TN-65-26
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April, 1965

FLUORESCENT RADIATION FROM BOMBARDING SYNCHROTRON RADIATION IN A STORAGE RING

I. INTRODUCTION

The necessary high vacuum for the SLAC storage ring is to be attained by cryogenic pumping. Since such a pumping system operates only at low temperatures, and since there is considerable power in the synchrotron radiation generated from the circulating beams, it is necessary to estimate the heat flow that the cryogenic pumps must handle. It is proposed to have a water cooled wall along the outer edge of the vacuum chamber to catch the synchrotron radiation. A cooled cryogenic baffle near the inner edge is to provide the cryogenic pumping. Some of the radiation which is reflected from the outer wall can reach the cryogenic baffle, supplying energy to the cryogenic pumping system which must be removed by it. The amount of this reflected radiation, or fluorescent radiation, is estimated here.

The method of the calculation consists of evaluating first the synchrotron power spectrum, then the absorption of the synchrotron radiation by the atoms of the wall material, the re-emission probability for these atoms (fluorescent radiation) and finally the transmission of the fluorescent x-rays as they leave the surface.

II. SYNCHROTRON RADIATION POWER SPECTRUM

Schwinger[1] has derived the energy spectrum $P(\omega)$ of the radiation from an electron in a synchrotron as

$$P(\omega)\, d\omega = \frac{3^{3/2}}{4\pi} \frac{e^2}{R} \left(\frac{E}{Mc^2}\right)^4 \omega_o \frac{\omega}{\omega_c^2} \int_{\omega/\omega_c}^{\infty} K_{5/3}(\eta)\, d\eta \cdot d\omega$$

where $P(\omega)\, d\omega$ is the power radiated at the frequency ω to $\omega + d\omega$.

$$\omega_c = \frac{3}{2} \omega_o \left(\frac{E}{Mc^2}\right)^3 \text{ is the critical frequency}$$

ω_o = is the orbital frequency

R = radius of curvature of the electron path

e = electron charge in esu (= 4.8×10^{-10})

E = energy of the electron

Mc^2 = rest mass of the electron

Associated with the critical frequency is the critical wavelength λ_c given by

$$\lambda_c = \frac{4\pi}{3} R \left(\frac{Mc^2}{E}\right)^3$$

or

$$\lambda_c = 5.59 \frac{R_{[\text{meters}]}}{E^3_{[\text{BeV}]}} \text{ Angstroms,}$$

and the critical energy

$$E_c = \frac{12.4}{\lambda_c} \text{ keV}$$

or

$$E_c = 2.22 \frac{E^3_{[\text{BeV}]}}{R_{[\text{meters}]}} \text{ keV .}$$

The total radiated power is given by

$$P = \frac{2}{3} \omega_o \frac{e^2}{R} \left(\frac{E}{Mc^2}\right)^4 .$$

For one revolution, the radiated energy is

$$\delta E_{[\text{keV}]} = 88.5 \frac{E^4_{[\text{BeV}]}}{R_{[\text{meters}]}}$$

and

$$P_{[\text{k watts}]} = I \, \delta E_{[\text{keV}]}$$

where I is the beam current in amperes.

The proposed storage ring has a bending radius $R = 10.8$ m. The power spectrum has been computed, and is shown in Fig. 1 in terms of the radiated photon energy per keV interval for a 1 ampere beam. Several electron energies E_o are used.

III. ABSORPTION COEFFICIENTS

Figure 2 gives the mass absorption coefficients (μ/ρ) as a function of photon energy E_γ for copper and aluminum. Both the incident synchrotron radiation and the emerging fluorescent radiation are attenuated according to

$$I = I_o \, e^{-\left(\frac{\mu}{\rho}\right) \rho x}$$

where

I_o = initial intensity of radiation

I = intensity at depth x (cm)

ρ = density of the material (gm/cm^3)

μ = total attenuation coefficient (cm^{-1}).

A similar expression holds for the photoelectric absorption of the incident radiation, except that μ is replaced by τ , the photoelectric absorption coefficient.

IV. FLUORESCENT COEFFICIENTS

When an atom has been excited by removal of a K shell electron, it can be de-excited to a lower excitation level by either emitting another electron, or by emitting a photon. The K shell fluorescent coefficient ω_K gives the probability for emission of the photon. Similar coefficients exist for L and M shell excitation levels. Figure 3 gives these coefficients as a function of atomic number Z . Analytical expressions are given in Burhop[2].

$$\omega_K = \frac{1}{1 + 1.12 \times 10^6 \, Z^{-4}}$$

$$\omega_L = \frac{1}{1 + 10^8 Z^{-4}}$$

V. CALCULATION FOR FLUORESCENT YIELD

The fraction R_K of incident intensity which is emitted as K shell fluorescent radiation from a thick plane absorber is given by Burhop[3] as

$$R_K = \omega_K \left(1 - \frac{1}{J_K}\right) \frac{d\Omega}{4\pi} \sum_i \sum_\eta \frac{r_i \, \tau_i \, Z_\eta \, \nu_\eta \operatorname{cosec}\theta}{\nu_i \, (\mu_i \operatorname{cosec}\theta + \mu_\eta \operatorname{cosec}\theta')}$$

where

ω_K = fluorescent coefficient for the K shell

J_K = jump factor at the K absorption edge (≈ 10)

$\left(1 - \frac{1}{J_K}\right)$ = fraction of incident radiation which excites K shell

$d\Omega$ = solid angle of emitting radiation

ν_i = frequency of incoming radiation

r_i = fraction of incident radiation having frequency ν_i

μ_i = total absorption coefficient for incident radiation

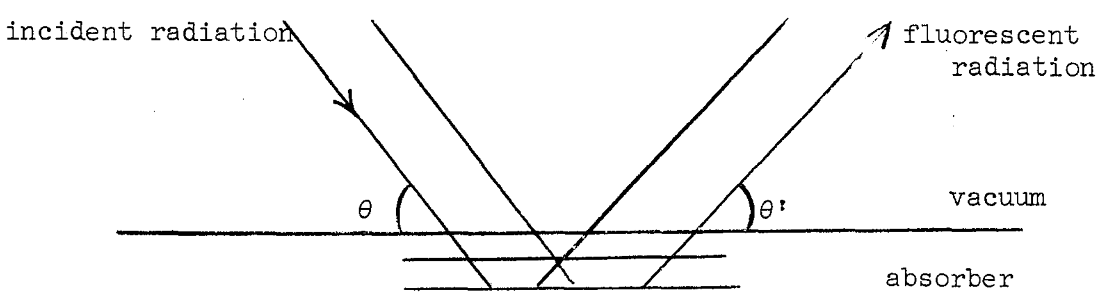
τ_i = photoelectric absorption coefficient for incident radiation

ν_η = frequency of emitted radiation

Z_η = fraction of emitted radiation having frequency ν_η

θ = angle of incidence

θ' = angle of emission



In order to obtain the total fluorescent radiation leaving the absorber, we need to integrate over θ' and φ' (azimuthal angle for emission). The integral is of the form

$$\int_{\varphi'=0}^{2\pi} d\varphi' \int_{\theta'=0}^{\pi/2} \frac{\cos\theta' \, d\theta'}{1 + \alpha \operatorname{cosec}\theta'} = 2\pi \int_0^1 \frac{x dx}{\alpha + x} = 2\pi \left[1 - \alpha \log\left(1 + \frac{1}{\alpha}\right)\right] .$$

Hence the total reflected K radiation $R_K^T = \iint R_K d\Omega$ can be put in the form

$$R_K^T = \frac{1}{2}\, \omega_K \left(1 - \frac{1}{J_K}\right) \sum_i \sum_\eta r_i \left(\frac{\tau_i}{\mu_i}\right) Z_\eta \left(\frac{\nu_\eta}{\nu_i}\right) \left[1 - \frac{\mu_\eta \sin\theta}{\mu_i} \log\left(1 + \frac{\mu_i}{\mu_\eta \sin\theta}\right)\right]$$

The photon energies in the synchrotron radiation are in the region where the photoelectric effect predominates, so that $\tau_i/\mu_i = 1$. Most of the fluorescent energy is in the K_α spectral lines, so we use

$$Z_\eta = 1, \; \nu_\eta = \nu_K$$

Expressing the frequencies in terms of photon energies ($E = h\nu$), and using mass absorption coefficients, then

$$R_K^T = \frac{1}{2}\, \omega_K \left(1 - \frac{1}{J_K}\right) \sum_i r_i \left(\frac{E_K}{E_i}\right) \left[1 - \frac{(\mu_K/\rho)\sin\theta}{(\mu_i/\rho)} \log\left(1 + \frac{(\mu_i/\rho)}{(\mu_K/\rho)\sin\theta}\right)\right]$$

This expression has been numerically summed over the synchrotron energy E_i in 1 keV steps, where r_i is now the synchrotron power spectrum in 1 keV bin widths and $E_i \geq E_K$. The value used for θ is 6°.

A similar treatment could be done for the L and M shells. However, if low Z materials are used, fluorescence from these levels is small, and only an estimate is necessary for the integration over E_i . The total fluorescent yield is then

$$R^T = R_K^T + R_L^T + R_M^T \;\ldots$$

$$4\rho x > 4.6$$

$$x > \frac{4.6}{4 \times 2.7} = 0.43 \text{ cm}$$

The film thickness d is given by

$$d = x \sin\theta \geq 0.43 \sin 6^{\circ}$$

$$\therefore \quad d \geq 0.045 \text{ cm} \cdot (= 0.018'')\ .$$

Hence an aluminum coating of 0.018" is required to absorb 99% of the synchrotron radiation. With such a film, the maximum fluorescent yield will be below 0.33% of the incident synchrotron power.

References

1. J. Schwinger, Phys. Rev. 75, 1912 (1949).
2. F. H. S. Burhop, The Auger Effect (Cambridge 1952).
3. Ibid; pg. 32.

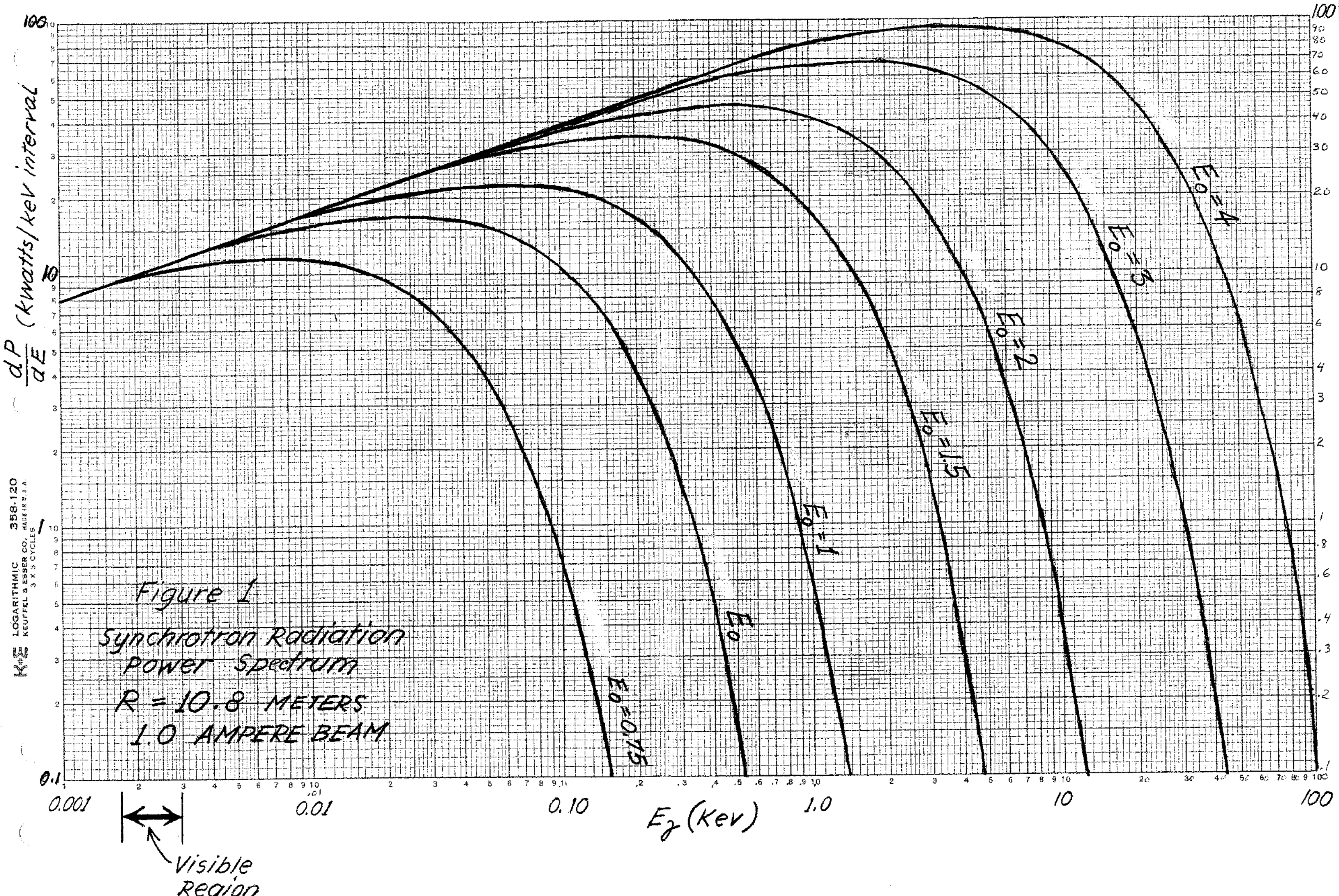
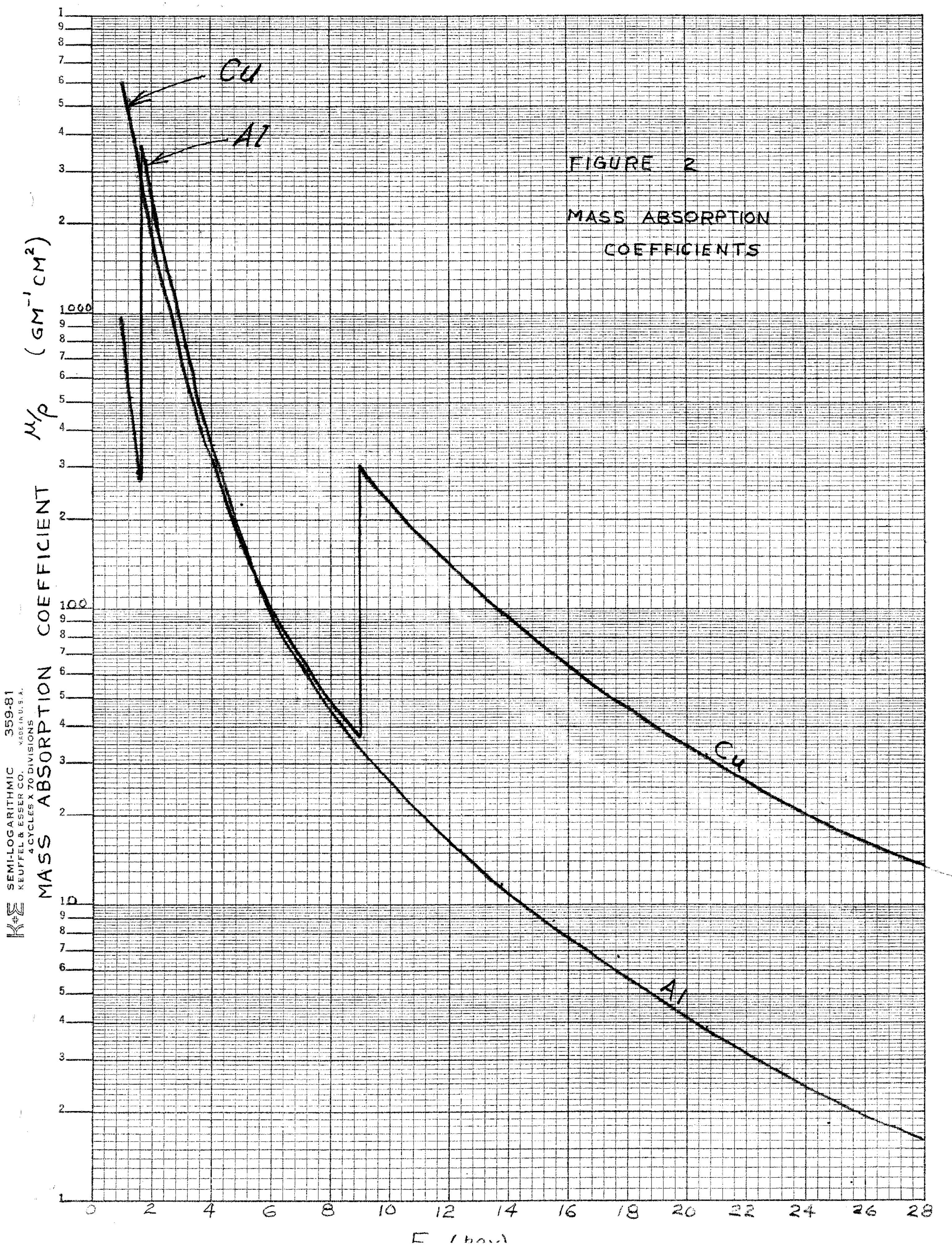


Figure 1
Synchrotron Radiation
Power Spectrum
R = 10.8 METERS
1.0 AMPERE BEAM
dP/dE (kwatts/keV interval)
Eγ (Kev)
0.001
0.01
0.10
1.0
10
100
0.1
1
10
100
E0 = 0.75
E0
E0 = 1
E0 = 1.5
E0 = 2
E0 = 3
E0 = 4
Visible Region
LOGARITHMIC 358-120
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 X 5 CYCLES



Cu
Al
FIGURE 2
MASS ABSORPTION COEFFICIENTS
MASS ABSORPTION COEFFICIENT μ/ρ (GM⁻¹ CM²)
1000
100
10
1
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
Cu
Al
E (kev)
K+E SEMI-LOGARITHMIC 359-81
KEUFFEL & ESSER CO. MADE IN U.S.A.
4 CYCLES X 70 DIVISIONS

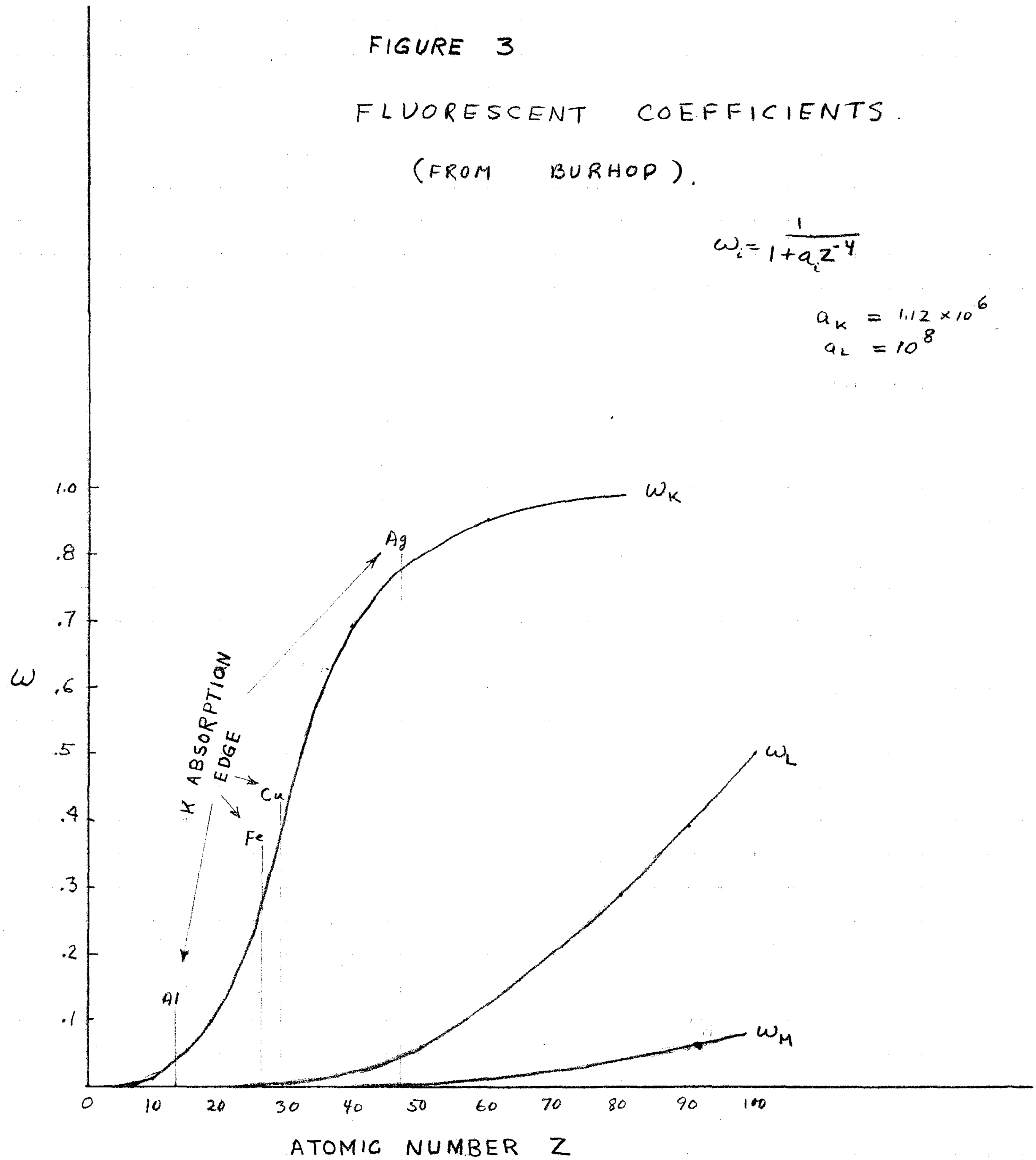


FIGURE 3
FLUORESCENT COEFFICIENTS.
(FROM BURHOP).
$\omega_i = \frac{1}{1 + a_i Z^{-4}}$
$a_K = 1.12 \times 10^6$
$a_L = 10^8$
ω_K
ω_L
ω_M
Ag
Cu
Fe
Al
K ABSORPTION EDGE
ω
1.0
.8
.7
.6
.5
.4
.3
.2
.1
0
10
20
30
40
50
60
70
80
90
100
ATOMIC NUMBER Z