A MAXIMAL MASS MODEL*

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Abstract

We investigate the possibility to construct a generalization of the Standard Model which we call the Maximal Mass Model because it contains a limiting mass M for its fundamental constituents. The parameter M is considered as a new universal physical constant of Nature and, therefore, is called the fundamental mass. It is introduced in a purely geometric way, like the velocity of light as a maximal velocity in the special relativity. If one chooses the Euclidean formulation of quantum field theory, the adequate realization of the limiting mass hypothesis is reduced to the choice of the de Sitter geometry as the geometry of the 4-momentum space. All fields defined in de Sitter p-space in configurational space obey five dimensional Klein-Gordon type equation with the fundamental mass M as a mass parameter. The role of dynamical field variables is played by the Cauchy initial conditions given at $x_5 = 0$, guaranteeing the locality and gauge invariance principles. The formulation of the theory of scalar and spinor fields corresponding to the geometrical requirements is considered in some detail. By a simple example it is demonstrated that the spontaneous symmetry breaking mechanism leads to renormalization of the fundamental mass M. A new geometric concept of the chirality of the fermion fields is introduced. It would be responsible for new measurable effects at high energies $E \geq M$. Interaction terms of a new type are revealed due to the existence of the Higgs boson. The most intriguing prediction of the new approach is the possible existence of exotic fermions with no analogues in the SM, which may be a candidate for dark matter constituents.

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1 Introductory remarks

For decades we have witnessed the impressive success of the Standard Model (SM) in explaining properties and regularities observed in experiments with elementary particles. The mathematical basis of the SM is local Lagrangian quantum field theory (QFT). The very concept of an elementary particle assumes that it does not have a composite structure. In agreement with the contemporary experimental, data this structure has not been disclosed for any fundamental particles of the SM, up to distances of the order of $10^{-16} - 10^{-17}$ cm. The adequate mathematical images of point like particles are the local quantized fields - boson and spinor. Particles are the quanta of the corresponding fields. In the framework of the SM these are leptons, quarks, vector bosons and the Higgs scalar, all characterized by certain values of mass, spin, electric charge, colour, isotopic spin, hypercharge, etc.

Intuitively it is clear that an elementary particle should carry small enough portions of different "charges" and "spins". In the theory this is guaranteed by assigning the local fields to the lowest representations of the corresponding groups.

As for the mass of the particle m, this quantity is the Casimir operator of the ***noncompact*** Poincaré group and in the unitary representations of this group, used in QFT, they may have arbitrary values in the interval $0 \leq m < \infty$. In the SM one observes a great variety in the mass values. For example, t-quark is more than 300000 times heavier than the electron. In this situation the question naturally arises: up to what values of mass one may apply the concept of a local quantum field? Formally, the contemporary QFT remains a logically perfect scheme and its mathematical structure does not change at all up to arbitrarily large values of masses of quanta. For instance, the free Klein-Gordon equation for the one component real scalar field $\varphi(x)$ has always the form

$$(\Box + m^2)\varphi(x) = 0. \tag{1}$$

Hence, after standard Fourier transform

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int e^{-ip_\mu x^\mu} \; \varphi(p) \, d^{\,4}p \qquad (p_\mu x^\mu = p^0 x^0 - \boldsymbol{p}.\boldsymbol{x}) \tag{2}$$

we find the equation of motion in the Minkowski momentum 4-space:

$$(m^2 - p^2)\varphi(p) = 0, \qquad p^2 = p_0^2 - \boldsymbol{p}^2. \tag{3}$$

From a geometric point of view m is the radius of the "mass shell" hyperboloid

$$m^2 = p_0^2 - \boldsymbol{p}^2, \tag{4}$$

where the field $\varphi(p)$ is defined and in the Minkowski momentum space one may embed hyperboloids of type (4) of an arbitrary radius.

In 1965 M. A. Markov [1] pioneered the hypothesis according to which the mass spectrum of the elementary particles should be cut off at the Planck mass $m_{Planck} = 10^{19} GeV$:

$$m \leq m_{Planck}. \tag{5}$$

The particles with the limiting mass $m = m_{Planck}$, named by the author "maximons", should play a special role in the world of elementary particles. However, Markov's original condition (5) was purely phenomenological and he used standard field theoretical techniques even for describing the maximon.

In [2] - [8] a more radical approach was developed. Markov' s idea of the existence of a maximal value for the masses of elementary particles was understood as a new fundamental principle of Nature, which similarly to the relativistic and quantum postulates should underlie QFT. Doing this the condition of finiteness of the mass spectrum should be introduced by the relation:

$$m \leq M, \tag{6}$$

where the maximal mass parameter M called the "***fundamental mass***" is a ***new universal physical constant***.

A ***new concept*** of a local quantum field has been developed on the basis of (6) and on simple geometric arguments the corresponding Lagrangians were constructed and an adequate formulation of the principle of local gauge invariance was found. It was also demonstrated that the fundamental mass M in the new approach plays the role of an independent universal scale in the region of ultrahigh energies $E \geq M$.

It is worth emphasizing that here, due to eq(6), the Compton wave length of a particle $\lambda_C = \hbar/mc$ cannot be smaller than the "***fundamental length***" $l = \hbar/Mc$. According to Newton and Wigner [15], the parameter λ_C characterizes the dimensions of the region of space in which a relativistic particle of mass m can be localized. Therefore, the fundamental length l introduces into the theory a universal limit on the accuracy of localization in space of elementary particles.

The objective of the present work, in few words, is to include the principle of maximal mass (6) into the basic principles of the Standard Model. The new scheme appearing in this way, which we called the Maximal Mass Model, from our point of view is interesting already because in it the trusted methods of the local gauge QFT are organically bound to the elegant, though not as popular, geometric ideas.

2 Boson fields in de Sitter momentum space

Let us go back to the free one component real scalar field we considered above (1 - 3). We shall suppose that its mass m satisfies the condition (6). How should one modify the equations of motion in order that the existence of the bound (6) should become as evident as it is the limitation $v \leq c$ in the special theory of relativity? In the latter case everything is explained in a simple way: the relativization of the 3-dimensional velocity space is equivalent to transition in this space from Euclidean to Lobachevsky geometry realized on the 4-dimensional hyperboloid [1](4). Let us act in a similar way and substitute the 4-dimensional Minkowski momentum space, which is used in the standard QFT, by the anti de Sitter momentum space realized on the 5-hyperboloid:

$$p_0^2 - \boldsymbol{p}^2 + p_5^2 = M^2. \tag{7}$$

We shall suppose that in the p-representation our scalar field is defined just on the surface (7), i.e., it is a function of five variables $(p_0, \boldsymbol{p}, p_5)$, which are connected by the relation (7):

$$\delta(p_0^2 - \boldsymbol{p}^2 + p_5^2 - M^2)\varphi(p_0, \boldsymbol{p}, p_5). \tag{8}$$

The energy p_0 and the 3-momentum $\boldsymbol{p}$ here preserve their usual meaning and the mass shell relation (4) is satisfied as well. Therefore, for the field considered $\varphi(p_0, \boldsymbol{p}, p_5)$ the condition (6) is always fulfilled.

Clearly in eq. (8) the specification of a single function $\varphi(p_0, \boldsymbol{p}, p_5)$ of five variables (p_μ, p_5) is equivalent to the definition of two independent functions $\varphi_1(p)$ and $\varphi_2(p)$ of the 4-momentum p_μ:

$$\varphi(p_0, \boldsymbol{p}, p_5) \equiv \varphi(p, p_5) = \begin{pmatrix} \varphi(p, |p_5|) \\ \varphi(p, -|p_5|) \end{pmatrix} = \begin{pmatrix} \varphi_1(p) \\ \varphi_2(p) \end{pmatrix}, |p_5| = \sqrt{M^2 - p^2}. \tag{9}$$

The appearance of the new discrete degree of freedom $p_5/|p_5|$ and the associated doubling of the number of field variables is important feature of the new approach. It must be taken into account in the search for the equation of motion for the free field in de Sitter momentum space. Due to the mass shell relation (4) the Klein - Gordon equation (3) should also be satisfied by the field $\varphi(p_0, \boldsymbol{p}, p_5)$:

$$(m^2 - p_0^2 + \boldsymbol{p}^2)\varphi(p_0, \boldsymbol{p}, p_5) = 0. \tag{10}$$

From our point of view this relation is unsatisfactory for two reasons:

1. It does not reflect the bounded mass condition (6).

[1] To be exact on the upper sheet of this hyperboloid.

2. It can not be used to determine the dependence of the field on the new quantum number $p_5/|p_5|$ in order to distinguish between the components $\varphi_1(p)$ and $\varphi_2(p)$.

Here we notice that, because of (7), eq.(10) can be written as:

$$(p_5 + M\cos\mu)(p_5 - M\cos\mu)\varphi(p, p_5) = 0, \qquad \cos\mu = \sqrt{1 - \frac{m^2}{M^2}}. \tag{11}$$

Now, following the Dirac trick we postulate the equation of motion under question in the form:

$$2M(p_5 - M\cos\mu)\varphi(p, p_5) = 0. \tag{12}$$

Clearly, eq. (12) has none of the enumerated defects of the standard Klein-Gordon equation (3). However, equation (3) is still satisfied by the field $\varphi(p, p_5)$.

From eqs. (12) and (9) it follows that

$$\begin{aligned} 2M(|p_5| - M\cos\mu)\varphi_1(p) &= 0, \\ 2M(|p_5| + M\cos\mu)\varphi_2(p) &= 0, \end{aligned} \tag{13}$$

and we obtain:

$$\begin{aligned} \varphi_1(p) &= \delta(p^2 - m^2)\widetilde{\varphi}_1(p) \\ \varphi_2(p) &= 0 \end{aligned} \tag{14}$$

Therefore, the free field $\varphi(p, p_5)$ defined in the anti de Sitter momentum space (7) describes the same free scalar particles of mass m as the field $\varphi(p)$ in the Minkowski p-space, with the only difference that now we necessarily have $m \leq M$. The two-component structure (9) of the new field does not manifest itself on the mass shell, owing to (14). However, it will play an important role when the fields interact, i.e., off the mass shell.

Now we face the problem of constructing the action corresponding to eq. (12) and transforming it to the configuration representation.

Due to mainly technical reasons [2] in the following we shall use the Euclidean formulation of the theory, which appears as an analytical continuation to purely imaginary energies:

$$p_0 \to ip_4. \tag{15}$$

In this case, instead of the anti de Sitter p-space (7), we shall work with de Sitter p-space

$$-p_n^2 + p_5^2 = M^2, \qquad n = 1, 2, 3, 4. \tag{16}$$

[2]The corresponding comments on the topic will be given a bit later.

Obviously,

$$p_5 = \pm\sqrt{M^2 + p^2}. \tag{17}$$

If one uses eq. (16), the Euclidean Klein-Gordon operator $m^2 + p^2$ may be written, similarly to (11), in the following factorized form:

$$m^2 + p^2 = (p_5 + M\cos\mu)(p_5 - M\cos\mu). \tag{18}$$

Clearly, the nonnegative functional

$$S_0(M) = \pi M \times$$

$$\int \frac{d^4p}{|p_5|}\left[\varphi_1^+(p)2M(|p_5| - M\cos\mu)\varphi_1(p) + \varphi_2^+(p)2M(|p_5| + M\cos\mu)\varphi_2(p)\right], \tag{19}$$

$$\varphi_{1,2}(p) \equiv \varphi(p, \pm|p_5|), \tag{20}$$

plays the role of the action integral of the free Euclidean field $\varphi(p, p_5)$. The action may be written also as a 5 - integral:

$$S_0(M) = 2\pi M \times$$

$$\int \varepsilon(p_5)\delta(p_L p^L - M^2)d^5p\left[\varphi^+(p, p_5)2M(p_5 - M\cos\mu)\varphi(p, p_5)\right], \tag{21}$$

$$L = 1, 2, 3, 4, 5,$$

where

$$\varepsilon(p_5) = \frac{p_5}{|p_5|}. \tag{22}$$

The Fourier transform and the configuration representation have a special role in this approach. First, we note that in the basic equation (16) which defines de Sitter p-space, all the components of the 5-momentum enter on equal footing. Therefore, the expression $\delta(p_L p^L - M^2)\varphi(p, p_5)$, which now replaces (8), may be Fourier transformed

$$\frac{2M}{(2\pi)^{3/2}}\int e^{-ip_K x^K}\delta(p_L p^L - M^2)\varphi(p, p_5)d^5\,p = \varphi(x, x_5), \quad K, L = 1, 2, 3, 4, 5. \tag{23}$$

This function obviously satisfies the following differential equation in the ***5-dimensional configuration space***:

$$\left(\frac{\partial^2}{\partial x_5^2} - \Box + M^2\right)\varphi(x, x_5) = 0. \tag{24}$$

Integration over p_5 in (23) gives:

$$\varphi(x,x_5)=\frac{M}{(2\pi)^{3/2}}\int e^{ip_nx^n}\frac{d^4p}{|p_5|}\left[e^{-i|p_5|x^5}\varphi_1(p)+e^{i|p_5|x^5}\varphi_2(p)\right],$$
$$\varphi^+(x,x_5)=\varphi(x,-x_5), \tag{25}$$

from which we get:

$$\frac{i}{M}\frac{\partial\varphi(x,x_5)}{\partial x_5}=\frac{1}{(2\pi)^{3/2}}\int e^{ip_nx^n}d^4p\left[e^{-i|p_5|x^5}\varphi_1(p)-e^{i|p_5|x^5}\varphi_2(p)\right], \tag{26}$$

The four dimensional integrals (25) and (26) transform the fields $\varphi_1(p)$ and $\varphi_2(p)$ to the configuration representation. The inverse transforms have the form:

$$\varphi_1(p)=\frac{-i}{2M(2\pi)^{5/2}}\int e^{-ip_nx^n}d^4x\left[\varphi(x,x_5)\frac{\partial e^{i|p_5|x^5}}{\partial x_5}-e^{i|p_5|x^5}\frac{\partial\varphi(x,x_5)}{\partial x_5}\right],$$

$$\varphi_2(p)=\frac{i}{2M(2\pi)^{5/2}}\int e^{-ip_nx^n}d^4x\left[\varphi(x,x_5)\frac{\partial e^{-i|p_5|x^5}}{\partial x_5}-e^{-i|p_5|x^5}\frac{\partial\varphi(x,x_5)}{\partial x_5}\right]. \tag{27}$$

We note that the independent field variables

$$\varphi(x,0)\equiv\varphi(x)=\frac{M}{(2\pi)^{3/2}}\int e^{ip_nx^n}d^4\,p\frac{\varphi_1(p)+\varphi_2(p)}{|p_5|} \tag{28}$$

and

$$\frac{i}{M}\frac{\partial\varphi(x,0)}{\partial x_5}\equiv\chi(x)=\frac{1}{(2\pi)^{3/2}}\int e^{ip_nx^n}d^4p\left[\varphi_1(p)-\varphi_2(p)\right] \tag{29}$$

can be treated as initial Cauchy data on the surface $x_5=0$ for the hyperbolic-type equation (24).

Now substituting eq.(27) into the action (19) we obtain

$$S_0(M)=\frac{1}{2}\int d^4\,x\left[\left|\frac{\partial\varphi(x,x_5)}{\partial x_n}\right|^2+m^2|\varphi(x,x_5)|^2+\left|i\frac{\partial\varphi(x,x_5)}{\partial x_5}-M\cos\mu\varphi(x,x_5)\right|^2\right.$$
$$\equiv\int L_0(x,x_5)d^4\,x. \tag{30}$$

It is easily verified that due to eq. (24) the action (30) is independent of x_5:

$$\frac{\partial S_0(M)}{\partial x_5}=0. \tag{31}$$

Therefore the variable x_5 may be arbitrarily fixed and $S_0(M)$ may be viewed as a functional of the corresponding initial Cauchy data for the equation (24).

For example, for $x_5 = 0$ we have:

$$S_0(M) = \frac{1}{2}\int d^4\, x \left[\left(\frac{\partial\varphi(x)}{\partial x_n}\right)^2 + m^2(\varphi(x))^2 + M^2 \left(\chi(x) - \cos\mu\varphi(x)\right)^2 \right] \equiv$$

$$\equiv \int L_0(x, M) d^4\, x. \tag{32}$$

We have thus shown that in the developed approach the property of locality of the theory does not disappear, moreover it becomes even deeper, as it is extended to dependence on the extra fifth dimension x_5.

The new Lagrangian density $L_0(x, x_5)$ [see (30)] is a Hermitian form constructed from $\varphi(x, x_5)$ and the components of the 5-component gradient $\frac{\partial\varphi(x)}{\partial x_L}, (L = 1, 2, 3, 4, 5)$. It is clear that although $L_0(x, x_5)$ depends explicitly on x_5, the theory essentially remains ***four-dimensional*** [see eq. (31) and (32)].

As may be seen from the transformations which have been made, the dependence of the action (32) on the two functional arguments $\varphi(x)$ and $\chi(x)$ is a direct consequence of the fact that in momentum space the field has a doublet structure $\begin{pmatrix} \varphi_1(p) \\ \varphi_2(p) \end{pmatrix}$ due to the two possible values of p_5. However, the Lagrangian $L_0(x, M)$ does not contain a kinetic term corresponding to the field $\chi(x)$. Therefore, this variable is just auxiliary.

The special role of the 5-dimensional configuration space in the new formalism is determined by the fact that the gauge symmetry transformations are localized now in it. The initial data for the equation (24)

$$\begin{pmatrix} \varphi(x, x_5) \\ \\ \frac{i}{M}\frac{\partial\varphi(x, x_5)}{\partial x_5} \end{pmatrix}_{x_5 = fixed\ \ value} \tag{33}$$

are subject to these transformations.

Let us now discuss this point in more detail, supposing that the field $\varphi(x, x_5)$ is not Hermitian and some internal symmetry group is associated with it:

$$\varphi' = U\varphi. \tag{34}$$

Upon localization of the group in the 5-dimensional x-space:

$$U \to U(x, x_5), \tag{35}$$

the following gauge transformation law arises for the initial data (33) on the plane $x_5 = 0$:

$$\varphi'(x) = U(x,0)\varphi(x), \qquad \chi'(x) = \frac{i}{M}\frac{\partial U(x,0)}{\partial x_5}\varphi(x) + U(x,0)\chi(x). \tag{36}$$

The group character of the transformations (36) is obvious. The specific form of the matrix $U(x, x_5)$ can be determined in the new theory of vector fields, which is a generalization of the standard theory in the spirit of our approach (see [5]).

It is clear that the equation (24) may be represented as a system of two equations of first order in the derivative $\frac{\partial}{\partial x_5}$ [10]:

$$\left\{ \frac{i}{M}\frac{\partial}{\partial x_5} - \left[\sigma_3 \left(1 - \frac{\Box}{2M^2} \right) - i\sigma_2 \frac{\Box}{2M^2} \right] \right\} \phi(x.x_5) = 0, \tag{37}$$

where

$$\phi(x,x_5) = \begin{pmatrix} \frac{1}{2}\left[\varphi(x,x_5) + \frac{i}{M}\frac{\partial \varphi(x,x_5)}{\partial x_5} \right] \\ \frac{1}{2}\left[\varphi(x,x_5) - \frac{i}{M}\frac{\partial \varphi(x,x_5)}{\partial x_5} \right] \end{pmatrix} \equiv \begin{pmatrix} \phi_I(x,x_5) \\ \phi_{II}(x,x_5) \end{pmatrix}, \tag{38}$$

($\sigma_i, i = 1, 2, 3$ are the Pauli matrices). If we compare (38) with (28) and (29) we find relations between the initial Cauchy data for the equation (24) and the system (37):

$$\phi(x,0) = \begin{pmatrix} \phi_I(x,0) \\ \phi_{II}(x,0) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\varphi(x) + \chi(x)) \\ \frac{1}{2}(\varphi(x) - \chi(x)) \end{pmatrix} \equiv \phi(x). \tag{39}$$

It easy to show that in the basis (39) the Lagrangian $L_0(x, M)$ from (32) looks like

$$L_0(x,M) = \frac{\partial \phi(x)}{\partial x_n}(1+\sigma_1)\frac{\partial \phi(x)}{\partial x_n} + 2M^2\phi(x)(1 - \cos\mu\,\sigma_3)\phi(x). \tag{40}$$

Let us discuss now the question about the conditions for the transition of the new scheme into the standard Euclidean QFT (the so called "correspondence principle"). The Euclidean momentum 4-space is the "flat limit" of the de Sitter p-space and may be associated with the approximation

$$\begin{aligned} |p_n| &\ll M \\ p_5 &\simeq M \end{aligned} \tag{41}$$

In the same limit, in the configuration space we have

$$\begin{aligned} \varphi(x, x_5) &= e^{-iMx_5}\varphi(x) \\ \chi(x) &= \varphi(x) \end{aligned} \tag{42}$$

or

$$\phi(x) = \begin{pmatrix} \varphi(x) \\ 0 \end{pmatrix} \tag{43}$$

With the help of (37) it is not difficult to obtain [11, 12] the corrections of the order of $O(\frac{1}{M^2})$ to the zero approximation (43)

$$\phi(x) = \begin{pmatrix} \left(1 - \frac{\Box}{4M^2}\right)\varphi(x) \\ \\ \frac{\Box}{4M^2}\varphi(x) \end{pmatrix} \tag{44}$$

from which (see eq. (39)) we have

$$\varphi(x) - \chi(x) = \frac{\Box\varphi(x)}{2M^2} \tag{45}$$

Taking into account (45) and (11) one may conclude that in the "flat limit" (formally when $M \to \infty$) the Lagrangian $L_0(x, M)$ from (32) coincides with its Euclidean counterpart.

A key role in the SM belongs to the scalar Higgs field, the interactions with which allow the other fields to get masses. As far as in our model the masses of all particles, including the mass of the Higgs boson itself, should obey the condition (6), one would presume that there exists a deep internal connection between the Higgs field and the fundamental mass M. As a matter of fact, before the Higgs mechanism is switched on, all fields by definition are massless [3] and because of that the bound (6) at this stage has no physical meaning. Only, together with the appearance of the mass spectrum of the particles the condition (6) makes sense and, therefore, the magnitude of M should be essentially fixed by the same Higgs mechanism.

In order to get some orientation in this situation, let us consider in the framework of our approach the example of the simplest mechanism, connected with the spontaneous breaking of a discrete symmetry. At the beginning, in order to describe the scalar field, let us use the doublet (39). The total Lagrangian $L_{tot}(x)$, in analogy with the traditional approach, will include a free part (40) at $\mu = 0$ and the well known interaction Lagrangian:

$$L_{int}(x) = \frac{\lambda^2}{4}(\phi^2 - v^2)^2. \tag{46}$$

[3] Higgs boson, as it is known, at this stage is with mass of a tachyon.

Therefore, we have:

$$L_{tot}(x) = \frac{\partial\phi(x)}{\partial x_n}(1+\sigma_1)\frac{\partial\phi(x)}{\partial x_n} + 2M^2\phi(x)(1-\cos\mu\,\sigma_3)\phi(x) + \frac{\lambda^2}{4}(\phi^2 - v^2)^2. \tag{47}$$

Here we used the field $\phi(x)$ only to write the interaction (46) in the known symmetric form. Now in (47) we may go back to the variables $\varphi(x)$ and $\chi(x)$ (see (39)):

$$L_{tot}(x) = \frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_n}\right)^2 + \frac{M^2}{2}\left(\varphi(x)-\chi(x)\right)^2 + \frac{\lambda^2}{4}\left(\frac{\varphi^2(x)+\chi^2(x)}{2} - v^2\right)^2 \tag{48}$$

The Lagrangian (48) remains invariant under the transformation

$$\begin{aligned}\varphi(x) &\to -\varphi(x)\\ \chi(x) &\to -\chi(x)\end{aligned} \tag{49}$$

However, this symmetry is spontaneously broken. The transition to a stable "vacuum" is realized by the transformations

$$\begin{aligned}\varphi(x) &= \varphi'(x) + v\\ \chi(x) &= \chi'(x) + v\end{aligned} \tag{50}$$

In the new variables $\varphi'(x)$ and $\chi'(x)$ the part of the Lagrangian (48) quadratic in the fields takes the form:

$$\frac{1}{2}\left(\frac{\partial\varphi'(x)}{\partial x_n}\right)^2 + \frac{1}{2}(M^2+\frac{\lambda^2v^2}{2})\left(\varphi'^2(x)+\chi'^2(x)\right) - (M^2-\frac{\lambda^2v^2}{2})\varphi'(x)\chi'(x). \tag{51}$$

Comparing (51) and (32) we may conclude that

1. As a result of the spontaneous breaking of the symmetry (49) the fundamental mass M experiences renormalization:

$$M^2 \to M^2 + \frac{\lambda^2v^2}{2} \tag{52}$$

2. The considered scalar particle acquires mass:

$$m = \sqrt{2}\lambda v\frac{1}{\sqrt{1+\frac{\lambda^2v^2}{2M^2}}}, \tag{53}$$

which satisfies the condition [4]:

$$m \le \sqrt{M^2 + \frac{\lambda^2v^2}{2}}. \tag{54}$$

[4] Let us note that (54) is equivalent to the inequality $\left(1-\frac{\lambda v}{\sqrt{2}M}\right)^2 \ge 0$.

Therefore, if we, in advance, take into account the renormalization (52) due to the Higgs mechanism we may write the Lagrangian (48) in the form [5]:

$$L_{tot}(x) = \frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_n}\right)^2 + \frac{1}{2}(M^2 - \frac{\lambda^2 v^2}{2})\left(\varphi(x) - \chi(x)\right)^2 + \\ + \frac{\lambda^2}{4}(\frac{\varphi^2(x)+\chi^2(x)}{2} - v^2)^2. \tag{55}$$

In this way instead of (53)we have

$$m = \sqrt{2}\lambda v\sqrt{1 - \frac{\lambda^2 v^2}{2M^2}} \equiv m_0\sqrt{1 - \frac{m_0^2}{4M^2}} \tag{56}$$

The quantity $m_0 = \sqrt{2}\lambda v$ is the maximal value of the mass of the considered scalar particle. It may be reached only in the "flat limit" $M \to \infty$, when the Lagrangian (55) because of (42) and (45) takes the usual form:

$$L_{tot}(x) = \frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_n}\right)^2 + \frac{\lambda^2}{4}\left(\varphi^2(x) - v^2\right)^2. \tag{57}$$

At the end of this section, we would like to explain why we prefer to develop our approach in Euclidean terms and pass from the anti de Sitter p-space (7) to the the de Sitter p-space (16).

Let us apply to (8) the 5-dimensional Fourier transform (compare with (23))

$$\varphi(x, x_5) \equiv \frac{2M}{(2\pi)^{3/2}}\int e^{-ip_0x_0+\boldsymbol{px}-ip_5x_5}\delta(p_0^2 - \boldsymbol{p}^2 + p_5^2 - M^2)\varphi(p, p_5)d^5\,p. \tag{58}$$

From here we find (compare with (28) and (29))

$$\varphi(x,0) \equiv \varphi(x) = \frac{M}{(2\pi)^{3/2}}\int_{p^2\le M^2} e^{-ipx}d^4p\frac{\varphi(p,|p_5|)+\varphi(p,-|p_5|)}{|p_5|} \\ \frac{i}{M}\frac{\partial\varphi(x,0)}{\partial x_5} \equiv \chi(x) = \frac{1}{(2\pi)^{3/2}}\int_{p^2\le M^2} e^{-ipx}d^4p\left[\varphi(p,|p_5|) - \varphi(p,-|p_5|)\right]. \tag{59}$$

The principal difference of these expressions in comparison with (28) and (29) is that in (59) there is a limitation on the integration region: $p_0^2 - \boldsymbol{p}^2 \le M^2$. This fact sharply restricts the class of functions $\varphi(x)$ and $\chi(x)$ and does not allow, in particular, to construct from them local Lagrangians or to apply to them local gauge transformations. Rigorously speaking eqs. (59) can not be treated

[5]In order the Lagrangian (47) remains positively definite, it is natural to suppose that $M^2 > \frac{\lambda^2 v^2}{2}$.

(without special reservations) as Cauchy data for the "ultra-hyperbolic" equation:

$$\left(\frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial x_5^2} - \frac{\partial^2}{\partial \boldsymbol{x}^2} + M^2\right)\varphi(x, x_5) = 0, \tag{60}$$

which is satisfied by the field (58). In mathematical physics there are developed methods which allow one to use partial differential equations of ultra-hyperbolic type with Cauchy initial data. From a technical point of view we consider this a more complicated procedure, than to work in the framework of Euclidean QFT. Moreover, thanks to the locality of the Euclidean formulation, coming back to the relativistic description is not a problem.

3 De Sitter fermion fields

As far as the new QFT is elaborated on the basis of the de Sitter momentum space (16) it is natural to suppose that in the developed approach the fermion fields $\psi_\alpha(p, p_5)$ have to be de Sitter spinors, i.e., to transform under the four dimensional representation of the group SO(4, 1). Further on we shall use the following γ - matrix basis ($\gamma^4 = i\gamma^0$):

$$\gamma^L = (\gamma^1, \gamma^2, \gamma^3, \gamma^4, \gamma^5)$$

$$\left\{\gamma^L, \gamma^M\right\} = 2g^{LM},$$

$$g^{LM} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{61}$$

Obviously we have:

$$\begin{aligned} M^2 - p_L p^L = M^2 + p_n^2 - p_5^2 = (M - p_L\gamma^L)(M + p_L\gamma^L) = \\ = (M + p^n\gamma^n - p^5\gamma^5)(M - p^n\gamma^n + p^5\gamma^5). \end{aligned} \tag{62}$$

In the "flat limit" $M \to \infty$ the quantities $\psi_\alpha(p, p_5)$ become Euclidean spinor fields which are used in the construction of different versions of the Euclidean QFT for fermions.

It is clear that the relations (23) - (29) , which we considered in the theory of boson fields, exist also in its fermion version. Let us write some of them without comments

$$\psi(x, x_5) = \frac{2M}{(2\pi)^{3/2}} \int e^{-ip_K x^K} \delta(p_L p^L - M^2)\psi(p, p_5) d^5\, p, \tag{63}$$

$$\left(\frac{\partial^2}{\partial x_5^2} - \Box + M^2\right)\psi(x,x_5) = 0, \tag{64}$$

$$\begin{aligned}\psi(x,0) \equiv \psi(x) &= \tfrac{M}{(2\pi)^{3/2}} \int e^{ip_n x^n} d^4\, p \tfrac{\psi_1(p)+\psi_2(p)}{|p_5|} = \\ &= \tfrac{1}{(2\pi)^{3/2}} \int e^{ip_n x^n} \psi(p) d^4\, p\end{aligned} \tag{65}$$

$$\begin{aligned}\tfrac{i}{M}\tfrac{\partial\psi(x,0)}{\partial x_5} \equiv \chi(x) &= \tfrac{1}{(2\pi)^{3/2}} \int e^{ip_n x^n} d^4 p \left[\psi_1(p) - \psi_2(p)\right] = \\ &= \tfrac{1}{(2\pi)^{3/2}} \int e^{ip_n x^n} \chi(p) d^4\, p.\end{aligned} \tag{66}$$

The next step is the construction of the action integral for the fermion field $\psi_\alpha(p, p_5)$. Here we will not follow our work [6], where this problem was solved in the spirit of the Schwinger's approach [13] with the use of 8-component real spinors and preserving the reality of the action. Now we shall follow the formulation of Osterwalder and Schrader [14] and write the Euclidean fermion Lagrangian in the form:

$$\begin{aligned}&L_E(x) = \overline{\zeta}_E(x)\left(-i\gamma_n \tfrac{\partial}{\partial x^n} + m\right)\psi_E(x), \\ &\{\gamma^n, \gamma^m\} = -2\delta^{nm} \quad (m,n = 1,2,3,4).\end{aligned} \tag{67}$$

Here the spinor fields $\overline{\zeta}_E(x) = \zeta_E^+(x)\gamma^4$ and $\psi_E(x)$ are independent Grassmann variables, which are not connected between themselves by Hermitian or complex conjugation. Correspondingly, the action is not Hermitian. The Osterwalder and Schrader approach has been widely discussed in the literature [14][6] and here we shall not go into details. It is easy to convince oneself that the expression $2M(p_5 - M\cos\mu)$, which in our approach substitutes (see eq.(32)) the Euclidean Klein-Gordon operator $p_n^2 + m^2$, may be represented as

$$\begin{aligned}&2M(p_5 - M\cos\mu) = \\ &= \left[p_n\gamma^n - (p_5 - M)\gamma^5 + 2M\sin\tfrac{\mu}{2}\right]\left[-p_n\gamma^n + (p_5 - M)\gamma^5 + 2M\sin\tfrac{\mu}{2}\right]\end{aligned} \tag{68}$$

In the Euclidean approximation (41) the relation (68) takes the form:

$$p_n^2 + m^2 = (p_n\gamma^n + m)\,(-p_n\gamma^n + m)\,. \tag{69}$$

[6]By the way, in the paper [15] the so called Wick rotation is interpreted in terms of the 5-dimensional space.

Therefore, we may use the expression

$$\mathcal{D}(p, p_5) \equiv p_n\gamma^n - (p_5 - M)\gamma^5 + 2M\sin\frac{\mu}{2} \tag{70}$$

like the new Dirac operator.

As a result, we come to an expression for the action of the Fermion field in the de Sitter momentum space

$$\begin{gathered} S_0(M) = 2\pi M \int \varepsilon(p_5)\delta(p_L p^L - M^2) d^5p \times \\ \times \left[\ \overline{\zeta}(p, p_5)(p_n\gamma^n - (p_5 - M)\gamma^5 + 2M\sin\tfrac{\mu}{2})\psi(p, p_5)\ \right], \end{gathered} \tag{71}$$

In the integral (71) it is possible to pass to the field variables

$$\begin{gathered} \psi(p) = \tfrac{M}{|p_5|}(\psi(p, |p_5|) + \psi(p, -|p_5|)) \equiv M\tfrac{\psi_1(p)+\psi_2(p)}{|p_5|} \\ \chi(p) = \psi_1(p) - \psi_2(p) \\ \overline{\zeta}(p) = M\tfrac{\overline{\zeta}_1(p)+\overline{\zeta}_2(p)}{|p_5|} \\ \overline{\xi}(p) = \overline{\zeta}_1(p) - \overline{\zeta}_2(p), \end{gathered} \tag{72}$$

which are the Fourier amplitudes of the local fields $\psi(x), \chi(x), \overline{\zeta}(x)$ and $\overline{\xi}$ (x) (compare with (65) and (66)). As a result, we get:

$$\begin{gathered} S_0^{\mathcal{D}} = -\pi \int d^4p \left(M + \tfrac{p_n^2}{M}\right) \overline{\zeta}(p)\gamma^5\psi(p) + \\ +\pi \int d^4p\overline{\zeta}(p) \left(\not{p} + M\gamma^5 + 2Msin\tfrac{\mu}{2}\right) \chi(p) + \\ +\pi \int d^4p\overline{\xi(p)} \left(\not{p} + M\gamma^5 + 2Msin\tfrac{\mu}{2}\right) \psi(p) - \\ -\pi \int d^4pM\overline{\xi(p)}\gamma^5\chi(p) \end{gathered} \tag{73}$$

In the configuration space we have, correspondingly,

$$S_0^{\mathcal{D}} = \int L_0^{\mathcal{D}}(x,M) d^4x =$$
$$= \frac{1}{2}\int d^4x \overline{\zeta}(x)\left(\frac{\Box}{M^2} - 1\right)\gamma^5\psi(x) +$$
$$+\frac{1}{2}\int d^4x \overline{\zeta}(x)\left(i\gamma^n \frac{\partial}{\partial x^n} + M\gamma^5 + 2M\sin\frac{\mu}{2}\right)\chi(x) + \tag{74}$$
$$+\frac{1}{2}\int d^4x \overline{\xi(x)}\left(i\gamma^n \frac{\partial}{\partial x^n} + M\gamma^5 + 2M\sin\frac{\mu}{2}\right)\psi(x) -$$
$$-\frac{1}{2}\int d^4x \overline{\xi(x)}\gamma^5\chi(x).$$

Hence, the modified Dirac Lagrangian $L_0^{\mathcal{D}}(x,M)$ is a local function of the spinor field variables $\psi(x), \chi(x), \overline{\zeta}(x)$ and $\overline{\xi}$ (x). Here there is an obvious analogy with the boson case (compare with (32) and (**??**)).

However, the fermion Lagrangian $L_0^{\mathcal{D}}(x,M)$ may be represented in a different form, if one uses the relations (62). Indeed, let us put

$$\begin{aligned} \frac{1}{2M}(M - p_K\gamma^K)\psi(p,p_5) \equiv \Pi_L\psi(p,p_5) \equiv \psi_L(p,p_5) \\ \frac{1}{2M}(M + p_K\gamma^K)\psi(p,p_5) \equiv \Pi_R\psi(p,p_5) \equiv \psi_R(p,p_5) \end{aligned} \tag{75}$$

Due to (16) the operators Π_L and Π_R are projectors:

$$\begin{aligned} \Pi_L + \Pi_R = 1, \\ \Pi_L^2 = \Pi_L \qquad \Pi_R^2 = \Pi_R, \\ \Pi_L\Pi_R = \Pi_R\Pi_L = 0. \end{aligned} \tag{76}$$

On the other hand they are the 5- analogue of the Dirac operator, and the fields $\psi_L(p,p_5)$ and $\psi_R(p,p_5)$ obviously satisfy the corresponding 5-dimensional Dirac equations

$$\begin{aligned} (M + p_K\gamma^K)\psi_L(p,p_5) = 0, \\ (M - p_K\gamma^K)\psi_R(p,p_5) = 0. \end{aligned} \tag{77}$$

Therefore, in this way the fermion field $\psi(p,p_5)$, given in the de Sitter momentum space (16), may be presented as a sum of two fields $\psi_L(p,p_5)$ and $\psi_R(p,p_5)$

$$\psi(p,p_5) = \psi_L(p,p_5) + \psi_R(p,p_5), \tag{78}$$

which obey the 5-dimensional Dirac equations (77). Obviously, the decomposition (78) is ***de Sitter invariant procedure***.

It is easy to verify that in the flat limit (41)

$$\Pi_{L,R} = \frac{1 \mp \gamma^5}{2}, \tag{79}$$

This is the reason that we consider the fields $\psi_L(p, p_5)$ and $\psi_R(p, p_5)$ as the "***chiral***" components in the developed approach [12]. The new operator of chirality $\frac{p_L \gamma^L}{M}$, similarly to its "***flat counterpart***", has eigenvalues equal to ± 1, but ***depends on the energy and momentum***. The last circumstance, as we hope, should be revealed experimentally (see section 4).

It is worthwhile to pass in (77) to the configurational representation. Applying (63) we get :

$$\begin{aligned} \psi_L(x, x_5) &= \tfrac{1}{2}\left(1 - \tfrac{i\gamma^n}{M}\tfrac{\partial}{\partial x^n} - \tfrac{i\gamma^5}{M}\tfrac{\partial}{\partial x^5}\right)\psi(x, x_5) \\ \psi_R(x, x_5) &= \tfrac{1}{2}\left(1 + \tfrac{i\gamma^n}{M}\tfrac{\partial}{\partial x^n} + \tfrac{i\gamma^5}{M}\tfrac{\partial}{\partial x^5}\right)\psi(x, x_5) \end{aligned} \tag{80}$$

Setting in (80) $x_5 = 0$ and taking into account (65) and (66) we shall have:

$$\begin{aligned} \psi_L(x, 0) \equiv \psi_{(L)}(x) &= \tfrac{1}{2}\left(1 - \tfrac{i\gamma^n}{M}\tfrac{\partial}{\partial x^n}\right)\psi(x) - \tfrac{\gamma^5}{2}\chi(x), \\ \psi_R(x, 0) \equiv \psi_{(R)}(x) &= \tfrac{1}{2}\left(1 + \tfrac{i\gamma^n}{M}\tfrac{\partial}{\partial x^n}\right)\psi(x) + \tfrac{\gamma^5}{2}\chi(x). \end{aligned} \tag{81}$$

As far as the field $\psi(x, x_5)$ obeys equation (24), the relations, we obtained for the scalar field in the "flat" approximation and in particular (45), may be applied to it. Taking this into account, we find that in this approximation the equalities (81) become

$$\begin{aligned} \psi_{(L)}(x) &= \tfrac{1}{2}(1 - \gamma_5)\psi(x) - \tfrac{i\gamma^n}{2M}\tfrac{\partial}{\partial x^n}\psi(x) + \tfrac{\gamma^5}{2}(\psi(x) - \chi(x)) \simeq \\ &\simeq \tfrac{1}{2}(1 - \gamma_5)\psi(x) - \tfrac{i\gamma^n}{2M}\tfrac{\partial}{\partial x^n}\psi(x) + \tfrac{\gamma^5}{4M^2}\Box\psi(x), \\ \psi_{(R)}(x) &\simeq \tfrac{1}{2}(1 + \gamma_5)\psi(x) + \tfrac{i\gamma^n}{2M}\tfrac{\partial}{\partial x^n}\psi(x) - \tfrac{\gamma^5}{4M^2}\Box\psi(x). \end{aligned} \tag{82}$$

Representation, analogous to (78), may be introduced for the field $\overline{\zeta}(p, p_5)$ appearing in (71)

$$\overline{\zeta}(p, p_5) = \overline{\zeta}_L(p, p_5) + \overline{\zeta}_R(p, p_5), \tag{83}$$

where

$$\begin{aligned} \overline{\zeta}_L(p, p_5) &= \overline{\zeta}(p, p_5)\Pi_R, \\ \overline{\zeta}_R(p, p_5) &= \overline{\zeta}(p, p_5)\Pi_L. \end{aligned} \tag{84}$$

Further it is not difficult to obtain relations similar to (80) - (82) for the fields $\overline{\zeta}_L(x)$ and $\overline{\zeta}_R(x)$:

$$\begin{aligned} \overline{\zeta}_{(L)}(x) &= \tfrac{1}{2}\overline{\zeta}(x) + \tfrac{i}{2M}\tfrac{\partial\overline{\zeta}(x)}{\partial x^n}\gamma^n + \overline{\xi}(x)\tfrac{\gamma^5}{2}, \\ \overline{\zeta}_{(R)}(x) &= \tfrac{1}{2}\overline{\zeta}(x) - \tfrac{i}{2M}\tfrac{\partial\overline{\zeta}(x)}{\partial x^n}\gamma^n - \overline{\xi}(x)\tfrac{\gamma^5}{2}, \end{aligned} \qquad (85)$$

$$\begin{aligned} \overline{\zeta}_{(L)} &\simeq \overline{\zeta}(x)\tfrac{1}{2}(1+\gamma_5) + \tfrac{i}{2M}\tfrac{\partial}{\partial x^n}\overline{\zeta}(x)\gamma^n - \tfrac{\Box}{4M^2}\overline{\zeta}(x)\gamma^5, \\ \overline{\zeta}_{(R)} &\simeq \overline{\zeta}(x)\tfrac{1}{2}(1-\gamma_5) - \tfrac{i}{2M}\tfrac{\partial}{\partial x^n}\overline{\zeta}(x)\gamma^n + \tfrac{\Box}{4M^2}\overline{\zeta}(x)\gamma^5. \end{aligned} \qquad (86)$$

Now substituting (81) and (85) in the action integral (74) we may pass to new variables $\psi_{(L)}(x), \psi_{(R)}(x), \overline{\zeta}_L(x)$ and $\overline{\zeta}_R(x)$:

$$\begin{aligned} S_0^{\mathcal{D}} &= \int L_0^{\mathcal{D}}(x,M)d^4x = \\ &= \int d^4x \left[\overline{\zeta}_{(L)}(x)i\gamma^n\tfrac{\partial}{\partial x^n}\psi_{(L)}(x) + \overline{\zeta}_{(R)}(x)i\gamma^n\tfrac{\partial}{\partial x^n}\psi_{(R)}(x)\right] + \\ &+ \int d^4x\overline{\zeta}_{(L)}(x)\left[i\gamma^n\tfrac{\partial}{\partial x^n} + M(1-\gamma^5)\right]\psi_{(R)}(x) + \\ &+ \int d^4x\overline{\zeta}_{(R)}(x)\left[i\gamma^n\tfrac{\partial}{\partial x^n} - M(1+\gamma^5)\right]\psi_{(L)}(x) + \\ &+ 2M\sin\tfrac{\mu}{2}\int d^4x\left[\overline{\zeta}_{(L)}(x)\gamma^5\psi_{(R)}(x) - \overline{\zeta}_{(R)}(x)\gamma^5\psi_{(L)}(x)\right] \end{aligned} \qquad (87)$$

The obtained expression is the basis for constructing a gauge theory of interacting fermion field. This topic will shortly be discussed in the next section. Concluding this part we would like to make one important remark [6].

The point is that for the quantity $2M(p^5 - M\cos\mu)$, which substituted in our approach the Euclidean Klein-Gordon operator together with (68) there exists ***one more decomposition to matrix factors***:

$$\begin{aligned} &2M(p^5 - M\cos\mu) = \\ &= (p_n\gamma^n - \gamma^5(p^5+M) + 2M\cos\tfrac{\mu}{2})(p_n\gamma^n - \gamma^5(p^5+M) - 2M\cos\tfrac{\mu}{2}) \end{aligned} \qquad (88)$$

Therefore, if our approach is considered to be realistic, it may be assumed that in Nature there exists some ***exotic*** fermion field whose free action integral has the form

$$\begin{aligned} &S_0^{(exotic)}(M) = 2\pi M\int\varepsilon(p_5)\delta(p_Lp^L - M^2)d^5p\,\times \\ &\times\left\{\ \overline{\xi}_{exotic}(p,p_5)\left[p_n\gamma^n - (p_5+M)\gamma^5 + 2M\cos\tfrac{\mu}{2}\right]\psi_{exotic}(p,p_5)\ \right\} \end{aligned} \qquad (89)$$

Applying the above developed procedure it is easy to obtain $S_0^{(exotic)}(M)$ in a form analogous to (87). However, in contrast to $S_0^{\mathcal{D}}$ this quantity does not have a limit as $M \to \infty$, which justifies the name chosen by us for this field. The polarization properties of the exotic field, evidently, differ sharply from standard ones.

We would like to conjecture that the quanta of the exotic fermion field have a direct relation to the structure of the "dark matter."

4 The new geometrical approach to the Standard Model

To the complete formulation of the Standard Model, consistent with the principle of maximal mass (6) and its geometrical realization in terms of de Sitter momentum space [7] (16) we shall devote a separate paper. Now we intend to make only several remarks important for the understanding of our general strategy.

1. $SU_L(2) \bigotimes U_Y(1)$ - ***symmetry***

The gauge $SU_L(2) \bigotimes U_Y(1)$ - symmetry is one of the most important elements of the SM which guaranteed its success. This is why it should be assumed as necessary to apply it also in our approach, taking into account our new definition of the chiral fields. However, in the new fermion Lagrangian $L_0^{\mathcal{D}}$ (see (87)) even for $m = 0$ there are crossed terms:

$$\begin{aligned} &\overline{\zeta}_{(L)} \left[i\gamma^n \tfrac{\partial}{\partial x^n} + M(1-\gamma^5) \right] \psi_{(R)}(x) + \\ &+ \overline{\zeta}_{(R)} \left[i\gamma^n \tfrac{\partial}{\partial x^n} - M(1+\gamma^5) \right] \psi_{(L)}(x) \end{aligned} \tag{90}$$

which, at first glance, are a insurmountable obstacle for the use of the group $SU_L(2) \bigotimes U_Y(1)$. The solution of this difficulty is to make the expression (90) invariant form with the help of the Higgs field. In this way, considering as before the Higgs boson to be a $SU_L(2)$-doublet, introducing the doublet structure for the L-component of the fermion field and passing to covariant derivatives with the rules of the SM, we may write (90) in the form:

$$\begin{aligned} &\tfrac{1}{v} \left(\overline{\zeta}_{(L)} . H(x) \right) \left[i\gamma^n D_n^R + M(1-\gamma^5) \right] \psi_{(R)}(x) + \\ &+ \tfrac{1}{v} \overline{\zeta}_{(R)} \left\{ H^+(x) . \left[i\gamma^n D_n^L - M(1+\gamma^5) \right] \psi_{(L)}(x) \right\} + conj., \end{aligned} \tag{91}$$

where $H(x)$ is the SM Higgs doublet and D^R and D^L are the SM covariant derivatives. After the Higgs mechanism is switched on from (91) separate our

[7] Let us recall that namely this ***geometrized*** SM is called in advance the Maximal Mass Model.

cross terms (90) and appear terms with interactions which are not present in the SM. Together with the corrections, caused by the difference between the new and old definitions of chirality (see (82) and (86)) they may be the ground for predictions which may be verified experimentally.

2. ***Chirality*** In the SM it is prescribed that the boson fields transform as representations of the group $SU_L(2)$, which for the vector fields is three-dimensional and two-dimensional for the Higgs scalar. Naively reasoning one may ask himself how the mentioned bosons should know about the existence of the 4×4 matrix γ^5 one of the eigenvalues of which corresponds to the index L ? In our approach all fields, boson and fermion, are given in the de Sitter p-space on equal footing, with the only difference that the boson fields obey the 5-equation of Klein-Gordon (see (24)), and the fermion 5-equations of Dirac (77). There is nothing strange that the field $\psi_{(L)}(x)$ and the Higgs scalar $\varphi(x)$ simultaneously have a doublet structure with respect to the $SU_L(2)$-symmetry. This has already happened in the old isospin symmetry. Let us recall the nucleon doublet and the K-meson doublet.

The new geometrical concept of chirality allows us to think that the parity violation in weak interactions discovered fifty years ago was a manifestation of the de Sitter nature of momentum 4-space.

3. ***Higgs mechanism***

This important element of the SM, as we can see already now, is conserved in the generalized SM without considerable changes. The role of the spontaneous symmetry breaking mechanism in the formation of the fundamental mass M has been studied by a simple example in section 2.

5 Concluding remarks

Concluding this article, we would like to pay attention to one peculiarity of the developed here approach. All fields, independently of their spins, charges, masses etc. satisfy the free 5-equation of hyperbolic type, and the role of "time" is played by the coordinate "x_5". The interaction between the fields is realized at the level of the Cauchy data given on the plane $x_5 = 0$, i.e., in the four-dimensional (Euclidean) world. Only the elementary particles, described by local fields and with masses, obeying the limitation $m \leq M$ have the right of such a "***free gliding***" in the 5-space.

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