

α_s WITH LATTICE QCD

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The status of determinations of the strong coupling constant based on lattice QCD is reviewed.

1 Introduction and Motivation

At present, the QCD coupling, α_s , is determined from many different experiments, performed at energies ranging from a few to more than 100 GeV.¹⁾ In most cases perturbation theory is used to extract α_s from the experimental information. Experimental and theoretical progress over the last few years has made these determinations increasingly precise. However, all determinations, including those based on lattice QCD, rely on phenomenologically-estimated corrections and uncertainties from non-perturbative effects. These effects will eventually (or already do) limit the accuracy of the coupling constant determination. When lattice QCD is used the limiting uncertainty comes from the (total or partial) omission of sea quarks in numerical simulations.

The determination of the strong coupling, α_s , proceeds in three steps:

1. The first step is always an experimental measurement. In α_s determinations based on perturbative QCD this might be a cross section or (ratio of) decay rates. In determinations based on lattice QCD this is usually a hadron mass or mass splitting, for example the mass of the ρ meson, or a better choice, spin-averaged splittings in the charmonium and bottomonium system. In "lattice language" this step is often referred to as "setting the scale" (see section 2).
2. The second step involves a choice of renormalization scheme. In perturbative QCD the standard choice is the $\overline{\text{MS}}$ scheme, which is only defined perturbatively. With lattice QCD a non-perturbative scheme may be chosen, and there are many candidates. In order to compare with perturbative QCD, any such scheme should be accessible to perturbative calculations (without excessive effort).
3. Finally, the third step is an assessment of the experimental and theoretical errors associated with the strong coupling determination. This is of course the most important (and sometimes also the most controversial) step as it allows us to distinguish and weight different determinations. As a theorist, I don't have much to say about experimental errors, other than that they should be small and controlled. The experimental errors on hadron masses are negligibly small in lattice determinations of α_s at this point. The theoretical errors that are part of α_s determinations based on perturbative QCD include higher order terms in the truncated perturbative series and the associated dependence on the renormalization scale, and hadronization or other generic non-perturbative effects. In lattice QCD the theoretical errors include (but are not limited to) discretization errors (due to the finite lattice spacing, $a \neq 0$), finite volume effects, and errors associated with the partial or total omission of sea quarks.

The consideration of systematic uncertainties should guide us towards quantities where these uncertainties are controlled, for a reliable determination of α_s . As has been argued by

Lepage,²⁾ quarkonia are among the easiest systems to study with lattice QCD, since systematic errors can be analyzed easily with potential models if not by brute force.

Finite-volume errors are much easier to control for quarkonia than for light hadrons, since quarkonia are smaller. Lattice-spacing errors, on the other hand, can be larger for quarkonia and need to be considered. An alternative to reducing the lattice spacing in order to control this systematic error is improving the action (and operators). For quarkonia, the size of lattice-spacing errors in a numerical simulation can be *anticipated* by calculating expectation values of the corresponding operators using potential-model wave functions. They are therefore ideal systems to test and establish improvement techniques.

A lot of the work of phenomenological relevance is done in what is generally referred to as the “quenched” (and sometimes as the “valence”) approximation. In this approximation gluons are not allowed to split into quark - anti-quark pairs (sea quarks). In the case of quarkonia, potential model phenomenology can be used to estimate this systematic error.

I shall have neither the time nor the space to give an introduction to lattice QCD. Instead, I refer the reader to a number of pedagogical introductions and reviews in the literature.³⁾

2 Determination of the Lattice Spacing and the Quarkonium Spectrum

The experimental input to the strong coupling determination is a mass or mass splitting, from which by comparison with the corresponding lattice quantity the lattice spacing, a , is determined in physical units. For this purpose, one should identify quantities that are insensitive to lattice errors. In quarkonia, spin-averaged splittings are good candidates. The experimentally observed 1P-1S and 2S-1S splittings depend only mildly on the quark mass (for masses between m_b and m_c). Figure 1 shows the observed mass dependence of the 1P-1S splitting in a lattice QCD calculation. The comparison between results from different lattice actions illustrates that higher-order lattice-spacing errors for these splittings are small.^{4,5)}

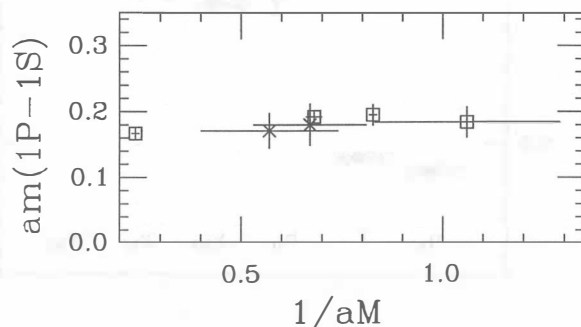


Figure 1: The 1P-1S splitting as a function of the 1S mass (statistical errors only) from Ref. [5]; \square : $\mathcal{O}(a^2)$ errors; \times : $\mathcal{O}(a)$ errors.

Two different formulations for fermions have been used in lattice calculations of the quarkonia spectra. In the non-relativistic limit the QCD action can be written as an expansion in powers of v^2 (or $1/m$), where v is the velocity of the heavy quark inside the boundstate,⁶⁾ I shall henceforth refer to this approach as NRQCD. Lepage and collaborators⁷⁾ have adapted this formalism to the lattice regulator. Several groups have performed numerical calculations of quarkonia in this approach. In Refs. [8,9] the NRQCD action is used to calculate the $b\bar{b}$ and $c\bar{c}$ spectra, including terms up to $\mathcal{O}(mv^4)$ and $\mathcal{O}(a^2)$. In addition to calculations in the quenched approximation, this group is also using gauge configurations that include 2 flavors of sea quarks with mass $m_q \sim \frac{1}{2}m_s$ to calculate the $b\bar{b}$ spectrum.^{4,10)} The leading order NRQCD action is used in Ref. [11] for a calculation of the $b\bar{b}$ spectrum in the quenched approximation.

The Fermilab group¹²⁾ developed a generalization of previous approaches, which encompasses the non-relativistic limit for heavy quarks as well as Wilson's relativistic action for light quarks. Lattice-spacing artifacts are analyzed for quarks with arbitrary mass. Ref. [5] uses this approach to calculate the $b\bar{b}$ and $c\bar{c}$ spectra in the quenched approximation. We considered the effect of reducing lattice-spacing errors from $\mathcal{O}(a)$ to $\mathcal{O}(a^2)$. The SCRI collaboration¹³⁾ is also using this approach for a calculation of the $b\bar{b}$ spectrum using the same gauge configurations as the NRQCD collaboration with $n_f = 2$ and an improved fermion action (with $\mathcal{O}(a^2)$ errors).

All but one group use gauge configurations generated with the Wilson action, leaving $\mathcal{O}(a^2)$ lattice-spacing errors in the results. The lattice spacings, in this case, are in the range $a \simeq 0.05 - 0.2$ fm. Ref. [14] uses an improved gauge action together with a non-relativistic quark action improved to the same order (but without spin-dependent terms) on coarse ($a \simeq 0.4 - 0.24$ fm) lattices. The results for the $b\bar{b}$ and $c\bar{c}$ spectra from all groups are summarized in Figures 2 and 3.

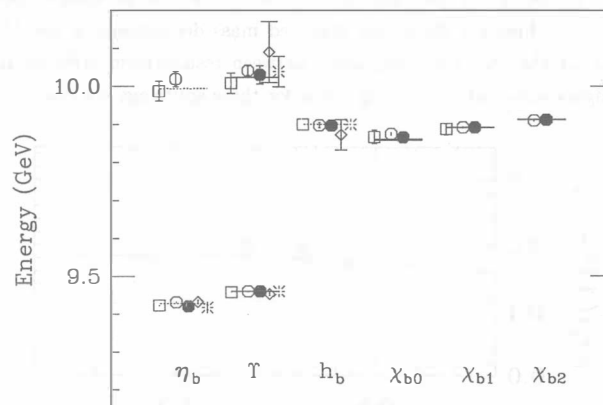


Figure 2: A comparison of lattice QCD results for the $b\bar{b}$ spectrum (statistical errors only). --: Experiment; \square : FNAL [5]; \circ : NRQCD ($n_f = 0$) [8]; \bullet : NRQCD ($n_f = 2$) [4]; \diamond : UK(NR)QCD [11]; $*$: SCRI [13].

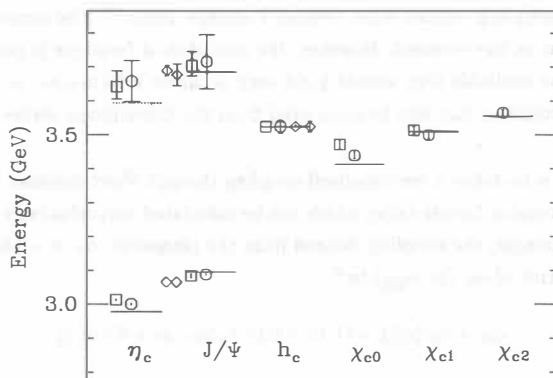


Figure 3: A comparison of lattice QCD results for the $c\bar{c}$ spectrum (statistical errors only). - : Experiment; \square : FNAL [5]; \circ : NRQCD ($n_f = 0$) [9]; \diamond : ADHLM [14].

The agreement between the experimentally-observed spectrum and lattice QCD calculations is impressive. As indicated in the preceding paragraphs, the lattice artifacts are different for all groups. Figures 2 and 3 therefore emphasize the level of control over systematic errors.

Results with 2 flavors of degenerate sea quarks have now become available from a number of groups,^{4,15,16,10} with lattice-spacing and finite-volume errors similar to the quenched calculations, significantly reducing this systematic error. However, several systematic effects associated with the inclusion of sea quarks still need to be studied further. They include the dependence of the quarkonium spectrum on the number of flavors of sea quarks, and the sea-quark action (staggered vs. Wilson). The inclusion of sea quarks with realistic light-quark masses is very difficult. However, quarkonia are expected to depend only very mildly on the masses of the light quarks.

3 Definition of a Renormalized Coupling

Within the framework of lattice QCD the conversion from the bare to a renormalized coupling can, in principle, be made non-perturbatively. In the definition of a renormalized coupling, systematic uncertainties should be controllable, and at short distances, its (perturbative) relation to other conventional definitions calculable. For example, a renormalized coupling can be defined from the non-perturbatively computed heavy-quark potential (α_V).¹⁷ An elegant approach has been developed in Ref. [18], where a renormalized coupling is defined non-perturbatively through the Schrödinger functional. The authors compute the evolution of the coupling non-perturbatively using a finite size scaling technique, which allows them to vary the momentum scales by an order of magnitude. The same technique has also been applied to

the renormalized coupling defined from twisted Polyakov loops.¹⁹⁾ The numerical calculations include only gluons at the moment. However, the inclusion of fermions is possible. Once such simulations become available they should yield very accurate information on α_s and its evolution. The strong coupling can also be computed from the three-gluon vertex, suitably defined on the lattice.²⁰⁾

An alternative is to define a renormalized coupling through short distance lattice quantities, like small Wilson loops or Creutz ratios which can be calculated perturbatively and by numerical simulation. For example, the coupling defined from the plaquette, $\alpha_P = -3 \ln \langle \text{Tr } U_P \rangle / 4\pi$, can be expressed in terms of α_V (or $\alpha_{\overline{\text{MS}}}$) by²¹⁾

$$\alpha_P = \alpha_V(q) [1 - (1.19 + 0.07 n_f) \alpha_V(q) + \mathcal{O}(\alpha_V^2)] \quad (1)$$

at $q = 3.41/a$, close to the ultraviolet cut-off. α_V is related to the more commonly used $\overline{\text{MS}}$ coupling by

$$\alpha_{\overline{\text{MS}}}(Q) = \alpha_V(e^{5/6} Q) (1 + \frac{2}{\pi} \alpha_V + \dots) \quad (2)$$

The size of higher-order corrections associated with the above defined coupling constants can be tested by comparing perturbative predictions for short-distance lattice quantities with non-perturbative results.²¹⁾ The comparison of the non-perturbatively calculated coupling of Ref. [18] with the perturbative predictions for this coupling using Eq. (1) is an additional consistency test.

The relation of the plaquette coupling in Eq. (1) to the $\overline{\text{MS}}$ coupling has recently been calculated to 2-loops^{22,23)} in the quenched approximation (no sea quarks, $n_f = 0$). The extension to $n_f \neq 0$ will significantly reduce the uncertainty due to the use of perturbation theory.

4 Sea Quark Effects

Calculations that properly include all sea-quark effects do not yet exist. If we want to make contact with the "real world", these effects have to be estimated phenomenologically or extrapolated away.

The phenomenological correction necessary to account for the sea-quark effects omitted in calculations of quarkonia that use the quenched approximation gives rise to the dominant systematic error in this calculation.^{24,25)} By demanding that, say, the spin-averaged 1P-1S splitting calculated on the lattice reproduce the experimentally observed one (which sets the lattice spacing, a^{-1} , in physical units), the effective coupling of the quenched potential is in effect matched to the coupling of the effective 3 flavor potential at the typical momentum scale of the quarkonium states in question. The difference in the evolution of the zero flavor and 3,4 flavor couplings from the effective low-energy scale to the ultraviolet cut-off, where α_s is determined, is the perturbative estimate of the correction.

For comparison with other determinations of α_s , the $\overline{\text{MS}}$ coupling can be evolved to the Z mass scale. An average¹⁾ of Refs. [24,25] yields for α_s from calculations in the quenched

approximation:

$$\alpha_{\overline{\text{MS}}}^{(5)}(m_Z) = 0.110 \pm 0.006 \quad . \quad (3)$$

The phenomenological correction described in the previous paragraph has been tested from first principles in Ref. [15]. The 2-loop evolution of $n_f = 0$ and $n_f = 2$ $\overline{\text{MS}}$ couplings – extracted from calculations of the $c\bar{c}$ spectrum using the Wilson action in the quenched approximation and with 2 flavors of sea quarks respectively – to the low-energy scale gives consistent results. After correcting the 2 flavor result to $n_f = 3$ in the same manner as before and evolving $\alpha_{\overline{\text{MS}}}$ to the Z mass, they find ¹⁵⁾

$$\alpha_{\overline{\text{MS}}}^{(5)}(m_Z) = 0.111 \pm 0.005 \quad (4)$$

in good agreement with the previous result in Eq. (3). The total error is now dominated by the rather large statistical errors and the perturbative uncertainty.

The most accurate result to date comes from the NRQCD collaboration.^{4,10)} They use results for α_s from the $b\bar{b}$ spectrum with 0 and 2 flavors of sea quarks to extrapolate the inverse coupling to the physical 3 flavor case directly at the ultraviolet momentum, $q = 3.41/a$. They obtain a result consistent with the old procedure. Recently, they have begun to study the dependence of α_s on the masses of the sea quarks. Their preliminary result is:

$$\alpha_V^{(3)}(8.2 \text{ GeV}) = 0.195 \pm 0.003 \pm 0.001 \pm 0.004 \quad . \quad (5)$$

The first error is statistics, the second error an estimate of residual cut-off effects and the third (dominant) error is due to the quark mass dependence. The conversion to $\overline{\text{MS}}$ (including the 2-loop term of Refs. [22,23]) and evolution to the Z mass then gives:

$$\alpha_{\overline{\text{MS}}}^{(5)}(m_Z) = 0.118 \pm 0.003 \quad , \quad (6)$$

where the error now also includes the perturbative uncertainty from eq. (2). A similar analysis is performed in Ref. [16] on the same gauge configurations but using the Wilson action for a calculation of the $c\bar{c}$ spectrum. The result for the coupling is consistent with Refs. [4,15].

The preliminary calculation of the SCRI collaboration³⁾ ($n_f = 2$) can be combined with the result of Ref. [5]. Using the same analysis as in Ref. [4] gives ¹⁰⁾

$$\alpha_{\overline{\text{MS}}}^{(5)}(m_Z) = 0.116 \pm 0.003 \quad , \quad (7)$$

nically consistent with eqn. (6). Clearly, more work is needed to confirm the results of eqns. (6) and (7), especially in calculations with heavy quark actions based on Ref. [12]. In particular, the systematic errors associated with the inclusion of sea quarks into the simulation have to be checked, as outlined in section 2.

5 Conclusions

Phenomenological corrections are a necessary evil that enter most coupling constant determinations. In contrast, lattice QCD calculations with complete control over systematic errors

will yield truly first-principles determinations of α_s from the experimentally observed hadron spectrum.

At present, determinations of α_s from the experimentally measured quarkonia spectra using lattice QCD are comparable in reliability and accuracy to other determinations based on perturbative QCD from high energy experiments. They are therefore part of the 1995 world average for α_s .¹⁾ The phenomenological corrections for the most important sources of systematic errors in lattice QCD calculations of quarkonia have already been replaced by first principles calculations. This has led to a significant increase in the accuracy of α_s determinations from quarkonia.

Still lacking for a first-principles result is the proper inclusion of sea quarks. A difficult problem in this context is the inclusion of sea quarks with physical light quark masses. At present, this can only be achieved by extrapolation (from $m_q \simeq 0.3 - 0.5m_s$ to $m_{u,d}$). If the light quark mass dependence of the quarkonia spectra is mild, as indicated by the NRQCD collaboration, the associated systematic error can be controlled. First-principles calculations of quarkonia could then be performed with currently available computational resources.

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