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DEBUNCHING OF HIGH ENERGY BEAMS

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Generally speaking, there are two reasons why one would want to debunch. If one is using a linac for an injector (or just to match between sections of a linac) one would want to be able to stretch or compress a beam in order to get better matching. Also, if one uses a linac for doing certain experiments, people would like longer duty factors.

One can classify duty factors into two categories. There is the macroscopic duty factor, which depends on the linac pulse rate, that one can't do much about except by increasing the linac pulse rate. The microscopic duty factor is caused by the rf structure, and that is what is being discussed here. In either of the two reasons for debunching, there is little difference in the nature of the method to be used, except that for intermediate matching one is much more interested in precise control, whereas for experimental purposes what is wanted is only to stretch the beam out.

Now let us examine some numbers. Assume a 938 MeV linac: I am told that the energy spread would be about 0.1%. Thus:

$$\frac{\Delta\gamma}{\gamma} = 5 \times 10^{-4}$$

$$\frac{\Delta\beta}{\beta} = 1.67 \times 10^{-4}$$

$$\frac{\Delta\eta}{\eta} = 6.67 \times 10^{-4}$$

(calling $\eta = \beta\gamma$)

Now consider a linac of frequency 1000 Mc/sec for convenience and assume that the beam is infinitely tightly bunched; therefore, we would like to spread a bunch over one wavelength. The velocity is $\beta c = 2.598 \times 10^{10}$ cm/sec, the distance over which one wishes to spread the beam is $\Delta\ell = \beta c\tau = 26$ cm. For the value of $\Delta\beta/\beta$ (1.67×10^{-4}) with which we want to stretch an infinitely tightly bunched beam, the length turns out to be 1.6 kilometers. That is a pretty large distance to use in a straightforward drift debuncher.

If you're depending on a debuncher to give an infinitely tightly bunched beam an energy spread, the inherent energy spread required is much larger than anything you can give it with a single gap buncher and the debuncher becomes something like a linac itself.

There are two proposals of which I am aware for doing this more easily. One comes from the fact that although $\Delta\beta/\beta$ is small, $\Delta\eta/\eta$ is fairly large, especially when you go to higher energies, so that a magnet bending the beam includes a change in orbit length of the different particles going through the magnet, and the difference in orbit length is proportional to $\Delta\eta/\eta$ rather than $\Delta\beta/\beta$.

The simplest arrangement one can think of is shown in Fig. 1.

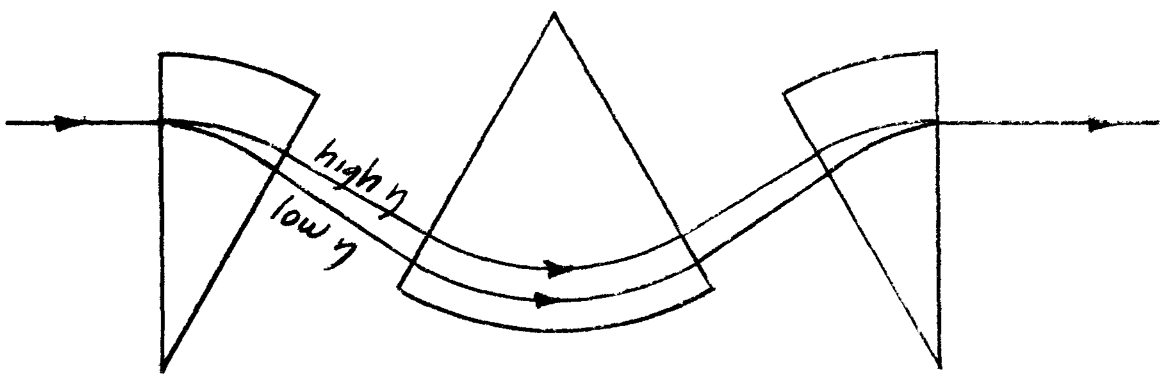

FIG. 1

Three magnets are arranged so that the low momentum particles move on the outside and high momentum particles on the inside in the way shown. (I shall talk about a symmetric system because it's easier to calculate.) As shown, the low momentum orbit is longer, the high momentum orbit is shorter, and the pulse is stretched out. In order to calculate something like this for a symmetric system, one takes half of it and does the first order matrix calculation and then the second half is derivable from the first half. You use the vector (x, x', ϵ, ℓ) in both halves, $\epsilon = \Delta\eta/\eta$ and ℓ is the orbit length. We need $\Delta\eta/\eta$ and ℓ, and also the x and x' optics of the beam. The matrix for the first half is:

$$\begin{bmatrix} x \\ x' \\ \Delta\eta/\eta \\ \ell \end{bmatrix} = \begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ 0 & 0 & 1 & 0 \\ g & h & k & 1 \end{bmatrix} \begin{bmatrix} x_o \\ x_o' \\ \Delta\eta/\eta \\ \ell_o \end{bmatrix}$$

The matrix in (a,b,d,e) is just the usual optics transformation and (c,f,1) represent the dispersion terms. (g,h,k, and 1) represent the change in length of the orbit. The second half of the matrix is quite simple. It is just the inverse of the matrix of the first half:

$$\begin{bmatrix} e & b & bf-ce & 0 \\ d & a & af-cd & 0 \\ 0 & 0 & 1 & 0 \\ eg-dh & bg-ah & k^{-1} & 1 \end{bmatrix}$$

where

$$k^{-1} = f(bg-ah) - c(eg-dh) + k$$

The simplest form may be obtained by setting some conditions. I chose a = f = 0:

(1) a = 0 means there is a cross-over at the half-way point.

(2) f = 0 means the whole system is achromatic.

This makes the system quite simple. The matrix of the first half becomes ($d = b^{-1}$) by the unitary condition:

$$\begin{bmatrix} 0 & b & c & 0 \\ 1/b & e & 0 & 0 \\ 0 & 0 & 1 & 0 \\ g & h & k & 1 \end{bmatrix}$$

and that of the second half is

$$\begin{bmatrix} e & b & -ce & 0 \\ -1/b & 0 & e/b & 0 \\ 0 & 0 & 1 & 0 \\ eg+\frac{h}{b} & bg & -c(eg+hb)+k & 1 \end{bmatrix}$$

and for the whole system

$$\begin{bmatrix} -1 & 2eb & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 2(beg+h) & 2k & 1 \end{bmatrix}$$

The system is achromatic, the optics are those of a drift space of length 2eb and the "isochronism" is given by the term 2k. We assume that the term representing the effects of x' on ℓ can be ignored because the beam is axial but that $\Delta\eta/\eta$ is not zero. Then one can write

$$\frac{\Delta\ell}{\ell} = \frac{2k}{\ell}\,\frac{\Delta\eta}{\eta}\,.$$

Let $K = (2k/\ell)$ which is the inverse of the so-called momentum compaction, α:

$$\alpha = \frac{(\Delta\eta/\eta)}{(\Delta\ell/\ell)} = \frac{1}{K}$$

One would like K to be negative: so that the higher momentum particles would go through a shorter distance in order to spread them out, and also one would like K to be large. For a small $\Delta\eta/\eta$, we want a large value of $\Delta\ell$. I don't have any numbers to substantiate this, but an offhand guess is that K can be made as large as 10 without too much difficulty. (All these matrix elements are fairly easily worked out for n = 0 magnets and straight sections.) If K is 10, ℓ will be 39 m for $\Delta\ell$ = 26 cm and the given $\Delta\eta/\eta$. I'm still not sure about using this system, for we need such a long magnet.

Another way of doing this is to use the linac as a debuncher. The idea is the following: On Fig. 2 the region of stable phase oscillation centers on C', and D is the unstable point. The curves about D are hyperbolas and with proper adjustment can be made into right hyperbolas. So, when at some stage of the acceleration, the beam is bunched into a very small region, you can, in the next linac section, jump phase and put the beam on the unstable phase point D. And then after a few linac sections, the beam will be stretched out into something like the curve BCDA.

Essentially, the major term in this operation is given by the matrices for the phase motion

$$\begin{bmatrix} \cos\Omega z & \frac{1}{\Omega}\sin\Omega z \\ -\Omega\sin\Omega z & \cos\Omega z \end{bmatrix} \quad \text{about } C'$$

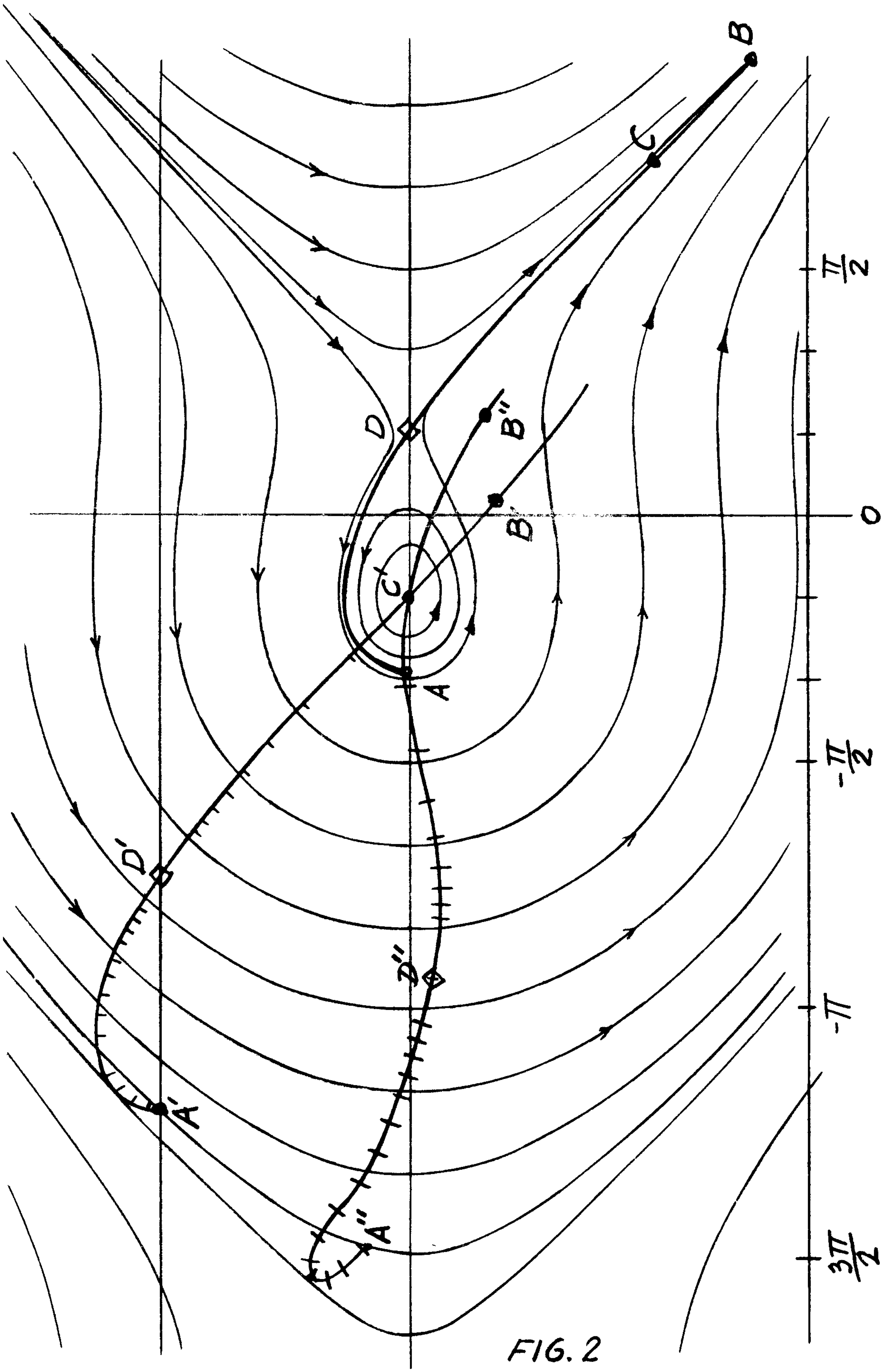

FIG. 2

and

$$\begin{bmatrix} \cosh \Omega z & \frac{1}{\Omega} \sinh \Omega z \\ \Omega \sinh \Omega z & \cosh \Omega z \end{bmatrix} \quad \text{about D}$$

which operate on the (γ,φ) vector (Ω is the phase oscillation frequency). After the bunch has been shaped into BCDA by the phase defocussing matrix, C can be recentered on C' as shown by A'D'C'B' on Fig. 2. Then after a short distance it takes the shape given by A", D", C' B". Of course, this takes "dead reckoning" and everything has to be exactly right.

There are several things wrong with this formulation. One is that this doesn't include the damping term, which can be easily fixed. Instead of Ωz use $\int \Omega\, dz$, and include the damping factor: (Ω_o/Ω) where $\Omega = \Omega_o$ at $z = 0$.

Terms like

$$\begin{bmatrix} \sqrt{\frac{\Omega_0}{\Omega}} \cos \int \Omega\, dz & \frac{1}{\sqrt{\Omega\Omega_o}} \sin \int \Omega\, dz \\ -\sqrt{\Omega\Omega_o} \sin \int \Omega\, dz & \sqrt{\frac{\Omega}{\Omega_o}} \cos \int \Omega\, dz \end{bmatrix}$$

appear, for example, in the phase focussing matrix. That is a first order approximation of phase amplitude but it can be used to get a rough idea of how long the unstable phase operation part of the linac must be. The precise calculation has to be done numerically.

The second difficulty is that this is only for linear motion as in the neighborhood of point D when the motion of a particle is still approximately a straight line, or as at point C', when it is still approximately a circle. We know that this is not going to be true because as you can see, this gets to be a curved line like ADCB: I understand that some people at CERN made some numerical calculations about the degree of the nonlinearity. I think that with their particular criteria, the region for linearity came out to be rather small. If it moves away from either C' or D just a little bit, it's non-linear. However, especially for doing counter experiments you really don't care too much about linear debunching.

In order to do some kind of a quick calculation, I assumed an infinitely tightly bunched beam, put it on D for a while and let it be stretched out into the ADCB line. After a little while it turned over to become a crooked line along the axis. After stretching, I moved it over so that point C went to C'. It is essentially stretched out to occupy a phase of 2π, except for the tail. The energy range after acceleration around C' is rather large, almost as large as the whole range of the fish. If one were willing to take a slightly shorter segment of the line, it could be improved. Figure 2 corresponds to a stable phase angle of -30°, using linac terminology.

I have also another one for a typical case where the stable phase angle is -60° (again, using linac terminology), and it looks a lot better, as shown on Fig. 3. In other words, as you can see, you needn't confine the debunched beam inside of the stable bucket, for it can stick way out and still be all right, provided you're not too fussy about linearity. When I stretched this out and moved it back, there was a shift in the horizontal axis also, and that shift is equivalent to calling the energy of the C' particle the new synchronous energy. In this particular case it means that a section of the linac must be repeated. The length to be repeated depends on how much one wants to shift the horizontal axis, and in this case shown in Fig. 3 the shift was not very much, for the point D was originally on the horizontal axis and was shifted about 1/4 of the energy width of the bucket.

This sort of debunching may not be very desirable because part of the linac is fabricated to be "wrong". However, if it doesn't work, you can still remove a section and replace it with a normal section. Another difficulty with this calculation (so far) is that it doesn't take into account the coupling between the radial oscillation, which has to be done by computation. And then one has to investigate the effect of the errors, as Wheeler and Ludlam discussed this morning. The effects of errors are not very serious to the operation, I feel, because we really don't want the phase rotation to be large. We turn the phase bunch about 1/8 of a phase oscillation, in both the case of operation about the stable fixed point

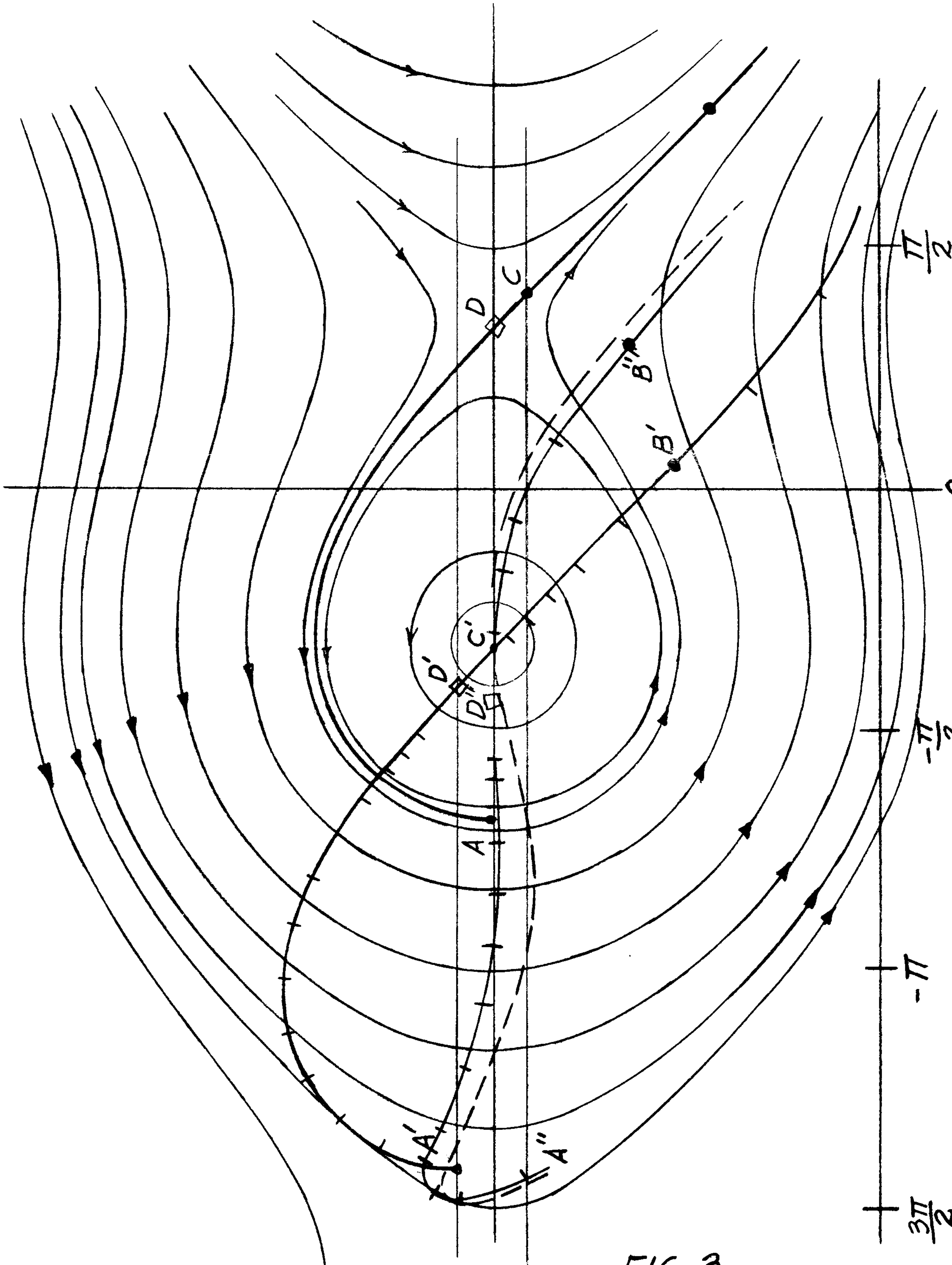

FIG. 3

and operation about the unstable fixed point. Certainly a lot of work remains to be done on this subject.

BLEWETT: Do you have some numbers for length?

TENG: In the report I wrote at Brookhaven there are numbers, and they are approximately right. If you want to design a debuncher, you are really going to give up acceleration over that length. What you would do is this. With debuncher you don't accelerate because you essentially operate on the -90° phase so the fish is somewhat like that in Fig. 3, only more so.

OHNUMA: We have made some calculations at 200 MeV for our particular machine. For example, if you use just one section without going to the unstable point, you just rotate it 90° and try to change the shape. You get about a 12 m section which seems too long to be practical. This is using about half of the phase period of the ordinary fish. On the other hand, if we wished to use Dr. Teng's suggestion, then we would rotate 45° around the different phase and then expand or shrink in one direction and come back again. Using our parameters we would have about 4 meters, which seems too small.

GLUCKSTERN: This is for matching longitudinal phase space at the frequency transition.

OHNUMA: Yes. Maybe we could live with this gadget, but the nonlinearity involved is very serious.

TENG: Yes, for matching inside you are much more critical about the shape. Hopefully it is better when it is properly bunched then if not at all. You certainly lose some particles.

BUTLER: Do you have a number for this capital K in the magnet system?

TENG: I didn't work out any specific case. Presumably there are enough parameters we can adjust. Suppose we use the three magnet system of Fig. 1. We know that the first bends through angle α and the first half of the second through minus α. Then one has the straight section length between, the gradients in the two magnets, and also one or two edge angles as additional variables. All you have to meet are two conditions: $a = f = 0$. There are a lot of parameters to juggle.

BUTLER: Something very close to $\rho(\alpha - \sin\alpha)$ is the value of K, without gradient.

TENG: But let's put in a gradient. Let's do all we can to increase K, adding quadrupoles if necessary.

QUESTION: But when you do you're simply trying to make a and f both zero.

TENG: Also to make K as large as possible. You would like the orbits to separate as much as you can during the first magnet. You would like the field to be higher at low momentum and lower at high momentum. So you want a negative gradient. When you go out to the next magnet, you want the high momentum orbit bent as sharply as you can to make its orbits as short as possible and to make the low momentum orbit go out. That again suggests a negative gradient toward the outside. I think that by using gradients, you can substantially increase K.

KNOWLES: You also need vertical focussing, which the gradients provide.

TENG: Yes, but you could do this with just an edge angle. I should really put down the other matrix for y. Suppose you would like a cross-over in the y direction also, so the optical part of the y comes out $a_y = 0$. You still only have three conditions ($a_x = a_y = f = 0$) and a lot of parameters to adjust, consistent with getting a K of about 10.