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MAGNET INSERTION CODE

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MAGNET INSERTION CODE (MAGIC)

A computer code (MAGIC) specially written for magnet lattice insertion design using thin-lens magnets has been developed. This development was initiated at the NAL 1973 Aspen summer study and was completed at the SLAC-LBL 1973 PEP summer study. This code enables the user to fit the values of the β, α, η, η' and ψ-functions[1] at both ends of an insertion and the values of the transport matrix elements. The fitting is done by using VMM,[2] a least square fitting code from ANL. Since MAGIC is a special purpose code and since it uses thin-lens magnets, the computation time and the rate of convergence are faster than those of a general purpose code such as TRANSPORT.[3] Because of the simplicity of this code it can be easily modified to perform other desired tasks. The method of computation used in this code is presented in this report.

METHOD

Consider an insertion composed of a number of quadrupole magnets, bending magnets and drift lengths. The transport matrix for either horizontal or vertical motion for the insertion is given by

$$T = M_n M_{n-1} \cdots M_2 M_1 \qquad (1)$$

where M_k is the transport matrix for the k^{th} element in the insertion and n is the total number of elements.

For a focusing quadrupole magnet

$$M_k = \begin{pmatrix} 1 & 0 & 0 \\ -x_k & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (2)$$

where x_k = 1/focal length;

for a bending magnet

$$M_k = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & x_k \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

where x_k = bend angle;

for a drift space

$$M_k = \begin{pmatrix} 1 & x_k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

where x_k = drift length.

The values of the β, α, η, η', and ψ-function at the exit of the insertion are given by:

$$\begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} T_{11}^2 & -2T_{11}T_{12} & T_{12}^2 \\ -T_{11}T_{21} & 1+2T_{12}T_{21} & -T_{22}T_{12} \\ T_{21}^2 & -2T_{22}T_{21} & T_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \eta_2 \\ \eta_2' \\ 1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_1' \\ 1 \end{pmatrix} \tag{6}$$

where
$$\psi_2 = \tan^{-1}\left(\frac{T_{12}}{\beta_1 T_{11} - \alpha_1 T_{12}}\right) - \psi_1 \tag{7}$$

$$\gamma = (1 + \alpha^2)/\beta, \tag{8}$$

and the values of these functions at the entrance (with subscript 1) equal to the values desired.

Let f_i be the value of the i^{th} function and $\bar{f}_i$ be the corresponding desired value with i = 1,3,..., 21 refer to β_2, α_2, η_2, η_2', ψ_2, T_{11}, T_{12}, T_{13}, T_{21}, T_{22}, T_{23} for the x-motion, and i = 2, 4, ..., 22 refer to these functions for the y-motion.

Consider the function

$$F = \sum (f_i - \bar{f}_i)^2 \tag{9}$$

with summation over the indexes of those functions whose values are to be fitted. The fitting is done via a least square minimization of F. In particular, a solution is obtained if the minimum value of F is zero, i.e., $f_i = \bar{f}_i$ for all the desired functions. If the minimum value of F is not zero we fail to find a solution.

For this minimization procedure we need to compute the derivatives of F respect to each parameter

$$\frac{\partial F}{\partial x_k} = \sum 2(f_i - \bar{f}_i) \frac{\partial f_i}{\partial x_k} \tag{10}$$

From the differentiation of Eqs. (5) to (7) we can express $\partial f_i/\partial x_k$ in terms of T_{ij} and $(D_k T)_{ij}$,

where
$$D_k T = M_n M_{n-1} \cdots (DM_k) \cdots M_1, \tag{11}$$

DM_k is the derivative of the M_k matrix with respect to x_k. It may be noted that the DM_k matrix has only one nonzero element. In addition, we need a guess solution (x_{ko}'s). Constraints may be imposed upon the values of system parameters by the conditions

$$\sum c_k x_k = \sum c_k x_{ko} \tag{12}$$

where c_k's are constants whose values may be specified by the user. For example, $c_k = \delta(k-p)$ keeps $x_p = x_{po}$; $c_k = 1$ for all drift spaces and $c_k=0$ for the other elements keeps the length of the insertion fixed.

APPLICATIONS

This code has been used for the initial design of low-beta and zero dispersion insertions for the proposed colliding beam facilities at NAL.[5] At SLAC, it was recently required for the design of the proposed 15 GeV e^+e^- ring to find an insertion which has the properties that the value of $T_{11}(x)$ varies over a large range while $T_{11}(y)$ = constant, and $T_{12} = T_{21} = 0$ for both x and y motion.[6] For this problem we modified the code so that it can search automatically for solutions corresponding to different values of $T_{11}(x)$ within the desired range of values. In this modified version first we look for a solution corresponding to some value of $T_{11}(x)$, say T_o. If we succeeded, the code will use this solution as a guess solution for finding the solution corresponding to the next value of $T_{11}(x) = T_o + DT$, for some small value DT. If we succeeded, the code will use this new solution as a starting point for finding a solution corresponding to $T_{11}(x) = T_o + 2\,DT$. This procedure continues until the entire range of the values of $T_{11}(x)$ is searched. Some of the usefulness of having a special purpose insertion design code such as MAGIC can be seen from these applications.

References

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