

CHROMATIC CORRECTIONS AND DYNAMIC APERTURE IN THE HERA  
ELECTRON RING

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Abstract and Conclusion

The dynamic aperture of various sextupole correction schemes for the asymmetric HERA  $e^-$  lattice is investigated. Results show that it is important to restore the supersymmetry at least approximately by optics manipulations. Satisfactory sextupole configurations can be found for the whole range of lattices between  $60^\circ$  and  $90^\circ$  per FODO cell. They provide good chromatic behaviour. The beta functions change by less than 5% and tunes change less than 0.01 in the momentum range between -1% and +1%. The dynamic aperture considerably exceeds the necessary 6.5 standard deviations at 35 GeV for the whole range of solutions investigated.

Introduction

As in every large colliding beam storage ring, carefully optimized sextupole correction schemes are required in the HERA electron ring. In HERA, there are additional aspects:

The head-on-interaction scheme requires a special design for the straight section which is complicated by the need to have longitudinally polarized beams in the electron ring<sup>1</sup>. It became necessary to reserve one of the four straight sections for utilities like injection, proton dump e t c. This results in a special optics in that quadrant. Owing to the broken supersymmetry, structure resonances are not suppressed automatically. This results in enhanced nonlinear effects in the beam dynamics.

Another aspect is that it is very desirable to be able to vary the electron beam emittance. This is achieved by variable focussing in the arcs: for optimum beam-beam interaction at high luminosity the transverse beam dimensions at the interaction point have to be the same for electron and proton beams. The proton beam emittance however is not easy to adjust and predict. Also there is not much freedom left for changing the beta functions at the interaction point. Therefore a variable electron beam emittance is necessary to achieve optimum conditions for colliding

beams.

The adjustable optics in the arcs requires variable sextupole correction schemes to correct the chromaticity and the half integer off-momentum stop-band without exciting strong nonlinear resonances.

In this report we compare various optics which approximately restore the supersymmetry with respect to chromatic corrections and dynamic aperture. Furthermore we present various sextupole correction schemes for betatron phase advances between  $60^\circ$  and  $90^\circ$  per FODO cell.

Linear Optics and Sextupole Distributions

The basis of the solutions discussed here is the linear lattice design presented in ref.<sup>1,2</sup>.

There are two possibilities to restore approximately the symmetry in the HERA e-ring by means of the linear optics:

The first possibility is to give the modified insertion the same betatron phase advances as the supersymmetric ones. This restores 4 fold supersymmetry and suppresses structure resonances if the contributions to the chromaticity in the modified and in the standard insertion are not very different and if the nonlinear fields are located outside the insertion. However in HERA- $e^-$ , the contributions to chromaticity from an interaction straight section are much larger than the contributions from the utility straight section. Furthermore sextupole magnets are needed also in the matching sections at the ends of the arcs where the optics are different for utility and interaction straight sections.

Therefore an alternative solution has been investigated: the quadrant containing the utility straight section has an integer phase advance  $N$  in both planes and the chromatic effects are compensated separately for interaction and utility quadrants. Thus, the beam transport matrix for the utility quadrant is a momentum independent unit matrix and the rest of the ring has a quasi 3-fold super-periodicity with tunes of  $(Q-N)/3$  per period. This solution is

only satisfactory if the contributions to the driving terms of nonlinear resonances coming from the modified subperiod are small compared with the contributions from the other periods. This condition is in fact fairly well met in the modified utility quadrant: the short matching section between the periodic FODO structure of the arc and the straight section has rather smooth lattice functions so that sextupoles are placed essentially in a periodic FODO structure only. For certain phase advances/FODO cell it is then possible to arrange sextupoles so that there are no contributions to the driving terms of the lowest order nonlinear resonances no matter how the different sextupole families are excited. (This is explained in the next sections).

The chromatic corrections are achieved by an interleaved sextupole scheme as proposed in ref.<sup>3</sup>. A horizontally focussing sextupole and a vertically focussing sextupole is placed in each FODO cell in the arcs. The members of the different families are distributed uniformly over the lattice.

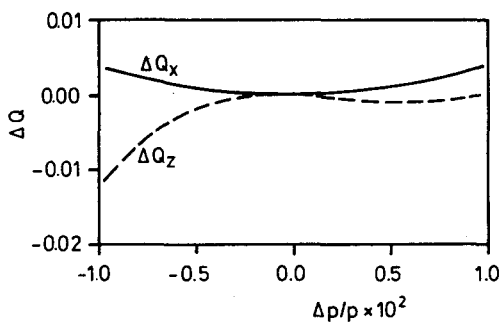
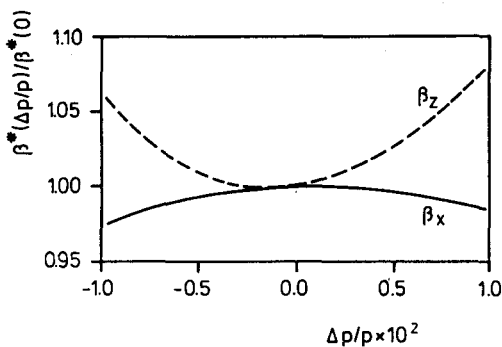


Fig. 1 a) tune change vs  $\Delta p/p$  for the  $60^\circ$  quasi 3-fold symmetric solution  
solid line: horizontal tune, broken line: vertical tune



b) change of  $\beta$  functions at interaction point, same optics  
solid line: horizontal, broken line: vertical

In order to compensate the chromaticities in both planes and the off momentum  $\beta$  and  $\alpha$ -beat factors (half integer stopband), at least 3 sextupole families per oscillation plane are needed. Because the beat factors have an amplitude and a phase which propagates with twice the betatron phase advance around the machine, we must provide two orthogonal correction circuits (if possible spaced by  $45^\circ$  phase advance) for each plane in addition to the chromaticity correction circuits.

The nonlinear effects of the sextupoles on the beam dynamics should be as small as possible. We expect that this will be the case if the driving terms of the lowest order nonlinear resonances (up to order 3) are zero. For a periodic FODO structure this is accomplished without any further sextupole circuits if the phase advance  $\phi_c$  per FODO cell is an even fraction of an integer

$$\phi_c = n/m \cdot 2\pi \quad (m \text{ even})$$

and if the sextupoles are arranged in closed blocks extending over  $m$  FODO cells which have an integer phase advance  $n \cdot 2\pi$  and which are composed of two identical parts each spanning a phase advance  $n\pi$ .

The intrinsic cancellation of driving terms is achieved for any excitation of the different families, because the sextupoles in a block are paired with other sextupoles of the same family which are spaced by  $n \cdot \pi$  phase advance so that distortions cancel in first order.

We investigated the sextupole configurations for phase advances between  $60^\circ$  and  $90^\circ$ /FODO cell (equal phase advances in both planes are considered only).

Consider for example a phase advance of  $60^\circ$ /FODO cell: 6 FODO cells form a sextupole block with  $\Delta\phi=2\pi$ . A HERA octant accommodates 4 sextupole blocks. The arrangement of the 3 families/oscillation plane is:

$$A - B - C - A - B - C$$

(where A includes two half FODO cells with a horizontally focussing sextupole family, "HA", and a vertically focussing sextupole family, "VA").

Orthogonality of the circuits for the compensation of beat factors is not perfect but it nevertheless allows the compensation of beat factors while deviations from the mean sextupole strength are smaller than 31%. The driving terms for the nonlinear resonances up to the order 3 are not excited in the periodic part of the arcs.

In a  $90^\circ$  lattice, the 4-family sextupole blocks are as short as 4 cells. The configuration is  $A - B - A - B$ . For this phase advance only one of the two orthogonal circuits is available for stopband com-

pensation. Therefore the off momentum  $\beta$ -beats excited in the straight section can only be compensated if they arrive with a phase of zero or  $180^\circ$  at the first sextupoles in the periodic arc structure. This has been achieved by a proper distribution of phase advances in the straight sections. The lowest order driving terms are intrinsically compensated, but there is some concern about higher order effects. Contributions to the fourth order resonance driving terms from each cell build up over the whole HERA quadrant so that these resonances are expected to be strong (see below).

Therefore we also investigated a nearby solution, a  $88^\circ$ , 4-family lattice where first and third integer resonances are not completely suppressed. The higher order resonances however are expected to be reduced. As an example for phase advances in between  $60^\circ$  and  $90^\circ$  we present  $67.5^\circ$ :

The sextupole block includes 16 FODO cells ( $\Delta\phi = 3 \cdot 2\pi$ ). The arrangement of the 5 families per plane is

A - B - C - B - D - B - E - B - A ....

This configuration provides two perfectly orthogonal circuit combinations which allow correction of beat factors with only 25% deviation from the mean sextupole strength. Stopband and chromaticity compensation are decoupled. The "B" families act on the chromaticities only, the A and D families form one of the orthogonal circuit combinations and C and E families the other one. Deviations of A and D from B have the same size but different signs thus  $m_A + m_D = m_C + m_E = 2 \cdot m_B$  ( $m$ : sextupole strength). Resonances up to 3rd order are suppressed intrinsically. The contributions to 4th order resonance driving terms from each cell have a tendency to cancel so that we only expect small contributions from higher order effects on the beam dynamics.

Excellent chromatic properties are achieved for all phase advances. In the momentum range of

$$-1\% < \Delta p/p < +1\%,$$

the tune change does not exceed 0.01 and the  $\beta$ -functions at the interaction point change by less than 5% (see fig. 1).

#### Dynamic Aperture

The dynamic aperture is determined by particle tracking using the computer code RACETRACK<sup>4</sup>. Tracking is performed with synchrotron oscillations in the approximation of externally modulated momentum deviations. The program estimates the phase space areas

in both planes which correspond to the maximum stable starting amplitude. These maximum stable phase space areas will be referred to as acceptances. For a more detailed discussion see ref.<sup>5</sup>. We always start with an initial emittance ratio of  $\epsilon_z/\epsilon_x = 10\%$ . As stability criterion, survival of a particle for 1000 machine turns within a given large aperture is assumed.

Firstly, we compare the dynamic aperture for the two quasi symmetric optical solutions for a phase advance of  $60^\circ$ /FODO cell. Sextupole fields are the only nonlinearities taken into account. In Fig.2 we present the available number of standard deviations (square root of horizontal acceptance divided by equilibrium emittance) as a function of the momentum amplitude of the synchrotron oscillation. The shaded area represents the space needed by a 35 GeV electron beam.

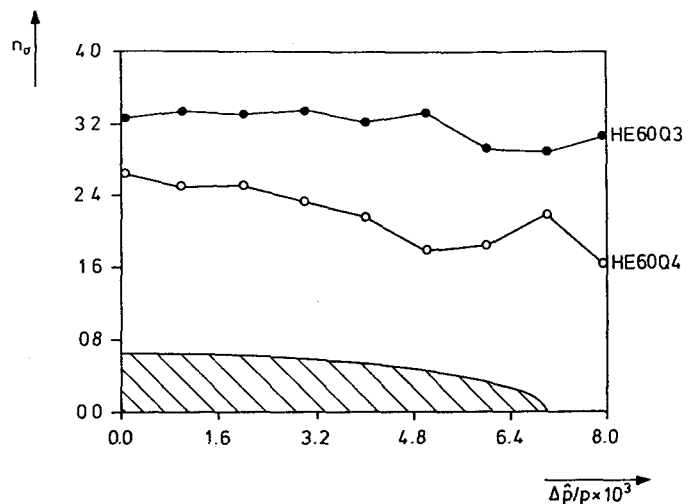


Fig.2 Sextupole acceptance  $A_x$  for quasi 4-fold (o) and quasi 3-fold (●) symmetric solutions.

Plotted is  $n_\sigma = \sqrt{A_x/\epsilon_x}$  vs the amplitude of momentum oscillations  $\Delta p/p$

We recognize a significant advantage of the quasi 3fold-symmetric solution. The reason is that the quasi 4fold-symmetric solution does not suppress sufficiently the lowest order resonances. The resonance widths for  $Q_x=47$  and  $Q_x+2Q_x=142$  are 5 times and twice as large respectively as the width in the 3-fold symmetric solution. In investigating the dynamic aperture in the vicinity of resonances we find a drastic reduction for both optics near  $4Q_x=189$  (which is a structure resonance for the 3fold quasi symmetric case). Near the third integer resonance however we find stability for the quasi 3-fold symmetric solution and an acceptance drop for the quasi 4-fold symmetric solution.

Because of the advantage of the quasi 3-fold symmetric solution we restrict the investigation of the acceptance for different phase advances in the arcs to this case. In Fig.3, the number of available standard deviations  $n_\sigma = (A_x/\epsilon_x)^{1/2}$  ( $E=35$  GeV, on momentum) is plotted for different phase advances (dots). Acceptances are obtained for optimized sextupole correction schemes according to the description in the previous section.

The quality of the various sextupole correction schemes can be discussed by comparing the tracking results with an analytic scaling law: we take  $60^\circ$  as a reference point and scale the sextupole strengths to compensate the phase dependent chromaticity contributions  $\xi^{\text{cell}} = \tan \phi_c/2$ , produced in the arcs. From the equation of motion of a particle in sextupole fields it follows that for a sextupole distribution which is scaled only quantitatively but which is not changed qualitatively  $n_\sigma$  scales simply as

$$n_\sigma \propto (m\epsilon)^{-k/2}.$$

The parameter  $k=1$  if the acceptance is restricted by effects linear in the sextupole strength  $m$ ,  $k=2$  for quadratic effects and so on.

Expressing the equilibrium emittance  $\epsilon_x$  and sextupole strength  $m$  in terms of the phase advance/cell  $\phi_c$  and taking the fixed chromaticity contributions from the straight sections into account we obtain the following scaling law:

$$n_\sigma \propto \{\sin^3/2(\phi_c/2) \cdot (1+1.26 \cot(\phi_c/2))\}^{-k}$$

This is represented by the solid lines in Fig.3. Results of tracking are close to the  $k=1$  curve. This shows that the quality of the excellent  $60^\circ$  sextupole configuration can be maintained for larger phase advances.

The only exception is the  $90^\circ$  solution where 2nd and higher order effects in the sextupole strengths are enhanced due to the special phase advance. For example, we found a strong acceptance reduction near the 5th order resonance  $3Q_x + 2Q_z = 236$  which is excited by terms cubic in the sextupole strengths. Small deviations from  $90^\circ$  however are enough to reduce

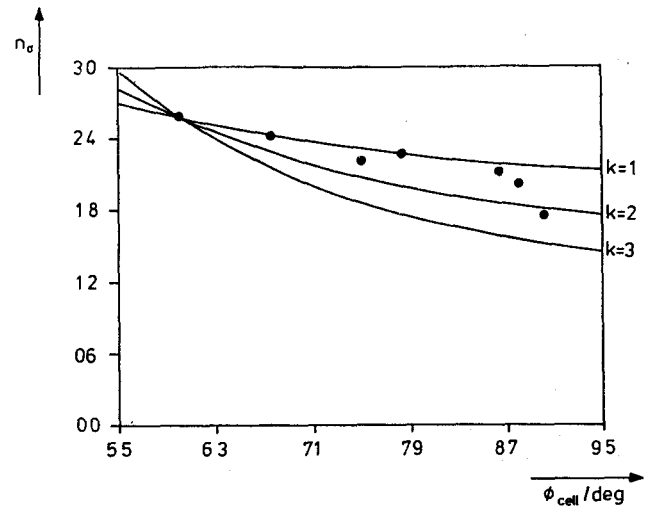


Fig. 3 Available standard deviations for different phase advances/FODO cell. The solid lines represent a scaling law for a nearly ideal sextupole distribution

these lattice resonances as is demonstrated by the  $88^\circ$  point.

For all the optical solutions the dynamic aperture considerably exceeds the minimum required 6.5 standard deviations. Thus we may conclude that the lifetime in the HERA electron ring will not be restricted by the sextupole acceptance.

#### References

1. D. Barber, R. Brinkmann, R. Kose, J. Roßbach, K. Steffen and F. Willeke: "HERA Straight Sections for Head-On Electron-Proton Interactions", IEEE Trans. Nucl. Sc. Vol. NS32-5, 1985
2. D. Barber, R. Brinkmann, R. Kose, J. Roßbach, K. Steffen and F. Willeke: The HERA Straight Sections, this conference
3. A. Wrulich: "Various Sextupole Schemes for the HERA Electron Ring", DESY HERA 85-14, 1985.
4. A. Wrulich: "RACETRACK: A Computer Code for the Simulation of Nonlinear Particle Motion in Accelerators", DESY 84-026, 1984.
5. R. Brinkmann and F. Willeke: Chromatic Corrections and Dynamic Aperture in the HERA Electron Ring, Part I, DESY 86 - 79