Empirical p-n interactions, the synchronized filling of Nilsson orbitals, and emergent collectivity

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Abstract. The onset of collectivity and deformation, changes to the single particle energies and magic numbers and so on are strongly influenced by, for example, proton (p) and neutron (n) interactions inside atomic nuclei. Experimentally, using binding energies (or masses), one can extract an average p-n interaction between the last two protons and the last two neutrons, called δV_{pn}. We have studied δV_{pn} values using calculations of spatial overlaps between p and n Nilsson orbitals, considering different deformations, for the Z= 50-82, N= 82-126 shells, and comparison of these theoretical results with experimental δV_{pn} values. Our results show that enhanced *valence* p-n interactions are closely correlated with the development of collectivity, shape changes, and the saturation of deformation in nuclei. We note that the difference of the Nilsson quantum numbers of the last filled Nilsson p and n orbitals, has a special relation, 0[110], in which they differ by only a single quantum in the z–direction, for those nuclei where δV_{pn} is largest for each Z in medium mass and heavy nuclei. The synchronised filling of such orbital pairs correlates with the emergence of collectivity.

1. Introduction and empirical proton-neutron (p-n) interactions

Magic numbers, the onset of deformation, configuration mixing, single particle energies etc can be interpreted in terms of proton(p)-neutron(n) interactions. Heavy nuclei with many possible interactions between nucleons are more complicated than lighter nuclei. Thus, to simplify this, it is often easier to consider only (number of) valence nucleons outside of a closed shell, which is a good approximation to gain an idea about the origins of the structure of nuclei. In these proceedings, empirical p-n interactions for heavy nuclei will mainly be discussed by considering only the number of valence nucleons. Before this, how one can extract empirical p-n interactions using masses (binding energies) will briefly be explained.

Equation (1), a double difference of binding energies, filters most of the interactions between nucleons and gives an average p-n interaction between the last two protons and the last two neutrons, called δV_{pn}. Its interpretation close to closed shells is obtained by looking at the Shell Model orbits for the unpaired last proton and the last neutron, and the overlap between their wave functions by considering their shell model quantum numbers, nl_j. For more details see Refs. [1,2].

$$\left| \delta V_{pn}(Z,N) \right| = 1/4\ [\ ((B(Z,N) - B(Z,N-2)) - (B(Z-2,N) - B(Z-2,N-2))\ ] \qquad (1)$$

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where B is the binding energy. To interpret δV_{pn} for deformed nuclei (near mid-shell), one has to take into account the deformed shell model, that is, Nilsson orbitals (see below).

2. The synchronized filling of Nilsson orbitals, and emergent collectivity

Figure 1 shows experimental δV_{pn} values against neutron number for light nuclei (left) and heavy nuclei (middle) and a color coded $R_{4/2}$ plot (right) for the Z=50-82 and N=82-126 shells. It is clearly seen that δV_{pn} has spikes at N=Z in the left panel where both proton and neutron orbits fill exactly the same orbits. These spikes with large p-n interactions have been interpreted [3] by Wigner's SU(4) spin-isospin symmetry [4]. Such a large p-n interaction is not expected in heavy nuclei due to the Coulomb force and spin-orbit force which brings unique parity orbits into each shell. However, although p-n interactions in heavy nuclei are reduced in magnitude, one can still see large p-n interactions at the same (or similar) number of valence protons (Z_{val}) and valence neutrons (N_{val}). One example is ^{168}Er which has 18 valence protons and 18 valence neutrons and it has a maximum in δV_{pn} in the N~100 region (see Refs. [5,6] for details and for more examples). Figure 1 (middle) has some sharp zig-zags. These comprise nuclei with both $N_{val} = Z_{val}$ and odd-Z, odd-N values due to the absence of pairing effects on the last nucleons. One example here is N=97 (N_{val}=15) for $_{65}$Tb (Z_{val}=15). The average p-n interaction has a peak when $N_{val} = Z_{val}$. Similar examples are also seen for Ho and other nuclei. The right panel of fig. 1 shows the locus of the maximum δV_{pn} values shown with white circles on an Z-N contour plot for $R_{4/2}$ which indicates the changes in structure from spherical (blue) through deformed (red). The black line is placed to indicate $N_{val} = Z_{val}$. As seen in this plot, the nuclei for which we have peaks in the experimental δV_{pn} values, lie where the saturation of collectivity starts so these maxima in δV_{pn} have a clear link to the evolution of collectivity [6].

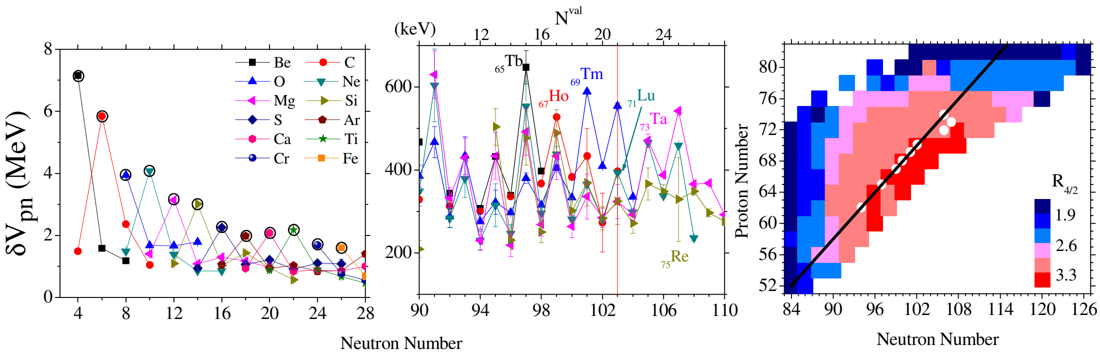

Figure 1. Experimental δV_{pn} values for light nuclei (Left) and heavy nuclei (Middle). (Right): Color coded $R_{4/2}$ values. White circles are shown for the nuclei for which there is a maximum in δV_{pn} in the middle panel. Based on Refs. [6,7].

As mentioned above, for these heavy nuclei, Nilsson orbitals, $K[N,n_z,\Lambda]$, are considered for the interpretation. Quantum numbers for a given Nilsson orbital are K projection of the total angular momentum, N oscillator quantum number, n_z number of quanta in the z-direction and Λ projection of the orbital angular momentum. Reference [5] points out that large δV_{pn} values with known data in heavy nuclei appear when neutron and proton Nilsson orbital quantum number differences $\delta K[\delta N, \delta n_z, \delta\Lambda]$= 0[110]. In the same example as above, ^{168}Er is filling the 7/2[523] proton orbital and 7/2[633] neutron Nilsson orbitals. The difference between these orbitals is 0[110]. This occurs for almost all of these nuclei with the largest δV_{pn} values [5,6]. This 0[110] combination is special since proton and neutron Nilsson orbitals differ by only a single quantum in the z –direction.

The most important point is that this special 0[110] relation of proton and neutron Nilsson orbitals is (this has not been explicitly noted before) satisfied for each pair of orbits near the beginning of the shell as proton and neutron orbitals fill sequentially. This can be seen in fig. 2 where Nilsson orbitals are shown for protons (left) in the Z =50-82 shell and neutrons (right) in the N =82-126 shell. Since

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the overall shapes or patterns of the Nilsson diagrams are so similar this sequential filling of highly overlapping pairs of orbitals satisfying 0[110] gives large δV_{pn} values (see fig. 1 (middle)) along the line corresponding to saturation in $R_{4/2}$ (right) and following $N_{val} \sim Z_{val}$ line. even as the deformation increases. As a result, this synchronized filling of 0[110] relation is directly related to onset of deformation.

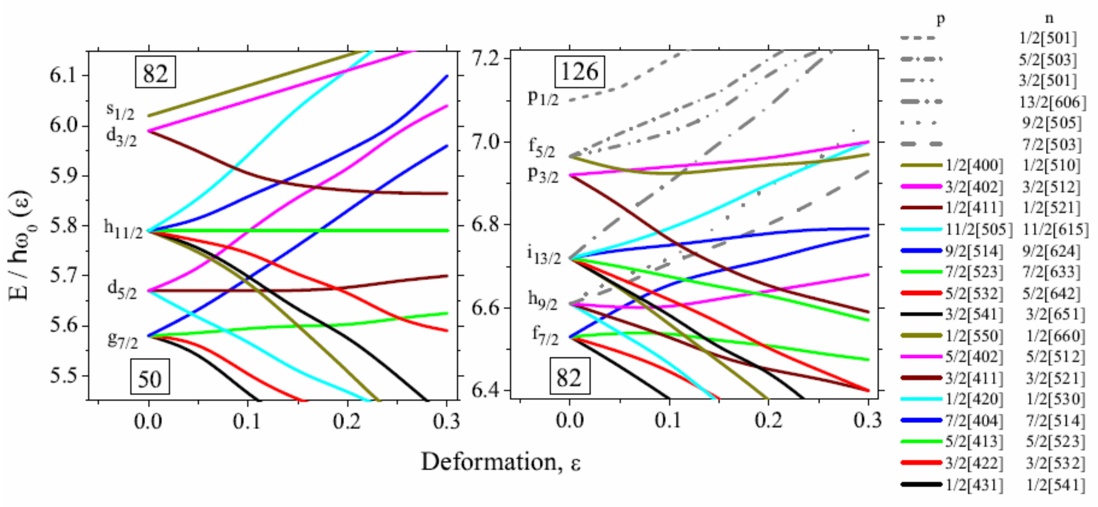

Figure 2. Nilsson diagrams for the proton Z= 50-82 and neutron N= 82-126 shells. Based on Refs. [6,7].

In order to understand these interactions in a simple way, we calculated [6] spatial overlaps of proton and neutron Nilsson wave functions with three deformations, namely ε= 0.05, 0.22, 0.3 [6,7]. The calculated overlaps are compared with the experimental δV_{pn} results in fig. 3. The left panel is experimental δV_{pn} values, theoretical spatial overlaps for the nuclei where the data exist are shown in the middle panel. All the calculated nuclei are illustrated in the right panel of fig. 3.

Since the proton shell has 32 nucleons and the neutron shell 44 nucleons, the figures are designed as square. Thus, along the diagonal line in fig. 3, nuclei will fill similar orbitals so large overlaps (large δV_{pn}) are expected along this line. The upper black line corresponds to $N_{val} = Z_{val}$ nuclei.

Overall agreement between the data and theory is very good. There are some disagreements such as in the upper right in fig. 3. However, this is not surprising because this disagreement may happen due to a γ-soft structure that our calculations do not take into account [7]. In addition, we obtain large δV_{pn} values with a 0[101] relation (not 0[110]) around Z~ 52-64 with N~ 92-108 (5/2[413] p - 5/2[512] n and 1/2[420] p - 1/2[521] n orbitals). Mass measurements in this region will help us to understand the p-n interactions more, in particular in the future at facilities such as FAIR or FRIB.

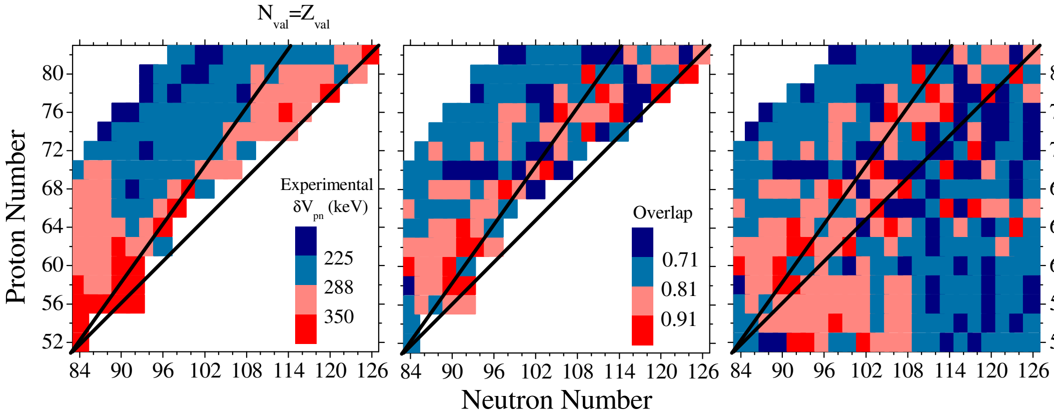

Figure 3. (Left): Color coded empirical δV_{pn} values for the Z= 50-82 and N= 82-126 shells. Large values have redder colors. (Middle): Similar as (Left) but for calculated overlaps for nuclei where empirical values of δV_{pn} are known. (Right): Calculated overlaps for the full major shells (excluding the nuclei beyond the proton dripline at far upper left). (see text for details). Based on Ref. [6,7].

3. Conclusion

Double differences of binding energies, δV_{pn}, extracted from high-precision mass measurements, are discussed from both experimental and theoretical perspectives by focusing on the Z= 50-82, N= 82-126 shells. Reflecting the clear spikes in δV_{pn} for light nuclei, studies of δV_{pn} for heavy nuclei are discussed in terms of the number of valence nucleons. We obtained a systematic behavior (large δV_{pn} values) when a nucleus has equal (or almost equal) numbers of valence protons and neutrons and noticed a consistent difference between proton and neutron Nilsson orbital quantum numbers, namely 0[110]. Calculated spatial overlaps between proton and neutron Nilsson orbitals are presented and compared with the experimental results. The sequential filling of orbitals with the 0[110] relation for equal numbers of valence protons and neutrons leads to a maximizing in the growth of collectivity along the $N_{val} \sim Z_{val}$ line.

4. References

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