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HIGHER DIMENSIONAL PERFECT FLUID COLLAPSE WITH COSMOLOGICAL CONSTANT

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In this paper, we investigate higher dimensional spherically symmetric perfect fluid collapse with positive cosmological constant. We take higher dimensional spherically symmetric metric in the interior region and higher dimensional Schwarzschild–de Sitter metric in the exterior region. The junction conditions between interior and exterior space-times are derived. We discuss the apparent horizons and their physical significance and conclude that the cosmological constant slows down the collapse of matter and hence limits the size of the black hole. This analysis gives the generalization of the four-dimensional perfect fluid collapse to higher dimensional perfect fluid collapse. We recover the results of the higher dimensional dust case ($p=0$).

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1. Introduction

The cosmological constant denoted by the Greek letter Λ was first introduced by Einstein to obtain a static, homogeneous cosmological model in 1917. Einstein included this term in his equations for General Relativity because he was not sure that his equations allow for a static universe. Gravity affects the universe to contract. To avoid this possibility, he introduced a term called the cosmological constant that would act as a repulsive form of gravity to balance the attractive nature of gravity. Einstein rejected his introduction of the cosmological constant after the expansion was discovered by Hubble.

Over the past decade, it was discovered that the expansion of the universe is accelerating. This was first observed from type Ia supernova [1, 2] results. To include this acceleration, one needs to add a cosmological constant term to the Einstein field equations. The study [3] of the peculiar

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motion of low-red shift galaxies give further support for the possibility of finite cosmological constant. Since the Einstein field equations relate the metric of the space-time and the stress-energy tensor of matter. Thus the effects of a cosmological constant can be analyzed if one can specify the metric and the stress-energy tensor and then relate them through the Einstein field equations.

Gravitational collapse is one of the key issues in General Relativity. This results in formation of compact stellar objects such as white dwarf and neutron star. It is interesting to investigate this issue by considering the appropriate geometry of interior and exterior regions and determine proper junction conditions which allow the matching of these regions. The pioneering work on gravitational collapse was first started by Oppenheimer and Snyder [4]. They studied collapse of dust by considering static Schwarzschild solution in the exterior and Friedman like solution in the interior. Since then many people [5–9] have studied gravitational collapse by taking an appropriate geometry of interior and exterior regions.

There is a large body of literature available [10–16] which shows a keen interest in this issue with cosmological constant. The work done by Oppenheimer and Snyder [4] was generalized by Markovic and Shapiro [17] in the presence of positive cosmological constant. Later, Lake [18] extended this work both for positive and negative cosmological constant. Cissoko *et al.* [19] discussed explicitly gravitational dust collapse with positive cosmological constant. Recently, Ghosh and Deshkar [20] have extended this work for higher dimensional space-times.

Motivated by string theory and other field theories, some recent investigations [21–26] show keen interest to study gravitational collapse in more than four-dimensions. Recently, we have extended the analysis [27] on gravitational perfect fluid collapse with cosmological constant to five-dimensional space-times [28]. In this paper, we extend this study to higher dimensional space-times. The paper is organized as follows. In next section, we derive junction conditions between static and non-static spherically symmetric space-times. Section 3 is devoted to discuss spherically symmetric perfect fluid solution of the Einstein field equations with a positive cosmological constant. This solution is specialized in Section 4. In Section 5, we discuss the apparent horizons and the role of the cosmological constant. The summary of the results is presented in the last section.

2. Junction conditions

To discuss junction conditions, we assume that the given $n+2$-dimensional space-time is divided by a time-like $n+1$-dimensional hypersurface Σ, into two regions interior and exterior space-times, denoted by V^- and

V^+, respectively. The interior space-time is described by $n+2$-dimensional spherically symmetric metric [21, 23, 25]

$$ds_-^2 = dt^2 - X^2 dr^2 - Y^2 d\Omega^2\,, \tag{1}$$

where X and Y are functions of t and r only and

$$\begin{aligned} d\Omega^2 &= \sum_{i=1}^{n}\left[\prod_{j=1}^{i-1}\sin^2\theta_j\right] d\theta_i^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1\sin^2\theta_2 d\theta_3^2 \\ &\quad + \ldots + \sin^2\theta_1\sin^2\theta_2\ldots\sin^2\theta_{n-1}d\theta_n^2\,, \end{aligned} \tag{2}$$

is the metric on n-sphere and $n = D-2$ (where D is the total number of dimensions). For the exterior space-time, we take the $(n+2)$-dimensional Schwarzschild–de Sitter metric,

$$ds_+^2 = AdT^2 - \frac{1}{A}dR^2 - R^2 d\Omega^2\,, \tag{3}$$

where

$$A(R) = 1 - \frac{2M}{(n-1)R^{n-1}} - \frac{2\Lambda R^2}{n(n+1)}\,, \tag{4}$$

M is a constant and Λ is the cosmological constant. The junction conditions [29, 30] demand that the first and second fundamental forms from the interior and the exterior space-times are the same. These conditions can be defined as follows:

(i) The continuity of the first fundamental form over Σ gives

$$(ds_-^2)_\Sigma = (ds_+^2)_\Sigma = ds_\Sigma^2\,. \tag{5}$$

(ii) The continuity of the second fundamental form over Σ gives

$$[K_{ab}] = K_{ab}^+ - K_{ab}^- = 0\,, \qquad (a,b = 0,2,3 \ldots n+1)\,, \tag{6}$$

where K_{ab}, the extrinsic curvature, is given by

$$K_{ab}^{\pm} = -n_\sigma^{\pm}\left(\frac{\partial^2 x_\pm^\sigma}{\partial\xi^a\partial\xi^b} + \Gamma_{\mu\nu}^{\sigma}\frac{\partial x_\pm^\mu}{\partial\xi^a}\frac{\partial x_\pm^\nu}{\partial\xi^b}\right), \qquad (\sigma,\mu,\nu = 0,1,2 \ldots n+1)\,, \tag{7}$$

$\Gamma_{\mu\nu}^{\sigma}$ are the Christoffel symbols calculated from the interior or exterior metrics (1) or (3), respectively, $n_\mu^{\pm}$ the unit normal vectors to Σ, $x_\pm^\sigma$ are the coordinates of the interior and exterior space-times and ξ^a are the coordinates of hypersurface Σ. The equations of hypersurface Σ in the coordinates $x^\sigma_\pm$ are written as

$$f_-(r,t) = r - r_\Sigma = 0\,, \tag{8}$$
$$f_+(R,T) = R - R_\Sigma(T) = 0\,, \tag{9}$$

where r_Σ is a constant.

Using Eq. (8) the interior metric on Σ can be written as

$$\left(ds^2_-\right)_\Sigma = dt^2 - [Y(r_\Sigma,t)]^2\, d\Omega^2\,. \tag{10}$$

Similarly, using Eq. (9) the exterior metric on Σ takes the form

$$\left(ds^2_+\right)_\Sigma = \left[A(R_\Sigma) - \frac{1}{A(R_\Sigma)}\left(\frac{dR_\Sigma}{dT}\right)^2\right] dT^2 - {R_\Sigma}^2 d\Omega^2\,, \tag{11}$$

where we assume that

$$A(R_\Sigma) - \frac{1}{A(R_\Sigma)}\left(\frac{dR_\Sigma}{dT}\right)^2 > 0\,, \tag{12}$$

so that T is a time-like coordinate. From Eqs. (5), (10) and (11), it follows that

$$R_\Sigma = Y(r_\Sigma,t)\,, \tag{13}$$

$$\left[A(R_\Sigma) - \frac{1}{A(R_\Sigma)}\left(\frac{dR_\Sigma}{dT}\right)^2\right]^{\frac{1}{2}} dT = dt\,. \tag{14}$$

The outward unit normals to Σ in V^- and V^+ follows from Eqs. (8) and (9)

$$n^-_\mu = (0, X(r_\Sigma,t), 0, 0, 0, ...0)\,, \tag{15}$$
$$n^+_\mu = (-\dot{R}_\Sigma, \dot{T}, 0, 0, 0, ...0)\,, \tag{16}$$

where dot means differentiation with respect to t. The components of the extrinsic curvature $K^\pm_{ab}$ are

$$K^-_{00} = 0\,, \tag{17}$$
$$K^-_{22} = \left(\frac{YY'}{X}\right)_\Sigma = \csc^2\theta_1 K^-_{33} = \csc^2\theta_1\csc^2\theta_2 K^-_{44} = \ldots$$
$$= \csc^2\theta_1\csc^2\theta_2 ... \csc^2\theta_{n-1} K^-_{n+1n+1}\,, \tag{18}$$

$$K_{00}^{+} = \left(\dot{R}\ddot{T} - \dot{T}\ddot{R} - \frac{A}{2}\frac{dA}{dR}\dot{T}^3 + \frac{3}{2A}\frac{dA}{dR}\dot{T}\dot{R}^2\right)_{\Sigma}, \tag{19}$$

$$\begin{aligned} K_{22}^{+} &= \left(AR\dot{T}\right)_{\Sigma} = \csc^2\theta_1 K_{33}^{+} = \csc^2\theta_1 \csc^2\theta_2 K_{44}^{+} = \dots \\ &= \csc^2\theta_1 \csc^2\theta_2 \dots \csc^2\theta_{n-1} K_{n+1n+1}^{+}, \end{aligned} \tag{20}$$

where prime denotes differentiation with respect to r. The continuity of the extrinsic curvature gives

$$K_{00}^{+} = 0, \tag{21}$$

$$K_{22}^{-} = K_{22}^{+}. \tag{22}$$

When we use Eqs. (17)–(22) along with Eqs. (13), (14) and (4), the junction conditions turn out to be

$$(X\dot{Y}' - \dot{X}Y')_{\Sigma} = 0, \tag{23}$$

$$\begin{aligned} M = &\left[\frac{n-1}{2}Y^{n-1} - \frac{(n-1)}{n(n+1)}\Lambda Y^{n+1}\right. \\ &\left. + \frac{(n-1)}{2}Y^{n-1}\dot{Y}^2 - \frac{(n-1)}{2X^2}Y^{n-1}Y'^2\right]_{\Sigma}. \end{aligned} \tag{24}$$

3. Solution of the field equations

The Einstein's field equations for perfect fluid with cosmological constant are given by

$$G_{\nu}^{\mu} - \Lambda\delta_{\nu}^{\mu} = 8\pi[(\rho + p)u^{\mu}u_{\nu} - p\delta_{\nu}^{\mu}], \tag{25}$$

where ρ is the energy density, p is the pressure and $u_{\mu} = \delta_{\mu}^{0}$ is the n+2-dimensional velocity in co-moving coordinates. For the metric (1), these equations can be written in component form as

$$\begin{aligned} G_0^0 &= -\frac{n(n-1)}{2Y^2}\left(\frac{Y'^2}{X^2} - \dot{Y}^2 - 1\right) + \frac{n}{XY}\left(\dot{X}\dot{Y} + \frac{X'}{X^2}Y'\right) - \frac{n}{X^2}\frac{Y''}{Y} \\ &= \Lambda + 8\pi\rho, \end{aligned} \tag{26}$$

$$G_1^1 = -\frac{n(n-1)}{2Y^2}\left(\frac{Y'^2}{X^2} - \dot{Y}^2 - 1\right) + n\frac{\ddot{Y}}{Y} = \Lambda - 8\pi p, \tag{27}$$

$$\begin{aligned} G_2^2 = &\frac{(n-1)(n-2)}{2Y^2}\left(\frac{Y'^2}{X^2} - \dot{Y}^2 - 1\right) + \frac{(n-1)}{XY}\left(\dot{X}\dot{Y} + \frac{X'}{X^2}Y'\right) \\ &+ \frac{(n-1)}{Y}\left(\ddot{Y} - \frac{Y''}{X^2}\right) + \frac{\ddot{X}}{X} = \Lambda - 8\pi p, \end{aligned} \tag{28}$$

$$G_3^3 = G_4^4 = \ldots = G_{n+1}^{n+1} = G_2^2 = \Lambda - 8\pi p\,, \tag{29}$$

$$G_1^0 = -n\frac{\dot{Y}'}{Y} + n\frac{\dot{X}}{X}\frac{Y'}{Y} = 0\,. \tag{30}$$

To solve these equations, we first integrate Eq. (30) w.r.t. t and get

$$X = \frac{Y'}{W}\,, \tag{31}$$

where $W = W(r)$ is an arbitrary function of r. The energy conservation equation for perfect fluid with interior metric $T^{\nu}_{\mu;\nu} = 0$ implies that the isotropic pressure is a function of t only *i.e.*, $p = p(t)$. From Eqs. (27) and (31), it follows that

$$n\frac{\ddot{Y}}{Y} + \frac{n(n-1)}{2}\left(\frac{\dot{Y}}{Y}\right)^2 + \frac{n(n-1)}{2}\frac{(1-W^2)}{Y^2} = \Lambda - 8\pi p(t)\,. \tag{32}$$

To obtain the explicit solution of this equation, we consider p in the following form [31]

$$p(t) = p_0 t^{-s}\,, \tag{33}$$

where p_0 and s are positive constants. Further, we integrate Eq. (32) by considering $s = 0$ for simplicity *i.e.*, $p(t) = p_0$ and get

$$\dot{Y}^2 = W^2 - 1 + \frac{2m}{Y^{n-1}} + \frac{2(\Lambda - 8\pi p_0)}{n(n+1)}Y^2\,, \tag{34}$$

where $m = m(r)$ is an arbitrary function of r and is related to the mass of the collapsing system. Using Eqs. (31) and (34) in (26), it turns out that

$$m' = \frac{8\pi}{n}(\rho + p_0)Y^n Y'\,. \tag{35}$$

For physical reasons, we assume that density and pressure are non-negative. Integrating Eq. (35) w.r.t. r, we obtain

$$m(r) = \frac{8\pi}{n}\int_0^r (\rho + p_0)Y^n Y' dr\,. \tag{36}$$

Here we take constant of integration to be zero. Also, the function $m(r)$ must be positive as $m(r) < 0$ implies negative mass which is not meaningful. When we use Eqs. (31) and (34) into the junction condition (24), we obtain

$$M = m(n-1) - \frac{8\pi p_0(n-1)}{n(n+1)}Y^{n+1}\,. \tag{37}$$

Eq. (4) implies that the exterior space-time becomes the $n+2$-dimensional Schwarzschild space-time for $\Lambda = 0$ and M as the total energy inside the surface Σ. The total energy $\tilde{M}(r,t)$ up to a radius r at time t inside the hypersurface Σ can be calculated by using the definition of the mass function [5, 20]. For the metric (1), this is given by

$$\tilde{M}(r,t) = \frac{(n-1)}{2} Y^{n-1}(1 + g^{\mu\nu} Y_{,\mu} Y_{,\nu}) = \frac{(n-1)}{2} Y^{n-1} \left[1 - \left(\frac{Y'}{X} \right)^2 + \dot{Y}^2 \right] . \tag{38}$$

Using Eqs. (31) and (34) in Eq. (38), it follows that

$$\tilde{M}(r,t) = (n-1)m(r) + \frac{(n-1)}{n(n+1)} (\Lambda - 8\pi p_0) Y^{n+1} . \tag{39}$$

4. Solution with $W(r) = 1$

For $\Lambda - 8\pi p_0 > 0$, the analytic solution in closed form can be obtained from Eqs. (31) and (34) as follows

$$Y(r,t) = \left[\frac{n(n+1)m}{\Lambda - 8\pi p_0} \right]^{1/n+1} \sinh^{2/n+1} \alpha(r,t) , \tag{40}$$

$$X(r,t) = \left[\frac{n(n+1)m}{\Lambda - 8\pi p_0} \right]^{1/n+1} \left[\frac{m'}{(n+1)m} \sinh \alpha(r,t) + t_0{}' \sqrt{\frac{2(\Lambda - 8\pi p_0)}{n(n+1)}} \cosh \alpha(r,t) \right] \sinh^{(1-n)/(1+n)} \alpha(r,t) , \tag{41}$$

where

$$\alpha(r,t) = \sqrt{\frac{(n+1)(\Lambda - 8\pi p_0)}{2n}} [t_0(r) - t] . \tag{42}$$

Here $t_0(r)$ is an arbitrary function of r. In the limit $\Lambda \to 8\pi p_0$, the above solution corresponds to the $n+2$-dimensional Tolman–Bondi solution [32]

$$\lim_{\Lambda \to 8\pi p_0} Y(r,t) = \left[\frac{m}{2} (n+1)^2 (t_0 - t)^2 \right]^{1/n+1} , \tag{43}$$

$$\lim_{\Lambda \to 8\pi p_0} X(r,t) = \frac{m'(t_0 - t) + 2mt_0'}{[2(n+1)^{n-1} m^n (t_0 - t)^{n-1}]^{1/n+1}} . \tag{44}$$

5. Apparent horizons

When the boundary of trapped n spheres is formed, we obtain the apparent horizon. Here we find this boundary of the trapped n spheres whose outward normals are null. For Eq. (1), this is given as follows

$$g^{\mu\nu}Y_{,\mu}Y_{,\nu}=\dot{Y}^2-\left(\frac{Y'}{X}\right)^2=0\,. \tag{45}$$

Using Eqs. (31) and (34) in Eq. (45), we obtain

$$(\Lambda-8\pi p_0)Y^{n+1}-\frac{n(n+1)}{2}Y^{n-1}+n(n+1)m=0\,. \tag{46}$$

The solutions of the above equation for Y give the apparent horizons. For $\Lambda=8\pi p_0$, it becomes the Schwarzschild horizon, *i.e.*, $Y=(2m)^{1/n-1}$. When $m=0,\ p_0=0$, it yields the de Sitter horizon $Y=\sqrt{\frac{n(n+1)}{2\Lambda}}$. The approximate solutions of Eq. (46) up to first order in m and $\Lambda-8\pi p_0$, respectively, are given by

$$Y_1 = \left[\frac{n(n+1)}{2(\Lambda-8\pi p_0)}\right]^{1/2}-\left[\frac{2(\Lambda-8\pi p_0)}{n(n+1)}\right]^{(n-2)/2}m\ldots\,, \tag{47}$$

$$Y_2 = (2m)^{1/n-1}+\frac{2(\Lambda-8\pi p_0)}{n(n-1)(n+1)}\,(2m)^{3/n-1}\ldots\,. \tag{48}$$

For $m=0$, from Eqs. (47) and (48), it follows that $Y_1=\sqrt{\frac{n(n+1)}{2(\Lambda-8\pi p_0)}}$ and $Y_2=0$. For $m\neq 0$, Y_1 is called the generalized cosmological horizon and Y_2 is called the generalized black hole horizon for $\Lambda\neq 8\pi p_0$ [11]. From Eqs. (40) and (46), the time for the formation of apparent horizon is given by

$$t_s=t_0-\sqrt{\frac{2n}{(n+1)(\Lambda-8\pi p_0)}}\sinh^{-1}\left(\frac{{Y_s}^{n-1}}{2m}-1\right)^{1/2}\,,\qquad (s=1,2)\,. \tag{49}$$

This equation shows that the cosmological constant modifies the time of formation of apparent horizon. If the formation of the apparent horizon precedes the formation of the singularity then it will necessarily be covered, *i.e.*, it is a black hole. On the other hand, if apparent horizon forms after the singularity formation then it will be naked. In the limit $\Lambda\to 8\pi p_0$, we obtain the result corresponding to Tolman–Bondi [32]

$$t_{\rm ah}=t_0-\frac{(2^n m)^{1/n-1}}{n+1}\,. \tag{50}$$

Eq. (49) implies that both the black hole horizon and the cosmological horizon form earlier than the singularity $t = t_0$. This shows that the singularity is covered, *i.e.*, it is a black hole. The black hole is characterized by an event horizon. Eq. (50) gives the time for the formation of event horizon in higher-dimensional Tolman–Bondi space-time. For naked singularity, the necessary condition is $t_{\text{ah}} > t_0$.

6. Conclusion

In this paper, we have generalized the previous work on gravitational perfect fluid collapse with cosmological constant [27, 28] to $n+2$-dimensional space-times. We have taken $n+2$-dimensional spherically symmetric metric in the interior region and $n+2$- dimensional Schwarzschild–de Sitter metric in the exterior region. The exact solution for the interior space-time with perfect fluid is derived. The effects of the cosmological constant on gravitational collapse are discussed as follows.

The Newtonian potential $\phi = \frac{1}{2}(1 - g_{00})$ for the exterior metric can be found from Eqs. (13) and (37) as

$$\phi(R) = \frac{m}{R^{n-1}} + \frac{(\Lambda - 8\pi p_0)}{n(n+1)} R^2 \,. \tag{51}$$

The corresponding Newtonian force is given by

$$F = -\frac{(n-1)m}{R^n} + \frac{2(\Lambda - 8\pi p_0)}{n(n+1)} R \,. \tag{52}$$

This force vanishes for $R = \frac{1}{(\Lambda - 8\pi p_0)^{1/n}}$ and $m = \frac{2}{n(n-1)(n+1)(\Lambda - 8\pi p_0)^{1/n}}$ which implies that the force becomes repulsive/attractive for larger/smaller mass and radius, respectively, than these values. Thus the size of the black hole can be visualized by comparing the repulsive and attractive forces. The repulsive force generates from the cosmological constant for $\Lambda > 8\pi p_0$. From Eq. (34), the rate of collapse turns out to be

$$\ddot{Y} = -\frac{(n-1)m}{Y^n} + \frac{2(\Lambda - 8\pi p_0)}{n(n+1)} Y \,. \tag{53}$$

For collapsing process, the force should be attractive, *i.e.*, the acceleration should be negative which implies that $Y < \left[\frac{n(n-1)(n+1)m}{2(\Lambda - 8\pi p_0)}\right]^{1/n+1}$. This can be explained in terms of geodesics as follows. The geodesic deviation equation characterizes the coming together or moving away of space-time geodesics as a result of the space-time curvature [33]. This implies that the geodesics will move towards or away from each other depending whether the space-time curvature is negative or positive. The geodesic deviation becomes zero if and only if all the components of the curvature tensor vanish. Since in general relativity, force is described in terms of curvature. Thus it corresponds to the fact that for collapsing/expanding processes, force should be attractive/repulsive, *i.e.*, acceleration should be negative/positive. It follows from Eq. (53) that the cosmological constant slows down the collapsing process if $\Lambda > 8\pi p_0$. This means that, for $p_0 > \frac{\Lambda}{8\pi}$, the force becomes attractive and hence the cosmological constant does not slow down the collapsing process.

Also, due to the presence of the term $\Lambda - 8\pi p_0$, there are several apparent horizons but only two are physical. One is the black hole horizon and the other is the cosmological horizon. The time difference between the formation of the apparent horizon and singularity is affected by the presence of the cosmological constant. We conclude that the cosmological constant affects the process of collapse and hence it limits the size of the black hole. In perfect fluid case, these results are valid only for $\Lambda > 8\pi p_0$ while in dust case [19] these are valid for all $\Lambda > 0$. Thus the pressure term creates a bound for the cosmological constant to act as a repulsive force. It is worth mentioning here that for $n = 2$ and $n = 3$, we recover the results given in the papers [27] and [28], respectively. We also recover the results of higher dimensional dust case [20] for $p_0 = 0$. Thus our analysis gives the generalization of the earlier results.

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