RECHARGING LARGE CAPACITOR BANKS

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The power bill for a large linac such as that proposed for the AGS conversion would be in the order of $100,000 per year. This is for an 0.6 msec pulse length and 30 pps, and an 8,000 hour year. It also assumes fairly high efficiency in the rf power stages. A typical "meson factory" would involve a power bill ten times as great.

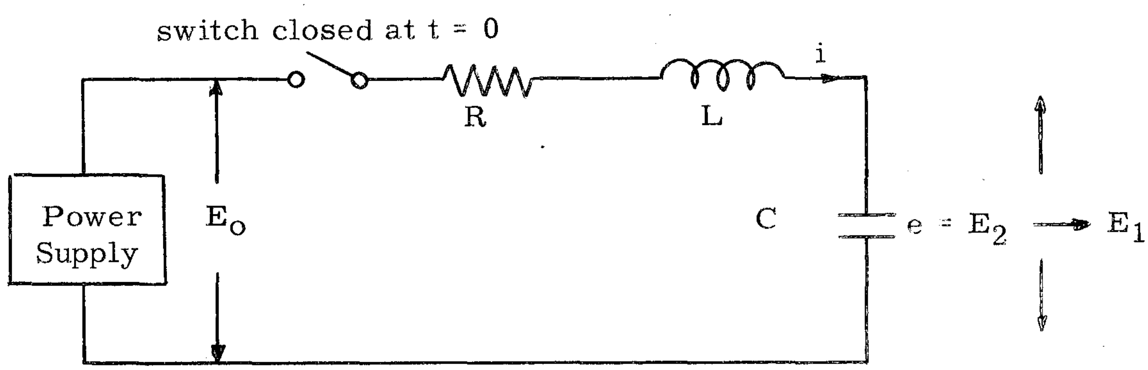
A simple method of recharging a modulator condenser bank or storage line from zero volts to a given voltage would be to use a standard dc power supply and a series resistance to limit the current. Such a circuit would dissipate as much energy in the resistance as it supplied to the condenser bank, and the power bill would then be doubled.

Clearly then it is very worthwhile to design condenser recharging circuits with view to achieving a high recharge efficiency, any improvement of a few per cent or more will be significant.

The modulator energy storage system usually takes the form either of a delay line or a single capacitor bank. After considering the alternatives it was decided that the single capacitor bank is the most suitable alternative when (a) the pulse length was too long to permit the use of a pulse transformer, and (b) it was necessary to modulate the dc plate voltage on the rf amplifier in order to control the rf amplitude in the linac tank, (c) the rf power amplifier was some sort of triode with a dc plate voltage around 30 kV. The reasons for this choice will not be enumerated here. Further practical considerations show that the optimum choice of capacitor bank size is not one which stores twice the energy delivered to the load, although this is the smallest bank in terms of energy stored. A capacitor bank that droops about 10% during the discharge, and stores about five times the energy delivered to the load is a better choice.

Recharge Analysis

The basic circuit upon which the analysis has been made is shown in the circuit diagram:



Circuit Diagram

The condenser is discharged by the load circuit (not shown) to voltage E_2. The recharge is effected by closing the switch at time $t = 0$. A variable current i flows and the condenser voltage e rises from E_2 at $t = 0$ to E_1 at $t = T$. At time T the current flow is interrupted either by opening the switch, or by some other means. The power supply voltage is E_o, and the energy dissipated in the circuit from time 0 to time T is U_R.

Arrangement of formula

In order to show that the analysis is dependent only upon the relative values of L, C and R, the results have been normalized. The capacitor current i becomes a dimensionless quantity

$$n_i = \frac{i}{(E_1 - E_2)} \frac{T}{C} ,$$

the capacitor voltage e becomes

$$n_e = \frac{e - E_2}{E_1 - E_2}$$

and the energy dissipated in the resistance U_R becomes

$$N_R = \frac{U_R}{(E_o - E_1)^2} \quad \frac{2}{C} \quad .$$

The efficiency of the recharging circuit can be obtaine directly from N_R since

$$\frac{\text{Energy dissipated}}{\text{Energy supplied to capacitor}} = N_R \frac{E_1 - E_2}{E_1 + E_2}$$

Thus for a capacitor with 10% voltage droop during discharge, an N_R value of 1 will give a 5% loss.

The circuit has been analyzed for the values of energy dissipated in R (N_R), the maximum capacitor voltage (N_C), the peak charging current (N_m), and the charging current at the end of the recharge period (N_T). The latter current is significant since it determines the rate of change of condenser voltage at switchoff, and hence the time tolerance requirements for a given voltage accuracy.

The analysis falls naturally into two cases, one when Q is greater than one-half, and the other when it is less. In the Q > 1/2 case, were it not for the power supply rectifiers, the capacitor voltage would be a damped periodic function of time; however, the decrease in capacitor voltage after the first peak is accompanied by a reversal of the charging current, and the rectifiers cut off at this point leaving the capacitor charged to the peak voltage. Thus, if the circuit is not interrupted by the switch, the recharge period will be one-half of the ringing period, the current at this time will be zero, and the capacitor voltage will be greater than (or in the limit equal to) the recharge voltage. The formula relating to the Q > 1/2 case assumes that the charge is terminated by current reversal at the end of the first half cycle, and shown in the first summary table.

SUMMARY

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} > \frac{1}{2}$$

$$\frac{\pi}{T} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad P = \sqrt{\frac{4L}{R^2C} - 1} = \sqrt{4Q^2 - 1}$$

$$n_i = \frac{\pi}{N_c}\left(\frac{1}{P^2} + 1\right) e^{\frac{-\pi}{P}\frac{t}{T}} \sin\frac{\pi t}{T}$$

$$i = \frac{(E_1 - E_2)C}{T} n_i \qquad N_T \equiv 0$$

$$n_e = \frac{1}{N_c}\left(1 - e^{\frac{-\pi}{P}\frac{t}{T}}\left(\cos\frac{\pi t}{T} + \frac{1}{P}\sin\frac{\pi t}{T}\right)\right) = \frac{e - E_2}{E_1 - E_2}$$

$$N_c = \frac{E_1 - E_2}{E_o - E_2} = 1 + e^{\frac{-\pi}{P}}$$

$$N_R = \frac{1}{N_c^2}\left(1 - e^{\frac{-2\pi}{P}}\right)$$

$$U_R = \frac{(E_1 - E_2)^2 C}{2} N_R$$

Here P is a function of Q, and T is defined as half the natural periodicity of the circuit.

N_T, the value of n_i at time T is zero. N_c is the recharge voltage ratio. Both current and voltage are damped trigonometrical functions of time. N_R is a function of P (and hence Q) only.

In the Q $<$ 1/2 case the capacitor voltage rises asymptotically towards the recharging voltage, so the recharge period is assumed to be terminated after time T by opening the switch. Thus there is a finite current flowing at time T and the final capacitor voltage is always less than the recharge voltage. The formula relating to the Q $<$ 1/2 case is shown in the second summary table.

In this case the recharge period T is not uniquely determined by Q and C as in the Q $>$ 1/2 case, so the period has to be specified. It has been normalized (somewhat arbitrarily) as the quantity $V = \frac{T}{RC}$. S is a function of Q, chosen so as to avoid the use of complex notation. The other symbols used are similar to the Q $>$ 1/2 case. N_T may be found by putting t = T in the n_i formula.

The current and voltage are exponential functions of time, and N_R is a function of Q and T.

Recharging Power Supply

A practical power supply would not deliver a smooth dc voltage as assumed in the analysis, but a rectified ac voltage. However, such a supply would be at least six phase and the ripple would be less than 15%, so results calculated for the dc case would be reasonably accurate.

The power supply would also have a finite internal impedance (generally inductive) and this impedance will, of course, add to any external resistance and inductance in the circuit.

The exact relationship of the power supply transformer leakage inductance and resistance to those values found as source impedance in a complete multiphase power supply is not clear, but it is probable that a resistance that would account for the supply dc regulation in series with the leakage inductance per rectifier phase would give the correct value of source impedance.

SUMMARY

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} < \frac{1}{2}$$

$$V = \frac{T}{RC} \qquad\qquad S = \sqrt{1 - \frac{4L}{R^2C}} = \sqrt{1 - 4Q^2}$$

$$n_i = \frac{V}{N_c S}\left(e^{\frac{-2V}{(1+S)T}t} - e^{\frac{-2V}{(1-S)T}t}\right)$$

$$i = \frac{(E_1 - E_2)C}{T}\, n_i$$

$$n_e = \frac{1}{N_c}\left[1 + \left(\frac{1}{S} - 1\right)\frac{e^{\frac{-2V}{(1-S)T}t}}{2} - \left(\frac{1}{S} + 1\right)\frac{e^{\frac{-2V}{(1+S)T}t}}{2}\right] = \frac{e - E_2}{E_1 - E_2}$$

$$N_c = \frac{E_1 - E_2}{E_o - E_2} = \left[1 + \left(\frac{1}{S} - 1\right)\frac{e^{\frac{-2V}{(1-S)}}}{2} - \left(\frac{1}{S} + 1\right)\frac{e^{\frac{-2V}{(1+S)}}}{2}\right]$$

$$N_R = \frac{1}{N_c^2 S^2}\left[(1 - S^2)\, e^{\frac{-4V}{(1-S^2)}} - (1 + S)\frac{e^{\frac{-4V}{(1+S)}}}{2} - (1 - S)\frac{e^{\frac{-4V}{(1-S)}}}{2} + S^2\right]$$

$$U_R = \frac{(E_1 - E_2)^2 C}{2}\, N_R$$

Evaluation of Results

Since the evaluation of formula by inspection is a difficult mental exercise, it was decided to program the Brookhaven computer to plot out the results. Copies of these results are shown in the next few diagrams.

Figure 1 shows the variation of current and voltage with time for Q values of 0.7 and 5. The $Q = 5$ case shows the current as a sine curve and the voltage as a cosine curve. The $Q = 0.7$ case shows heavily damped versions of the same curves. Figure 2 shows N_c, the recharge voltage ratio; N_m, the peak to average current ratio; and N_R, the loss factor; varying with Q for the $Q > 1/2$ case.

For the $Q < 1/2$ case, Figures 3, 4 and 5 show the variation of current with time. They show Q values of 0.5 and 0.12, and T/RC values of 0.5, 1 and 3. Note that the definition of T is quite different from that in the $Q > 1/2$ case, so the values of $\frac{t}{T}$ in the two cases have no correspondence. It will be seen that as the recharge period $\frac{T}{RC}$ is made shorter, the $Q = 0.12$ curve shapes approach the simple RC exponentials.

For the $Q < 1/2$ case the values of N_c, N_T, N_R and N_m are shown as functions of T/RC rather than Q, since the recharge period is the more significant parameter. Figure 6 shows N_c and N_T, and Figure 7 shows N_R and N_m.

It may be seen from the plots that the dissipation value N_R is always greater than one for $Q < 1/2$, and less than one for $Q > 1/2$. Two further points may be seen in the $Q > 1/2$ case; with increasing Q the dissipation decreases toward zero, and the maximum current during the recharge period I_m decreases toward $\pi/2$. This would seem to indicate that the highest attainable Q is the ideal situation. However, with high Q values the N_C ratio tends toward 2, and this leads to an unstable recharge voltage in that when the capacitor bank is discharged to an unusually low voltage (e.g. because of a partial breakdown) then the bank is recharged to a higher voltage than normal. In the extreme case consider a bank which normally has a 10% droop and would require a recharge voltage E_o of about 95% of the maximum capacitor voltage E_1. If such a circuit were discharged to zero voltage then the capacitor bank would theoretically recharge to 1.9 times the normal maximum voltage, which could be very dangerous.

A practical compromise would seem to be a circuit designed to give a Q a little above 0.5, say 0.75. This would give a dissipation value of 0.9, which means a power loss of 4.7% during the recharge of a 10% droop;

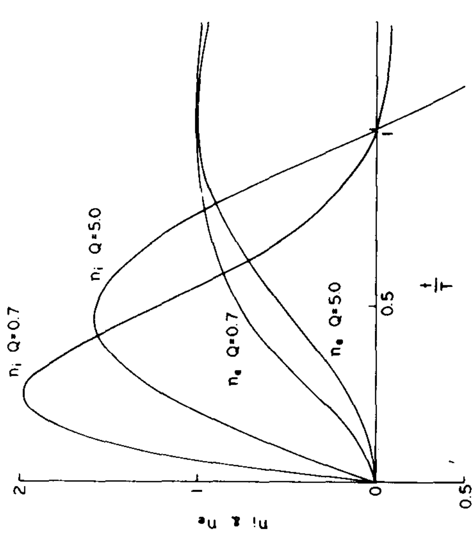


FIG. 1

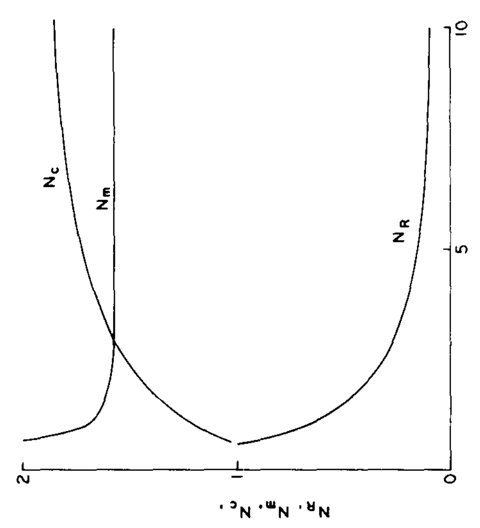


FIG. 2

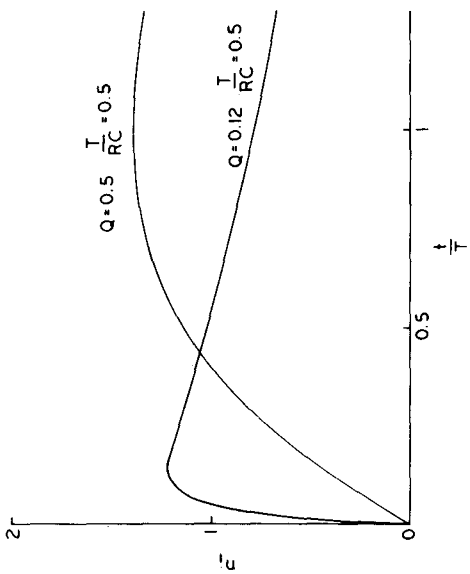


FIG. 3

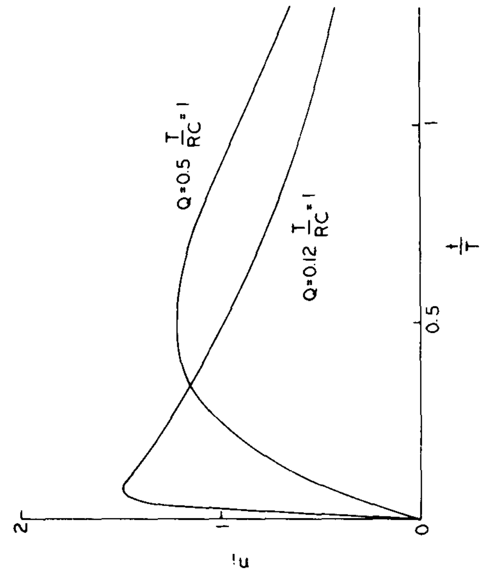


FIG. 4

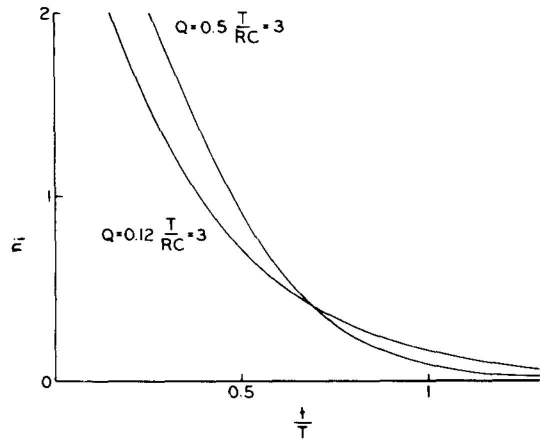


FIG. 5

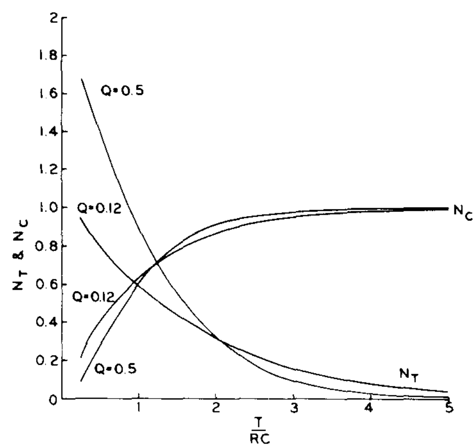


FIG. 6

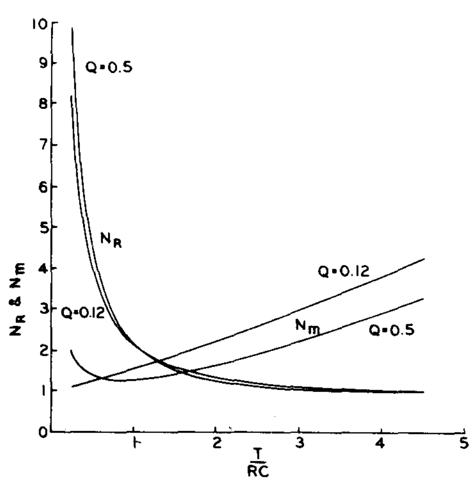


FIG. 7

and an N_C ratio of 1.06, so under conditions of complete discharge the condenser voltage would rise only 6% higher than normal. The maximum current for such a circuit would be 1.9 times the average current and the circuit would automatically cut off at zero current when the recharge is finished. Although the danger of over-volting the capacitors is now removed, there would still be the problem of high recharge current in the event of recharging from zero voltage, in which case the maximum current would tend to rise to 19 times the normal current. The recharge control equipment would have to have some provision for reducing such current in the event of a flashover or crowbar, and also at start-up times.

There seems to be no advantage in using the $Q < 1/2$ case, except on the grounds of simplicity, or perhaps to obtain a lower maximum recharge current at the expense of greater losses.

It is of interest to note that for the case of $Q < 1/2$ and $T/RC < 1$, N_R increases with increasing Q. This is because when the switch is opened with a finite current flowing, then the LI^2 energy in the inductance at that time is dissipated (presumably in the switching arc) and thus is lost to the circuit. When T/RC is less than one, the ratio of energy stored in the inductance to that dissipated in the resistance is such that increasing the Q also increases the total power loss.

Variable Resistance Recharging

Mr. J. F. Sheehan of Yale University has suggested a very interesting alternative to the LR current limiting network. This would replace the LR combination with a variable R, automatically controlled to keep the recharge current constant. The variable resistance would take the form of a hard tube.

The most useful case of this circuit would be when the recharge voltage E_o is equal to the maximum voltage E_1, and so the minimum variable resistance value is zero. This gives a dissipation value N_R of 1.0, so the power loss would be similar to the $Q = 0.75$ case outlined above. The maximum current ratio N_m is now the ideal value of 1.0, the N_C ratio is 1.0 and the current at the end of the charge is zero since there is no way for the capacitor voltage to rise above the recharge voltage.

Provided the current controlling action is automatic, the only effect of recharging a fully discharged bank would be to prolong the recharge period. The main disadvantage would be the complexity of such a circuit as compared with a passive LR combination.

Continuously Recharged Capacitor Systems

Some consideration has been given to the case which is often used; when recharge current is not zero at the start of the recharge period. This would occur if the switch was omitted in the $Q < 1/2$ case, or if the discharge repetition rate was greater than twice the ringing frequency in the $Q > 1/2$ case. An analysis of the recharge current in the latter case has been carried out.

The arrangement presents the difficulty that each recharge cycle depends upon the previous one, and there is no obvious indication as to how the current at the start of each recharge period will converge upon a steady value. Furthermore, this steady state will be upset by any irregularity in the pulsing rate. It is considered then that unless used with a $Q < 1/2$ and a large value of T/RC, in which case the initial recharge current will be very small, the continuous recharge system is likely to give less stable operation than one in which the recharge current is zero at the start of the recharge cycle. There would seem to be no real advantage of a continuous recharge circuit over one in which Q is a little greater than 1/2, and the recharge terminated by current reversal.

Conclusions

The optimum design of a circuit for recharging condenser banks of the type described above, would be one with a Q value about 0.75. If an active circuit is acceptable, then a constant current circuit would be preferable. It seems that a continuously recharged system would be less stable than one in which the recharge time is controlled.

WHEELER: If you look at the figure here, $I_m = 1.9$, and make a quick estimate of the peak power demand for a large installation, it turns out to be of the order of 60 MW for the instantaneous demand. This is reflected into the power line and the power companies will not be very happy about it. For this reason alone I think it may be necessary to go to a constant current-charging system.