

## TOPLESS MODEL FOR GRAND UNIFICATION

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### ABSTRACT

A gauge model unifying weak, electromagnetic, and strong interactions is presented. It is based on the group  $SU_L(3) \otimes SU_R(3) \otimes SU_C(3) \otimes P$  with a complete flavor-color permutation symmetry. All fermions are in one irreducible representation of this group. This representation contains a new heavy lepton  $\lambda$  and a new b-like quark  $h$  but no top quark besides  $u$  and  $c$ . The model has a global  $E_6 \otimes P$  symmetry before the local gauging i.e. in the limit of vanishing gauge coupling. A global  $SU(4)$  subgroup which may survive the spontaneous symmetry breaking leads to mass relations. The weak decays of  $b$  and  $h$  quarks can proceed through non-diagonal neutral currents providing for a specific signature. The weak mixing angle is in accord with experiments.

Local gauge invariance is now generally assumed to be the basis for all elementary particles forces /1/. The difference between strong and weak interactions lies mainly in their different coupling strengths. This difference is presumably a result of a spontaneous symmetry breaking which also leads to the confining phase for quarks and gluons on the one hand and to the plasma phase in the case of color singlet hadrons, leptons, and intermediate vector bosons on the other. The minimal gauge group describing these interactions is

$$G_{\text{minimal}} = SU_L(2) \otimes U(1) \otimes SU_C(3) \quad (1)$$

which involves 3 different gauge coupling constants.

It is well known that the group  $SU(5)$  can be used to unite all three interactions simultaneously  $/2,3/$ . This is the minimal group for a grand unification (the flavor subgroup of  $SU(5)$  is just  $SU(2) \otimes U(1)$ ). The  $5$  and  $10^*$  representations of  $SU(5)$  if taken together represent particle families with 15 members, for instance

$$\begin{aligned} \tilde{5} &= \nu_e, e^-, \hat{d} & 10^* &= e^+, (\begin{smallmatrix} u \\ d \end{smallmatrix}), \hat{u} & \text{electron family} & (2) \\ \tilde{5} &= \nu_\mu, \mu^-, \hat{s} & 10^* &= \mu^+, (\begin{smallmatrix} c \\ s \end{smallmatrix}), \hat{c} & \text{myon family} & . \end{aligned}$$

For convenience one deals in grand unified models with two component (left handed) Weyl spinor fields only. In this notation the Dirac-field for the up quark, e.g., can be written  $(\begin{smallmatrix} u \\ \sigma_2 \hat{u}^* \end{smallmatrix})$  where  $\hat{u}$  is an antiquark (left handed) Weyl spinor. If the heavy lepton  $\tau$  (1.8 GeV) and the constituent of the ypsilon  $b$  ( $\approx 4.7$  GeV) also belong to such a reducible multiplet of  $SU(5)$ , a top quark  $t$  is necessary to complete the  $\tau$ -family. To have all 15 particles in one irreducible representation, one needs to extend  $SU(5)$  to  $SO(10)/4,3/$ . Here the lowest non-trivial representations are 10 and 16 which decompose with respect to the  $SU(5)$  subgroup according to

$$\tilde{10} = \tilde{5} + \tilde{5}^* \quad \tilde{16} = \tilde{1} + \tilde{5} + \tilde{10}^* \quad (3)$$

The 16 representation can thus be used to describe a lepton-quark family with one additional neutral lepton. However, it is not clear why nature favors the 16 representation of  $SO(10)$  instead of the lower 10 representation. Also, these models give no explanation why at least three such families of fermions exist.

In this talk we present an alternative to the  $SU(5) - SO(10)$  scheme. In particular, we find it attractive to perform a full unification of the flavor interactions (weak and electromagnetic) independent of the strong forces. The final grand unification will then combine the flavor and color dynamics. In this case, the new flavor group should give the correct electromagnetic charges of quarks  $(2/3, -1/3)$  and leptons  $(0, \pm 1)$  naturally. An obvious way to do this is to use an  $SU(3)$  group under which the quarks transform as flavor triplets and the leptons as flavor octets and singlets. It was recently shown by Okubo  $/5/$  that this is actually the only possibility. The minimal model of this type  $/5,6/$  which contains the observed currents is based on the flavor group  $/8/$

$$G_{\text{flavor}} = SU_L(3) \otimes SU_R(3) \otimes P_{LR} \quad (4)$$

$P_{LR}$  is a parity operation which interchanges the left (L) and right (R)  $SU(3)$  groups. Because of parity invariance there is only one gauge coupling constant. The observed weak and electromagnetic currents generate the  $SU_L(2) \otimes U(1)$  subgroup of (4). The charges are defined by the diagonal sum  $Q = Q_L + Q_R$ . The left-handed quarks are taken to be the following flavor triplets:

$$\begin{array}{cc}
 q = (\underline{3}, \underline{1}) & Q = (\underline{3}, \underline{1}) \\
 \begin{array}{c} d \quad u \\ \diagdown \quad \diagup \\ \quad b \end{array} & \begin{array}{c} s \quad c \\ \diagdown \quad \diagup \\ \quad h \end{array}
 \end{array} \quad (5)$$

This will lead to a "topless" model with two  $SU_L(2)$  singlet quarks of charge  $-1/3$ . One of these could be the constituent of the ypsilon. Let us note that the new hypercharge of  $SU_L(3)$  and  $SU_R(3)$  is not related to the usual hypercharge quantum number of the strange quark.

The parity transformation  $P_{LR}$  operates on the quark fields  $q(x)$  as follows

$$P_{LR} q(x) P_{LR}^{-1} = q_R(\bar{x}) \equiv \sigma_2 \hat{q}^*(\bar{x}) \quad (6)$$

where  $q_R$  is a right-handed Weyl spinor and  $\bar{x} = (x_0, -\vec{x})$ . Since  $P_{LR}$  also exchanges  $SU_L(3) \leftrightarrow SU_R(3)$ , the antiquarks  $\hat{q}(x)$  are  $(1, 3^*)$  states in the flavor space.

The gauge group that contains the strong interaction as well is now the group /9/

$$G = SU_L(3) \otimes SU_R(3) \otimes SU_C(3) \otimes P \quad (7)$$

The color part of this group brings about a second gauge coupling constant. In view of the symmetric form (7) it is, however, very suggestive to make the three  $SU(3)$  groups equivalent before spontaneous symmetry breaking.  $P$  will then denote a generalized parity operation which includes, besides the parity operation  $P_{LR}$ , all permutations of the three  $SU(3)$  groups with the property  $P^2 = +1$ . With only one gauge coupling constant (7) unifies the three interactions in a remarkable flavor-color symmetric way. In the following we will explore the implications of the symmetric grand unification group (7).

Of course, as in all grand unification models, a superstrong breaking is required to explain the large difference between the actual

observed values of the strong and electromagnetic coupling constants /10,1/\*. This superstrong breaking is in fact welcome since it renormalizes the weak interaction angle from the large canonical value  $(\sin^2\theta_W)_0 = 3/8$ , down to a value compatible with the experimental findings.

The generators of the group G and thus also the  $2^4$  gauge bosons show again the permutation symmetry inherent in (7):

$$\begin{array}{ccc} (8, \underset{\sim}{1}, \underset{\sim}{1}) & (1, \underset{\sim}{8}, \underset{\sim}{1}) & (1, \underset{\sim}{1}, \underset{\sim}{8}) \\ W_L & W_R & W_C = \text{gluons} \end{array} \quad (8)$$

In order that the total Lagrangian will be invariant, the fermion fields occurring in the Lagrangian must be a representation of the SU(3) groups and of this additional permutation symmetry. We will restrict ourselves to SU(3) singlet and triplets because these are the only ones needed in the color sector. From this condition and the requirement of integer charges for leptons follows the existence of  $5^4$  fermion Weyl fields:

$$\begin{aligned} \ell(x) &= (\underset{\sim}{3}^*, \underset{\sim}{3}, \underset{\sim}{1}) & \ell_R(\bar{x}) &= (\underset{\sim}{3}, \underset{\sim}{3}^*, \underset{\sim}{1}) \\ q(x) &= (\underset{\sim}{3}, \underset{\sim}{1}, \underset{\sim}{3}^*) & q_R(\bar{x}) &= (\underset{\sim}{1}, \underset{\sim}{3}^*, \underset{\sim}{3}) \\ \hat{Q}(x) &= (\underset{\sim}{1}, \underset{\sim}{3}^*, \underset{\sim}{3}) & \hat{Q}_R(\bar{x}) &= (\underset{\sim}{3}, \underset{\sim}{1}, \underset{\sim}{3}^*) \end{aligned} \quad (9)$$

where  $P_{LR} \ell(x) P_{LR}^{-1} = \ell_R(\bar{x})$  etc.

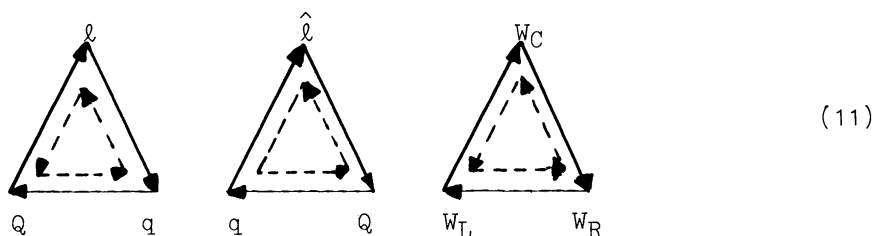
The right-handed spinors  $\ell_R$ ,  $q_R$ ,  $\hat{Q}_R$  can be expressed via conjugate fields of left-handed Weyl spinors. In this way Weyl spinor fields of the corresponding antiparticles are defined:

$$\ell_R \equiv \sigma_2 \hat{\ell}^*, \quad q_R \equiv \sigma_2 \hat{q}^*, \quad \hat{Q}_R \equiv \sigma_2 \hat{Q}^* . \quad (10)$$

The total Lagrangian which contains the  $5^4$  massless Weyl spinors  $\ell$ ,  $q$ ,  $\hat{Q}$ ,  $\hat{\ell}$ ,  $\hat{q}$ ,  $Q$  and the  $2^4$  massless gauge bosons (8) is invariant under the generalized parity transformation P as required. Odd permutations of the three SU(3) groups involve the space reflected fields. Even permutations perform the cyclic change

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\*In the group G (7) the baryonic number is conserved in contrast to the situation in other unifying groups /1/.



In addition to P a charge conjugation operator C which leaves the Lagrangian invariant can be defined\*. Besides the gauge symmetry and the discrete symmetries P, C the Lagrangian has an additional global  $(SU(2))^3$  symmetry. The corresponding symmetry transformations act on the doublets

$$\begin{pmatrix} \ell \\ \hat{\ell} \end{pmatrix} \quad \begin{pmatrix} q \\ Q \end{pmatrix} \quad \begin{pmatrix} \hat{Q} \\ \hat{q} \end{pmatrix} \quad (12)$$

One also finds that the color current is automatically a pure vector current. Formally, the fermion fields, the gauge fields, and the Higgs fields to be discussed later on can all be constructed from the set of elementary subquarks

$$\zeta_L = (\underline{3}, \underline{1}, \underline{1}), \quad \zeta_R = (\underline{1}, \underline{3}, \underline{1}), \quad \zeta_C = (\underline{1}, \underline{1}, \underline{3})$$

and their antiparticles.

The 54 Weyl spinors can be combined into 27 Dirac-spinors consisting of a nonet of leptons with integer charges and two flavor triplets of quarks. The precise way this combination occurs will

\*C changes particle in antiparticle fields, turns upper into lower indices and vice versa, and also interchanges L and R indices. For instance

$$C \ell^i_k(x) C^{-1} = \hat{\ell}^k_i(x)$$

$$P_{LR} \ell^i_k(x) P_{LR}^{-1} = \sigma_2(\hat{\ell}^k_i(\bar{x}))^*$$

(the first index is here an L index, the second index an R index)

of course depend on the mass matrix arising from Higgs-field couplings\*. Thus, the unified gauge group (7) requires the existence of two quark multiplets as it is needed phenomenologically (see (5)).

To name the 54 fermion fields we make the following preliminary identification (before the mixing due to spontaneous symmetry breaking is taken into account)

$$q = \begin{pmatrix} u \\ d \\ b \end{pmatrix} \quad \hat{Q} = (\hat{c}, \hat{s}, \hat{n}) \quad \ell = \begin{pmatrix} N & \lambda^- & \mu^- \\ \tau^+ & \nu_\lambda & \nu_\mu \\ e^+ & \hat{\nu} & M \end{pmatrix} \quad (13)$$

$$Q = \begin{pmatrix} c \\ s \\ h \end{pmatrix} \quad \hat{q} = (\hat{u}, \hat{d}, \hat{b}) \quad \hat{\ell} = \begin{pmatrix} \hat{N} & \tau^- & e^- \\ \lambda^+ & \nu_\tau & \nu_e \\ \mu^+ & \hat{\nu}_\mu & \hat{M} \end{pmatrix} \quad (14)$$

In this presentation  $SU_L(3)$  acts vertically,  $SU_R(3)$  horizontally. Color indices have been suppressed. As anticipated we have no top quark apart from  $u$  and  $c$ . Instead, there are two  $SU_L(2)$  singlet quarks  $b$  and  $h$ . The lepton matrix contains besides  $\mu$  and  $e$  two charged leptons  $\tau$  and  $\lambda$  with a V-A coupling to their neutrinos and a V+A coupling (via conventional W's) to a (heavy?) neutral lepton  $N$ . We note that the quarks  $Q$  have the opposite intrinsic parity compared to the quarks  $q$  and the same is true for the leptons  $e^-$ ,  $\tau^-$  when compared with  $\mu^-$ ,  $\lambda^-$ .

The minimal simple group which incorporates the group  $G$  of (7) is obtained by adding to the 24 generators given in (8) new ones which perform the cyclic change (see (11))

$$\ell \rightarrow q \rightarrow \hat{Q} \rightarrow \ell, \quad \hat{\ell} \rightarrow Q \rightarrow \hat{q} \rightarrow \hat{\ell}$$

and vice versa. The simplest step operators of this kind have the following transformation property /9/

$$(\underline{3}, \underline{3}, \underline{3}) \quad (\underline{3}^*, \underline{3}^*, \underline{3}^*) \quad (15)$$

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\* Some of the neutral leptons could also form Majorana instead of Dirac particles.

and are permutation-symmetric. The 54 new generators together with the 24 generators of the three  $SU(3)$ 's form a closed algebra. The corresponding Lie group is the exceptional group  $E_6$ .  $E_6$  has been much discussed by Gürsey and coworkers /11/ as a candidate for a local gauge symmetry. In our case we must take  $E_6 \otimes P$  with the lowest 54 representation (9)(10): The fermions  $\ell, q, \hat{Q}$  form a 27 representation of  $E_6$  (without parity), the fermions  $\tilde{\ell}, \hat{q}, Q$  another one. Both 27 representations transform into each other by the parity operation. The accidental global  $(SU(2))^3$  symmetry is reduced to the global diagonal  $SU(2)$  acting on the doublets shown in (12). In case one uses the group  $E_6 \otimes P$  as a local gauge group, one obtains according to (15) 54 lepto-quark gauge bosons which have to be made superheavy by a spontaneous symmetry breaking to avoid a fast decay of the proton. A parity invariant superstrong breaking with a Higgs field in the adjoint representation does this job and gives

$$E_6 \otimes P \xrightarrow{H^{78}} SU_L(2) \otimes SU_R(2) \otimes U_L^Y(1) \otimes U_R^Y(1) \otimes P_{LR} \otimes SU_C(3) \quad (16)$$

The direction of breaking defines the new hypercharge  $Y$ . This breaking does not lead to superheavy fermions and gives automatically a superstrong breaking of  $SU_{L,R}(3)$  which is a necessary condition for a large difference between the strong and electromagnetic fine structure constants at ordinary energies. In fact any breaking that gives mass to all lepto-quarks breaks  $SU_L(3) \otimes SU_R(3)$  as well. If no further superstrong breakings occur, one finds for the renormalized weak interaction angle, using the method of Georgi, Quinn and Weinberg /10/

$$\sin^2 \theta_W = \frac{3}{8} \left( 1 - \frac{1}{3} \left( 1 - \frac{8}{3} \frac{\alpha}{\alpha_{\text{strong}}} \right) \right) \approx \frac{1}{4} \quad (17)$$

If one includes an additional superstrong breaking down to the Weinberg-Salam group one obtains values for  $\sin^2 \theta_W$  in the range /10,3,9/

$$0.2 \lesssim \sin^2 \theta_W \lesssim \frac{1}{4} \quad . \quad (18)$$

Using Higgs fields in the adjoint representation alone is not sufficient to reduce  $E_6 \otimes P$  or  $G$  down to the Weinberg-Salam group. One needs in addition a 27 or a  $(3^*, 3, 1)$  field to complete the local breaking which may also be responsible for the suppression of right-handed charged currents.

The local gauging of  $E_6 \otimes P$  is not necessary to achieve the results (16) to (18). One may just as well use the gauge group (7) with the corresponding Higgs fields. In particular, a breaking through the adjoint representation of this group leads again to the subgroup which occurs in (16) and to the result (17). In the following we will use the gauge group (7) as before and take  $E_6 \otimes P$  only as a global classification symmetry before gauging, valid only in the high energy limit where the gauge coupling constant is negligible. (Local  $E_6$  has more problems than the group (7). In particular, it is difficult to avoid sizeable neutrino masses caused by radiative corrections via lepton-quark exchanges.)

With this choice the proton remains absolutely stable. Quark masses are generated by Higgs fields  $(L, R) = (3^*, 3)$ . Lepton masses are induced by Higgs fields transforming as  $(3^*, 3)$ ,  $(6, 3)$ ,  $(3^*, 6^*)$ ,  $(6, 6^*)/12/$ . The irreducible Higgs representations of the global  $E_6$  group which are responsible for the fermion mass terms and contain the above fields appear in the decomposition

$$(27 \quad 27')^* = (27 + 351^*)_{\text{symm.}} + 351'^*_{\text{antisymm.}} \quad (19)$$

Because the superstrong breaking via the adjoint representation is huge compared to the masses of the known fermions, the Higgs representations (19) will be influenced dynamically, and most of their neutral and color singlet members will develop vacuum expectation values which give rise to fermion masses. Simple relations between these masses will exist if there remains a global residual symmetry in the fermion sector. Suggestive candidates for such a residual symmetry are groups which leave some or all of the neutral and color singlet members of the scalar and pseudoscalar 27 Higgs representation invariant /9/. (The 27 is taken because of its relevance for the local breaking of  $G$  and because it is the classifying representation which determines the directions of charge, hypercharge, isospin, and color.) The assumption is that such a residual global symmetry is respected also by the remaining part of the Higgs Lagrangian.

The global group which leaves all the 5 neutral and color singlet member of a 27 Higgs field unchanged is a specific  $SU(4)$  subgroup of  $E_6$  /9,14/. We call it the "charge group". It is generated by the operators

$$SU(4): \{Q_{\text{el.mag.}}, (\underline{1}, \underline{1}, \underline{8}), (\text{up}, \text{up}, \underline{3}), (\text{up}^*, \text{up}^*, \underline{3}^*)\} \quad (20)$$

and contains the locally conserved subgroup  $U_Q(1) \otimes SU_C(3)$ . One finds



$$E_6 \otimes P \supset SU(4) \otimes SU_L^U(2) \otimes SU_R^U(2) \otimes P_{LR} \quad (21)$$

where U denotes the U-spin of SU(3). Decomposing the two "27" in (13), (14) one finds

$$\tilde{27} = (\tilde{6}^*, \tilde{1}, \tilde{1}) + (\tilde{4}, \tilde{1}, \tilde{2}) + (\tilde{4}^*, \tilde{2}, \tilde{1}) + (\tilde{1}, \tilde{2}, \tilde{2}) + (\tilde{1}, \tilde{1}, \tilde{1})$$

$$(u, \hat{c}) \quad ((\lambda^-, \hat{n}), (\mu^-, \hat{s})) \begin{pmatrix} (e^+, d) \\ (\tau^+, b) \end{pmatrix} \begin{pmatrix} v_\lambda & v_\mu \\ \hat{v}_e & M \end{pmatrix} \quad N \quad (22)$$

$$(c, \hat{u}) \quad ((\tau^-, \hat{b}), (e^-, \hat{d})) \begin{pmatrix} (\mu^+, s) \\ (\lambda^+, h) \end{pmatrix} \begin{pmatrix} v_\tau & v_e \\ \hat{v}_\mu & \hat{M} \end{pmatrix} \quad \hat{N}$$

If the fermion couplings to the Higgs field are indeed left SU(4) invariant by the spontaneous symmetry breaking, one can form SU(4) invariant mass terms and obtains after diagonalization the following mass relations

$$\frac{m_d}{m_e} = \frac{m_s}{m_\mu} = \frac{m_h}{m_\lambda} = \frac{m_b}{m_\tau} = 1 \quad . \quad (23)$$

The masses of the u and c quarks remain unrelated. A parity-invariant Higgs breaking would, however, give  $m_u = m_c$ . Thus, the experimental mass relation  $m_c \gg m_u$  is due to spontaneous parity non-conservation and may therefore have the same origin as the relation  $m_{WR} \gg m_{WL}$  which we need in order to obtain the dominance of left-handed charged currents over right-handed charged currents at present energies.

The mass relations (23) are only valid at extreme energies where the gauge coupling constant is very small. Nevertheless, these relations can - with some optimism - be extrapolated down to ordinary energies using the renormalization group techniques [10, 3]. Estimates similar to those performed by Buras et al. [3] for SU(5) simply change the one on the right-hand side of (23) to a larger value:

$$\frac{m_d}{m_e} \approx \frac{m_s}{m_\mu} \approx \frac{m_h}{m_\lambda} \approx \frac{m_b}{m_\tau} \approx 2.6 \quad . \quad (24)$$

Thus, if a new heavy lepton  $\lambda$  is found the quark  $h$  should have a mass roughly 2.6 times as big.

As long as we do not impose further symmetries, in particular discrete ones, the remaining masses and Cabibbo-type angles are free parameters. The fact that in (22)  $\nu_\tau$  occurs together with  $\nu_e$  makes it plausible that the mass of  $\nu_\tau$  is small as it is required experimentally /13/. The lepton  $N$ , on the other hand, may acquire a high mass. The specific form of the  $\tau$ -neutrino decay amplitude depends on the mass of  $N$ . If  $N$  and  $\nu_\tau$  are massless, the  $\tau$ -neutrino vertex will have a pure vector form. If  $N$  has a mass  $m_N \gtrsim m_\tau$ , the  $\tau$ -decay proceeds by a V-A interaction.

An important difference between our scheme and more conventional models appears in the weak decays of particles which contain  $b$  or  $h$  quarks. These decays should proceed through mixings of  $b$  and  $h$  with  $d$  and  $s$  quarks. Consequently, transitions originating from non-diagonal neutral currents are expected, for instance the process

$$B \rightarrow (\pi, K) + e^+ + e^- \quad . \quad (25)$$

Here  $B$  denotes a  $b\hat{q}$  meson. Processes of the type (25), if observed, would strongly support /15/ the "topless" model.

Before concluding one should point out that a very different particle spectrum from the one just discussed could arise even within the framework of the groups  $G$  or  $E_6 \otimes P$  /9/. This happens if a particular vacuum expectation value of a  $27$  Higgs field causes a very strong or superstrong breaking. The maximal subgroup of  $E_6$  which leaves just one vacuum expectation value in  $27$  invariant, is  $SO(10)$ .  $E_6 \otimes P$  is then reduced as follows:

$$E_6 \otimes P \supset SO(10) \otimes U(1) \otimes P_{LR} \quad (26)$$

$$27 \sim 1 + 10 + 16$$

In this case the quarks and leptons in the 10 representation obtain very high and roughly equal masses. We cannot identify them with  $\tau$ ,  $b$ ,  $\nu_\tau$  any more. The particles in the 16 representation, on the other hand, are still massless at this stage and can be identified with the conventional fermions as in (2). We see that for a suitable breaking pattern  $SO(10)$  and  $SU(5)$  are submodels of  $E_6 \otimes P$ , in which the "16" representation has the lowest energy. To incorporate the constituent of the  $\nu_\tau$  and the heavy lepton  $\tau$ , a new fermion representation is needed, however. In this circumstance a top quark is required. Our global charge subgroup  $SU(4)$  which should now govern the residual breakings and masses leads again to the mass relations (25).

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