\sqrt{\tau/t}$. In terms of k, (3.1) becomes

$$\rho(t) = \frac{4\pi f_a^2}{t^2} \int_{\frac{\Omega}{t}}^{\frac{\Omega}{\sqrt{tt^*}}} [\ln(\Omega^2/tk^2\delta) - 1/2]\frac{dk}{k}. \tag{3.3}$$

This formula is good for all times up to when $\Omega/t = m(t)$ is satisfied, that is, when the axion mass is equal to the energy of the lowest energy axion.

We will need to know this time, so I will calculate it here. Recall that $\tilde{t}$ is defined so that $\tilde{m} \equiv m(\tilde{t}) = 1/\tilde{t}$. Instanton calculations[13] imply that for three quark flavors

$$m(t) \sim 7.3(\Lambda^6/\pi^4 f_a)(m_u m_d m_s/\Lambda^3)^{1/2} m_{pl}^2 \ln(\pi^2 m_{pl}/\Lambda^2 t)\frac{1}{t^2},$$

where Λ is the QCD scale factor, m_{pl} is the Planck mass and $m_{u,d,s}$ are the quark masses. Since the logarithm evolves so slowly we may write

$$m(t) = t^2/\tilde{t}^3$$

as long as t does not stray too far from $\tilde{t}$. Setting this equal to Ω/t we find that the latest time for which eq.(3.3) is valid is $\Omega^{\frac{1}{3}}\tilde{t}$.

After the axion mass has turned on the picture is entirely different. Any string must be attached to a domain wall. The wall-bounded-by-string system breaks up into loops spanned by walls; they oscillate and decay radiatively; but since the axion is now massive the radiation is gravitational, at least until the wall area shrinks to a size of a few meters-squared, when axion emission can again take over. The cosmology of the string-wall system is very interesting,[14] but here we are concerned only with the effects of those axions radiated before $\Omega^{1/3}\widetilde{t}$.

After $\Omega^{1/3}\widetilde{t}$ the energy density of these axions has the form [See A1]

$$\rho_a(t) = \frac{4\pi f_a^2}{t^2} \int_{\Omega^{2/3}/\widetilde{t}}^{\Omega^{5/6}/\sqrt{\widetilde{t}t_*}} \sqrt{\frac{k^2 + \frac{t}{\widetilde{t}}m_a^2}{k^2 + \widetilde{m}^2}} [\ln(\Omega^{5/3}/\widetilde{t}k^2\delta) - 1/2] \frac{dk}{k}.$$

Notice that this has the appropriate redshifting properties: $\sim t^{-2}$ for large k modes and $\sim t^{-3/2}$ for low k modes. Looking at this spectrum we see that as long as Ω is not too large there is a significant contribution from very low values of k, down to $k = \Omega^{2/3}/\widetilde{t} = \Omega^{2/3}\cdot\widetilde{m}$, where the effect of the WKB insert is important. So, contrary to the previous example, there should be a large enhancement when the mass turns on.

Only an approximate solution can be found. First, we can set $\widetilde{m} = 0$ in the denominator, introducing an error of at most a few percent. Second, we can underestimate ρ_a by truncating the upper limit of integration down to $k = m_a$. Third, we can put $k = m_a$ in the logarithm and pull it out of the integral, obtaining

$$\rho_a(t) \approx \frac{4\pi f_a^2}{t^2} \ln(\Omega^{5/3}/\widetilde{t}m_a^2\delta) \int_{\Omega^{2/3}/\widetilde{t}}^{m_a} \sqrt{1 + \frac{tm_a^2}{\widetilde{t}k^2}} \frac{dk}{k}$$

$$\approx \frac{4\pi f_a^2}{t^2} \ln[\Omega^{5/3}/\widetilde{t}m_a^2\delta] \left\{ \sqrt{1 + \frac{t\widetilde{t}m_a^2}{\Omega^{4/3}}} - \sqrt{1 + \frac{t}{\widetilde{t}}} \right\} \approx \frac{4\pi f_a^2}{t^2} \ln[\Omega^{5/3}/\widetilde{t}m_a^2\delta] \sqrt{\frac{t\widetilde{t}m_a^2}{\Omega^{4/3}}}.$$

At this point it is fruitful to cast our result into a form that can be easily compared to previous calculations. In the following I will be closely paralleling the work in ref.[6]. The energy density can be written

$$\rho_a(T) \approx 4\pi \ln[\Omega^{5/3}/\tilde{t}m_a^2\delta] \times \frac{1}{\Omega^{2/3}} \times \frac{1}{.3}\left[\frac{\mathcal{N}(T)^3}{\mathcal{N}(\tilde{T})}\right]^{1/4} \times \left[\frac{f_a}{m_{pl}}\right]\left[\frac{f_a m_a}{\tilde{T}}\right]T^3,$$

where I have used the relation $tT^2 = .3m_{pl}/\sqrt{\mathcal{N}}$. Next, using $\tilde{T}$=800Mev and dividing by the critical density of the universe, we get

$$\rho/\rho_{cr} = 4\pi \ln[\Omega^{5/3}/\tilde{t}m_a^2\delta]\frac{1}{\Omega^{2/3}} \times \frac{1}{.3}\left[\frac{\mathcal{N}(T)^3}{\mathcal{N}(\tilde{T})}\right]^{1/4} \times \frac{f_a}{10^{13}\text{Gev}}. \qquad (3.4)$$

Of course the bound on the axion scale comes from the condition that this quantity be less than 1.

4. Conclusion

$\mathcal{N}(\tilde{T}) = 61.75$, which includes the helicity states of photons, gluons, three neutrinos, two charged leptons, and three quark flavors. At the present temperature $\mathcal{N}_{eff}(T) = 3.4$, which accounts for the neutrino decoupling before e^+e^- annihilation. Taking for now $\Omega = 2\pi$, $f_a \sim 10^{10}$Gev, $m_a = [(m_u m_d)^{1/2}/(m_u + m_d)](m_\pi f_\pi/f_a) \sim 6 \times 10^{-4}$ev and $\delta = (1/f_a)$, eq.(3.4) gives

$$\rho/\rho_{cr} \approx 4.5 \times \frac{f_a}{10^{11}\text{Gev}},$$

which implies

$$f_a \lesssim 2 \times 10^{10}\text{Gev}, \qquad (4.1)$$

a factor of 200 lower than the bound previously obtained by looking only at the zero-momentum mode of the axion field.

Evidently the bound (4.1) depends critically on the value of Ω, as well as on the assumption that one cubic cell of size t contains one string of length t. If we assume that the wavelength of the radiation accurately follows the scale of the string network then a larger value of Ω requires more length of string per cell. In fact, since Ω scales as the string step-length, the length per cell goes as $\sim \Omega^3$. This appears in the numerator of eq.(3.4) so the choice of $\Omega = 2\pi$ in eq.(4.1) truly would give an upper bound to f_a. Another class of uncertainties going into eq.(4.1) have to do with the nature of the quark-hadron phase transition. These were considered in refs.[6-9]. It suffices to say that the effects considered tend to strengthen the bound, except for ref.[9], which used the coherence of the zero-momentum mode to argue that the bound may be weakened. Since cosmic string axions are not coherent this argument should not apply.

Because there appears to be some confusion, I would like to make some remarks about the effect of inflation on the cosmological bound for f_a. First, cosmological difficulties with the zero-momentum mode of the axion field are most serious without inflation. This is because the horizon at the quark-hadron phase transition contained many, many regions which were causally disconnected at the PQ transition, but were brought together in the subsequent history of the universe. Causality requires us to suppose that the value of the zero-momentum mode was assigned randomly over all of these regions, and to imagine that all of these combined to result in a value which is zero to great precision is very difficult. Furthermore, if this did occur, then the axion mass turn-on would have been safe at the quark-hadron transition, but it would have had to have happened in nearly every region in the present horizon that was disconnected at then. Thus we can only take $a_0 \sim o(f_a)$ at the quark-hadron transition. As long as $f_a \lesssim 4 \times 10^{12}$Gev then normal damping effects in an expanding universe are sufficient to render this mode of the axion harmless. Note that the bound is saturated for $\rho_a = \rho_{cr}$, the closure density of the universe. In the absence of inflation we have no reason to insist on a flat universe; it is more reasonable to assume that there is no more mass than is consistent with observations. This is $\rho/\rho_{cr} \approx .4 - .1$, and the bound on f_a is correspondingly tightened.

Inflation ameliorates the axion problem because everything in the present horizon was causally connected at the PQ transition, so a_0 could have been close to zero in that tiny region which has become the present universe. In this case the PQ transition could have been at the GUT scale. To many people, though, it seems very unnatural for our universe to be singled out as a region where $a_0 = 0$ when it could only have happened by chance. To these skeptics it is more comfortable to suppose that $a_0 \sim o(f_a)$ at the quark-hadron transition, with or without inflation, obtaining once again $f_a \lesssim 4 \times 10^{12}$Gev. If so, the post-inflationary reheating required by GUT baryosynthesis brings the temperature above f_a. The universe then cools normally, so except for the fact that inflation requires $\rho/\rho_{crit} = 1$ the effects of inflation are irrelevant.

This letter shows that axion strings strengthen the bound when there is no inflation. With inflation it is possible remove the strings from the observable universe, and as long as the post inflationary reheating does not bring the universe above $T_{PQ} = f_{PQ}$ then the result presented here does not apply. Suppose f_{PQ} satisfies the zero-momentum cosmological bound. Since GUT baryosynthesis requires reheating to $\sim 10^{14}$Gev strings are unavoidable in these models. It is only possible to get rid of strings if baryon number is generated below T_{PQ}.

Besides solving the strong CP problem, axions have been proposed as the missing mass of the universe. Efforts have been undertaken to detect these axions exploiting the coherence of the zero-momentum mode. This letter indicates that the cosmic axion field is dominated by incoherent axions, so these efforts may be in vain. On the other hand, if the axion were discovered by some other method then the tightness of the bound implies it is more likely to contribute significantly to the density of the universe.[7]

In summary, I cannot argue that the assumptions going into (4.1) are anything but reasonable. To be certain of this bound would require detailed knowledge of the evolution of the axion string system. Though computer simulations have been done in the case of gauge strings, to my knowledge no one has attempted similar calculations for strings from global symmetries. The problem is inherently more difficult because the global strings have long range interactions. What this letter suggests unambiguously is that is that axions from strings are of cosmological importance. To go further, and accept the bound (4.1), would leave scant room for the axion to exist.

APPENDIX

The WKB Approximation for Non-Zero Momentum Modes

We want to know the enhancement an energy density initially given by

$$\rho = \int f(k)dk, \tag{A1}$$

when the axion mass turns on.

For an axion mode which at some reference time t_1, before the mass turn-on, has the form $a = A(t)\cos kx$, the equation of motion is

$$\frac{d^2}{dt^2}A + [\frac{t_1}{t}k^2 + m^2(t)]A + 3H(t)\frac{d}{dt}A = 0.$$

H is the Hubble constant and m is the time dependent axion mass. $\tilde{t}$ and $\tilde{m}$ are defined by the equation

$$\tilde{t} = 1/m(\tilde{t}) = 1/\tilde{m}.$$

This is roughly the time at which the axion Compton wavelength crosses the horizon. The axion mass starts at zero for $t \ll \tilde{t}$ and rises to its final value m_a for $t \gg \tilde{t}$. If $\frac{1}{m}\frac{dm}{dt} < m$ and $H < m$ then the adiabatic condition is met. Since $m(t) \propto t^2$ (neglecting a logarithmic factor), the time that marks the onset of the adiabatic regime is just $2\tilde{t}$. Let us take $t_1 = \tilde{t}$. The solution is then

$$A(t) \approx \bar{A}(t)\cos\int^t \sqrt{\frac{\tilde{t}}{\tau}k^2 + m^2(\tau)d\tau},$$

with the condition that the number of axions per comoving volume is constant. Thus

$$\sqrt{\frac{\tilde{t}}{t}k^2 + m_a^2}\bar{A}(t)^2 = \sqrt{k^2 + \tilde{m}^2}\tilde{\bar{A}}^2(\tilde{R}/R(t))^3,$$

and the energy density of this mode is

$$\frac{1}{2}(\frac{\tilde{t}}{t}k^2 + m_a^2)\bar{A}(t)^2 = \left\{\sqrt{\frac{(k^2 + \frac{t}{\tilde{t}}m_a^2)}{(k^2 + \tilde{m}^2)}}\right\}(\tilde{t}/t)^2 \times \frac{1}{2}(k^2 + \tilde{m}^2)\tilde{\bar{A}}^2.$$

Finally, since $(\tilde{t}/t)^2 \times \frac{1}{2}(k^2 + \tilde{m}^2)\tilde{\bar{A}}^2$ is the energy density of the mode at $\tilde{t}$, and the spectrum at this time is eq.(A1), we can simply insert the term in braces into the integral to obtain

$$\rho = \frac{1}{t^2} \int \sqrt{\frac{(k^2 + \frac{t}{t} m_a^2)}{(k^2 + \widetilde{m}^2)}} f(k) dk. \tag{A2}$$

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