ROTATING AND VIBRATING SKYRMIONS

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Abstract

The stability of rotating solitons is analyzed. It is found that both the linear σ-model and the chiral Skyrme Lagrangian (Skyrmion) yield unstable solutions with respect to pion emission. Introducing a symmetry breaking pion mass term stable solutions for the nucleon and the $\Delta(1232)$ are obtained in the Skyrme model. Furthermore with spherical symmetry, no parameter set is found which yields stable rotating solutions for both the nucleon and the delta, with correct masses. When parameters from earlier literature are used, the nucleon is stable but not the delta. To describe baryon excited states small amplitude fluctuations around the rotating solution are considered. The calculated P_{11} phase shift to the "breathing mode" excitation of the nucleon is compared to earlier results neglecting rotations and it is found that rotation-vibration coupling leads to sizable changes.

I. Introduction

Except for a very short time period after the "big bang" the world of strong interactions is in the "confined phase" in which quarks and gluons are clustered into colorless hadrons. There is strong evidence that strong interactions can be described by quantum chromodynamics (QCD), the quantum field theory of colored quarks and gluons. The mere possibility of phase transitions in QCD demands for non-perturbative treatment of the theory. This is particularly important for the calculation of the hadron mass spectrum. Such a treatment is provided by the lattice simulations of QCD which have progressed quite far over the last few years. However such simulations, even though leading to exact results in principle, are very time consuming and often do not provide a simple description of low-energy hadron and nuclear physics.

However it was noted by 't Hooft already in 1974[1)] and later on substantiated by Witten[2)] that in the limit of a large number of colors ($N_c \rightarrow \infty$) QCD turns into an effective field theory of meson fields only in which baryons emerge as solitary waves. Solitary waves are defined as waves for which the energy density is localized at all times.

There are several advantages to describe confined strong interaction physics with effective meson field Lagrangians. First of all they may be derivable from QCD as conjectured by 't Hooft and Witten. In fact there are recent efforts to construct renormalizable meson field theories[3] which, when put on a lattice, can be compared directly with QCD simulations. In that way the parameters of the effective theory could be determined. Secondly a unified description of meson and baryon dynamics is provided, i.e. explicit introduction of fermion fields is avoided. Thirdly there is obviously a great simplicity to the description because the number of parameters can be kept fairly limited, as we shall discuss. Last not least such theories are potentially useful in low and medium energy nuclear physics where new insights into the two-nucleon[4] and many nucleon problem may be obtained[5].

II. The Linear σ-Model

a) The σ-model Lagrangian

A possible candidate for an effective meson field theory is the linear σ-model of Gell-Mann and Levi[6]. It is an SU(2)xSU(2) chiral model which describes the low energy behaviour of pions as Goldstone bosons of spontaneous chiral-symmetry breaking. The Lagrange density can be written in a compact way by introducing a unitary SU(2) field $U(\vec{r},t)$ as

$$\mathcal{L}_0(x) = \frac{F_\pi^2}{16}\,\mathrm{Tr}(\partial_\mu U \partial^\mu U^+) \tag{1}$$

The model is specified by a single parameter F_π the pion decay constant which provides a length scale. U can be reexpressed by a vector field in isospin $\vec{\phi}$ space as

$$U(x) = e^{i\vec{\tau}\vec{\phi}(x)} = \cos\phi + i\vec{\tau}\,\frac{\vec{\phi}}{\phi}\sin\phi \tag{2}$$

where $\vec{\tau}$ are Pauli matrices which form the generators of the SU(2) group. The connection to the sigma and pion field representation of the model is made by the identification

$$\sigma(x) = \cos\phi(x) \tag{3.a}$$

$$\vec{\pi}(x) = \frac{\vec{\phi}}{\phi}\sin\phi(x) \tag{3.b}$$

$\vec{\pi}$ characterizes an isotriplet of massless pions. In this representation $\mathcal{L}_0$ takes the familiar form

$$\mathcal{L}_0(x) = -\frac{F_\pi^2}{8}\{(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2\} \tag{4}$$

The unitarity condition $U^+U = 1$ leads to a normalization condition for σ and $\vec{\pi}$

$$\sigma^2 + \vec{\pi}^2 = 1 \tag{5}$$

which introduces interactions among the fields.

A field configuration of finite energy must satisfy the boundary condition

$$U(\vec{r},t) \to 1 \qquad |\vec{r}| \to \infty \tag{6}$$

Such configurations fall into classes of solutions of the field equations which are characterized by an integer valued index

$$B = \frac{1}{24\pi^2}\varepsilon^{ijk}\int d\vec{r}\ \mathrm{Tr}(U^+\partial_i U\partial_j U^+\partial_K U) \tag{7}$$

This index has been identified by Skyrme as the baryon number which has later on been proven to be the correct interpretation. It is a constant of the motion. B=1 corresponds to a single baryon, B=2 to two baryons etc. B=0 describes mesons.

b) Static field configurations

To study the solutions to the field equations of the linear σ-model it is convenient to start with static classical configurations. The lowest classical energy in the $B\neq0$ sector is attained by the "hedgehog" form of the U-field

$$U_0(\vec{r}) = A\, e^{i\vec{\tau}\cdot\hat{r}F_0(r)}\, A^+ \tag{8}$$

in which the isospin points in the radial direction and A is any constant SU(2) matrix. These solutions are "spherically symmetric" in the sense that a coordinate space rotation is equivalent to an isospin rotation of the matrix A. This configuration corresponds to a mapping of the internal symmetry group SU(2) onto $\mathbb{R}^3$. Since for finite energy the "chiral angle" $F_0(r)$ has to vanish asymptotically, all points at infinity are equivalent. Therefore the mapping reduces to a mapping of SU(2) onto the unit sphere S_3 embedded in $\mathbb{R}^3$. The topological index n which characterizes the number of times SU(2) is wrapped around the sphere S_3 (winding number) is identical to the baryon number B.

The function $F_0(r)$ is subject to a second boundary condition. At r=0 it has to be an integer multiple of π: $F_0(0) = B\pi$. The field equations in the static case are easily obtained from eq. (1) via the principle of least action and reduce to a second order differential equation for F_0 $(\tilde{r} = F_\pi r)$

$$\frac{d^2F_0}{d\tilde{r}^2} + \frac{2}{\tilde{r}}\frac{dF_0}{d\tilde{r}^2} - \frac{\sin 2F_0}{\tilde{r}^2} = 0 \tag{9}$$

This equation is nonlinear and therefore in principle has solitary wave solutions. However if one tries to solve the equation numerically, for instance via relaxation, one finds that for any $B \neq 0$ F_0 shrinks to a point. This can be understood from the behaviour of the classical energy functional

$$M_0[F_0] = -\int d\vec{r}\,\mathcal{L}_0(x) \tag{10}$$

under dilatation transformations $F_0(r) \to F_0(\lambda r)$. One verifies that M_0 scales as $1/\lambda$ tending to collapse the soliton.

c) Rotating field configurations

Baryons are fermions with half integer spin and isospin. These properties have to be constructed from the time dependence of the U-field. According to eq. (8) the static hedgehog configuration is deformed in the intrinsic SU(2) space, since the scalar product $\vec{\tau}\cdot\hat{r}$ fixes a direction. In other words, there is a finite moment of inertia associated with rotations in this space. Since the σ-field transforms like a scalar under SU(2) rotations it cannot contribute to the moment of inertia. Rotations however add a time dependence to the pion field given by the "cranking" expression

$$\dot{\vec{\pi}} = \vec{\omega} \times \vec{\pi} \tag{11}$$

Here $\vec{\omega}$ denotes the angular velocity of the rotation. Inserting this time dependence into the Lagrange density (1) or (4) one obtains the expected form for the Lagrangian

$$L = -M_0 + 1/2\ \omega_i I_{ij} \omega_j \tag{12}$$

where the second term is just the rotational kinetic energy. Introducing the angular moment $\vec{T}$ in the usual way

$$T_i = \frac{\partial L}{\partial \omega_i} \tag{13}$$

the Lagrange density $\mathcal{L}_T$ for the rotating fields is given as a sum of the static part $\mathcal{L}_0$ and the rotational kinetic energy density

$$\mathcal{L}_T(x) = \mathcal{L}_0(x) + \frac{\vec{T}^2}{2I} \tag{14}$$

This expression is given in the rotating frame in which the moment of inertia tensor I_{ij} becomes diagonal: $I_{ij} = I\delta_{ij}$. The constant I is a functional of the rotating solutions characterized by F_T and is calculated as

$$I[F_T] = \frac{2\pi}{3F_\pi} \int d\tilde{r}\tilde{r}^2 \sin^2 F_T(\tilde{r}) \ . \tag{15}$$

As noted earlier there is an intimate connection between isospin rotations and coordinate space rotations. In fact the rotating field is represented in analogy to eq. (8) by

$$U_T(\vec{r},t) = A(t)\ e^{i\vec{\tau}\hat{r}F_T(r)}\ A^+(t) \tag{16}$$

where now A is a time dependent SU(2) matrix. Any isospin rotation is equivalent to a coordinate rotation. On the basis of this one can show that the spatial angular momentum $\vec{J}$ and the isospin angular momentum $\vec{T}$ have to be equal and opposite.

The quantum mechanical treatment of the spin and isospin is straightforward and proceeds in analogy to the quantization of the rigid rotor[7]. The wave function is composed of products of two $\mathcal{D}$-functions

$$\mathcal{D}^{(J)}_{M_J I_J}(\alpha,\beta,\gamma)\mathcal{D}^{(T)}_{M_T I_T}(\alpha',\beta',\gamma') = \langle\alpha\beta\gamma|JM_J I_J\rangle\langle\alpha'\beta'\gamma'|TM_T I_T\rangle \tag{17}$$

where α,β,γ and α',β',γ' denote sets of Euler angles in coordinate and isospin space and I_J and I_T are projections of J and T onto the body axis. From the relation $\vec{T}=-\vec{J}$ one has $I_J=-I_T=I$. The total spin-isospin wave function $|JM_JTM_T\rangle$ of the rotating solution is now obtained as a linear superposition of the $\mathcal{D}$-functions with different I and weights determined by Clebsch-Gordon coefficients. One has

$$|JM_JTM_T\rangle = \sum_I (JIT{-I}|00)|JM_JI\rangle|TM_T{-I}\rangle \tag{18}$$

Because of the "spherical symmetry" of the ansatz (16) only rotational states with J=T, for instance the nucleon and the Δ can be obtained. To generate $J\neq T$ states also spatially deformed fields have to be allowed[8].

As discussed above the static solutions of the linear σ-model collapse to zero size. Rotations add a centrifugal term to the energy which could prevent the time dependent solution from collapsing. To see whether stability with $B\neq 0$ can be reached one has to solve the equations of motion for U_T. They reduce to a differential equation for the "rotating chiral angle" F_T

$$\frac{d^2F_T}{d\tilde{r}^2} + 2\,\frac{dF_T}{d\tilde{r}} - \sin 2F_T\left(\frac{1}{\tilde{r}^2} - P\right) = 0 \tag{19}$$

$$P = \frac{T(T+1)}{3F_\pi^2 I^2[F_T]} \tag{20}$$

Except for the P term this form is identical to eq. (9). Also the expression for the moment of inertia is the same as eq. (15). Eq. (19) has to be solved numerically. Starting with some initial guess for F_T in P the differential equation is iterated until P becomes selfconsistent. One finds however that the selfconsistency requirement cannot be obtained. The reason becomes clear from the asymptotic form of F_T. This has to be of the form

$$F_T(r) \underset{r\to\infty}{\to} \frac{c}{r}\, e^{-\sqrt{-2P}\, r} \tag{21}$$

which, since P>0, is oscillatory at infinity. Therefore the moment of inertia diverges! Another way of looking at the problem is the behaviour of the energy functional

$$M_T[F_T] = M_o[F_T] + \frac{T(T+1)}{2I[F_T]} \tag{22}$$

under the scale transformation $F_T(r) \to F_T(\lambda r)$. While M_0 goes like $1/\lambda$ the kinetic energy scales as λ^3. Thus the rotations can prevent collapse, introduce however another instability due to emission of pions. This phenomenon is similar to the electromagnetic case of a rotating classical charge which radiates off photons.

We conclude that the σ-model, even though the equations of motion are nonlinear, does not support soliton solutions in 3+1 dimensions.

III. The Skyrme Model

a) Nonrotating stable solitons

In order to prevent the collapse of classical solutions in the σ-model higher derivatives to $\mathcal{L}_0$ have to be added. A minimal extension involves at least four derivatives. A particular choice of such fourth order terms has been introduced by Skyrme[9]. The Lagrange density in the Skyrme model takes the form

$$\mathcal{L}_{SK}(x) = \mathcal{L}_0(x) + \frac{1}{32e^2}\,\mathrm{Tr}([(\partial_\mu U)U^+,(\partial_\nu U)U^+])^2 \tag{23}$$

One additional parameter e is needed to specify the dynamics. Of course $\mathcal{L}_{SK}$ is not the only possible choice consistent with chiral symmetry.

In analogy to the σ-model the lowest energy classical solution in the $B\neq 0$ sector is obtained by the "hedgehog" (eq. (8)). Due to the fourth order term the field equations are somewhat more complicated

$$\frac{dF_o}{d\tilde{r}^2}\{1+\frac{8}{e^2}\frac{\sin^2 F_o}{\tilde{r}^2}\}+\frac{2}{\tilde{r}}\frac{dF_o}{d\tilde{r}}+(\frac{dF_o}{d\tilde{r}})^2\frac{4}{e^2}\frac{\sin 2F_o}{\tilde{r}^2}$$
$$-\frac{\sin 2F_o}{\tilde{r}^2}\{1+\frac{4}{e^2}\frac{\sin 2F_o}{\tilde{r}^2}\}=0\ . \qquad (24)$$

but nevertheless this equation is easily solved numerically. Eq. (9) is recovered in the limit $e\to\infty$. The solution is spatially extended and is indicated in Fig. 1 for the single baryon case (B=1). Choosing the parameter set of Adkins et al.[10] given as F_π = 129 MeV and e = 5.45 the energy of the hedgehog is 864 MeV.

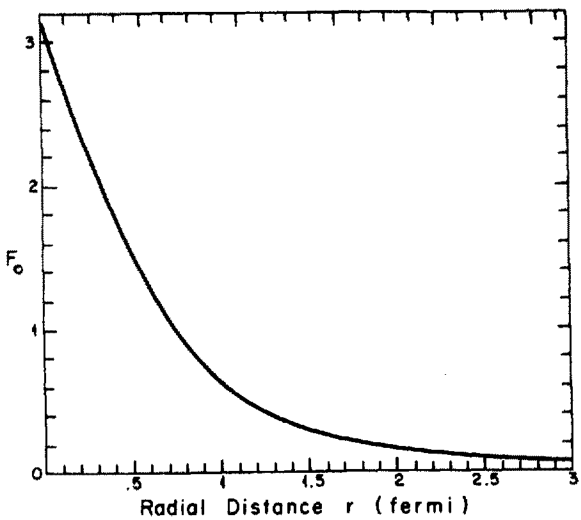

Fig. 1: Static B=1 solution for the chiral angle $F_o(r)$ in the Skyrme model.

b) Rotating Skyrmions

To project out the proper spin-isospin states rotated fields according to eq. (16) have to be obtained. Adkins et al.[10] proceed by replacing the rotating chiral angle $F_T(r)$ by the static angle $F_o(r)$. This choice yields a finite moment of inertia for the Skyrmion given by

$$I[F_o]=\frac{2\pi}{3F_\pi}\int d\tilde{r}\tilde{r}^2\sin^2 F_o\{1+\frac{4}{e^2}[(\frac{dF_o}{d\tilde{r}})^2+\frac{\sin^2 F_o}{\tilde{r}^2}]\} \qquad (25)$$

and a nonzero Δ-N mass splitting is obtained. The experimental splitting energy of 293.1 MeV is used to adjust F_π and e. This adjustment yields the parameter values quoted above. Many static properties of the nucleon and the isobar like rms radii, magnetic moments g_A etc. can be calculated. Some results are listed in Table 1. The agreement with experiment is quite remarkable particularly in view of the fact that only two parameters are involved.

Quantity	Prediction	Experiment
M_N	938.9 MeV (input)	938.9 MeV
M_Δ	1232 MeV (input)	1232 MeV
$(r^2)^{1/2}$ (isoscalar)	0.59 fm	0.72 fm
μ_p	1.87	2.79
μ_n	-1.24	-1.91
μ_p/μ_n	1.43	1.46
g_A	0.61	1.23
$g_{\pi NN}$	8.9	13.5
$g_{\pi N\Delta}$	13.2	20.3
$\mu_{N\Delta}$	2.3	3.3

Table 1: Static properties of nucleon and isobar in the Skyrme model as calculated in Ref. 10.

In the procedure of Adkins et al.[10], replacing F_T by F_0, the resulting U-field does, however, not satisfy the Euler-Lagrange equations. The function $F_0(r)$ only minimizes $M_0[F]$ but not the full functional $M_T[F_T]$ (eq. (22)). Although for $F_T(r) \approx F_0(r)$, $M_0(F_T)$ and $M_T(F_T)$ may not differ too much for low values of T, the extrema of these two functionals may be quite different. Keeping the rotational part in extremizing the energy the Skyrme equation of motion is modified to give

$$\frac{d^2F_T}{d\tilde{r}^2}\{1+\frac{8}{e^2}\sin^2F_T\,(\frac{1}{\tilde{r}^2}-P)\}+\frac{dF_T}{d\tilde{r}}\{\frac{2}{\tilde{r}}-\frac{16P}{e^2\tilde{r}}\sin^2F\}$$
$$+(\frac{dF}{d\tilde{r}})^2\{\frac{4}{e^2}\sin^2F\,(\frac{1}{\tilde{r}^2}-P)\}-\sin^2F\,\{\frac{1}{\tilde{r}^2}-P+\frac{4}{e^2}\frac{\sin^2F}{\tilde{r}^2}[\frac{1}{\tilde{r}^2}-2P]\}=0 \qquad (26)$$

where P is defined as in eq. (20). As in the rotating σ-model this equation does not have a solution which yields a localized energy density. The asymptotic form of F_T is in fact identical in both cases. This desease of the rotating Skyrmion has been noted by Bander and Hayot[11] and independently by Braaten and Ralston[12]. The reason for the instability is again easily understood from the behaviour of the Skyrmion energy under scale transformations. The fourth order term adds a contribution proportional to λ which is sufficient to stabilize the classical solution, but there is no term to offset the λ^3 instability from the rotational kinetic energy. Therefore there is no stable rotating Skyrmion.

c) Skyrmions with finite pion mass

In the real world chiral SU(2)xSU(2) is only an approximate symmetry good at the 10 % level. We are therefore allowed to explicitly break the symmetry by adding a pion mass term to the Skyrme Lagrange density

$$\mathcal{L}(x) = \mathcal{L}_{SK}(x) + \frac{m_\pi^2 F_\pi^2}{8}\,[\mathrm{TrU}-2] \tag{27}$$

In the energy this mass term adds a $1/\lambda^3$ contribution which can offset the λ^3 instability from the rotations. In the presence of a finite pion mass an extra term $-m_\pi^2/F_\pi^2 \sin F_T$ is added to the Euler-Lagrange equation (eq. (26)). This modifies the large distance behaviour of F_T to give

$$F_T(\tilde{r}) \underset{r\to\infty}{\to} \frac{c}{\tilde{r}}\, e^{-\sqrt{\frac{m_\pi^2}{F_\pi^2} - 2P}\;\tilde{r}} \quad . \tag{28}$$

Without rotations (P=0) one therefore obtains the correct asymptotic Yukawa form of the pion field. In the presence of rotations the stability however is controlled by the magnitude of P. Only for $P < m_\pi^2/2F_\pi^2$ one obtains a stable soliton. This condition clearly depends on the choice of parameters and has to be explored by solving the Euler-Lagrange equations explicitly[13]. It turns out that the solutions only depend on two independent quantities m_π/F_π and e such that a two parameter space has to be explored. The numerical stability limits are summarized in Fig. 2. We observe that the parameter space is divided into two regions: a stable region of localized solitons ($P < m_\pi^2/2F_\pi^2$) and an unstable region ($P > m_\pi^2/2F_\pi^2$) in which the moment of inertia diverges. The boundary, obtained numerically, shows polynomial behaviour up to large values of m_π/F_π. Since P depends on the value of the isospin the stable Δ-region is naturally smaller than the nucleon region. One may ask if it is possible to fit the Δ-N mass split with a combination m_π/F_π and e for which both the nucleon and the Δ is stable. The answer is no. As seen from Fig. 2 the lines of constant m_Δ and m_N do not cross in the allowed region for the delta. Using the quantization procedure in Ref. 10 such a crossing can be found (dotted lines in Fig. 2) for the values quoted by Adkins and Nappi[14] (m_π/F_π = 1.277 and e = 4.84). The crossing point is however in the unstable region for the delta. This result is not necessarily a bad feature of the model since we know that the delta is not a stable particle.

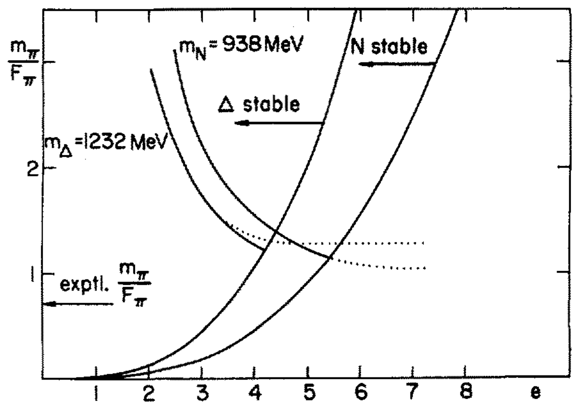

Fig. 2: Stability limits of the broken SU(2)xSU(2) parameter space including rotations. The lines of constant nuclear mass m_N and delta mass m_Δ in the presence of rotations are also indicated (solid lines). The results of the quantization procedure used in Ref. 10 are indicated by dotted lines.

IV. Excited States of the Nucleon and the Delta in the Skyrme Model

Excited states of the nucleon and the delta are observed for instance as resonances in the πN- and $\pi\Delta$-system. To describe the scattering problem within the Skyrme model we have to go back to the general expression of the unitary field U (eq. (2)). The $\vec{\Phi}$-field is expanded around the stable rotating B=1 soliton as

$$\vec{\Phi}(\vec{r},t) = F_T(r)\hat{r} + \vec{\eta}(\vec{r},t) \ . \qquad (29)$$

$\vec{\eta}$ characterizes fluctuations around the soliton which represent the pion-soliton scattering states and carry baryon number B=0. We consider here the simplified case in which the amplitude is small, such that $\eta^2 >> \eta^4$. Substituting the expansion of $\vec{\Phi}$ into the broken SU(2) Skyrme Lagrange density (eq. (27)) retaining only terms quadratic in η one readily obtains

$$L = - M_{SK} + \frac{T(T+1)}{2I_o} + 1/2 \int d\vec{r} \ [\dot{\eta}_i B_{ij} \dot{\eta}_j - \eta_i A_{ij} \eta_j] + T_{rot-vib} \qquad (30)$$

It can be shown that all terms linear in $\dot{\eta}$ and η vanish using the equation of motion for F_T given in the last section. M_{SK} denotes the classical rotating Skyrmion mass and $I_o[F_T]$ is the moment of inertia in the absence of fluctuations. The vibrational part is contained in the third term of the Lagrangian and has the familiar form. The restoring force tensor is a second order differential operator acting on the three components η_i and B_{ij} denotes the inertial mass. Both A_{ij} and B_{ij} are complicated functions of F_T. Their calculation, though tedious, is straightforward. It should be noted that in a spherical basis B_{ij} and A_{ij} become diagonal. In addition there is a kinetic energy contribution from the rotation-vibration coupling $T_{rot-vib}$ which will be analyzed below for monopole vibrations.

Expanding the fluctuations into normal modes

$$\vec{\eta}(\vec{r}) = \sum_n c_n(r)\vec{\eta}^{(n)}(\vec{r}) \tag{31}$$

the vibrations are quantized as harmonic oscillators in the usual way to give

$$L = - M_{SK} + \frac{T(T+1)}{2I_o} + \sum_n (N_n+1/2)\hbar\omega_n + T_{rot-vib} \,. \tag{32}$$

Here N_n denotes the phonon number operator. To order $\hbar$ the phonon zero-point energies contribute to the baryon energies. Summing over all modes this contribution becomes infinite. In the absence of renormalizability of the Skyrme Lagrangian zero point corrections to the mass are ignored[10,14], as will be done here also.

In order to avoid complicated angular momentum algebra we shall limit the discussion to radial oscillations only. They carry phonon angular momentum zero. Such "breathing modes" are observed in p-wave pion scattering as the $P_{11}(1440)$-resonance in the πN-system and the $P_{33}(1600)$-resonance in the $\pi\Delta$-system. The partial wave expansion of the nth normal mode is in general given by

$$\vec{\eta}^{(n)}(\vec{r}) = \sum_{I\ell M} f_{I\ell M}(r)\vec{Y}_{I\ell M}(\hat{r}) \tag{33}$$

where ℓ is the pion orbital angular momentum. For p-wave scattering to the breathing mode ℓ=1 and I=0, i.e. the vector spherical harmonic $\vec{Y}_{I\ell M}(\hat{r})$ is proportional to $\hat{r}$. In this case the expression for the $\vec{\phi}$-field given in eq. (29) simplifies to give

$$\vec{\phi}(\vec{r},t) = (F_T(r)+\xi(r)e^{i\omega t})\hat{r} \tag{34}$$

i.e. the vector $\vec{\eta}$ only points in the radial direction. $\xi(r)$ is the radial part of the scattered wave which determines the phase shifts. In the monopole case $T_{rot-vib}$ is easily obtained. Physically it comes from a change I_1 in the moment of inertia

$$I[\phi] = I_0[F_T] + I_1[F_T,\xi] \tag{35}$$

as the Skyrmion oscillates. To second order in ξ one finds[15)]

$$T_{rot-vib} = -\frac{T(T+1)}{2I_0^3}\, I_1^2 \tag{36}$$

where

$$I_1[F_T,\xi] = \frac{8\pi}{3}\int_0^\infty dr\Big\{\frac{F_\pi^2}{4}\, r^2\sin 2F_T + \frac{1}{e^2}\Big[r^2\sin(2F_T)\Big(\frac{\partial F_T}{\partial r}\Big)^2 - 4r\sin^2F_T\,\frac{dF_T}{dr} - 2r^2\sin^2F_T\,\frac{d^2F_T}{dr^2} + 2\sin(2F_T)\sin^2F_T\Big]\Big\}\xi(r) \tag{37}$$

The wave equation for ξ is determined from the least action principle which leads to the equation of motion

$$\frac{\partial \mathcal{L}}{\partial \xi} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\xi}} = 0 \tag{38}$$

As a result one obtains an integro-differential equation of the following structure

$$\frac{d^2\xi}{dr^2} + B(r)\frac{d\xi}{dr} + C(r,\omega^2)\xi + D(r,\xi) = 0 \tag{39}$$

where the coefficients B,C,D are complicated functions of F_T not listed here and D involves an integral over ξ. The integral emerges from the rotation-vibration coupling. To obtain the phase shifts we have to impose two boundary conditions on ξ. Near the origin the regular solution of eq. (39) behaves as

$$\xi(r) \underset{r\to 0}{\propto} r \tag{40a}$$

and the asymptotic form

$$\xi(r) \underset{r\to\infty}{\propto} j_1(\omega r)\cos\delta_1 - n_1(\omega r)\sin\delta_1 \tag{40b}$$

is a linear combination of regular and irregular spherical Bessel functions with orbital angular momentum $\ell=1$. Numerically δ_1 is obtained by integrating the wave equation (39) out to some radius R where the solution is matched to the asymptotic form.

The preliminary results[15] given in Fig. 3 are compared to earlier calculations by Walliser and Eckart[16] in which rotations and finite pion mass have been neglected. We conclude that the inclusion of rotations leads to sizable changes in the P_{11} phase shift and, therefore, cannot be ignored.

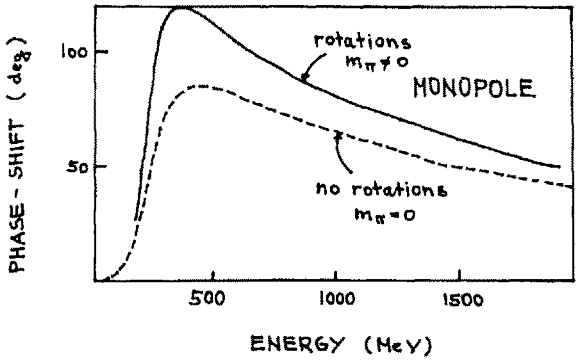

Fig. 3: P_{11} phase shift for πN scattering including rotations and finite pion mass as compared to results from Ref. 16.

V. Summary

In summary, the discussion given above, suggests the following conclusions:

(1) The linear σ-model of Gell-Mann and Levy, a possible candidate for an effective meson field theory in the large N_c limit, does not support stable solutions of the field equation. The solitons collapse to zero size. Including a kinetic energy due to rotations the collapse is offset but a new instability with respect to pion emission is introduced.

(2) Adding higher derivatives to the σ-model Lagrangian as in the Skyrme model stable classical solutions are obtained but in the presence of rotations the same instability as in the σ-model is found in the chiral limit ($m_\pi = 0$).

(3) To offset this instability chiral SU(2)xSU(2) has to be broken explicitly by introducing a finite pion mass. Whether stability is obtained depends on the parameter set m_π/F_π and e. The parameter space is divided into stable and unstable regions separated by a boundary which depends on spin and isospin and shrinks as S and T increase.

(4) In the allowed region rotational energies are quite small. The maximum value for the Δ is 167.8 MeV and for the nucleon 47.96 MeV. No parameter set can be found which yields stable solutions as well as the correct masses for both the nu-

cleon and the delta. With the parameters of Adkins and Nappi[14] the nucleon is stable but the delta is unstable.

(5) Baryon excited states can be described as fluctuations of baryon number zero around the rotating field configuration. A rotation-vibration coupling term in the kinetic energy is introduced which has been analyzed for breathing mode excitations of the nucleon[15]. The predicted P_{11}-phase shift is quite different with and without rotations.

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