Generalized Second Law and Brane Cosmological Model with Phantom Dominated Bulk

BALENDRA KR. DEV CHOUDHURY[1]
[1]*Deptt. of Physics, Pub Kamrup College, Baihata Chariali, Assam(india)*
bdevchoudhury@yahoo.com
DOI: 10.7529/ICRC2011/V05/0087

Abstract: Non-empty bulk consideration in brane-world cosmological set-up is welcome from different needs. But still the basis of taking unconstrained non-empty bulk is phenomenological. So one is bound to be cautious to consider anything in bulk. With the present accelerated expansion of the universe, we judge the legitimacy of taking modified chaplygin gas, as a phantom candidate in bulk. Moreover, we check the validity of Generalised Second Law of thermodynamics in the model with cosmological event horizon and apparent horizon envelope.

Keywords: Phantom, Modified Friedmann Equation, GSL

1 Introduction

From different observational supports, it is now on firm footing that our present universe is expanding with an acceleration [1]. Though the immediate response towards this unusual finding was a unthinkable shock to 'Big-Bang-Cosmologists', the finding naturally finds its explanation within Einstein's equation of gravity. In addition to standard cosmology based on Einstein equation, string cosmology/brane-based cosmology is in the list of active pursuit.

We intend to investigate the thermodynamical status of phantom dominated brane-world cosmology. As a phantom candidate, Modified Chaplygin Gas (also called as Generalized Chaplygin Gas) will be considered satisfying the condition, the state parameter $w < -1$. However Chaplygin cosmology has been extensively studied in the context of late-time acceleration of the universe [2]. In brane approach, generally empty bulk is considered for its relative simplicity. But there are different urgent needs to investigate non-empty bulk [3], though we will not take these issues in the present work. Bulk field is constrained to correspond to string fields, however, constraints may be relaxed from a phenomenological point of view. Thus, one is free to consider any matter in the bulk[4]. In this perspective modified chaplygin gas has been already considered in the bulk, and resulting cosmological consequences are discussed [5]. We also consider modified chaplygin gas constrained to be phantom in the bulk but with different aim. Firstly, we intend to investigate the constraints imposed by Generalised Second Law (GSL) of thermodynamics in our proposed brane-world cosmological set-up. Secondly, we want to see whether modified chaplygin gas can be considered in bulk. The phantom idea threatens the basic modern physics [6]. So the idea of effective state parameter w_{eff} is forwarded. Validity of GSL in phantom cosmology again upholds the legitimacy of phantom consideration. Through such type of approaches, undeniable role of GSL has already been established to impose bounds on astrophysical and cosmological models [7]. In our approach, we find exact solution of approximate Friedmann-like equation. Derived scale factor is used to find out the radius of cosmological event horizon. Through the Gibb's equation, validity of GSL is checked not only with cosmological event horizon envelope but with apparent horizon envelope also.

The plan of the work is outlined as follows: the section II is the preliminary. It provides all the basic mathematical expressions already obtained. These expressions are extensively used to carry out our work. Our own work is presented in section III. The last section covers the discussion and the conclusion.

2 Preliminaries

Brane-world perspective can be represented by five dimensional spacetime. Usual 5-D spacetime considered here is generally takes the form[5]

$$ds^2 = -n^2\,(t,y)\,dt^2 + a^2\,(t,y)\,\delta_{ij}dx^i dx^j + b^2\,(t,y)\,dy^2 \tag{1}$$

where δ_{ij} is a maximally symmetric 3-D metric and y is the fifth coordinate.

The 5-D Einsteins's equation for gravity read as

$$G_{\alpha\beta} = \kappa^2 T_{\alpha\beta} \tag{2}$$

where α, β = 0, 1, 2, 3, 4. κ is related to 5-D Newton's constant, $G(5)$.

In equation (2), $T_{\alpha\beta}$ represents total energy-momentum tensor. In the proposed work, it has two parts:

1. The energy-momentum tensor of bulk

$$T^{\alpha}{}_{\beta}|_B = diag\left(-\rho_B, P_B, P_B, P_B, P_5\right) \quad (3)$$

where ρ_B, P_B, and P_5 give energy density, and pressure component respectively. They are independent of fifth coordinate.

2. The energy-momentum tensor of brane

Assuming homogeneous and isotropic geometry inside the brane, it is given as

$$T^{\alpha}{}_{\beta}|_{br} = \frac{\delta(y)}{b} diag\left(-\rho_b, p_b, p_b, p_b, 0\right) \quad (4)$$

where ρ_b and p_b are the energy density and pressure in brane, and it is assumed there is no transfer of energy from bulk to brane.

Using equation (1) and equation (2), components of Einstein's tensor are evaluated. By imposing Z_2- symmetry and going through the usual steps, conservation relation on the bulk can be obtained as

$$\dot{\rho_B} + 3\frac{\dot{a}}{a}\left(\rho_B + P_B\right) + \frac{\dot{b}}{b}\left(\rho_B + P_B\right) = 0 \quad (5)$$

Moreover, choosing modified chaplygin gas for bulk, another two important relations are obtained:

$$\rho_b = \frac{\rho_0}{a_b(t)} \quad (6)$$

where ρ_0 is an integration constant, and $a_b(t)$ is the brane scale factor.

And

$$\dot{a_b^2} = \frac{\kappa^4}{36}\rho_0^2 + \frac{\kappa^2}{6}\rho_B a_b^2 - \frac{\zeta}{a_b^2} - \kappa \quad (7)$$

Hence, from equation (6) and equation (7), we can have

$$H^2 = \left(\frac{\dot{a_b}}{a_b}\right)^2 = \frac{\kappa^4}{36}\left(\frac{\rho_0^2}{a_b^2}\right) + \frac{\kappa^2}{6}\rho_B - \frac{\zeta}{a_b^4} - \frac{\kappa}{a_b^2} \quad (8)$$

3 Present WorK

3.1 a_b derivation

The equation of state (EOS) for modified chaplygin gas considered in bulk is

$$P_B = P_5 = \gamma\rho_B - \frac{A}{\rho_B^{\alpha}} \quad (9)$$

Here, γ and A are two positive constants, and $0 < \alpha \leq 1$

On substitution of this equation of state in the conservation equation (5), we can have

$$\rho_B^{\alpha+1} = \frac{A}{1+\gamma} + \frac{C}{(a^3 b)^{(\alpha+1)(1+\gamma)}} \quad (10)$$

where C is the constant of integration.

Taking present magnitude of state parameter $w = w_0 = \gamma - \frac{A}{\rho_{B0}^{\alpha+1}} = -1.06$ [8], where ρ_{B0} is the energy density for bulk corresponding to $w_0 = -1.06$ and on the basis of preliminaries, the expression (10) is led to

$$\rho_B^{\alpha+1} = \frac{\rho_{B0}^{(\alpha+1)}}{A - 0.06\rho_{B0}^{(\alpha+1)}}\left[A - \frac{0.06\rho_{B0}^{(\alpha+1)}}{(a_b^3 b_b)^{(\alpha+1)(1+A)}}\right] \quad (11)$$

Let us suppose

$$A = n\rho_{B0}^{\alpha+1} \quad (12)$$

where n is a numerical coefficient. This substitution can be justified as Guo and Zhang shows variable chaplygin gas has better data fitting [9]. There they considered $p_v = -\frac{A(a)}{\rho_v}$ where $A(a)$ is a positve function of the cosmological scale factor a. Again, discussing chaplygin gas dominated anisotropic brane world cosmological models, Mak and Harko obtains $A = \rho_{c0}^{(\alpha+1)}$ for specific condition, where ρ_{c0} is present critical value considered for dark energy density [10]. Setting $a_b = b_b$, we write equation (11) as

$$\rho_B^{\alpha+1} = \frac{\rho_{B0}^{(\alpha+1)}}{A - 0.06\rho_{B0}^{(\alpha+1)}}\left[A - \frac{0.06\rho_{B0}^{(\alpha+1)}}{a_b^{4(\alpha+1)(1+A)}}\right] \quad (13)$$

With the assumption (14) and $w_0 = -1.06$, we arrive

$$\rho_B^{\alpha+1} = \frac{1}{0.5}\left[0.56 - \frac{0.06}{a_b^{2(\alpha+1)}}\right]\rho_{B0}^{\alpha+1} \quad (14)$$

To evaluate a_b, we consider modified Friedmann equation (8) for brane set up in an approximate form as

$$H^2 = \left(\frac{\dot{a_b}}{a_b}\right)^2 = \frac{\kappa^4}{36}\left(\frac{\rho_0^2}{a_b^2}\right) + \frac{\kappa^2}{6}\rho_B \quad (15)$$

ignoring the other two terms since we are interested in late-time universe. The first term is still retained to see the effect of squarred ρ_b^2 (coming via ρ_0^2) which actually radically changes the standard cosmology.

Using equation (16) and equation (17), we have

$$\left(\frac{\dot{a_b}}{a_b}\right)^2 = \frac{\kappa^4}{36a_b^2}$$

$$\left[\rho_0^2 + \left(\frac{12}{\kappa^2}\right)^{\frac{1}{\alpha+1}}\{0.56a_b^{2(\alpha+1)} - 0.06\}^{\frac{1}{\alpha+1}}\rho_{B0}\right] \quad (16)$$

Then assuming

$$0.56a_b^{2(\alpha+1)} - 0.06 = z^{\alpha+1} \quad (17)$$

and

$$z^{\alpha+1} + 0.06 \cong z^{\alpha+1} \quad (18)$$

and integrating, we have

$$z|_{z*}^{z} = \frac{\kappa^4}{36}C^2{\rho_{B0}}^2\left(t - t_*\right)^2 \quad (19)$$

where we consider

$$\rho_0{}^2 + \left(\frac{12}{k^4}\right)^{\frac{1}{\alpha+1}} z \equiv Cz \qquad (20)$$

From equation (16) and equation (20), we can ultimately obtain

$$a_b = \left[\frac{3}{28} + \left(0.56a_*^{2(\alpha+1)} - 0.06\right)^{\frac{1}{\alpha+1}} + D^2\left(t - t_*\right)^2\right]^{\frac{1}{2}} \qquad (21)$$

where $D = \frac{\kappa^2}{6} C\rho_{B0}$

3.2 R_H, expression

Assuming that universe is enveloped by cosmological event horizon, the status of GSL has been discussed in standard cosmological set-up by various authors. On the basis of recent data, it is pointed out that GSL does not hold where the enveloping surface is cosmological event horizon. The doubt has been cast on its physical existence from the thermodynamical point of view [11]. We also want to investigate the status of GSL with event horizon envelope but in our proposed brane-cosmological set-up. Event horizon radius R_H is given as

$$R_H = a_b \int_{t*}^{t} \frac{dt'}{a_b(t')} \qquad (22)$$

Now let us substitute

$$\frac{3}{28} + \frac{25}{14}\left(0.56a_*^{2(\alpha+1)} - 0.06\right)^{\frac{1}{\alpha+1}} = M^2$$

and

$$D\left(t - t_*\right) = MSinh(x) \qquad (23)$$

Then we have

$$a_b = Mcosh(x) \qquad (24)$$

and event horizon becomes

$$R_H = \frac{M}{D} = \frac{6\left[\frac{3}{28} + \frac{25}{14}\left(0.56a_*^{2(\alpha+1)} - 0.06\right)^{\frac{1}{\alpha+1}}\right]^{\frac{1}{2}}}{\kappa^2 C\rho_{B0}} \qquad (25)$$

So it is very peculiar that R_H ceases to be time dependent for the particular observer. The observer becomes particular being defined by the magnitude of a_* and α.

It is to be noted that $\ddot{a_b}$ comes as

$$\ddot{a_b} = \frac{\frac{D^2}{M}}{\sqrt{\{1 + \frac{D^2}{M^2}(t - t_*)^2\}}}\left[1 + \frac{t(t - t_*)}{D\left(1 + \frac{D^2}{M^2}(t - t_*)^2\right)}\right] \qquad (26)$$

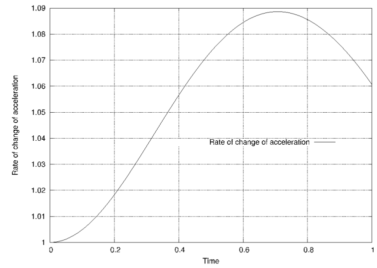

Figure 1: Rate of acceleration vs. time

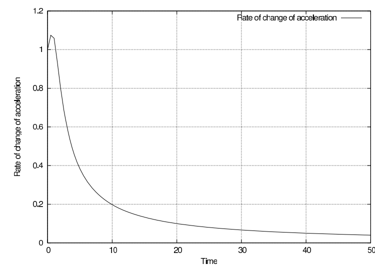

Figure 2: Rate of acceleration vs.time

3.3 status of GSL

Anyway, if we consider the case that the universe is enveloped by cosmological event horizon R_H, from Gibb's equation, we can obtain

$$TS = 4\pi R_H^3 \left\{1 - log(a_b)\right\}\left(\rho_b + p_b\right) + K \qquad (27)$$

And considering the event horizon temperature as $T = \frac{1}{2\pi R_H}$

$$S = 8\pi^2 R_H^4 \left\{1 - log(a_b)\right\}\left(\rho_b + p_b\right) + \frac{K}{T} \qquad (28)$$

where K is the constant of integration.

For present time, $t = t_*$, the equation (23) gives $x = 0$, and so the present scale factor $a_{b*} = M$ from equation (24). If we tune M as $Lt_{\to 0}$ M, then $log(a_b)|_{t=t_*} = log(M) \cong log0 = 1$, and so $\frac{K}{T} = S_*$, gives the present entropy of the universe, that is at $t = t_*$

Now taking the time derivative of equation (28), and considering the phantom case, that is, $(\rho_b + p_b) < 0$, we get

Instead of cosmological event horizon, if we consider apparent horizon, $R_A = \frac{1}{H}$, the rate of change of total entropy, that is $S_{tot} = S_{int} + S_H$ where S_H is the entropy associated with the apparent horizon becomes

$$\dot{S_{tot}} = \frac{2\pi^2}{\tau^{10}} log\left(1 + \tau^2\right) + \frac{\pi}{\tau^5} - \frac{4\pi^2}{\tau^3} - \frac{\pi^2}{4\tau^{10}} \qquad (29)$$

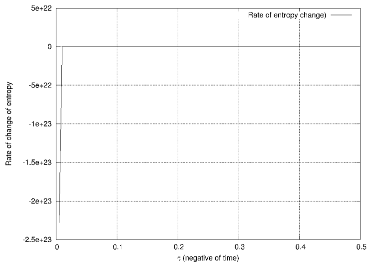

Figure 3: Rate of change of entropy vs.tau

where $\tau = -t$

4 Discussion and Conclusion

From the first sight on equation (28), it appears that GSL is always satisfied in future univesre from theoretical point of view as $t > t_*$. But from the observational point of view the situation is completely opposite. Whatever data sets are employed, data represent the state back in time, that is $t < t_*$. So on the observational ground, GSL is bound to be violated. This striking result upholds the already obtained different data analyses [11]. Moreover as the expression (25) for R_H is time independent, the point might be the indicative that cosmological event horizon is not the physical horizon from the thermodynamical point of view.

However in case of apparent horizon, it is clear from the graph (3), though GSL appears violated at very early universe but subsequently it may take some constant positive value. Here $\tau = -t$. The early universe result may be attributed to the need of consideration of quantum aspects or due to our approximation. Constant entropy suggests an adiabatic expansion (contraction).

But the result is also to face the legitimacy-test to consider modified chaplygin gas in the bulk. Since the basis of consideration of unconstrained non empty bulk is phenomenological. So parameters found in brane cosmological set up should be tested on the confirmed results already obtained in standard cosmology. The scale factor found in equation (21) is parabolic in nature and symmetrical about the vertical axis, that is $x = 0$. Its sharp asymptotic nature at very short interval of x looks very odd. But x does not follow linear evolution against time t as $x = Sinh^{-1}[D(t - t_*)]$. Now if we take the issue of acceleration, we have from the equation (26), the graph [2]. After being accelerated, it shows ultimately a contracting universe. It should be mentioned that $M = D = 1$ is taken for both the graphs. Then one can evaluate the range of a_* using the constraint of α imposed on the chaplygin model.

In summary it may be concluded that on obervational ground GSL would be always violated in the brane cosmological set up considered here if the universe is enveloped by cosmological event horizon. This result upholds the same result found in standrad cosmological set up. Moreover the indication is strong that envelope of cosmological event horizon may not be physical from thermodynamical point of view. Though the explanation regarding the result with apparent horizon looks plausible, graphical representation of $\ddot{a}_b$ makes the consideartion of modified chaplygin gas in bulk questionable. However the scale factor is found solving an approximate Friedmann like equation, so there needs further investigation.

References

[1] Perlmutter, S. J. et al. (1999), Astrophys. J. **517**, 565; (1998) astro-ph/9812133; Riess, A.G. et al. (1998), Astrophys. J. **116**, 1009; Riess, A.G. et al. (2004), Astrophys. J. **607**, 665; astro-ph/0402512; Garnavich,P.M. et al. (1998), Astrophys. J. **509**, 74; Spergel, D.N. et al. (2003), Astrophys. J. Suppl. **148**, 175; astro-ph/9812133 and references therein; Tonry, J.L. et al. (2003), Astrophys. J. **594**, 1; Wang, Y. and Tegmark, M. (2004) Phys. Rev. Lett. **92**, 241302

[2] Mak M. K., and Harko, gr-qc/0505034, (2005) and references therein

[3] Brax P., Bruck C. van de, Davis A C., Rhodes C S., hep -th/0210057, (2003)

[4] Mozumdar A., Mohapatra R. N., and Perez-Lorenzana A., JCAP, **0406** (2004) 004; Bogdanos C. and Tamvakis K., Phys. Lett. B, **646** (2007) 39

[5] Saaidi Kh and Mohammadi A.H., gr-qc/1006.1847 (2010)

[6] Cai R-G., Gong Y. and Wang B., hep-th/0511301 (2006) and references therein

[7] Radicella N., and Pavo'n D., gr-qc/1006.3745 (2010)

[8] Spergel D. N., et al, astro-ph/0603449 (2006)

[9] Guo Z-K. and Zhang Y-Z, astro-ph/0509790 (2005); Hirano K., Komiya Z., astro-ph/1006.3641 (2010)

[10] Guo Z-K., Zhang Y-Z., Phys. Lett. B, **645** (2007) 326

[11] Zhou J., Wang B., Gong Y., Abdalla E., gr-qc/0705.1264 (2007)