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Massive scalar field quasinormal modes of a black hole in the deformed Hořava-Lifshitz gravity

Research Article

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Abstract: We calculated the quasinormal modes of massive scalar field of a black hole in the deformed Hořava-Lifshitz gravity with coupling constant $\lambda = 1$, using the third-order WKB approximation. Our results show that when the scalar field mass increases, the oscillation frequency increases while the damping decreases. And we find that the imaginary parts are almost linearly related to the real parts, the behaviors are very similar to that in the Reissner-Nordström black hole spacetime. These information will help us understand more about the Hořava-Lifshitz gravity.

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1. Introduction

Recently, motivated by the Lifshitz model in condensed matter physics, Hořava [1] proposed a renormalization of the gravity theory in the (3+1)-dimensional quantum gravity. This theory is believed to be the potential ultraviolet (UV) completion of Einstein gravity, and it reduces to general relativity in the infrared (IR) limit (setting the dynamical coupling constant $\lambda = 1$ in the action). Since then, various aspects on this gravity theory [2–13] have been investigated, and some static spherically symmetric black hole solutions [14–17] have been given.

In order to reduce the number of independent coupling constants, the detailed balance condition was imposed in the Hořava-Lifshitz (HL) gravity. One result from the imposition is that the generic IR vacuum of the HL gravity is not a Minkowski vacuum but the one in an anti-de Sitter background. Besides, in the IR limit, both the Newton constant and the speed of light are dependent on the cosmological constant. For the aim to make the HL gravity recover the Minkowsi vacuum in the IR limit, the theory was modified by adding a relevant operator proportional to the Ricci scalar of the three-geometry "$\mu^4 R$" to the original action in [14]. Such a modification softly breaks the detailed balance condition and changes the IR properties, but the ones at the UV level are not affected. In terms of the IR modified HL gravity, a static, spherically symmetric black hole solution was presented.

On the other hand, black hole quasinormal modes (QNMs) govern the decay of perturbations at intermediate times and are important when studying the dynamics of black holes. The QNMs of a black hole are defined as proper

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solutions of the perturbation equations belonging to certain complex characteristic frequencies which satisfy the boundary conditions appropriate for purely ingoing waves at the event horizon and purely outgoing waves at infinity [24, 25]. The quasinormal modes of a black hole present complex frequencies, the real parts of QNMs represent the ring down frequency and the imaginary parts represent the decay time. The QNMs are independent of the initial perturbation, so, one can extract information about the physical parameters of the black hole, viz., mass, electric charge and angular momentum [18–26] through it. As a result, it is generally believed that QNMs carry a unique footprint to directly identify the existence of a black hole.
In this paper, we use the third-order Wentzel-Kramers-Brillouin (WKB) approximation method to calculate the QNMs of massive scalar field perturbation around a black hole in the deformed HL gravity, and give the effect of the scalar field mass on the QNMS.

2. Metric and massive scalar field perturbation

Using the Arnowitt-Deser-Misner (ADM) decomposition and introducing the lapse function N, the shift vector N_i and the spatial metric g_{ij}, we have [14]

$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (1)$$

and the IR modification HL action which reads

$$\begin{aligned} I = \int dtd^3x\sqrt{g}N\Big\{ &\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) \\ &+ \frac{\kappa^2\mu^2(\Lambda_W R - 3\Lambda_W^2)}{8(1-3\lambda)} + \frac{\kappa^2\mu^2(1-4\lambda)}{32(1-3\lambda)}R^2 \\ -\frac{\kappa^2}{2w^4}\left(C_{ij} - \frac{\mu w^2}{2}R_{ij}\right)&\left(C^{ij} - \frac{\mu w^2}{2}R^{ij}\right) + \mu^4 R\Big\}, \end{aligned} \quad (2)$$

where K_{ij} is extrinsic curvature

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i\right), \quad (3)$$

and C_{ij} is the Cotton tensor

$$C_{ij} = \epsilon^{ijk}\nabla_k(R_l^j - \frac{1}{4}R\delta_l^j), \quad (4)$$

where $\lambda, \mu, \kappa, w = \frac{8\mu^2(3\lambda-1)}{\kappa^2}$ are constant parameters, and R and R_{ij} are three-dimensional scalar curvature and Ricci tensor, respectively. The last term "$\mu^4 R$" in (2) is the added term, which softly violates the detailed balance condition in [1], modifies the IR properties of the HL gravity but not amend the UV behaviors.
From the metric in the (3+1) ADM formalism, the Einstein-Hilbert action can be expressed as

$$I_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{g}N(K_{ij}K^{ij} - K^2 + R - 2\Lambda_W), \quad (5)$$

where we have introduced the coordinate $x^0 = ct$.
In the IR limit, $\Lambda_W \to 0$, one compares the IR modified action (2) with the Einstein-Hilbert action and reads the parameter λ, the speed of light c, the Newton's constant G as

$$\Lambda_W = 0, \quad \lambda = 1, \quad c^2 = \frac{\kappa^2\mu^4}{2}, \quad G = \frac{\kappa^2}{32\pi c}. \quad (6)$$

In the background of deformed HL gravity, a spherically symmetric black hole solution that satisfies the full set of motion equations is presented as [14]

$$ds^2 = -\frac{2(r^2 - 2Mr + \alpha)}{r^2 + 2\alpha + \sqrt{r^4 + 8\alpha Mr}}dt^2$$

$$+\frac{r^2 + 2\alpha + \sqrt{r^4 + 8\alpha Mr}}{2(r^2 - 2Mr + \alpha)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (7)$$

where M is an integration constant related to the black hole mass, $\alpha = \frac{1}{2w} = \frac{\kappa^2}{32\mu^2}$ is the Hořava-Lifshitz parameter. This black hole is formally different from the usual Schwarzschild black hole in the Einstein's general relativity. If $\alpha = 0$, the metric (7) exactly coincides with the Schwarzschild black hole.
There are two event horizons

$$r_\pm = M \pm \sqrt{M^2 - \alpha} \quad (8)$$

are very similar with those of the Reissner-Nordström (RN) black hole. $M \geq \alpha$ to avoid the naked singularity, and $M = \alpha$ is the extremity condition. Some associated properties of the black hole (7) have been investigated in [27–33]. In this paper, our main purpose is to discuss the effect of the scalar field mass on the quasinormal modes of the deformed Hořava-Lifshitz black hole.
The massive scalar field in a curved background is governed by the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu})\Psi = u^2\Psi \quad (9)$$

where Ψ is the scalar field, u is the mass of scalar field.

After separate variables $\Psi(t,r,\theta,\varphi) = \frac{1}{r}\Phi(r)Y_{lm}(\theta,\varphi)e^{-i\omega t}$, the radial equation for the massive scalar field perturbation can be obtained, in the deformed Hořava-Lifshitz gravity spacetime

$$\frac{d^2\Phi(r)}{dr_*} + [\omega^2 - V(r)]\Phi(r) = 0 \tag{10}$$

where r_* is the tortoise coordinate

$$dr_* = \frac{r^2 + 2\alpha + \sqrt{r^4 + 8\alpha Mr}}{2(r^2 - 2Mr + \alpha)}dr \tag{11}$$

and the effective potential $V(r)$ is given by

$$V(r) = \frac{2(r^2 - 2Mr + \alpha)}{r^2 + 2\alpha + \sqrt{r^4 + 8\alpha Mr}} \times \left[\frac{l(l+1)}{r^2} + \frac{1}{\alpha} - \frac{r^3 + 2\alpha M}{\alpha r\sqrt{r^4 + 8\alpha Mr}} + u^2\right] \tag{12}$$

It is obvious that the effective potential can be reduced to the Schwarzschild black hole when $\alpha = 0$. In this paper, we only investigate the influence of the scalar field mass u to the quasinormal frequencies. In Fig. 1, we show how the variation of $V(r)$ depends on u in the deformed Hořava-Lifshitz gravity, and compare with that of Reissner-Nordström black hole. We find that the effective potential depends on the scalar field mass, which is in the form of a barrier. With other parameters fixed, the peak value of the barrier and the location of peak gets higher and higher as u increases, respectively. The behavior is very similar to that in Reissner-Nordström black hole background spacetime.

3. Massive scalar quasinormal frequencies

In this section, we evaluate the quasinormal frequencies for the massive scalar field in the background spacetime (1) by using the third-order WKB approximation [34, 35]. The WKB approach is based on the analogy with the problem of waves scattering near the peak of the potential barrier $V(r)$ in quantum mechanics, where ω^2 plays a role of energy. This semianalytic method originally developed by Schutz and Will [36] and has been extended to the sixth order [37]. This method has been found to be accurate for low-lying modes with $l > n$, while is not so good if $l = n$ and not at all applicable for $l < n$. It is very useful on calculating the quasinormal modes of various black hole cases, such as Schwarzschild [35], Reissner-Nordström [38], Kerr [39], Kerr-Newman [40] and so on. In this method, the formula for the complex quasinormal frequencies ω is

$$\omega^2 = [V_0 + (-2V_0'')^{1/2}\Lambda] - i(n+\frac{1}{2})(-2V_0'')^{1/2}(1+\Omega) \tag{13}$$

where

$$\Lambda = \frac{1}{(-2V_0'')^{1/2}}\left\{\frac{1}{8}\left(\frac{V_0^{(4)}}{V_0''}\right)\left(\frac{1}{4}+\alpha^2\right) - \frac{1}{288}\left(\frac{V_0'''}{V_0''}\right)^2(7+60\alpha^2)\right\} \tag{14}$$

$$\Omega = \frac{1}{-2V_0''}\left\{\frac{5}{6912}\left(\frac{V_0'''}{V_0''}\right)^4(77+188\alpha^2) - \frac{1}{384}\left(\frac{V_0'''^2V_0^{(4)}}{V_0''^3}\right)(51+100\alpha^2) + \frac{1}{2304}\left(\frac{V_0^{(4)}}{V_0''}\right)^2(67+68\alpha^2) + \frac{1}{288}\left(\frac{V_0'''V_0^{(5)}}{V_0''^2}\right)(19+28\alpha^2) - \frac{1}{288}\left(\frac{V_0^{(6)}}{V_0''}\right)(5+4\alpha^2)\right\}. \tag{15}$$

and

$$\alpha = n + \frac{1}{2}, \qquad V_0^{(n)} = \frac{d^nV}{dr_*^n}\bigg|_{r_*=r_*(r_p)} \tag{16}$$

where n is the overtone number and r_p is the value of polar coordinate r corresponding to the peak of the effective potential $V(r)$. Firstly, we need to find the value of r_p and then substituting (12) into the formula above, we can obtain the quasinormal modes frequencies of the massive scalar field perturbation in the deformed Hořava-Lifshitz black hole. Next, the values of quasinormal modes for $l = 1,2,3,4$ are listed in Tables and Figures, and compared the variation in the behaviour of the real parts and imaginary parts on the scalar field mass with Reissner-Nordström black hole. The case for $l = 0$ are not considered here, since the WKB approach accuracy is not sufficient in this case.

4. Summary and discussion

In the deformed Hořava-Lifshitz gravity, We have evaluated the massive scalar field quasinormal frequencies in a black hole spacetime by using the third-order WKB approximation. These results show that scalar field mass u plays an important role for the quasinormal frequencies.

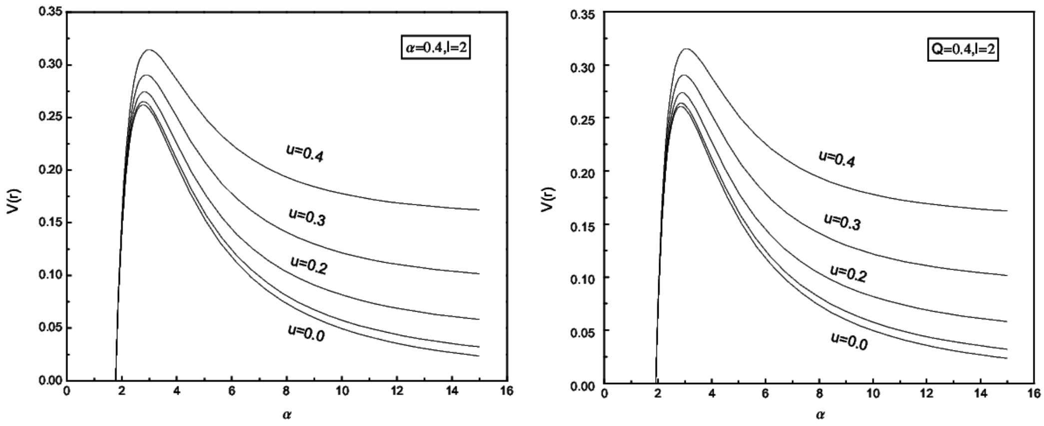

Figure 1. The variation of the effective potentials $V(r)$ versus radial coordinate r with the scalar field mass $u = 0.0, 0.1, 0.2, 0.3, 0.4$. The left is for the deformed Hořava-Lifshitz black hole with the parameter $\alpha = 0.4, l = 2$, and the right for the Reissner-Nordström black hole with the parameter $Q = 0.4, l = 2$.

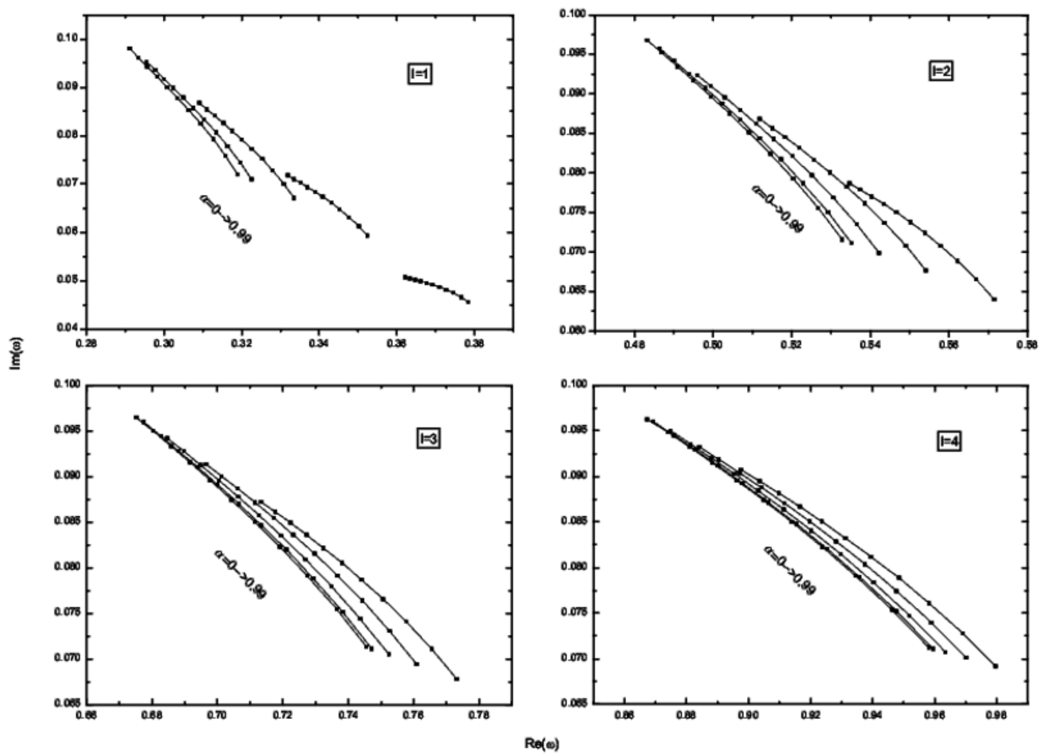

Figure 2. The relation of real parts and magnitudes of imaginary parts of the massive scalar quasinormal frequencies for the potential V : $l = 1, 2, 3, 4$; $n = 0$. The lines are drawn for $u = 0.0, 0.1, 0.2, 0.3, 0.4$.

The data of Table 1 is obtained by using the WKB method to a black hole in deformed Hořava-Lifshitz gravity, with the variation of scalar field mass u and the parameter α (for fixed $l = 2$ and $n = 0$), and Table 2 is under the Reissner-Nordström black hole background with different u and Q. Explicitly, in Fig. 2 and Fig. 3 we plot the relationship between the real and imaginary parts of quasinormal frequencies in deformed HL gravity. From the Tables and Figures, we find that the magnitudes of the imaginary parts of the quasinormal frequencies de-

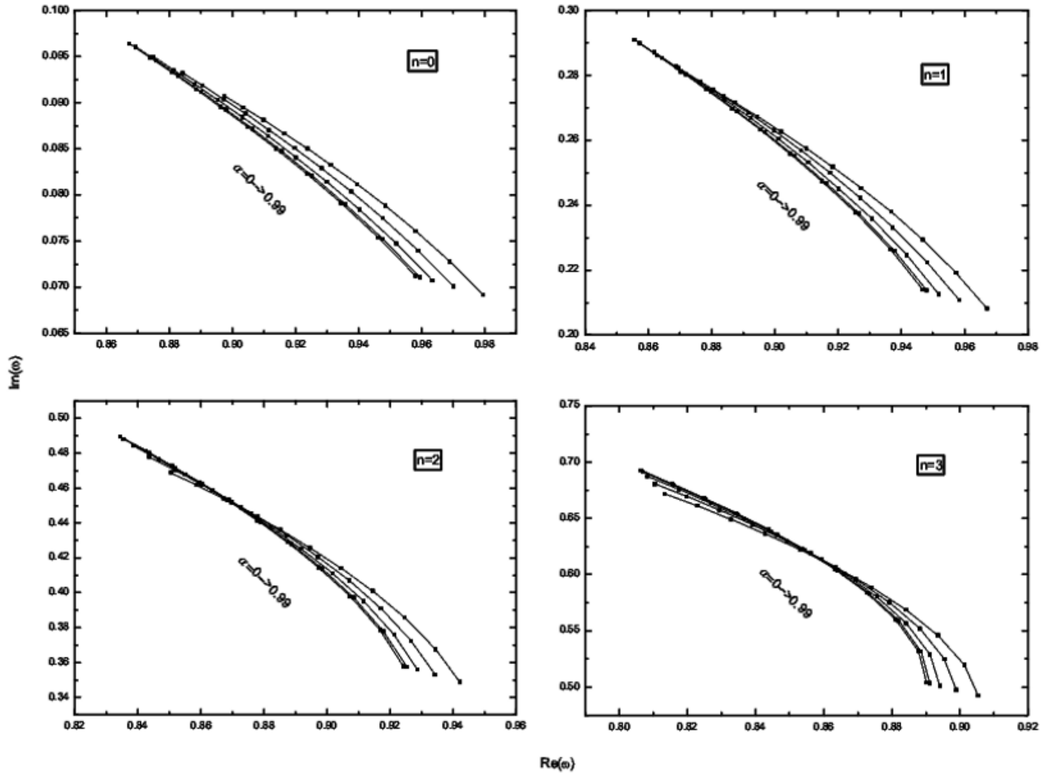

Figure 3. The relation of real parts and magnitudes of imaginary parts of the massive scalar quasinormal frequencies for the potential V : $l = 4$, $n = 0, 1, 2, 3$. The lines are drawn for $u = 0.0, 0.1, 0.2, 0.3, 0.4$.

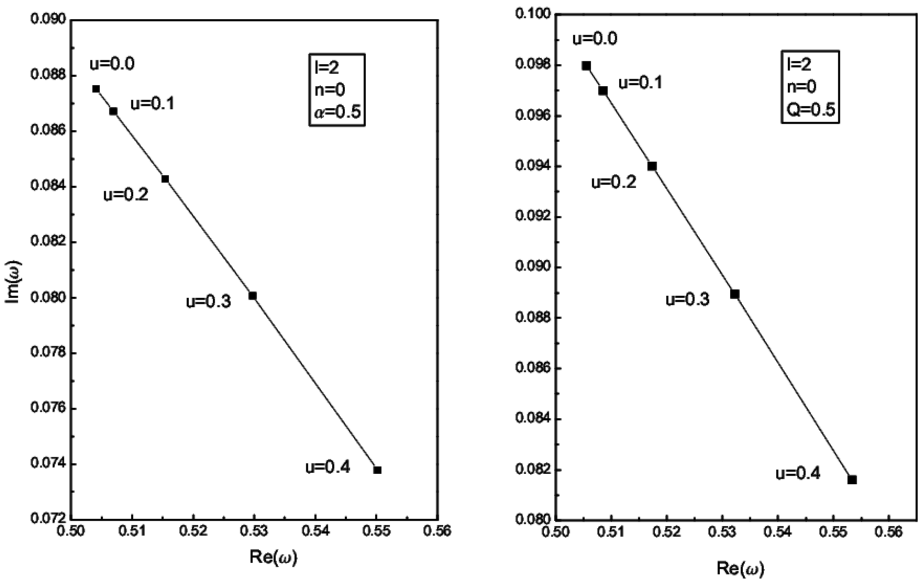

Figure 4. The relation of real parts and magnitudes of imaginary parts of the massive scalar quasinormal frequencies for the potential V : $l = 2$, $n = 0$. The left line is for the deformed HL black hole with $\alpha = 0.5$, and the right line for RN black hole with $Q = 0.5$.

Table 1. The quasinormal modes frequencies of massive scalar field for $l = 2$ and $n = 0$, in the deformed Hořava-Lifshitz gravity background.

α	$\omega(u = 0.0)$	$\omega(u = 0.1)$	$\omega(u = 0.2)$	$\omega(u = 0.3)$	$\omega(u = 0.4)$
0.0	$0.48321 - 0.09681i$	$0.48637 - 0.09572i$	$0.49589 - 0.09243i$	$0.51191 - 0.08683i$	$0.53466 - 0.07870i$
0.1	$0.48693 - 0.09521i$	$0.49002 - 0.09418i$	$0.49936 - 0.09105i$	$0.51507 - 0.08570i$	$0.53740 - 0.07791i$
0.2	$0.49085 - 0.09351i$	$0.49388 - 0.09253i$	$0.50302 - 0.08957i$	$0.51841 - 0.08449i$	$0.54030 - 0.07705i$
0.3	$0.49501 - 0.09186i$	$0.49797 - 0.09076i$	$0.50690 - 0.08796i$	$0.52195 - 0.08316i$	$0.54338 - 0.07609i$
0.4	$0.49942 - 0.08970i$	$0.50231 - 0.08884i$	$0.51102 - 0.08622i$	$0.52571 - 0.08169i$	$0.54665 - 0.07501i$
0.5	$0.50412 - 0.08752i$	$0.50693 - 0.08673i$	$0.51541 - 0.08429i$	$0.52972 - 0.08007i$	$0.55013 - 0.07379i$
0.6	$0.50915 - 0.08511i$	$0.51188 - 0.08438i$	$0.52011 - 0.08214i$	$0.53401 - 0.07824i$	$0.55387 - 0.07239i$
0.7	$0.51456 - 0.08239i$	$0.51719 - 0.08173i$	$0.52516 - 0.07970i$	$0.53862 - 0.07614i$	$0.55789 - 0.07077i$
0.8	$0.52039 - 0.07926i$	$0.52293 - 0.07868i$	$0.53060 - 0.07688i$	$0.54359 - 0.07371i$	$0.56222 - 0.06885i$
0.9	$0.52671 - 0.07557i$	$0.52915 - 0.07508i$	$0.53650 - 0.07354i$	$0.54898 - 0.07080i$	$0.56691 - 0.06654i$
0.99	$0.53285 - 0.07157i$	$0.53518 - 0.07117i$	$0.54223 - 0.06991i$	$0.55421 - 0.06763i$	$0.57147 - 0.06398i$

Table 2. The quasinormal modes frequencies of massive scalar field for $l = 2$ and $n = 0$, in the Reissner-Nordström black hole background.

Q	$\omega(u = 0.0)$	$\omega(u = 0.1)$	$\omega(u = 0.2)$	$\omega(u = 0.3)$	$\omega(u = 0.4)$
0.0	$0.48321 - 0.09681i$	$0.48637 - 0.09572i$	$0.49589 - 0.09243i$	$0.51191 - 0.08683i$	$0.53466 - 0.07870i$
0.1	$0.48403 - 0.09686i$	$0.48718 - 0.09577i$	$0.49667 - 0.09250i$	$0.51264 - 0.08691i$	$0.53533 - 0.07881i$
0.2	$0.48650 - 0.09701i$	$0.48963 - 0.09594i$	$0.49906 - 0.09270i$	$0.51489 - 0.08718i$	$0.53739 - 0.07916i$
0.3	$0.49076 - 0.09726i$	$0.49384 - 0.09621i$	$0.50313 - 0.09303i$	$0.51875 - 0.08761i$	$0.54094 - 0.07975i$
0.4	$0.49700 - 0.09759i$	$0.50002 - 0.09657i$	$0.50912 - 0.09348i$	$0.52442 - 0.08821i$	$0.54615 - 0.08056i$
0.5	$0.50556 - 0.09798i$	$0.50850 - 0.09700i$	$0.51735 - 0.09402i$	$0.53223 - 0.08895i$	$0.55335 - 0.08160i$
0.6	$0.51699 - 0.09835i$	$0.51982 - 0.09742i$	$0.52834 - 0.09461i$	$0.54267 - 0.08980i$	$0.56301 - 0.08282i$
0.7	$0.53217 - 0.09859i$	$0.53487 - 0.09772i$	$0.54297 - 0.09511i$	$0.55660 - 0.09065i$	$0.57593 - 0.08416i$
0.8	$0.55267 - 0.09834i$	$0.55518 - 0.09757i$	$0.56275 - 0.09522i$	$0.57547 - 0.09121i$	$0.59351 - 0.08537i$
0.9	$0.58154 - 0.09662i$	$0.58381 - 0.09597i$	$0.59066 - 0.09400i$	$0.60217 - 0.09064i$	$0.61851 - 0.08571i$
0.99	$0.62054 - 0.09023i$	$0.62251 - 0.08979i$	$0.62844 - 0.08841i$	$0.63841 - 0.08603i$	$0.65259 - 0.08250i$

crease as the scalar field mass u increases for fixed the other parameters, while the real parts increase. And, the more interesting feature is discovered in Fig. 4. The imaginary parts are almost linearly related to the real parts, and the behaviors are very similar to the usual RN black hole. The phenomenon for these two black holes come from the similar variation of the effective potentials with the scalar field mass displayed in Fig. 1. In addition, Chen and Jing in [32] have shown that the dependence of quasinormal modes of the black hole in deformed HL gravity on the parameter α is different from that on the charge Q of the RN black hole in Einstein gravity theory. Considering these similar and different informations between black holes will be helpful for the future work to detect the Hořava-Lifshitz gravity deeply.

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