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# FERMION MASSES IN THE STANDARD MODEL[⋆,◇]

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ABSTRACT

We consider a proposal that the masses of fermions in the Standard Model are determined by dynamical symmetry breaking. Rather than being introduced as arbitrary parameters in the Lagrangian, they are determined self-consistently by the requirement that the proper self-energy vanish on the fermion mass shell. We find in the one-loop approximation that it is possible to generate a heavy top quark mass dynamically, while the other fermions remain massless.

In Memoriam M. A. B. Bég

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We address the question of fermion masses in the Standard Model of quarks, leptons, gauge bosons, and elementary Higgs. The many successes of the Standard Model are accompanied with an appreciation that it also contains a large number of arbitrary constants—in particular, the three masses of the charged quarks and lepton in each of the three generations. At a deeper level one puzzles at the origin of the fermion masses which break the chiral gauge symmetry and for which no compelling—or even physically attractive—origin has been proposed analogous to the dynamical symmetry breaking leading to the masses of the $W^{\pm}$ and $Z^{\circ}$ gauge bosons.

In this paper we look into possible origins of the finite fermion masses starting with the assumption that the Standard Model Lagrangian, $\mathcal{L}_{SM}$, is an effective Lagrangian for sub-TeV physics. We use the Higgs mechanism to give the $W^{\pm}$ and $Z^{\circ}$ their observed masses and treat the Higgs as an elementary scalar for this study.

The simplest way to formulate our calculation is to assert that the fermion masses $m_f$ that appear in $\mathcal{L}_{SM}$ are not arbitrary parameters but are the *physical* masses themselves and their values are determined by the underlying dynamics. This means that the proper self-energy of the fermions vanishes on the mass shell:

$$\Sigma(m_f, Q_f) \ = \ 0 \ . \tag{1}$$

The fermion masses appearing in $\mathcal{L}_{SM}$ are thus not renormalized. Their values are determined by the solutions to Eq. 1 which also depend on the fermion charges $Q_f$.

In our calculations we apply Eq. 1 in the one-loop approximation to the third generation fermions $(\tau, t, b)$ to relate their masses to known $m_W$ and $m_Z$ for a range of masses of the Higgs, $m_H$, and of the effective cutoff $\Lambda \sim 1$ TeV. The neutrino mass is automatically zero since there is no right-handed neutrino in the Glashow-Salam-Weinberg model. We shall initially apply this approach to calculate masses of the third generation fermions because these masses are in the several to multi-GeV region and less sensitive to (unknown) mixing parameters and to any other smaller corrections on the scale of MeVs. The difference between current and constituent quark masses ($\sim 300$ MeV) is also ignored along with all contributions from QCD.

At a deeper level one can motivate this calculation in terms of dynamical symmetry breaking of the underlying chirally invariant gauge theory by applying self-consistency arguments. We start by assuming that the Higgs mechanism applies but that the basic Lagrangian remains chirally invariant. Denote this Lagrangian by $\mathcal{L}^{\circ}_{SM}$, i.e., the standard model, lacking the Yukawa terms, $\mathcal{L}_Y$, that couple right- and left-handed fermions and give rise to the fermion masses via the Higgs mechanism.

Due to the explicit chiral symmetry of $\mathcal{L}^{\circ}_{SM}$, the Yukawa couplings, as well as the fermion masses, will not be generated perturbatively. They may, however, be generated nonperturbatively if the chiral symmetry is dynamically broken. The self-consistency argument[1] we use goes as follows:

Suppose that the physical Yukawa couplings $\mathcal{L}_{Y,phys}$ are generated dynamically, giving rise to physical fermion masses, $m_f$. The Lagrangian

$$\mathcal{L} \ = \ \mathcal{L}^{\circ}_{SM} + \mathcal{L}_{Y,phys} \tag{2}$$

then constitutes an effective Lagrangian and provides a basis for perturbative calculation in terms of the physical, observed fermion masses. In the usual perturbative

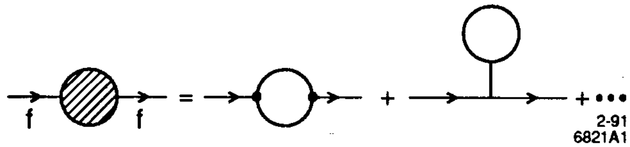


Fig. 1. Proper self-energy of the fermion. The unlabelled lines include all particles coupling to fermion $f$ in the self-energy bubble, and all particles coupling to the Higgs in the tadpole.

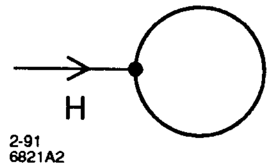


Fig. 2. Tadpole contribution to the vacuum expectation value, $v$, of the Higgs field. The bubble includes all particles that couple to the Higgs.

approach, if one studies $\mathcal{L}$ beyond the tree approximation, one will in general find self-mass corrections from loop diagrams as in Fig. 1. The amplitudes corresponding to Fig. 1 will correct the fermion propagator, leading to

$$m_f \rightarrow m_f + \Sigma(p)|_{m_f} \neq m_f \, . \tag{3}$$

Here, since we assume that $m_f$ is the physical fermion mass, we must impose the condition Eq. 1. Equation 1 is the basic equation for determining the fermion masses.

In either of its formulations, Eq. 1 is not a new idea to quantum field theory. However, it cannot be applied to generate mass in pure (Abelian) quantum-electrodynamics for which the second order self-energy contribution is

$$\Sigma(m) = \frac{3\alpha}{4\pi} \, m \left[ \ell n \, \frac{\Lambda^2}{m^2} + \mathcal{O}(1) \right] \, . \tag{4}$$

The unique solution to Eq. 1 based on Eq. 4 is $m = 0$.

As long ago as 1939, in the early days of QED, Stückelberg[2] conjectured that the self-energy of a Dirac electron could be made finite without disturbing its experimental successes if there were another short-range interaction that could cancel the logarithmic cutoff dependence of Eq. 4. Extensive studies along this line were pursued, initially by Pais and Sakata and Hara.[3] In the approach we adopt in this work, we find that the Standard Model, with quarks and leptons interacting via the electroweak forces, admits $m \neq 0$ solutions to Eq. 1, in contrast to QED. However, these solutions depend quadratically as well as logarithmically on the cutoff. Working in the one-loop approximation we find solutions that are insensitive to the actual value of the cutoff for $m < \Lambda \sim$ TeV.

The tadpole graphs in Fig. 1 must be included along with the self-energy bubble in a gauge invariant calculation of $\Sigma(m)$. However, these tadpoles also renormalize the vacuum expectation value of the Higgs field, $v$, corresponding to the graph in Fig. 2.

By itself Fig. 2 is not gauge invariant; gauge dependence appears in the parts that depend logarithmically on $\Lambda$, but not quadratically. We may choose the gauge so that the tadpole vanishes, thereby preserving the vacuum expectation value of the Higgs field, $v$, in the presence of dynamical symmetry breaking, or we may choose to renormalize $v$ in treating the Higgs. In either case, Eq. 1 is a gauge independent consistency condition and in the one-loop approximation gives rise to the following equations for fermions of flavor $f$ (including the leptons):

$$\begin{aligned} \Sigma(m_f, Q_f) &= -m_f \frac{\alpha}{4\pi} \frac{1}{\sin^2\theta_W} F_f(m_{f_i}, Q_f, m_H, \Lambda) \\ &= 0 \end{aligned} \tag{5}$$

with

$$\begin{aligned} &F_f(m_{f_i}, Q_f, m_H, \Lambda) \\ &= \left(\frac{3}{4} + \frac{3}{2}\frac{m_W^2}{m_H^2} + \frac{3}{4}\frac{m_Z^2}{m_H^2} - \sum_{f_i}\frac{m_{f_i}^2}{m_H^2}\right)\frac{\Lambda^2}{m_W^2} + \sum_{f_i}\frac{m_{f_i}^4}{m_H^2 m_W^2}\ln\frac{\Lambda^2}{m_{f_i}^2} \\ &+ \left\{\left[C_f\frac{m_f^2(3m_f^2 - 4m_Z^2)}{(m_f^2 - m_Z^2)^2} + \frac{m_f^4}{8(m_f^2 - m_Z^2)^2} - \frac{m_f^4(m_f^2 - 2m_Z^2)}{8m_Z^2(m_f^2 - m_Z^2)^2}\right]\frac{m_Z^2}{m_W^2}\right. \\ &\qquad \left. -3\sin^2\theta_W Q_f^2 + \frac{m_f^4(3m_f^2 - 2m_H^2)}{8m_W^2(m_f^2 - m_H^2)^2}\right\}\ln\frac{\Lambda^2}{m_f^2} \\ &+ \left[C_f\frac{m_Z^2(3m_Z^2 - 2m_f^2)}{(m_f^2 - m_Z^2)^2} - \frac{m_f^4}{8(m_f^2 - m_Z^2)^2} - \frac{m_f^2 m_Z^2}{8(m_f^2 - m_Z^2)^2} - \frac{3}{4}\frac{m_Z^2}{m_H^2}\right]\frac{m_Z^2}{m_W^2}\ln\frac{\Lambda^2}{m_Z^2} \\ &+ \sum_{f'} V_{ff'}^\dagger V_{f'f}\left[(2 + \frac{m_f^2}{m_W^2})\frac{m_{f'}^4}{8(m_{f'}^2 - m_W^2)^2} - \frac{m_{f'}^4(3m_{f'}^2 - 4m_W^2)}{8m_W^2(m_{f'}^2 - m_W^2)^2}\right]\ln\frac{\Lambda^2}{m_{f'}^2} \\ &+ \left\{\sum_{f'} V_{ff'}^\dagger V_{f'f}\left[(2 + \frac{m_f^2}{m_W^2})\frac{m_W^2(m_W^2 - 2m_{f'}^2)}{8(m_{f'}^2 - m_W^2)^2} - \frac{m_{f'}^2(3m_W^2 - 2m_{f'}^2)}{8(m_{f'}^2 - m_W^2)^2}\right]\right. \\ &\qquad \left. - \left(\frac{3}{2}\frac{m_W^2}{m_H^2} + \frac{1}{4}\right)\right\}\ln\frac{\Lambda^2}{m_W^2} \\ &+ \frac{m_H^2}{8m_W^2}\left[\frac{m_f^2(3m_H^2 - 4m_f^2)}{(m_f^2 - m_H^2)^2} - 3\right]\ln\frac{\Lambda^2}{m_H^2}\ . \end{aligned} \tag{6}$$

In writing Eq. 6 we have kept leading terms for large $\Lambda$, and dropped terms of order unity. For simplicity, $f_i$ is used to denote all the fermions. $C_f \equiv (I_{3_f} - Q_f\sin^2\theta_W)Q_f\sin^2\theta_W$, where $I_{3_f}$ is the third component of the weak isospin for fermions of flavor $f$. $\sum_{f'}$ denotes the summation over all fermions which are coupled to the fermion $f$ via the charged currents. The matrix $V$ is the Kobayashi-Maskawa matrix, $U_{KM}$, when $I_{3_f} = -1/2$, and $V = U_{KM}^\dagger$ when $I_{3_f} = 1/2$. $\theta_W$ is the Weinberg angle. There is one consistency equation for each fermion flavor, and all equations are coupled.

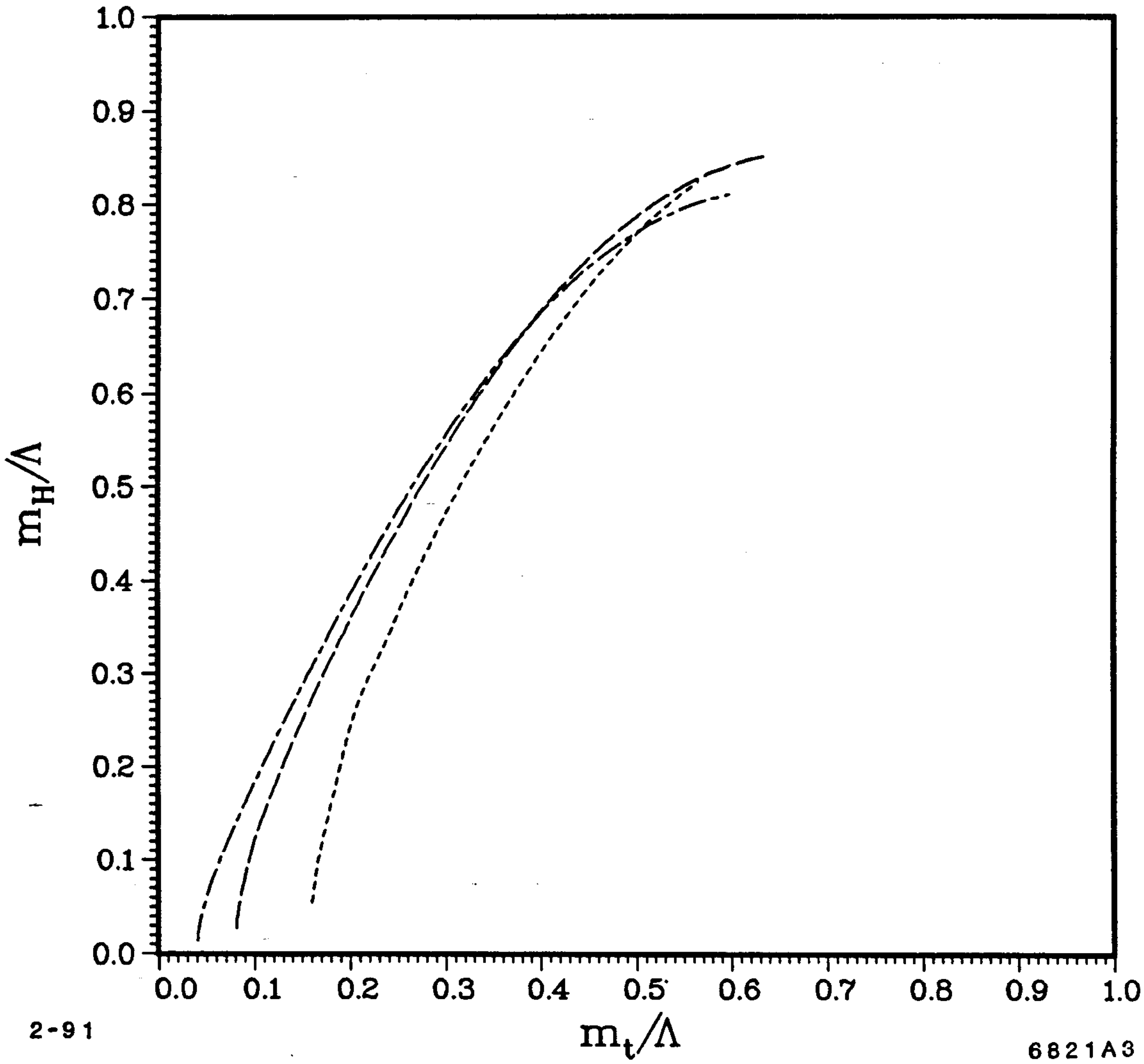


Fig. 3. Relation between $m_t$ and $m_H$ given by Eq. 5 for $f = t$. The dotted, dashed, and dotted-dashed lines correspond, respectively, to $\Lambda = 5\,m_Z$, $10\,m_Z$, and $20\,m_Z$.

Note that $m_f = 0$ is always a solution. To search for nontrivial solutions, we solve the consistency equation for the mass of the top quark, $m_t$, for three different cutoff scales: $\Lambda = 5\,m_Z$, $10\,m_Z$ and $20\,m_Z$, and for a range of Higgs mass, 25 GeV $\leq m_H < \Lambda$, assuming that all other fermions are massless. We find that nonzero solutions for $m_t$ do exist and they are always greater than 70 GeV for the range of cutoff and Higgs masses explored. These solutions are plotted in Fig. 3. This result implies that, without putting in the Yukawa couplings by hand, it is possible to generate a heavy top quark mass dynamically, while the other fermions remain massless.* Given the upper and lower bounds on $m_t$ which have been obtained from various experiments, including direct search and $\sin^2\theta_W$ measurements, one can also

* This result does not depend crucially on the charge of the top quark. More generally, we find that, if $m_{f_i} = 0$ for all $f_i \neq f_0$, then eq. 5 admits nonzero solutions for $m_{f_0}$ which are insensitive to the charge $Q_{f_0}$.

obtain bounds on the Higgs mass from Fig. 3. For 89 GeV $< m_t <$ 250 GeV, we find that 75 GeV $< m_H <$ 375 GeV, 75 GeV $< m_H <$ 475 GeV, and 100 GeV $< m_H <$ 500 GeV, for $\Lambda = 5\,m_Z$, $10\,m_Z$ and $20\,m_Z$, respectively.

It is clear that, for $f = t$, the contributions to the right-hand side of Eq. 6 from light fermions are negligible. We have checked that the solutions for $m_t$ differ by no more than 0.5% whether we use the "known" masses$^+$ for all the fermions other than the top quark, or assume that they are all massless. On the other hand, in order to have a nonzero mass for, say, the $b$-quark, as well as the $t$-quark, the consistency equations for both $f = t$ and $f = b$ have to be satisfied. We find that, for all three cutoff scales considered, there is no solution with $m_t > 80$ GeV and $m_b \approx 5$ GeV which satisfies Eq. 5 for $f = t$ and $f = b$ simultaneously.

One may thus be led to conclude that, while a heavy top quark mass can be generated dynamically, the observed masses of the light fermions cannot be generated in our approach. It should, however, be pointed out that this conclusion is based on the solutions to Eqs. 5 and 6, which are only a crude approximation to the true consistency equations, Eq. 1. We observe that, although $F_b$ does not vanish when $m_b = 5$ GeV and $m_t$ takes the value which satisfies the equation $F_t|_{m_b = 5\,\mathrm{GeV}} = 0$, $|\Sigma(m_b)|$ is always very small, less than $0.07\,m_b$ when $m_t < 250$ GeV. This example suggests the possibility that a more refined approximation to Eq. 1 may yield solutions with the observed $m_b$ and $m_\tau$, as well as a large $m_t$.

Concerning the contributions of QCD, it is commonly assumed that the Yukawa couplings in the Standard Model give rise to the current quark masses. The difference between the constituent quark mass and the current quark mass (of order 300 MeV) is attributed to the dynamics of QCD. In principle, we can check this assumption by solving an equation similar to Eq. 5 which includes the QCD contributions to the fermion self-mass, and compare the two solutions of $m_f$. We carry out such a check for the $t$-quark by including the one-loop QCD contribution. We find that where solutions exist for both equations, they are not as close to each other as 300 MeV. In most cases, nonzero solutions cannot be found when QCD contributions are included. This can mean either that the above common assumption is invalid, or, much more likely, that a one-loop QCD calculation is not adequate to represent the full QCD effects, even for energy as high as the $t$-quark mass.

Our main interest in the above calculations is in the underlying idea on which Eq. 1 is based rather than on attaching numerical significance to the resulting masses in view of the crudity of the approximation.

Similar considerations to what we are discussing here—but with different motivation and interpretation—have appeared in the literature. R. Decker and J. Pestieau[4] in 1980 and M. Veltman[5] in 1981 proposed cancellation of the quadratic divergences that appear in Eq. 6 so that what Veltman described as "naturalness" obtains in the Standard Model in the sub-TeV regime. In the one-loop approximation this condition for cancelling the quadratic divergences is:

$^+$For quarks, there is some uncertainty as to what exactly the current quark masses are.

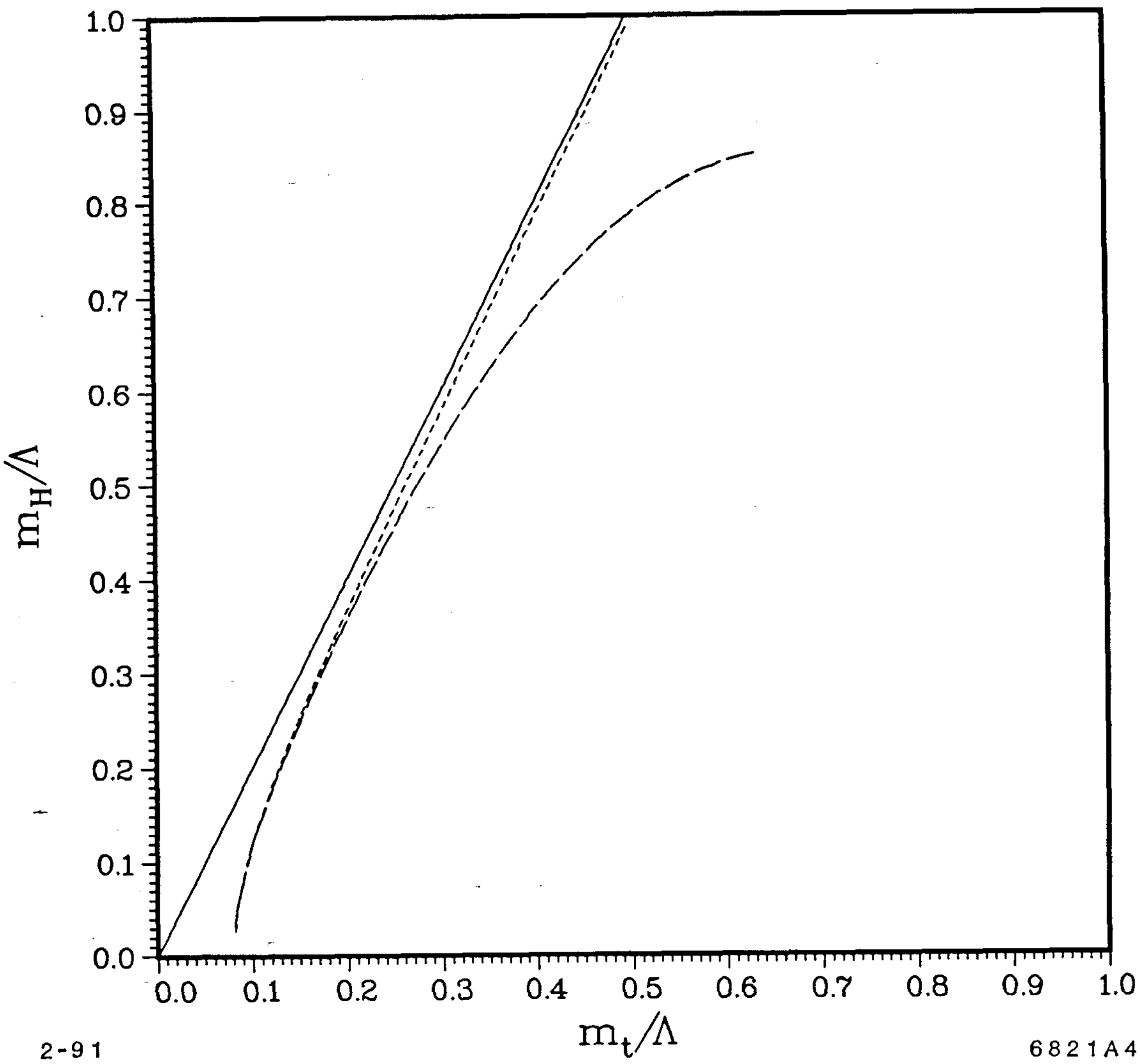


Fig. 4. Relation between $m_t$ and $m_H$. The dashed line is the solution to Eq. 5, with a cutoff $\Lambda = 10\, m_Z$. The dotted line corresponds to Eq. 7, and the solid one, $m_H = 2m_t$.

$$\sum_{\text{all } f} m_f^2 = \frac{3}{4}\left[m_H^2 + 2m_W^2 + m_Z^2\right] \; . \tag{7}$$

The solution to Eq. 7 is plotted in Fig. 4 relative to that for Eq. 5 for $\Lambda = 10\, m_Z$ from which it differs only slightly as the masses increase.

Recently Ruiz-Altaba, Gonzalez and Vargas[6] extended Eq. 7 to a two-loop calculation, predicting $m_t = 124$ GeV, $m_H = 234$ GeV and the weak mixing angle $\sin^2\theta_W \simeq .24$.

Decker and Pestieau[7] went further in 1989, insisting that the theory have neither quadratic nor logarithmic divergences. They solved for $m_t$ and $m_H$ by separately cancelling quadratic and logarithmic terms in the one-loop calculation. Here we do neither of these. Our physical motivation is to remove the fermion masses as arbitrary parameters of the Standard Model and treat them as parameters determined by the dynamics in the sub-TeV region. Rather than cancel the divergences, we impose Eq. 1 as a physical condition and interpret the Standard Model $\mathcal{L}$ as an "effective Lagrangian," in order to determine the physical masses.

## Acknowledgment

We dedicate this paper to the late Professor Mirza A. B. Bég, a close friend, a warm colleague, and a seminal contributor to understanding of many features of the Standard Model—and to particle physics more generally. We shall miss him. His hospitality, together with that of Professor T. D. Lee of the Columbia University Physics Department, enabled S.D.D. to spend a very stimulating six months in New York City and undertake this collaboration. We thank Professors Mahiko Suzuki and Robert Shrock for fruitful discussions, and also Professor Howard Haber for bringing to our attention pertinent references.

## References

1. Though very different from, this approach is clearly inspired by the pioneering work of Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122** (1961) 345.
2. E. C. G. Stückelberg, *Nature* **144** (1939) 118.
3. A. Pais, *Verh. Roy. Acad. Amsterdam* **10** (1946) 1; S. Sakata and O. Hara, *Prog. Theor. Phys.* **2** (1947) 30.
4. R. Decker and J. Pestieau, Lett. Nuovo Cimento **29** (1980) 560.
5. M. Veltman, *Acta Physica Polonica* **B12** (1981) 437.
6. M. Ruiz-Altaba, B. Gonzalez, and M. Vargas, CERN–TH 5558/89, UGVA–DPT 89/11-638 (unpublished); see also discussions in I. Jack and D. R. T. Jones, *Phys. Lett.* **B234** (1990) 321.
7. R. Decker and J. Pestieau, *Modern Phys. Lett.* A4 (1989) 2733.