Comparison of Grand Unified Theories with electroweak and strong Coupling Constants measured at LEP

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Abstract

Using the renormalization group equations one can evolve the electroweak and strong coupling constants, as measured at LEP, to higher energies in order to test the ideas of Grand Unified Theories, which predict that the three coupling constants become equal at a single unification point. With data from the DELPHI Collaboration we find that in the minimal *non-supersymmetric* Standard Model with one Higgs doublet a single unification point is excluded by more than 7 standard deviations. In contrast, the minimal *supersymmetric* Standard Model leads to good agreement with a single unification scale of $10^{16.0\pm0.3}$GeV. Such a large scale is compatible with the present lower limits on the proton lifetime. The best fit is obtained for a SUSY scale around 1000 GeV and limits are derived as function of the strong coupling constant. The unification point is sensitive to the number of Higgs doublets and only the minimal SUSY model with two Higgs doublets is compatible with GUT unification, if one takes the present limits on the proton lifetime into account.

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1 Definition of the coupling constants

In the SM based on the group $SU(3)_C \otimes SU(2)_L \otimes U(1)$ the usual definitions of the coupling constants are:

$$\alpha_1 = 5/3\, g'^2/(4\pi) = 5 \cdot \alpha/(3 \cdot \cos^2\theta_{\overline{MS}}), \tag{1}$$
$$\alpha_2 = g^2/(4\pi) = \alpha/\sin^2\theta_{\overline{MS}}, \tag{2}$$
$$\alpha_3 = g_s^2/(4\pi) \tag{3}$$

where g_s is the $SU(3)_C$ coupling constant. The factor 5/3 in the definition of α_1 has been included for the proper normalization at the unification point[1]. The running coupling constant $\alpha_i(\mu)$ is completely determined by the particle content and their couplings inside the loop diagrams of the gauge bosons, as expressed by the renormalization group equations. In second order the renormalization group equations can be written as:

$$\mu\frac{\partial}{\partial\mu}\alpha_i(\mu) = \frac{2}{4\pi}\left(b_i + \frac{b_{ij}}{4\pi}\alpha_j(\mu) + \frac{b_{ik}}{4\pi}\alpha_k(\mu)\right) \cdot \alpha_i^2(\mu) + \frac{2 \cdot b_{ii}}{(4\pi)^2}\alpha_i^3(\mu), \tag{4}$$

where μ is the energy at which the couplings are evaluated, $i,j,k = 1,2,3$ and $i \neq j \neq k$. The b_{ij}'s for the SM and for the SUSY model are given in Ref. [2]. The b_i's for the SM are[2]:

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam}\begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs}\begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}, \tag{5}$$

while for the minimal SUSY they have been calculated to be[2]:

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam}\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs}\begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}, \tag{6}$$

where N_{Fam} is the number of families of matter fields and N_{Higgs} is the number of Higgs doublets. In the minimal SM and in the minimal SUSY model $N_{Fam} = 3$ and $N_{Higgs} = 1$ and 2, respectively. Note that in the supersymmetric model the dominating first order coefficients lead to a much weaker running of α_3 than predicted by the standard model, while the running of α_2 has the opposite sign and α_1 runs somewhat faster.

2 Measurement of the coupling constants

Using a recent calculation of $\triangle r$ including the QCD and M_{top}^4 corrections[10], one can obtain limits on the top mass from the average value of $\sin^2\theta_W = 1 - M_W^2/M_Z^2 = 0.2290 \pm 0.0035$, obtained in neutrino scattering[5,6,7] and $p\overline{p}$ collisions[8,9]. We find for $M_{Higgs} = 45(1000)$ GeV a value of $M_{top} = 116 \pm 38(144 \pm 37)$ GeV. The errors include the uncertainty from the Z^0 mass and the vacuum polarization[11]. With the limits on M_{top} one can calculate the electroweak mixing angle in the $\overline{MS}$ scheme to be[12]:

$$\sin^2\theta_{\overline{MS}} = 0.2336 \pm 0.0018. \tag{7}$$

To define the electroweak coupling constants at a scale M_Z we use for the fine structure constant the parametrization of Ref. [13] and one gets $\alpha(M_Z) = \frac{1}{128.8}$ With the value of $\sin^2\theta_{\overline{MS}}$ given above one obtains:

$$\alpha_1(M_Z) = 0.016887 \pm 0.000040, \quad (8)$$
$$\alpha_2(M_Z) = 0.03322 \pm 0.00025. \quad (9)$$

The present analysis uses the two values of α_s from Refs. [14], which are based on the measurements of the differential jet rates and of the asymmetry of the energy energy correlation. After symmetrizing the theoretical errors, we obtain for the weighted average and its estimated 68% C.L. error:

$$\alpha_3(M_Z) = \alpha_s(M_Z) = 0.108 \pm 0.005. \quad (10)$$

This value of α_s agrees with the recent α_s determination from deep inelastic lepton nucleon scattering and single γ production ($\alpha_s(M_Z) = 0.109^{+0.004}_{-0.005}$)[15].

3 Comparison with Grand Unified Theories

The coupling constants should evolve smoothly until they become identical at the unification scale. Here we make the simplifying assumption that at the unification point the couplings cross without changing slopes.

The evolutions of the three coupling constants with the new data are shown in Fig. 1b using the minimal SM with 3 families and 1 Higgs doublet. Compared to the results of 1987 (Fig. 1a), the errors, indicated by the width of the lines, are considerably smaller.

It is clear that a single unification point cannot be obtained within the present errors: the α_3 coupling constant misses the crossing point of the other two by more than 7 standard deviations. Only with $\alpha_s(M_Z)$=0.07 one can force $1/\alpha_3$ to pass through the crossing point of the other two or, alternatively, if one leaves $\alpha_s(M_Z)$ at 0.108, one has to lower $\sin^2\theta_{\overline{MS}}$ to 0.21 in order to get a single unification point. These values are in disagreement with the experimental values quoted in the previous section.

In SUSY GUT's [16] we fitted both the unification scale M_{GUT} and the SUSY breaking scale M_{SUSY} , which is defined as the transition point where the slopes of the extrapolation change. The mass of the lightest Higgs doublet was chosen to be equal to M_Z and the mass of the heavier doublet was taken to be equal to M_{SUSY}. These choices have practically no influence on our conclusions as long as the Higgs masses are less than a few times M_{SUSY}.

The second evolutions give the results shown in Fig. 2a. The values of M_{GUT} and M_{SUSY} are correlated. By taking this correlation into account, one finds:

$$M_{SUSY} = 10^{3.0\pm1.0}\text{GeV}, \quad (11)$$
$$M_{GUT} = 10^{16.0\pm0.3}\text{GeV}, \quad (12)$$
$$\alpha_{GUT}^{-1} = 25.7 \pm 1.7. \quad (13)$$

We have repeated the fits for different values of $\alpha_3(M_Z)$ and the results are shown in Figs. 2b and 2c. One observes that M_{SUSY} is a steep function of α_s: for $\alpha_s(M_Z)$ between 0.10 and 0.12, M_{SUSY} varies between 30 TeV and 10 GeV.

Until now the assumption was made that the slopes change from SM values to SUSY values exactly at M_{SUSY}. This abrupt change is unphysical, not only because the particles are virtual, but also because different SUSY particles are likely to have different masses. To model the actual behaviour we have smeared this change over 1 to 3 orders of magnitude symmetrically around M_{SUSY} by taking the average of the SM and SUSY slopes in this interval. This smearing lowers the fitted value of M_{SUSY} and has little influence on M_{GUT}, as shown by the dashed and dotted lines in Figs. 3a and 3b.

4 Summary

It was shown that the evolution of the coupling constants within the minimal Standard Model with one Higgs doublet does not lead to Grand Unification,

On the contrary, the minimal supersymmetric extension of the Standard Model leads to unification at a scale of $10^{16.0\pm0.3}$ GeV. The best fit to the allowed minimal SUSY model, shown in Fig. 2, is obtained for a SUSY scale around 1000 GeV or, more precisely, $M_{SUSY} = 10^{3.0\pm1.0}$ GeV, where the error originates mainly from the uncertainty in the strong coupling constant. If this minimal supersymmetric GUT describes nature, SUSY particles, which are expected to have masses of the order of M_{SUSY}, could be within reach of the present or next generation of accelerators.

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Figure captions

Fig. 1. a) First order evolution of the three coupling constants in the minimal Standard model (world average values in 1987 from Ref. 1). The small figure is a blow up of the crossing area.

b) As above but using M_Z and $\alpha_s(M_Z)$ from DELPHI data. The three coupling constants disagree with a single unification point by more than 7 standard deviations.

Fig. 2. a) Second order evolution of the three coupling constants in the minimal SUSY model. M_{SUSY} has been fitted by requiring crossing of the couplings in a single point. The two lower plots show M_{SUSY} (b) and M_{GUT} (c) as function of $\alpha_3(M_Z)$. The uncertainties in M_{GUT} and M_{SUSY} from the errors in $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$ are small. The full line assumes that all SUSY particles have the mass of the SUSY scale. The dashed, dotted and dashed-dotted lines indicate the results if the SUSY particle spectrum is smeared over the range indicated in the figure.

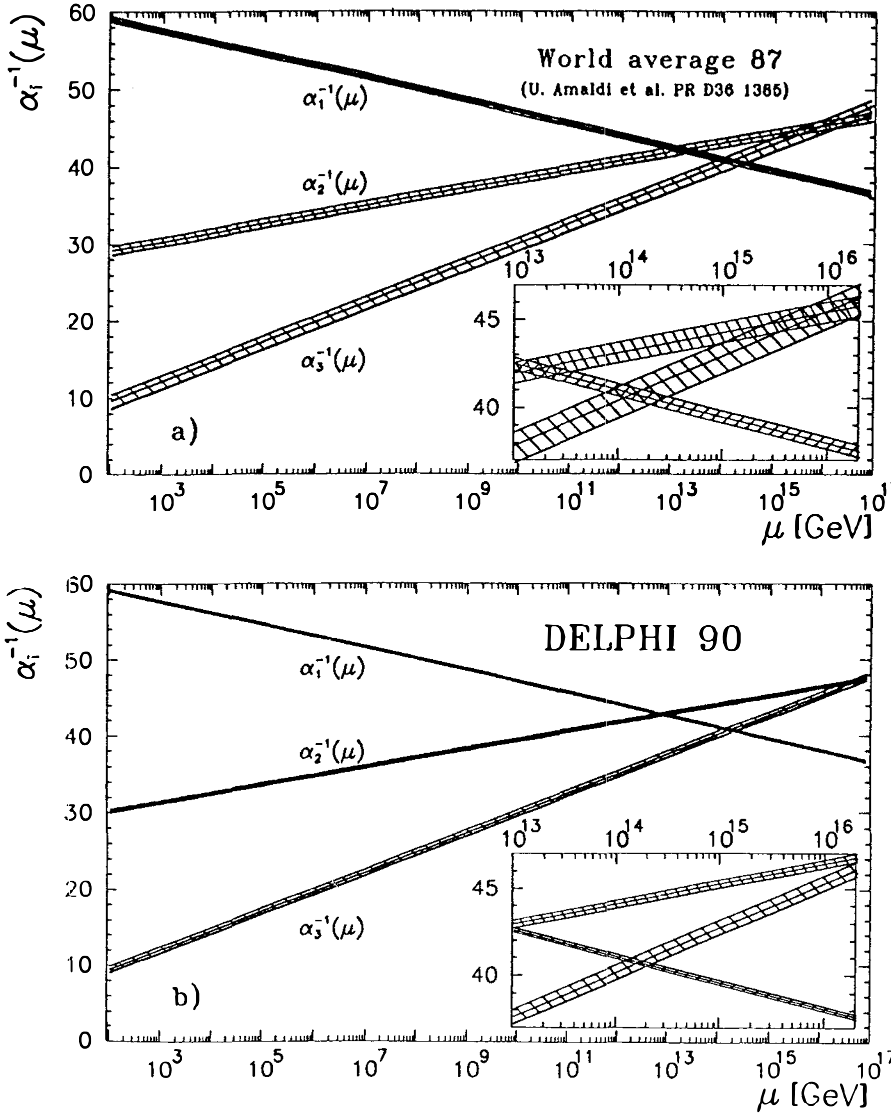
World average 87
(U. Amaldi et al. PR D36 1385)
$\alpha_i^{-1}(\mu)$
$\alpha_1^{-1}(\mu)$
$\alpha_2^{-1}(\mu)$
$\alpha_3^{-1}(\mu)$
a)
μ [GeV]
DELPHI 90
b)

Fig. 1

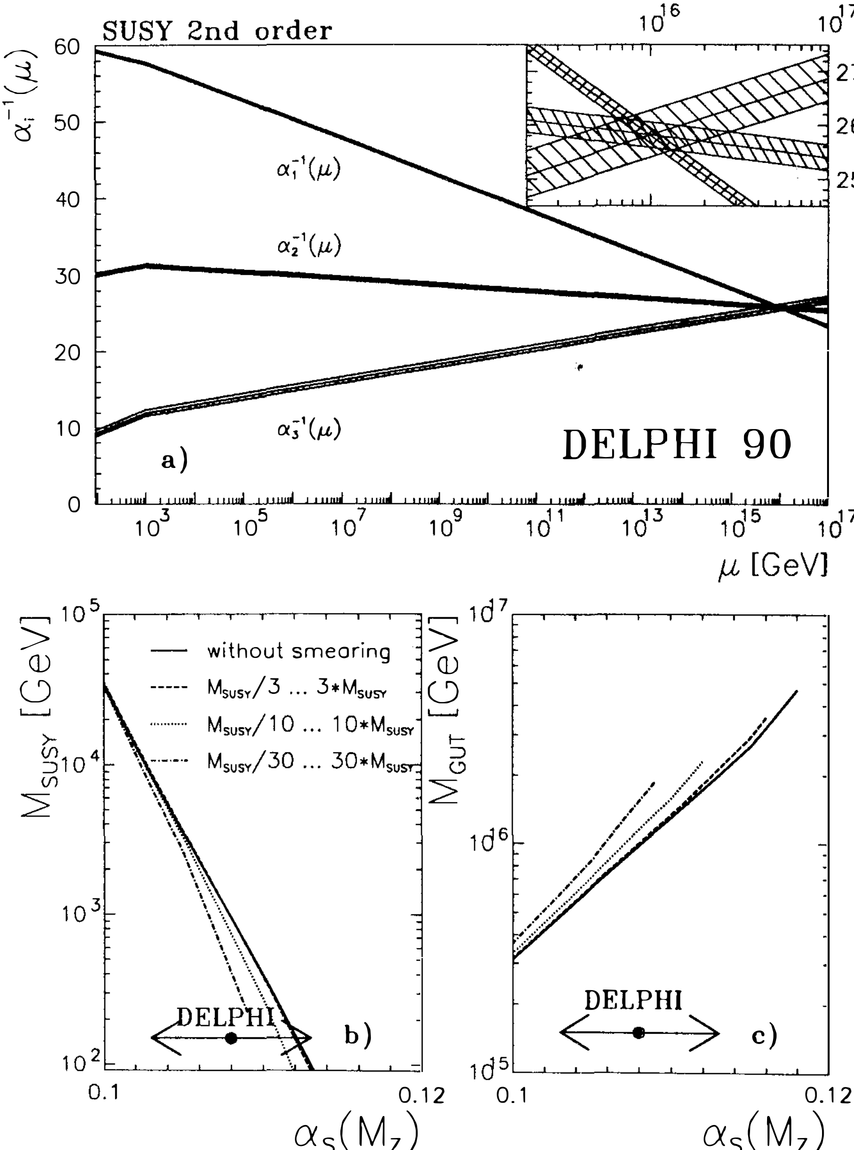
SUSY 2nd order
$\alpha_i^{-1}(\mu)$
$\alpha_1^{-1}(\mu)$
$\alpha_2^{-1}(\mu)$
$\alpha_3^{-1}(\mu)$
DELPHI 90
a)
μ [GeV]
M_{SUSY} [GeV]
without smearing
$M_{SUSY}/3$ ... $3*M_{SUSY}$
$M_{SUSY}/10$ ... $10*M_{SUSY}$
$M_{SUSY}/30$ ... $30*M_{SUSY}$
DELPHI
b)
$\alpha_s(M_Z)$
M_{GUT} [GeV]
DELPHI
c)
$\alpha_s(M_Z)$

Fig. 2