

Katowski-Sachs generalized ghost dark energy cosmological model in Saez-Ballester scalar -tensor theory

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Abstract. In this paper we have studied the Kantowski-Sachs (KS) space time filled with generalized ghost dark energy (GGDE) and dark matter in the Saez-Ballester theory of gravitation. The equation of state (EoS) parameter of dark energy (ω_λ) has been formulated as a function of cosmic time and shows quintom like behavior. The properties of density of dark energy (ρ_λ) and the density of dark matter (ρ_m) indicate the accelerated expansion of the universe. The pressure of the dark energy is negative at present and late time evolution of the universe indicating the accelerated expansion of our universe during these periods. The physical and geometrical aspects of the statefinder parameters (r, s), squared speed of sound (v_s^2) and $\omega_\lambda - \omega'_\lambda$ plane are also discussed.

1. Introduction

The acceleration of the cosmic expansion is among the most important discoveries in existing day cosmology [1–6]. This acceleration requires that nearly 75 % of the energy of the universe is a component with a negative pressure, namely dark energy (DE). Survey of cosmic microwave background radiation [7] and investigation of large-scale structures [8] have proved this discovery. Several models were introduced to study the accurate nature and evolution of DE which include cosmological constant model [9], quintessence models [10, 11], phantom dark energy models [12, 13], holographic dark energy models [14, 15], ghost dark energy model [16]. The cosmological constant model is unable to explain the cosmic coincidence of dark matter and dark energy densities and also faces the problem of fine tuning. The ghost field has no contribution to vacuum energy density in Minkowski space time, but in curved space time, it contributes to vacuum energy density $\sim (10^3 \text{eV})^4$ with Hubble parameter (H) $\sim 10^{-33}$ eV and Λ_{QCD} (QCD mass scale) $\sim 100 \text{eV}$ [17–19]. These numerical values indicate ghost dark energy has no fine-tuning problem. Modified ghost density proposed of the form $\rho_\lambda \cong \alpha H + \beta H^2$ [20], where H is the Hubble parameter, α and β are constant parameters. The constraints on α and β were developed using the recent observational data from Type Ia Supernovae (SNIa), the Cosmic Microwave Background (CMB), Baryon Acoustic Oscillation (BAO), Bayesian Neural Networks (BNN) and the Hubble parameter data. GGDE has attained significant attention and various aspects have been discussed such as interacting GGDE models. Some works on ghost dark energy can be found in [21–25].

The KS cosmologies have two symmetry properties, the spherical symmetry and the



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invariance under spatial translations. The vacuum solution for this line element is equivalent to the inner Schwarzschild space time. KS class of metric represents homogeneous but anisotropically expanding (contracting) cosmologies and provides models where the effect of anisotropy can be estimated and compared with FRW class of cosmologies [26]. Some works on KS cosmological model can be found in [27–30].

The equation of state (EoS) parameter ω_λ is important quantity in dark energy investigation, which relates pressure and density through an equation of state of the form $p_\lambda = \omega_\lambda \rho_\lambda$. Due to lack of observational evidence in making a distinction between constant and variable, usually the equation of state parameter is considered as a constant [31, 32] with values 0, $\frac{1}{3}$, -1 and 1 for dust, radiation, vacuum fluid and stiff fluid dominated universe respectively. But in general, ω_λ is a function of time or redshift [33, 34]. Some of quintessence models involving scalar fields give rise to time-dependent ω_λ are found in [35–37]. Hence, there is sufficient background for considering ω_λ as time-dependent for a better understanding of the cosmic evolution.

Alternative theories [38–41] become crucial in the field of cosmology, this is because of the fact that general relativity does not fully comprise Machs principle. There are two categories of alternative gravitational theories which involves a scalar field ϕ . The first category comprises of that in which the scalar field ϕ has the dimension as inverse of G , in which Brans-Dicke holds a place of prominence. The importance of Brans-Dicke is underlined by the fact that it introduces an additional scalar field ϕ besides metric tensor g_{ij} and $\bar{\omega}$ as a dimensionless coupling constant. The second category comprises of a dimensionless scalar field. The theory which has the coupling of dimensionless scalar field was proposed by Saez and Ballester, which in addition describes the weak fields. This theory has an answer to the question of missing matter in FRW universe (non-flat). The importance of scalar tensor theories is underlined by the fact that it solves the problem of the smooth exit in inflation era [42].

Saez-Ballester theory of gravitation with Bianchi type cosmological models can be found in [27, 43–47]. Motivated by the above works in the present paper we have studied KS cosmological model filled with ghost dark energy and dark matter in Saez-Ballester theory of gravitation. The plots of some of the cosmological parameters versus redshift are presented to study their physical properties.

The paper is organized as follows. In section 2, we discuss about the basic formulation of field equations followed by the solution of the field equations in section 3. In section 4, we discuss about some important properties of the model and we summarize the results in the last section.

2. Basic Formalism

The field equations of Saez and Ballester scalar-tensor theory ($8\pi G = 1$ and $c = 1$) [41] are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} + \bar{\omega}\phi^n \left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,\beta}\phi^{,\beta} \right), \quad (1)$$

and the scalar field ϕ satisfies the equation

$$2\phi^n \phi_{;i}^i + n\phi^{n-1}\phi_{,\beta}\phi^{,\beta} = 0, \quad (2)$$

where R_{ij} is the Ricci tensor, R is the scalar curvature, ϕ is a dimensionless scalar field which is a function of cosmic time t alone, $\bar{\omega}$ and n are constants. Comma and semicolon denote partial and covariant differentiation, respectively the energy-momentum tensor of the ghost dark energy is given by

$$T_{ij} = T_{ij}^\lambda + T_{ij}^m, \quad (3)$$

where,

$$T_{ij}^\lambda = \text{diag}[1, -\omega_\lambda, -(\omega_\lambda + \delta), -(\omega_\lambda + \gamma)]\rho_\lambda, \quad (4)$$

is the stress energy tensor of ghost dark energy. Here $\omega_\lambda = \frac{p_\lambda}{\rho_\lambda}$, ρ_λ , p_λ , δ , γ are equation of state(EoS) parameter, density of the ghost dark energy, pressure of the ghost dark energy, skewness parameters in the direction of y-axis, z-axis, respectively and

$$T_{ij}^m = \text{diag}(\rho_m, 0, 0, 0), \quad (5)$$

is the stress energy tensor of the matter, ρ_m is the density of dark matter. Also, we have energy conservation equation as

$$T_{;j}^{ij} = 0. \quad (6)$$

We consider the spatially homogeneous and anisotropic Kantowski-Sachs space-time given by [48] in the form

$$ds^2 = dt^2 - D^2(t)dr^2 - E^2(t) [d\theta^2 + \sin^2\theta d\psi^2], \quad (7)$$

where D and E are the functions of cosmic time t only.

For the metric given by equation (7) using equations (3) - (5) the field equations (1) and (2) takes the form

$$\frac{\dot{E}^2}{E^2} + 2\frac{\ddot{E}}{E} + \frac{1}{E^2} - \frac{1}{2}\bar{\omega}\phi^n\dot{\phi}^2 = -\omega_\lambda\rho_\lambda, \quad (8)$$

$$\frac{\ddot{D}}{D} + \frac{\ddot{E}}{E} + \frac{\dot{D}\dot{E}}{DE} - \frac{1}{2}\bar{\omega}\phi^n\dot{\phi}^2 = -(\omega_\lambda + \delta)\rho_\lambda, \quad (9)$$

$$\frac{\ddot{D}}{D} + \frac{\ddot{E}}{E} + \frac{\dot{D}\dot{E}}{DE} - \frac{1}{2}\bar{\omega}\phi^n\dot{\phi}^2 = -(\omega_\lambda + \gamma)\rho_\lambda, \quad (10)$$

$$2\frac{\dot{D}\dot{E}}{DE} + \frac{\dot{E}^2}{E^2} + \frac{1}{E^2} + \frac{1}{2}\bar{\omega}\phi^n\dot{\phi}^2 = \rho_m + \rho_\lambda, \quad (11)$$

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{D}}{D} + 2\frac{\dot{E}}{E} \right) + \frac{n}{2} \frac{\dot{\phi}^2}{\phi} = 0. \quad (12)$$

The energy conservation equation (6), yields

$$\dot{\rho}_\lambda + \dot{\rho}_m + 3(\rho_\lambda + \rho_m + p_\lambda)H = 0, \quad (13)$$

where H is Hubbles parameter and overdot(.) represents derivative with respect to cosmic time t .

We assume that the matter and holographic dark energy donot interact with each other hence both the components conserve separately, so that the continuity equation of ghost dark energy is

$$\dot{\rho}_\lambda + 3(\rho_\lambda + \omega_\lambda\rho_\lambda)H = 0, \quad (14)$$

and the continuity equation of matter is

$$\dot{\rho}_m + 3\rho_m H = 0. \quad (15)$$

3. Solutions of the field equations

From equations (9) and (10) we get,

$$\delta = \gamma, \quad (16)$$

As the result of equation (16) the field equations (8) to (12) reduced to four independent equations with seven unknowns D , E , ϕ , ρ_λ , ρ_m , ω_λ , δ . In order to find a deterministic solution

we take the following three physically valid conditions,

(i) We take the scale factor as given by hybrid expansion law [49],

$$a(t) = (t^b e^t)^{\frac{1}{l}}, \quad (17)$$

where b and l are positive constants.

(ii) We take the shear scalar σ in the model to be proportional to the expansion scalar θ , this condition leads to [26, 50]

$$D = E^k, \quad (18)$$

where $k > 0$ and $k \neq 1$ is a constant.

(iii) We define the generalized ghost dark energy as [20]

$$\rho_\lambda = \alpha H + \beta H^2. \quad (19)$$

The Hubble parameter H is given by,

$$H = \frac{\dot{a}}{a}, \quad (20)$$

Using equations (17) and (20) the Hubble parameter H is obtained as

$$H = \frac{1}{l} \left(\frac{b}{t} + 1 \right). \quad (21)$$

Using equation (18) we get,

$$\frac{\dot{D}}{D} = k \frac{\dot{E}}{E}, \quad (22)$$

The directional Hubble parameters are,

$$H_1 = \frac{\dot{D}}{D}, \quad H_2 = H_3 = \frac{\dot{E}}{E}, \quad (23)$$

The mean Hubble parameter is,

$$H = \frac{1}{3}(H_1 + 2H_2). \quad (24)$$

Then from equations (22)- (24) it follows that

$$H_1 = \frac{3k}{k+2}H, \quad H_2 = H_3 = \frac{3}{k+2}H. \quad (25)$$

From equations (20), (23) and (24) we get,

$$D = (t^b e^t)^{\frac{3k}{l(k+2)}}, \quad (26)$$

$$E = (t^b e^t)^{\frac{3}{l(k+2)}}. \quad (27)$$

Now the metric (7) with the help of equations (26) and (27) can be written as

$$ds^2 = dt^2 - (t^b e^t)^{\frac{6k}{l(k+2)}} dr^2 - (t^b e^t)^{\frac{6}{l(k+2)}} [d\theta^2 + \sin^2 \theta d\psi^2]. \quad (28)$$

From equations (12), (17), (20), (23) and (24) we find Sez-Ballester scalar field as

$$\phi = \left[\frac{n+2}{2} \int (ct^b e^t)^{\frac{-3}{l}} dt + \phi_0 \right]^{\frac{2}{n+2}}, \quad (29)$$

where c and ϕ_0 are constants of integration.

From equations (19) and (21) the density of ghost dark energy(ρ_λ) is obtained as

$$\rho_\lambda = \frac{\alpha}{l} \left(\frac{b}{t} + 1 \right) + \frac{\beta}{l^2} \left(\frac{b}{t} + 1 \right)^2. \quad (30)$$

From equations (15), (17) and (20) the dark matter (ρ_m) is obtained as,

$$\rho_m = c_1 (t^b e^t)^{-\frac{3}{l}}, \quad (31)$$

where c_1 is constant of integration.

Using equations (8), (9), (12), (16), (26) and (27) the skewness parameters obtained as

$$\delta = \gamma = \frac{\frac{9(1-k)}{l^2(k+2)} \left(\frac{b}{t} + 1 \right)^2 + \frac{3b(k-1)}{lt^2(k+2)} + \frac{1}{(t^b e^t)^{\frac{6}{k+2}}}}{\frac{\alpha}{l} \left(\frac{b}{t} + 1 \right) + \frac{\beta}{l^2} \left(\frac{b}{t} + 1 \right)^2}, \quad (32)$$

Using equations (8)-(10), (12), (16), (26), (27) and (30), the EoS parameter (ω_λ) for the Ghost dark energy is obtained as

$$\omega_\lambda = \frac{\frac{54}{l^2(k+2)^2} \left(\frac{b}{t} + 1 \right)^2 - \frac{12b}{lt^2(k+2)} + \frac{2}{(t^b e^t)^{\frac{6}{k+2}}} - \frac{\bar{\omega}}{(t^b e^t)^{\frac{6}{l}}}}{-\frac{2\alpha}{l} \left(\frac{b}{t} + 1 \right) - \frac{2\beta}{l^2} \left(\frac{b}{t} + 1 \right)^2}, \quad (33)$$

From equations (30) and (33) the pressure of ghost dark energy (p_λ) is obtained as

$$p_\lambda = \omega_\lambda \rho_\lambda = -\frac{27}{l^2(k+2)^2} \left(\frac{b}{t} + 1 \right)^2 + \frac{6b}{lt^2(k+2)} - \frac{1}{(t^b e^t)^{\frac{6}{k+2}}} + \frac{\bar{\omega}}{2(t^b e^t)^{\frac{6}{l}}}. \quad (34)$$

4. Some other important properties of the model

The spatial volume of the model (28) is given by

$$V = a^3 = (t^b e^t)^{\frac{3}{l}}. \quad (35)$$

The expansion scalar θ for the model is,

$$\theta = u_{;i}^i = 3H = \frac{3}{l} \left(\frac{b}{t} + 1 \right). \quad (36)$$

From equation (36) we observe that when $t \rightarrow 0$, $\theta \rightarrow \infty$ and this indicates the inflationary scenario at early stages of the universe.

The shear scalar σ for the model is,

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{1}{2} \left(\frac{\dot{D}^2}{D^2} + 2 \frac{\dot{E}^2}{E^2} \right) - \frac{1}{6} \theta^2 = 3 \left(\frac{k^2 - 2k + 1}{l^2(k+2)^2} \right) \left(1 + \frac{b}{t} \right)^2. \quad (37)$$

The average anisotropic parameter \mathcal{A} for the model is given by

$$\mathcal{A} = \frac{1}{3} \sum_{i=1}^3 \frac{(H_i - H)^2}{H^2} = \frac{2(k-1)^2}{l^2(k+2)^2}. \quad (38)$$

The value of the average anisotropic parameter \mathcal{A} is positive constant for $k \neq 1$ which shows that our model is anisotropic through out the evolution of the universe.

In graphical representations of physical parameters we constraint the constants as: $\alpha = 2$, $\beta = 0.3$, $k = 4$, $\bar{\omega} = 100$, $c_1 = 0.6$, $b = 5, l = 18.6 : b = 7, 17.6 : b = 8, 20.6$ and the cosmic time t in billion years. Here we plotted graphs by taking three different combinations of b and l other parameters being the same. We observed that the first combination is best.

From figure 1 we observed that the volume of the model is decreasing function of redshift and

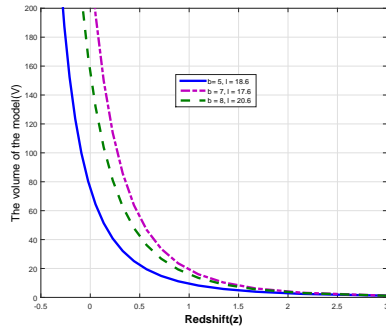


Figure 1. Plot of the volume V of the model versus redshift(z).

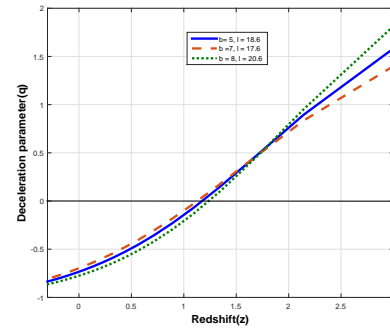


Figure 2. Plot of q versus redshift(z)

hence it shows the expansion of the universe.

The deceleration parameter (DP) q which measure the rate of slowing down of the expansion factor, for the model is given by

$$q = -1 + \frac{d}{dt} \left(\frac{1}{H} \right) = -1 + \frac{bl}{(b+t)^2}. \quad (39)$$

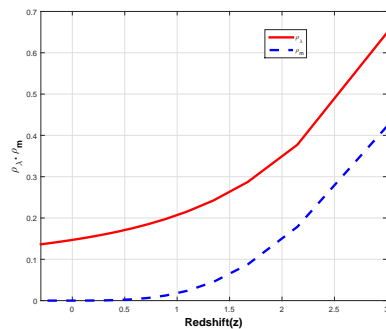


Figure 3. Plot of ρ_λ , ρ_m versus redshift(z).

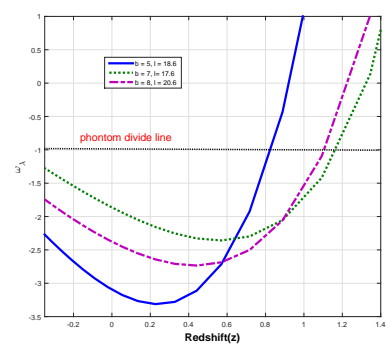


Figure 4. Plot of ω_λ versus redshift(z).

The universe decelerates for positive value of deceleration parameter whereas it accelerates for negative one [1,51]. From figure 2 we observed that the values of the deceleration parameter $q < 0$ for $z < 1.2$ indicating that the universe appears to be expanding in accelerating rate at present epoch and late time and $q > 0$ for $z > 1.2$ indicating that the model was decelerating

at early time. From figure 2 we can also observe that $q = 0$ when $z \approx 1.2$, therefore transition from early deceleration to late time inflation of the universe in our models occurs at $z \approx 1.2$. In our model the present value of deceleration parameter $q_0 \approx -0.73$, which match the observed value [52].

From figure 3 we observed that the density of dark energy (ρ_λ), the dark matter (ρ_m) are increasing functions of the red shift and remains positive throughout the evolution of the universe which shows that the universe is accelerating. From figure 3 we also observed that the ghost dark energy dominates the corresponding dark matter throughout the evolution of the universe. From equations (33), it is clear that the equation of state parameter of dark energy ω_λ is a function of time and from figure 4 we have observed that the equation of state of dark energy (ω_λ) is plotted as function of redshift and it crosses the phantom divide line ($\omega_\lambda = -1$). Thus ω_λ transits from quintessence to phantom so it has quintom like behavior and can explain the acceleration of the universe. From figure 5 we observed that at present and late time the dark energy pressure (p_{de}) is negative, which is the cause of the accelerated expansion of the universe.

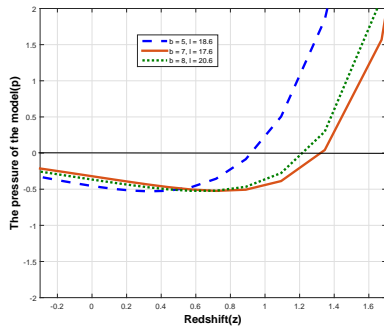


Figure 5. Plot of p versus redshift(z).

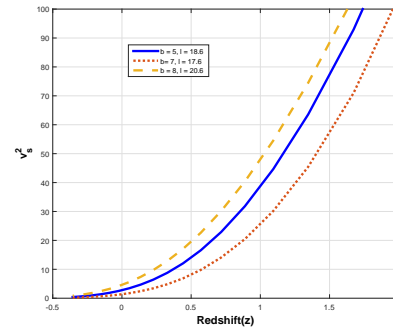


Figure 6. Plot of v_s^2 versus redshift(z).

4.1. Squared Speed of the Sound

We now consider and study an important quantity considered in cosmology in order to check the stability of any DE model and it is known as squared speed of sound, it is denoted with v_s^2 . The models with $v_s^2 > 0$ are stable where as models with $v_s^2 < 0$ are unstable. The squared speed of the sound is defined as follows [53]:

$$v_s^2 = \frac{\dot{p}_\lambda}{\dot{\rho}_\lambda}, \quad (40)$$

where \dot{p}_λ and $\dot{\rho}_\lambda$ are cosmic time derivatives of pressure and density of dark energy, respectively. Using equations (30) and (34) v_s^2 for model (28) is given by,

$$v_s^2 = \frac{\frac{54b}{(lt(k+2))^2}(\frac{b}{t} + 1) - \frac{12b}{lt^3(k+2)} + \frac{6}{(k+2)(t^b e^t)^{\frac{6}{k+2}}}}{-\frac{\alpha b}{lt^2} - \frac{2\beta b}{l^2 t^2}(\frac{b}{t} + 1)}. \quad (41)$$

The plot of squared speed of sound for the model with equations (28), is displayed against redshift in figures 6 and we have observed that the squared speed of sound is positive throughout the evolution of evolution of the universe and, hence the model is stable at all time evolution of the universe. Moreover the model satisfies the inequality $0 \leq v_s^2 < 1$ at late time evolution of the universe.

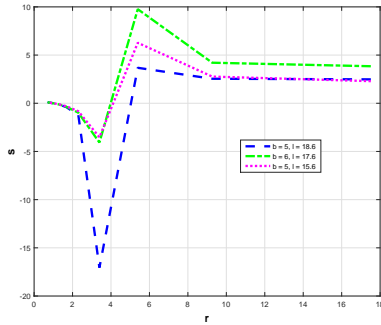


Figure 7. Plot of r versus s .

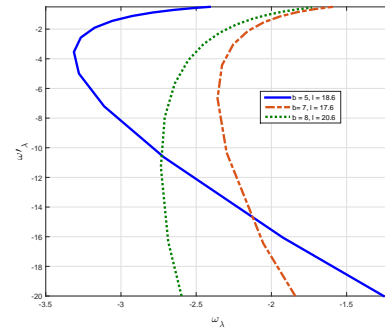


Figure 8. Plot of ω_λ versus ω'_λ .

4.2. State Finder Parameters

The statefinder pair is a geometrical diagnostic in the sense that it is constructed from a spacetime metric directly; it is more universal than physical variables that depend on the properties of physical fields describing DE, because physical variables are model dependent. Usually one can plot the trajectories corresponding to different DE models in the $\{r, s\}$ plane to see the qualitatively different behaviors. For flat the Λ CDM scenario the statefinder pair is $\{r, s\} = \{1, 0\}$. Taking a third order of derivatives with respect to the cosmic time t of the scale factor a , the state finder parameters are defined as [54,55]

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r-1}{3(q-\frac{1}{2})}. \quad (42)$$

Using equations (17) and (21) in (42) we get,

$$r = 1 - \frac{3lb}{(t+b)^2} + \frac{2l^2b}{(t+b)^3}. \quad (43)$$

From equations (39), (42) and (43) we get,

$$s = \frac{2(2bl^2 - 3lb(t+b))}{3(t+b)(2lb - 3(t+b)^2)}. \quad (44)$$

From figure 7 we have observed that the values of state finder pair becomes $r = 1$, $s = 0$ at late time and consistent with standard Λ CDM model.

4.3. $\omega_\lambda - \omega'_\lambda$ Plane Analysis

To differentiate different DE models through trajectories on its plane, the $\omega_\lambda - \omega'_\lambda$ was proposed by [58]. This plane analysis which is very useful tool in our modern days cosmological analysis. At the start, this method has been applied on quintessence DE model which leads to two classes of its plane the one with $\omega_\lambda < 0$ and $\omega'_\lambda < 0$ is called freezing region and the other with the property $\omega_\lambda < 0$ and $\omega'_\lambda > 0$ is known as thawing region. Differentiating EoS parameter $\omega_\lambda = \frac{p_\lambda}{\rho_\lambda}$ with respect to $(\ln a)$ we get,

$$\omega'_\lambda = \frac{\dot{p}_\lambda \rho_\lambda - p_\lambda \dot{\rho}_\lambda}{H \rho_\lambda^2} \quad (45)$$

For model (28)

$$\dot{p}_\lambda = \frac{54b}{(lt(k+2))^2} \left(\frac{b}{t} + 1 \right) - \frac{12b}{lt^3(k+2)} + \frac{6}{(k+2)(t^b e^t)^{\frac{6}{k+2}}} - \frac{6\bar{\omega}}{(t^b e^t)^{\frac{6}{i}}} \quad (46)$$

$$\dot{\rho}_\lambda = -\frac{\alpha b}{lt^2} - \frac{2\beta n}{l^2 t^2} \left(\frac{b}{t} + 1 \right), \quad (47)$$

and ρ_λ and p_λ is given by equation (30) and (34).

From figure 8 we have observed that our model lies in freezing region.

5. Conclusion

In this paper we have presented a spatially homogeneous anisotropic Kantowski-Sachs space time filled with ghost dark energy and dark matter in framework of Saez-Ballester theory of gravitation. The spatial volume V is decreasing function of redshift, indicating the accelerated expansion of the universe. The time dependent DP (q) is positive at early age of the universe and becomes negative at present and late time, showing that our models evolves from early decelerating phase to late time accelerating phase. We have found the present value ($z = 0$) of deceleration parameter as $q_0 \approx -0.73$, which match the observed value. We have also observed that the ghost dark energy density (ρ_λ) and the dark matter (ρ_m) are increasing with respect to redshift, representing accelerating universe. Moreover the ghost dark energy density dominates the corresponding dark matter throughout the evolution of the universe. The EoS parameter (ω_λ) for our model crosses the phantom divide line $\omega_{de} = -1$, thus it has quintom-like behavior. Since the squared speed of sound v_s^2 is positive for all z we observed the model is stable throughout the evolution of the universe. Moreover the model satisfies the inequality $0 \leq v_s^2 < 1$ at late time evolution of the universe so that the model do not admit superluminal fluctuations during late time. We have observed that the values of state finder pair becomes $r = 1$, $s = 0$ at late time and consistent with standard Λ CDM model. We have observed that our model lies in freezing region. We also observed that for present and late times the dark energy pressure (p_λ) is negative, which is the cause of the accelerated expansion of the universe. The model obtained and presented here represents an accelerating and expanding cosmological model of the universe. Thus our model is accord with present cosmological observations.

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