Dynamical Interpretation of Viscous Term of Chaplygin Gas Cosmology

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Abstract: The dark sector of the Universe is firmly based on the indirect observational findings. The physics related to this part of the Universe is still unsettled. Along with the different proposed candidates, Chaplygin gas which itself carries a history of modifying form offers an attracting feature of interpolating between dark matter to dark energy at different conditions. Just like the other extension, Chaplygin gas has been investigated endowed with viscous term to fit the experimental data by different research works. The evolution of the Universe finds appealing role of bulk viscosity. But the viscosity term in Chaplygin gas is put by hand considering the phenomenological aspects. In our work, we find a dynamical interpretation of the viscous term following the energy conservation equation. The dynamical equation of the scale factor, so obtained, relates the coefficient of viscosity to different parameters of Chaplygin gas cosmology.

Keywords: dark matter, dark energy, viscous chaplygin gas

1 Introduction

The dark sector of the Universe is not directly confronted entity by the probing agent. It is rather indirect inferences of various observational facts.It is, of course, not new in the history of advancement of natural sciences that the plethora of explanations might be forwarded to explain even one of the observed facts out of many.In case of late-time acceleration of the Universe, various dark energy candidates have been postulated to break the mistry; and side by side an active field of research has been capable of forwarding different ideas. It is not an exaggeration that the cosmological model is in one sense, the model of dark energy since the evolution of cosmology is supposed to be constrained almost entiely by the dark energy with its dominance by about 96% content out of total content of the Universe.

Out of different concepts getting importance in the active field of dark enegy research, cosmic viscosity is also one of these investigated by different angles. The concept is showing attracting feature since it is capable of avoiding some disturbing results.But the weakness of putting the concept in the model is its phenomenogical origin [1]. The classical viscous stress tensor appears in mathematical expression

$$\sigma_{vis;ij} = \xi \partial_k v_k \delta_{ij} + \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v_k \delta_{ij} \qquad (1)$$

where v_k is the k-dimensional component of velocity.

On consideration of incompatibility with the cosmological principle, effective viscosity expression reads

$$\sigma_{ij} = -\delta_{ij}(p - \zeta \partial_k v_k) \qquad (2)$$

The general relativistic form of energy-momentum tensor is found

$$T_{\mu\nu} = \rho U_\mu U_\nu + (p - 3H\xi)h_{\mu\nu} \qquad (3)$$

where ρ and p are respectively the energy density and isotropic pressure of perfect fluid. The four velocity of the cosmic fluid in comoving coordinates is represented by $U^\mu = (1,0,0,0)$ whereas the projection tensor is represented by $h_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$ and $3H = \partial_\mu U^\mu$

The effective pressure emerges from equation (3) which includes the contribution from bulk viscosity is

$$p = w\rho - 3H\xi \qquad (4)$$

where ξ is the bulk viscosity parameter and the equation of state (EOS) parameter for cosmic fluid without viscosity is w.

To find out the possible physical origin of cosmic viscosity different collision/interaction model in cosmology is being investigated [1] . On the basis of our own work for different context [2], we search an kinematical expression for viscous term which is capable of giving some indication for physical origin of viscous term.The discussion on the kinematic term might be the the guiding explanation for finding the dynamical origin.

2 Kinematical Expression of Viscous Term

Our previous work of other context which now suffices for present work was the studies generalised Chaplygin gas in persepective of Brane based concept. The studies begins from the work of Saaidi et al on brane cosmology with generalised Chaplygin gas in the bulk [3]. Five dimensional spacetime (5- D) serves the background for Brane-world scenario.Here the expression for scale factor and modified Friedmann equation read

$$\dot{a}_b^2 = \frac{\kappa^4}{36}\rho_0^2 + \frac{\kappa^2}{6}\rho_B a_b^2 - \frac{\zeta}{a_b^2} - \kappa \qquad (5)$$

And

$$H^2 = \left(\frac{\dot{a}_b}{a_b}\right)^2 = \frac{\kappa^4}{36}\left(\frac{\rho_0^2}{a_b^2}\right) + \frac{\kappa^2}{6}\rho_B - \frac{\zeta}{a_b^4} - \frac{\kappa}{a_b^2} \qquad (6)$$

where subscript b stands for brane and subscript B indicates bulk.

For the present aim, it becomes helpful to reproduce our previous mathematical findings to arrive at some new conclusion. It is almost straight-forward from the equation (5) that a kinematical viscous-like term can be obtained on direct derivation. We intend to investigate the scenario demanding the bulk candidate as modified Chaplygin gas, and hence first reproduce our previous result:

Remembering that the physical origin of cosmic viscosity somehow must be implicit in some interaction, we consider the interacting dark energy model [4]. We only assume self interacting dark energy.So the interaction term is $\Gamma = 3\varepsilon H\rho_B$, where ε ought to be non-negative and small. Now with interaction term, conservation relation on the bulk is

$$\dot{\rho}_B + 3\frac{\dot{a}}{a}(\rho_B + P_B) + \frac{\dot{b}}{b}(\rho_B + P_B) = 0 \tag{7}$$

After simplification, we obtain

$$\dot{\rho}_B + 3\frac{\dot{a}}{a}[(1+\varepsilon)\rho_B + P_B] + \frac{\dot{b}}{b}(\rho_B + P_B) = 0 \tag{8}$$

Since in our consideration, bulk candidate is modified chaplygin gas, we have

$$P_B = \gamma\rho_B - \frac{A}{\rho_B^{\alpha}} \tag{9}$$

Where γ and A are two positive constant, and $0 < \alpha \leq 1$
On substitution of this equation of state in the conservation equation, we obtain

$$\rho_B^{\alpha+1} = \frac{A}{1+\varepsilon+\gamma} + \frac{C}{(a^3 b)^{(\alpha+1)(1+\varepsilon+\gamma)}} \tag{10}$$

Where C is the constant of integration.

As modified chaplygin gas has the status perfect fluid, so

$$w = \gamma - \frac{A}{\rho_B^{\alpha+1}}$$

Where w is the state parameter. Taking the present magnitude of the state parameter as w_0, and already obtained relation in the basis of the mathematical structure [3]

$$\frac{\dot{a}(t,y)}{a(t,y)} = \frac{\dot{b}(t,y)}{b(t,y)} = \frac{\dot{a}_b(t)}{a_b(t)} \tag{11}$$

we have the expression

$$\rho_B^{\alpha+1} = \frac{\rho_{B0}^{\alpha+1}}{A+(1+\varepsilon+w_0)}\left[A + \frac{(1+\varepsilon+w_0)\rho_{B0}^{\alpha+1}}{a_b^{4(\alpha+1)(1+\varepsilon+\gamma)}}\right] \tag{12}$$

Now, let us suppose

$$A = n\rho_{B0}^{\alpha+1}$$

Where n some numerical number only. Substituting this for A in equation (15), we obtain

$$\rho_B^{\alpha+1} = \frac{1}{1+\varepsilon+n+w_0}\left[n + \frac{1+\varepsilon+w_0}{a_b^{4(\alpha+1)(1+\varepsilon+n+w_0)}}\right]\rho_{B0}^{\alpha+1} \tag{13}$$

Let $\frac{1}{1+\varepsilon+n+w_0} = K$, some constant, we can have

$$\rho_B^{\alpha+1} = K\left[n + \frac{1+\varepsilon+w_0}{a_b^{4(\alpha+1)(1+\varepsilon+n+w_0)}}\right]\rho_{B0}^{\alpha+1} \tag{14}$$

Therefore

$$\rho_B = K^{\frac{1}{\alpha+1}}\left[n + \frac{1+\varepsilon+w_0}{a_b^{\frac{4(\alpha+1)}{K}}}\right]^{\frac{1}{\alpha+1}}\rho_{B0} \tag{15}$$

The modified Friedmann equation (6) is considered in the approximate form as

$$H^2 = \left(\frac{\dot{a}_b}{a_b}\right)^2 = \frac{\kappa^4}{36}\left(\frac{\rho_0^2}{a_b^2}\right) + \frac{\kappa^2}{6}\rho_B \tag{16}$$

The other two terms are ignored as we are focused to late-time universe. The retained first term is very crucial as it radically changes the standard cosmology.
Now from equation (15) and (16)

$$H^2 = \left(\frac{\dot{a}_b}{a_b}\right)^2$$
$$= \frac{\kappa^4}{36}\left(\frac{\rho_0^2}{a_b^2}\right) + \frac{\kappa^2}{6}K^{\frac{1}{\alpha+1}}\left[n + \frac{1+\varepsilon+w_0}{a_b^{\frac{4(\alpha+1)}{K}}}\right]^{\frac{1}{\alpha+1}}\rho_{B0} \tag{17}$$

Now, let us follow some simplification:
Assuming, $\frac{4(\alpha+1)}{K} \cong 4(\alpha+1)$, we have

$$K^{\frac{1}{\alpha+1}}\left[n + \frac{1+\varepsilon+w_0}{a_b^{\frac{4(\alpha++1)}{K}}}\right] = (Kn)^{\frac{1}{\alpha+1}}$$

On consideration of this simplification, we arrive from equation (17)

$$(\dot{a}_b)^2 = \frac{\kappa^4}{36}\rho^2 + \frac{\kappa^2}{6}(Kn)^{\frac{1}{\alpha+1}}\rho_{B0}a_b^2 \tag{18}$$

And

$$\ddot{a}_b = \frac{\kappa^2}{6}(Kn)^{\frac{1}{\alpha+1}}\rho_{B0}\dot{a}_b \tag{19}$$

Moreover following some steps and assuming $\frac{\kappa^4}{36}\rho_o^2 = P^2$ and $\frac{\kappa^2}{6}(Kn)^{\frac{1}{\alpha+1}}\rho_{B0} = Q^2$, we have

$$\log\left[\frac{a_b}{\frac{P^2}{Q^2}} + \sqrt{1+\left(\frac{a_b}{\frac{P^2}{Q^2}}\right)^2}\,\right]\Bigg|_{t_*}^{t}$$
$$= \left(\frac{1}{1+n+\varepsilon+w_0}\right)^{\frac{1}{2(\alpha+1)}}\left[\frac{\kappa^2}{6}n^{\frac{1}{\alpha+1}}\right]^{\frac{1}{2}}(t-t_*) \tag{20}$$

3 Discussion

From equation (19), it is observed that a viscous -like force is acting on the brane, and it might be suggested $\xi \equiv Q = \frac{\kappa^2}{6}(Kn)^{\frac{1}{\alpha+1}}\rho_{B0}$.So the presence of the bulk candiadate here it is Chaplygin gas, creates some tension which is of viscous like nature. It is suggestive from equation (20) that viscous is not due from dark energy self interaction but ξ should have some exponential evolving behaviour.

References

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