SLAC-PUB-391

SECOND BORN CORRECTIONS TO WIDE ANGLE ELECTRON PAIR PRODUCTION AND BREMSSTRAHLUNG*

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(Submitted to Phys. Rev.)

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* Work supported by the U. S. Atomic Energy Commission.

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ABSTRACT

Cross sections, correct to second order, are derived for the photoproduction of wide angle electron pairs in a spherical, static, nuclear charge distribution. For typical double coincidence experiments it is found to be sufficient to treat only the convection current of the electron. The results obtained are essential to the interpretation of asymmetric electron pair production and may also be applied to wide angle electron bremsstrahlung. For non-coincidence experiments in which one lepton is detected with most of the available energy, the higher Born corrections are shown to be identical with those of electron scattering. Numerical results are presented. The large asymmetries which have been reported for coincident pairs produced on a lead target are satisfactorily explained by the second Born contribution. The predicted asymmetry for pairs produced in carbon with invariant pair mass near the ρ^{0} mass is small compared to the Compton contribution.

I. INTRODUCTION

The comparison of theory and experiment in wide angle electron pair production has reached a new stage of sophistication. Now that the basic predictions[1-3] of quantum electrodynamics have been vindicated by the high energy symmetric pair experiments[4,5], pair production has turned out to be an essential tool in the study of hadron processes, especially the nuclear Compton amplitude[6,7].

One of the uncertainties in comparing the theoretical cross section for wide-angle electron pair production with experimental measurements is the correction due to higher Born contributions. Such a correction would be expected to be important for experiments involving high Z targets or asymmetric detection kinematics for the electron and positron. The differential cross section for pair production in a Coulomb field was calculated without the use of Born approximation by Bethe and Maximon[8], but the wave functions used for evaluating matrix elements were only valid for small production angles ($\theta_{\pm} \sim m_e/E_{\pm}$), and the effect of finite nuclear size was not considered. In this paper we calculate the differential cross section for high energy, wide angle electron pairs produced in a static[9], spherically symmetric[10] nuclear charge distribution, correct through second Born approximation (two photon exchange). Our calculations and results are analogous to those given by R. R. Lewis, Jr.[11] for the potential scattering of high energy electrons in second Born approximation.

The higher Born corrections discussed in this paper turn out to be unimportant for the symmetric wide-angle pair production experiments[12] on carbon which have thus far established the validity of quantum electrodynamics for electron-positron invariant pair mass up to 550 MeV [4]. In these experiments the electron and positron are detected by spectrometers of symmetric acceptance, and contributions to the cross section which are odd in the nuclear charge Z, such as the interference of the first Born amplitude with the two photon exchange amplitude or the Compton contributions (see Fig. 1), must vanish[2].

In experiments which are designed to detect the interference of the Compton and Bethe-Heitler amplitudes, the effect of higher Born contributions must be considered. In the experiment of Asbury, et al.[7], coincidence measurements of near-symmetric wide-angle pairs have been used to determine the phase and magnitude of the virtual Compton amplitude at high energies on complex nuclei, and have verified the predictions of diffraction production of the ρ^0 vector meson[13]. This experiment determines quantities such as

$$\epsilon(\delta) = \frac{N_+(\delta) - N_-(\delta)}{N_+(\delta) + N_-(\delta)} \qquad (I.1)$$

where $N_+(\delta)$ is the production rate when the electron and positron are detected mirror-symmetrically in angle, but the positron has δ less momentum than the electron, and $N_-(\delta)$ is the corresponding rate when the electron has less momentum. The real part of both the Compton and even order Born amplitudes interfering with the first Born amplitude contribute to $\epsilon(\delta)$. The radiative corrections can be another source of asymmetry, but this contribution involves photon emission from the nucleus and is negligible.

The second Born contribution to $\epsilon(\delta)$ is given in Section IV, and is compared with experiment in Section VI. The calculations are based on a simplifying assumption that the correction to the pair production cross section due to higher Born contributions is independent of lepton spin to lowest order in the leptons' production angles. This assumption is justified in Section II where other applications of spin zero electrodynamics to electron processes are also discussed. The results are given for a general spherical charge distribution in terms of a spectral representation and are evaluated explicitly for point and Yukawa nuclear charge distributions. Although we have not carried out extensive numerical calculations, we find that the large asymmetries which have been reported[7] for pairs produced on a lead target are satisfactorily accounted for by the second Born contribution. On the other hand, the second Born asymmetry for carbon turns out to be small compared to the Compton contribution to the asymmetry, for electron pairs produced with invariant pair mass near the ρ^0 peak, and should not appreciably affect the recent determination[7] of the phase of the Compton amplitude.

We also discuss in this paper a second type of wide angle pair production experiment[14], which is sensitive to Compton effects[6], and, because it does not require a coincidence measurement, can be performed advantageously with linear accelerators. In this experiment one detects only one lepton (at a given angle and energy) and measures the difference in electron and positron production rates. The presence of higher Born amplitudes as well as the Compton amplitudes contributes to this difference. In these experiments the detected lepton takes nearly all the incident photon energy. As a consequence, we demonstrate in Section III that the higher Born contributions for this experiment can be calculated immediately from the corresponding contribution for ordinary electron or positron scattering.

We have also considered the second Born corrections to wide-angle electron bremsstrahlung. The discussion is given in Section V.

An alternative approach to the calculation of the higher Born corrections to pair production and bremsstrahlung could be based on distorted partial wave calculations in analogy with the treatment given by Yennie, Ravenhall, and Wilson[15,16] for high energy electron scattering by nuclei. The second Born corrections given in Section IV for the coincidence experiments should be sufficiently accurate, however, when the production angles are small ($\theta_{\pm}^2 \ll 1$), and the momentum transfer to the nucleus is smaller than the inverse of the nuclear radius. For the single lepton experiments (discussed in Section III), the higher Born corrections can be taken from the electron scattering partial wave results and are not restricted to second Born approximation.

We should emphasize that the calculation of Bethe and Maximon[8] gives the <u>total</u> cross section for pair production to all orders in a Coulomb field, but because of the approximations which were used, their results are not appropriate for the wide angle differential cross section. In particular, the Coulomb field (point nucleus) and forward angle approximations ($\theta_{\pm} \sim m/E_{\pm}$) lead to <u>no charge asymmetry</u> in the differential cross section. This can be understood by noting that at forward angles the Dirac equation for an electron in a Coulomb field can be effectively replaced by a Schrödinger equation for an electron (with its mass replaced by its energy) in a Coulomb field. Except for a phase, the pair production amplitude is then odd in $Z\alpha$, as can be seen from inspection of the scattering solutions of the Schrödinger equation. The charge asymmetric corrections to the differential cross section calculated in this paper are important at wide angles where the finite size of the nuclear charge distribution or the relativistic corrections to the potential in the equation of motion are important.

II. FIRST BORN APPROXIMATION AND SPIN INDEPENDENCE

Before proceeding to the calculation of the second Born corrections, we shall review the usual expressions for electron pair production in a static, spherically symmetric charge distribution. The Born approximation differential cross section for unpolarized electron pairs is[17]

$$\frac{d\sigma}{dE_1 dE_2 d\Omega_1 d\Omega_2} = \frac{Z^2\alpha^3}{2\pi^2} \left|\vec{p}_1\right| \left|\vec{p}_2\right| \frac{S(k)}{k} \frac{F^2(Q^2)}{Q^4} W_{\frac{1}{2}} \qquad \text{(II.1)}$$

where[18]

$$W_{\frac{1}{2}} = - \frac{(k\cdot p_2)^2 + (k\cdot p_1)^2 + (E_2^2 + E_1^2 + p_1\cdot p_2)Q^2}{(k\cdot p_1)(k\cdot p_2)} + \frac{m^2}{2}\left[\frac{Q^2 + 4E_2^2}{(k\cdot p_1)^2} + \frac{Q^2 + 4E_1^2}{(k\cdot p_2)^2}\right] \qquad \text{(II.2)}$$

Here $F(Q^2)$ is the elastic nuclear charge form factor and $S(k)$ is the energy spectrum of the incident bremsstrahlung. The electron, positron, and incident photon four-momentum are p_1, p_2 and k respectively, and the momentum transfer to the nucleus is $Q^\mu = k^\mu - p_1^\mu - p_2^\mu$, with $Q^0 = 0$.

It is interesting to compare this result with the corresponding cross section for pairs of spin zero. This cross section is given by

(II.1) with $W_{\frac{1}{2}}$ replaced by

$$W_0 = -\frac{(k\cdot p_1)(k\cdot p_2) + Q^2 E_1 E_2}{(k\cdot p_1)(k\cdot p_2)} - m^2 \left[\frac{E_1^2}{(p_2\cdot k)^2} + \frac{E_2^2}{(p_1\cdot k)^2}\right] . \qquad \text{(II.3)}$$

We notice that

$$W_{\frac{1}{2}} - 2W_0 = -\frac{[k\cdot(p_1-p_2)]^2 + Q^2[(E_1-E_2)^2 - \frac{1}{2}(p_1-p_2)^2]}{(k\cdot p_1)(k\cdot p_2)} + O(m^2), \qquad \text{(II.4)}$$

and hence for some kinematical conditions, electron pair production will be well-approximated by twice the spin zero result. In particular, this is true for the coincidence measurements of wide-angle electron pairs. For these experiments

$$\frac{m^2}{E_\pm^2} \ll \theta_\pm^2 \ll 1 , \qquad \text{(II.5a)}$$

the Q^2 term nearly always dominates (II.2)[19], and the electron mass can be ignored. Thus if

$$(E_1 - E_2)^2 \ll E_1^2 + E_2^2 , \qquad \text{(II.5b)}$$

the cross section for electron pairs is accurately given by twice[20] the spin zero result[21].

Thus the presence of spin interactions turns out to be inessential to the Born cross section. The interpretation of this is that where the spin zero pair production is not dynamically suppressed, the presence of spin flip channels does not markedly affect the cross section. From this standpoint, it is natural to assume that the correction to the Born cross section due to second and higher order Born contributions is independent of lepton spin to lowest order in the lepton production angle. This can be explicitly checked in the extreme situation where one lepton has nearly all the available incident photon energy (see Section III); the second Born correction to pair production then turns out to be identical to that for electron or positron scattering and is the same as the spin zero result to lowest order in the lepton angle. We can also support the spin independence assumption by considering a model in which the effect of higher Born contributions is a perturbation on the static Born approximation potential V acting on the leptons:

$$V \to V\left(1 + \tfrac{1}{2}\delta_S + \tfrac{1}{2}\delta_A\right) \text{ for the electron}$$

$$V \to V\left(1 + \tfrac{1}{2}\delta_S - \tfrac{1}{2}\delta_A\right) \text{ for the positron} \tag{II.6}$$

This model is especially well-adapted to represent the effect of the finite nuclear size in the higher Born terms[16] and still retains the magnetic contribution at the incident vertex. We than recalculate the Born cross sections using the modified potentials, keeping terms linear in the perturbation,

$$W_{\frac{1}{2}} \to W_{+}(1 + \delta_S) - \delta_A \frac{2(E_1^2 + E_2^2 + p_1\cdot p_2)(k\cdot p_2 - k\cdot p_1) + (k\cdot p_1)^2 - (k\cdot p_2)^2}{(k\cdot p_1)(k\cdot p_2)}$$

$$2W_0 \to 2W_0(1 + \delta_S) - \delta_A \frac{4E_1E_2(k\cdot p_2 - k\cdot p_1)}{(k\cdot p_1)(k\cdot p_2)} \tag{II.7}$$

Thus to lowest order in $\theta_{\pm}^2$, the higher Born contributions here are the same for the spin $\frac{1}{2}$ cross section and for twice the spin 0 cross section if $(E_1 - E_2)^2 \ll E_1^2 + E_2^2$. Aside from the restriction to $(m/E_{\pm})^2 \ll \theta_{\pm}^2 \ll 1$, the ratio of higher Born to the first Born contributions is independent of spin regardless of the detection kinematics in this model.

We thus have some assurance that our simplifying assumption of spin independence is correct, and probably is no more drastic than the approximations one must make concerning the nuclear inelastic processes. It has the advantage, however, of markedly simplifying the integrations associated with Figure 1(e), and enables us to write the pair production cross section through order $Z^3\alpha^4$ in a reasonably compact form. The calculation of the second Born contribution to the charge asymmetry in spin zero pair production is presented in Section IV.

III. THE SINGLE-ARM EXPERIMENTS

In typical experiments[14] where only one lepton is detected, the detected electron or positron has nearly all the available energy. We can then easily show that the higher Born corrections to pair production may be obtained simply from the higher Born corrections to electron or positron scattering.

We first note that for $E_1 \gg E_2$ and $\frac{m^2}{E_1^2} \ll \theta_1^2 \ll 1$, the Born approximation spin $\frac{1}{2}$ cross section is given by (II.1) where now[22]

$$W_{\frac{1}{2}} \cong - \frac{E_1^2\, Q^2}{(k\cdot p_1)(k\cdot p_2)} + \frac{2m^2\, E_1^2}{(k\cdot p_2)^2} \qquad \text{(III.1)}$$

Despite the appearance of the first term, this contribution comes entirely from the square of the matrix element of Fig. 1(b) [23]. Similarly, the higher Born contributions are dominated by diagrams such as 1(d) for which the lepton propagator is nearly on-shell:

$$\frac{(k-p_2)^2 - m^2}{E_1^2} = \frac{\vec{p}_1^{\,2} - (\vec{k}-\vec{p}_2)^2}{E_1^2} = O\left(\frac{E_2}{E_1}\theta_2^2\right) + O\left(\frac{m^2}{E_1^2}\right) \qquad \text{(III.2)}$$

The contribution of these diagrams is proportional to [24]

$$\sum_{\text{pol}} \text{Tr}\,\frac{[\not{p}_1 \Gamma(\not{p}_2-\not{k})\not{\epsilon}\;\not{p}_2\;\not{\epsilon}(\not{p}_2-\not{k})\tilde{\Gamma}]}{(k\cdot p_2)^2} = -4\,\text{Tr}\,\frac{[\tilde{\Gamma}\;\not{p}_1\Gamma\;\not{k}]}{(k\cdot p_2)} \cong -4\,\text{Tr}\,\frac{[\tilde{\Gamma}\;\not{p}_1\Gamma(\not{p}_1+\not{Q})]}{k\cdot p_2} \qquad \text{(III.3)}$$

where Γ is the entire electron-nucleus interaction for an electron to scatter from momentum $\vec{p}_1 + \vec{Q}$ to $\vec{p}_1$; because of (III.2) we can neglect off-shell effects.

Eq. (III.3) shows explicitly that the higher Born corrections for pair production are the same as those for electron scattering when $E_1 \gg E_2$. Thus if $\mathcal{R}$ represents the higher Born corrections to an electron scattering from momentum $\vec{p}_1 + \vec{Q}$ to $\vec{p}_1$:

$$d\sigma_{\text{elec-scatt}} \equiv d\sigma_{\text{Born}}^{(es)}(1+\mathcal{R}) \;, \qquad \text{(III.4)}$$

then the cross section for pair production with only the electron detected is

$$d\sigma_{(-)} = d\sigma_{\text{Born}}^{(pp)}(1+\mathcal{R}) \qquad \text{(III.5)}$$

if $E_1 \gg E_2$.

Similarly, the cross section for positron detection is

$$d\sigma_{(+)} = d\sigma^{(pp)}_{\text{Born}}(1 + \tilde{\mathcal{R}}) \tag{III.6}$$

if $E_2 \gg E_1$. Here $\tilde{\mathcal{R}}$ is obtained from $\mathcal{R}$ by interchanging $\vec{p}_1$ with $\vec{p}_2$ and Z with -Z:

$$\tilde{\mathcal{R}} = \mathcal{R}(\vec{p}_1 \longleftrightarrow \vec{p}_2,\ Z \longleftrightarrow -Z) \tag{III.7}$$

Lewis[11] has given $\mathcal{R}$ in second Born approximation for an arbitrary spherical charge distribution. Distorted wave calculations suitable for high Z targets have been given by Yennie, Ravenhall and Wilson and others[15]. The second Born result for pair production in a pure Coulomb potential is[25]

$$\frac{d\sigma_- - d\sigma_+}{d\sigma_- + d\sigma_+} = \frac{\pi\ Z\alpha \sin\frac{\theta}{2}}{1 + \sin\frac{\theta}{2}} \tag{III.8}$$

where θ is the laboratory angle of the detected lepton which is assumed to have nearly all the available energy.

IV. THE SECOND BORN CALCULATION

As has been discussed in Section II, electron pair production is well-approximated by spin zero pair production (multiplied by a phase space factor of two) for the usual experimental kinematics of the double coincidence experiments. The spin zero pair production Feynman diagrams are shown in Fig. (2).

The matrix elements corresponding to Fig. (2) may be written down directly from the Feynman rules; the results for the static nucleus are then obtained by passing to the limit $M \to \infty$. Taking crossed and uncrossed diagrams together, the surviving contributions correspond to multiple Coulomb exchange with no energy transfer. Alternatively, one can obtain the matrix elements from propagator perturbation theory for the Klein-Gordon equation for a potential-scattered spin zero particle interacting once with the radiation field.

If we cancel out common factors from the nuclear phase space and the matrix elements, the cross section through second Born approximation is

$$\frac{d\sigma}{d\Omega_1 d\Omega_2 dE_1 dE_2} = \frac{Z^2\alpha^3}{2\pi^2} \left|\vec{p}_1\right| \left|\vec{p}_2\right| \frac{S(k)}{k} \sum_{\text{pol}} \left| M^{(1)} + Z\alpha\, M^{(2)} \right|^2 \qquad \text{(IV.1)}$$

where the first Born contribution is

$$M^{(1)} = \frac{F(Q^2)}{\left|\vec{Q}\right|^2} \left[\frac{E_2 \epsilon \cdot p_1}{k \cdot p_1} + \frac{E_1 \epsilon \cdot p_2}{k \cdot p_2} - \epsilon_0 \right] . \qquad \text{(IV.2a)}$$

Here

$$M^{(2)} = M_S + M_R \qquad \text{(IV.2b)}$$

where we have grouped the second Born contributions from diagrams (2d) and (2e) into M_S and the remainder into M_R. Here

$$M_S = 2\pi \left[- \frac{\epsilon \cdot p_1}{k \cdot p_1} + \frac{\epsilon \cdot p_2}{k \cdot p_2} \right] \int \frac{d^3 q}{(2\pi)^3} \frac{F(\vec{q}^{\,2}) F(|\vec{Q}-\vec{q}|^2)}{\vec{q}^{\,2} (\vec{Q}-\vec{q})^2} \qquad \text{(IV.3a)}$$

$$M_R = 8\pi \int \frac{d^3q}{(2\pi)^3} \frac{F(\vec{q}^{\,2})F(|\vec{Q}-\vec{q}|^2)}{\vec{q}^{\,2}(\vec{Q}-\vec{q})^2} \quad .$$

$$\left\{ \left[\frac{\epsilon\cdot p_1 E_2^2}{k\cdot p_1} - \epsilon_0 E_2\right] \frac{1}{(p_2+q)^2-m^2} - \left[\frac{\epsilon\cdot p_2 E_1^2}{k\cdot p_2} - \epsilon_0 E_1\right]\frac{1}{(p_1+q)^2-m^2} \right. \qquad \text{(IV.3b)}$$

$$\left. - \frac{E_1E_2\epsilon\cdot(p_1+q)}{k\cdot(p_1+q)}\left[\frac{1}{(p_1+q)^2-m^2} - \frac{1}{(p_1+q-k)^2-m^2}\right]\right\}$$

and $q_0 \equiv 0$. The amplitudes M_s and M_R are separately invariant with respect to gauge changes for the incident photon. They are also appropriately odd under interchange of electron and positron four-momenta.

If the nuclear charge distribution has a Yukawa structure, $F(\vec{q}^{\,2}) = \mu^2/(\vec{q}^{\,2}+\mu^2)$, then the second Born matrix elements will involve

$$\frac{F(\vec{q}_1^{\,2})}{\vec{q}_1^{\,2}}\frac{F(\vec{q}_2^{\,2})}{\vec{q}_2^{\,2}} = \frac{1}{\vec{q}_1^{\,2}}\frac{1}{\vec{q}_2^{\,2}} - \frac{1}{\vec{q}_1^{\,2}+\mu^2}\frac{1}{\vec{q}_2^{\,2}} - \frac{1}{\vec{q}_2^{\,2}}\frac{1}{\vec{q}_1^{\,2}+\mu^2} + \frac{1}{(\vec{q}_1^{\,2}+\mu^2)(\vec{q}_2^{\,2}+\mu^2)} \qquad \text{(IV.4)}$$

We shall define $M_R(\mu_1,\mu_2)$ and $M_S(\mu_1,\mu_2)$ from Eq. (IV.3) by replacing (IV.4) with

$$\frac{1}{\vec{q}_1^{\,2}+\mu_1^2}\;\frac{1}{\vec{q}_2^{\,2}+\mu_2^2} \;;$$

the final results for the Yukawa distribution are then obtained by successively replacing the photon "masses" μ_1 and μ_2 by μ or 0 according to

$$M^Y \equiv M(0,0) - M(\mu,0) - M(0,\mu) + M(\mu,\mu) \tag{IV.5}$$

More general distributions can be represented by a superposition of Yukawa forms or a spectral representation such as that given by Lewis[11].

The amplitude M_s is readily evaluated:

$$M_s(\mu_1,\mu_2) = \left[-\frac{\epsilon\cdot p_1}{k\cdot p_1} + \frac{\epsilon\cdot p_2}{k\cdot p_2}\right]\frac{1}{2|\vec{Q}|}\tan^{-1}\left(\frac{|\vec{Q}|}{\mu_1+\mu_2}\right); \tag{IV.6}$$

for a Yukawa charge distribution this yields

$$M_s^Y = \left[-\frac{\epsilon\cdot p_1}{k\cdot p_1} + \frac{\epsilon\cdot p_2}{k\cdot p_2}\right]\frac{1}{2|\vec{Q}|}\left\{\frac{\pi}{2} - 2\tan^{-1}\frac{|\vec{Q}|}{\mu} + \tan^{-1}\frac{\vec{Q}}{2\mu}\right\} \tag{IV.7}$$

One can recognize M_s as the amplitude corresponding to the $-V^2$ term in the Klein-Gordon equation for a spin zero electron in the static potential $V(r)$.

For the case of a point charge distribution $(\mu \to \infty)$, M_s is finite and we shall show that the real, interfering, part of M_R vanishes. (This also follows from our discussion in Section I, since M_R corresponds to the second Born corrections to the Schrödinger equation.)

The interference of second and first Born amplitudes in the point nucleus limit is then

$$\frac{d\sigma_{int}}{d\Omega_1 d\Omega_2 dE_1 dE_2} = \frac{Z^3\alpha^4}{8\pi}\left|\vec{p}_1\right|\left|\vec{p}_2\right|\frac{S(k)}{k}\frac{1}{|\vec{Q}|^3}\left[-\frac{(E_2-E_1)Q^2 + 2E_2 k\cdot p_2 - 2E_1 k\cdot p_1}{k\cdot p_1\; k\cdot p_2}\right], \tag{IV.8}$$

where we have averaged over the initial photon polarization and neglected the lepton mass. The correction to Born approximation is

$$\mathcal{R} \equiv \frac{d\sigma_{int}}{d\sigma_{Born}} = \frac{Z\alpha\pi|\vec{Q}|}{4}\left[\frac{(E_2-E_1)Q^2 + 2E_2 k\cdot p_2 - 2E_1 k\cdot p_1}{E_1E_2Q^2 + (k\cdot p_1)(k\cdot p_2)}\right] + O(Z\alpha)^3 \quad . \tag{IV.9}$$

(spin zero, point nucleus)

It is interesting to apply (IV.9) to the case where one lepton has most of the available energy. Even though (IV.8) and (II.3) are then poor approximations to the spin $\frac{1}{2}$ cross sections, the ratio (IV.9) gives the same result for second Born corrections to lowest order in θ as the spin $\frac{1}{2}$ results (III.5)-(III.8). As noted in Section II, this is an indication that the spin zero calculations give a very good estimate of the correction factor due to higher Born terms, even when condition (II.5b) is not satisfied and even though the spin zero and spin $\frac{1}{2}$ cross sections are quite different.

The reader should note that the point distribution results may not be accurate even if $\vec{Q}^2 \ll \mu^2$, where $6/\mu^2$ is the mean square nuclear radius. As will be evident from our results (IV.25) for M_R, the finite size effects are modified if $k\cdot p_1$ or $k\cdot p_2$ is comparable with μ^2.[26]

For the calculation of M_R it is convenient to use the radiation gauge: $\epsilon_0 = 0$, $\vec{\epsilon}\cdot\vec{k} = 0$. We introduce a Feynman parameter x to combine the photon propagators and, for the third line of (IV.3b), use another parameter λ to combine the two lepton propagators. We then obtain

$$M_R(\mu_1,\mu_2) = -8\pi\int_0^1 dx\int_0^1 d\lambda\int\frac{d^3q}{(2\pi)^3}\cdot\left[\frac{\epsilon\cdot p_1E_2^2}{k\cdot p_1}\frac{1}{\vec{q}^{\,2}-\vec{p}_2^{\,2}}\frac{1}{[(\vec{q}-\vec{r}_{xo})^2+\tau^2]^2}\right.$$

$$\left.-\frac{\epsilon\cdot p_2E_1^2}{k\cdot p_2}\frac{1}{\vec{q}^{\,2}-\vec{p}_1^{\,2}}\frac{1}{[(\vec{q}-\vec{r}_{x1})^2+\tau^2]^2}+\frac{2\vec{\epsilon}\cdot\vec{q}\;E_1E_2}{(\vec{q}^{\,2}-E_\lambda^2)^2[(\vec{q}-\vec{r}_{x\lambda})^2+\tau^2]^2}\right] \qquad \text{(IV.10)}$$

where we have defined

$$\begin{aligned}\vec{r}_{x\lambda} &= x[\vec{p}_1-(1-\lambda)\vec{k}] - (1-x)[\vec{p}_2-\lambda\vec{k}]\\ &= \lambda\,\vec{r}_{x1} + (1-\lambda)\vec{r}_{xo}\\ \vec{r}_{x1} &= \vec{p}_1 + (1-x)\vec{Q}\\ \vec{r}_{xo} &= -\vec{p}_2 - x\,\vec{Q}\\ \tau^2 &= x\,\mu_1^2 + (1-x)\,\mu_2^2 + x(1-x)\,\vec{Q}^{\,2}\\ E_\lambda^2 &= [\lambda\,E_1-(1-\lambda)E_2]^2\end{aligned} \qquad \text{(IV.11)}$$

and we have shifted the integration variable $\vec{q}$ by $-\vec{p}_2$, $-\vec{p}_1$, and $-\vec{p}_1+(1-\lambda)\vec{k}$ in the three terms of M_R respectively. The electron mass has been ignored in the definition of E_λ^2.

The momentum integration can now be done using differentiated forms of the basic integral[27,28]

$$\int\frac{d^3q}{[(\vec{q}-\vec{r})^2+\tau^2][\vec{q}^{\,2}-E^2-i\epsilon]} = \frac{i\pi^2}{|\vec{r}|}\log\left\{\frac{\tau-i|E|-i|\vec{r}|}{\tau-i|E|+i|\vec{r}|}\right\} \qquad \text{(IV.12)}$$

where the contour prescription is determined by the Feynman propagator for the electron. Thus M_R becomes

$$M_R(\mu_1,\mu_2) = -\int_0^1 dx \int_0^1 d\lambda \left[\frac{\epsilon\cdot p_1 E_2^2}{k\cdot p_1} \frac{1}{\tau b^*} - \frac{\epsilon\cdot p_2 E_1^2}{k\cdot p_2} \frac{1}{\tau a} - \frac{2E_1E_2\epsilon\cdot[xp_1-(1-x)p_2]}{[(\tau-i|E_\lambda|)^2+\vec{r}_{x\lambda}^{\,2}]^2} \right] \tag{IV.13a}$$

where

$$a = (\tau-iE_1)^2 + [(1-x)\vec{Q} + \vec{p}_1]^2$$

$$b = (\tau+iE_2)^2 + [x\,\vec{Q} + \vec{p}_2]^2 \tag{IV.13b}$$

The denominator of the last term in M_R is linear in λ:

$$d \equiv (\tau-i|E_\lambda|)^2 + \vec{r}_{x\lambda}^{\,2} = \begin{cases} \lambda a + (1-\lambda)b & \lambda \geq E_2/(E_1+E_2) \\ \lambda a^* + (1-\lambda)b^* & \lambda < E_2/(E_1+E_2) \end{cases} \tag{IV.14}$$

and hence the λ integral in (IV.13) can be done simply:

$$\int_0^1 \frac{d\lambda}{d^2} = \left[\frac{E_2}{b^*} + \frac{E_1}{a}\right] \frac{1}{aE_2 + bE_1} \tag{IV.15}$$

where the coefficient $aE_2 + bE_1 \equiv xg + (1-x)h$ is real:

$$g = 2E_1 k\cdot p_1 + (E_1+E_2)\mu_1^2$$

$$h = 2E_2 k\cdot p_2 + (E_1+E_2)\mu_2^2 \tag{IV.16}$$

For the remaining integration we shall apply a series of variable changes used by Lewis for an integral of a similar form[27]:

1) Let $z = \mu_1 x/\mu_2(1-x)$

2) Then let $u^2 = z_1(z + z_2)/(z+z_1)$ where

$$z_1 = \frac{e-\sqrt{e^2-4}}{2}\ ,\ z_1 z_2 = 1,$$

$$e = [|\vec{Q}|^2 + \mu_1^2 + \mu_2^2]/\mu_1\mu_2$$

3) Finally let $v = \sqrt{z_2}\,(u-\sqrt{z_1})/(\sqrt{z_2}-u)$. We then obtain

$$M_R(\mu_1,\mu_2) = -2\int_0^\infty dv\left[\frac{\epsilon\cdot p_1 E_2^2}{k\cdot p_1}\,\frac{1}{\alpha v^2 + 2\beta v + \gamma} - \frac{\epsilon\cdot p_2 E_1^2}{k\cdot p_2}\,\frac{1}{\tilde{\gamma} v^2 + 2\tilde{\beta} v + \tilde{\alpha}}\right.$$

$$\left. - 2E_1E_2\left(\frac{E_2}{\alpha v^2 + 2\beta v + \gamma} + \frac{E_1}{\tilde{\gamma} v^2 + 2\tilde{\beta} v + \tilde{\alpha}}\right)\frac{N}{D}\right] \qquad (IV.17)$$

where

$$N(v) = \mu_2\epsilon\cdot p_1(1+2fv) - \mu_1\epsilon\cdot p_2(2fv+v^2)$$

$$D(v) = \mu_2 g(1+2fv) + \mu_1 h(2fv+v^2)$$

$$\alpha = \mu_1(\mu_2^2 - 2i\mu_2E_2)/f$$

$$\gamma = \mu_2(\mu_1^2 - 2i\mu_1E_2 + 2k\cdot p_1)/f \qquad (IV.18)$$

$$\beta = \mu_1\mu_2(\mu_1+\mu_2) + 2\mu_2 k\cdot p_1 - iE_2\mu_1\mu_2/f^2$$

$$f^2 = (e+2)^{-1} = \mu_1\mu_2/[|\vec{Q}|^2 + (\mu_1+\mu_2)^2]$$

and $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ are obtained from α, β, γ, respectively, by the interchange

$p_1 \longleftrightarrow p_2$. The amplitude M_R is then odd under $p_1 \longleftrightarrow p_2$ with $v \longleftrightarrow 1/v$ and $\mu_1 \longleftrightarrow \mu_2$. After some rearrangement, Eq. (IV.17) may also be written as

$$M_R(\mu_1,\mu_2) = -\, 2E_1E_2 \int_0^\infty dv \left[\frac{E_2F(v)}{\alpha v^2 + 2\beta v + \gamma}\,\frac{1}{D} - (p_1 \longleftrightarrow p_2,\ \mu_1 \longleftrightarrow \mu_2)\right] \quad \text{(IV.18a)}$$

where

$$F = \left[\frac{\epsilon\cdot p_1}{E_1 k\cdot p_1} + \frac{\epsilon\cdot p_2}{E_2 k\cdot p_2}\right]\mu_1 h(v^2 + 2fv)$$

$$+ \mu_1\mu_2(E_1 + E_2)\left[\frac{\epsilon\cdot p_1}{E_1 k\cdot p_1}\,\mu_1(1 + 2fv) - \frac{\epsilon\cdot p_2}{E_2 k\cdot p_2}\,\mu_2(v^2 + 2fv)\right]. \quad \text{(IV.18b)}$$

The second term in (IV.18a) is obtained from the first term by interchanging the lepton four-momentum, μ_1 with μ_2, and changing the integration variable v to $1/v$. The v integration is of the form

$$\int_0^\infty dv\, \frac{F(v)}{(v-v_1)(v-v_2)(v-v_3)(v-v_4)} = \frac{F(v_1)\log(1/v_1)}{(v_1-v_2)(v_1-v_3)(v_1-v_4)} +$$

$$\frac{F(v_2)\log(1/v_2)}{(v_2-v_1)(v_2-v_3)(v_2-v_4)} + \frac{F(v_3)\log(1/v_3)}{(v_3-v_1)(v_3-v_2)(v_3-v_4)} + \frac{F(v_4)\log(1/v_4)}{(v_4-v_1)(v_4-v_2)(v_4-v_3)} \quad \text{(IV.19)}$$

where the roots $v_1 \ldots v_4$ are non-positive, non-degenerate and $F(v)$ is a second order polynomial.

We shall not restrict our attention to the real (interfering) part of M_R. It may then be verified that the factor

$$\mathrm{Re}\left[\frac{E_2}{\alpha v^2 + 2\beta v + \gamma} + \frac{E_1}{\tilde{\gamma} v^2 + 2\tilde{\beta} v + \tilde{\alpha}}\right]$$

in Eq. (IV.17) vanishes at the two roots of D[29]. The roots that do contribute to Re M_R are

$$v_{1,2} = [-\beta \pm iR]/\alpha \qquad \text{(IV.20)}$$

or

$$\tilde{v}_{1,2} = [-\tilde{\beta} \pm i\tilde{R}]/\tilde{\gamma}$$

where $R^2 \equiv \alpha\gamma - \beta^2$ and $\tilde{R}^2 = \tilde{\alpha}\,\tilde{\gamma} - \tilde{\beta}^2$ can be shown to be real and positive. Performing the final integration, we obtain

$$\begin{aligned}
\mathrm{Re}\, M_R(\mu_1,\mu_2) = &\left[\frac{E_2 \epsilon\cdot p_1}{k\cdot p_1} + \frac{E_1 \epsilon\cdot p_2}{k\cdot p_2} - \epsilon_o\right] \mu_1 E_2 h\ I_2 \\
&+ \left[\frac{(E_1+E_2)\epsilon\cdot p_1}{k\cdot p_1} - \epsilon_o\right] E_2^2 \mu_1^2 \mu_2\ I_1 \\
&- \left[\frac{(E_1+E_2)\epsilon\cdot p_2}{k\cdot p_2} - \epsilon_o\right] E_1 E_2 \mu_2^2 \mu_1\ I_2 \\
&- (p_1 \longleftrightarrow p_2,\ \mu_1 \longleftrightarrow \mu_2)
\end{aligned} \qquad \text{(IV.21a)}$$

where

$$I_n = \mathrm{Re}\left[\frac{1}{iR}\left(\log v_1 \frac{N_n(v_1)}{D(v_1)} - \log v_2 \frac{N_n(v_2)}{D(v_2)}\right)\right]$$

$$(n = 1,2) \qquad \text{(IV.21b)}$$

$$N_1(v) = 1 + 2fv$$

$$N_2(v) = v^2 + 2fv$$

We have used the gauge-invariance of (IV.3b) to rewrite our result in the Lorentz gauge.

For a general nuclear charge distribution one must make a spectral sum over μ_1 and μ_2 in $M_s + \mathrm{Re}M_R$ and substitute the result into Eq. (IV.1) to obtain the cross sections through second Born approximation. For a Yukawa charge distributions we require the substitutions indicated in Eq. (IV.5). The limits μ_1, $\mu_2 \to 0$ must be taken with care. We find

$$\mathrm{Re}M_R(0,0) = 0, \qquad \text{(IV.22a)}$$

$$\mathrm{Re}M_R(0,\mu) + \mathrm{Re}M_R(\mu,0) = -\left[\frac{E_2\epsilon\cdot p_1}{k\cdot p_1} + \frac{E_1\epsilon\cdot p_2}{k\cdot p_2} - \epsilon_o\right] G \qquad \text{(IV.22b)}$$

$$\begin{aligned}\mathrm{Re}M_R(\mu,\mu) = &\left[\frac{E_2\epsilon\cdot p_1}{k\cdot p_1} + \frac{E_1\epsilon\cdot p_2}{k\cdot p_2} - \epsilon_o\right] C \\ &+ \left[\frac{(E_1+E_2)\epsilon\cdot p_1}{k\cdot p_1} - \epsilon_o\right] B_1 \\ &- \left[\frac{(E_1+E_2)\epsilon\cdot p_2}{k\cdot p_2} - \epsilon_o\right] B_2\end{aligned} \qquad \text{(IV.22c)}$$

where

$$C = \mu(E_2 h I_2 - E_1 g \tilde{I}_2)$$

$$B_1 = \mu^3(E_2 I_1 + E_1 \tilde{I}_2)E_2 \qquad \text{(IV.23)}$$

$$B_2 = \mu^3(E_2 I_2 + E_1 \tilde{I}_1)E_1$$

$$G = \frac{1}{|\vec{Q}|^2 + \mu^2}\left\{\tan^{-1}\left[\frac{2k.p_2 + \mu^2}{2E_1\mu}\right] - \tan^{-1}\left[\frac{2k.p_1 + \mu^2}{2E_2\mu}\right]\right\}$$

and $\tilde{I}_n$ is obtained from I_n by the interchange of four vectors p_1 and p_2. The parameters, α, β, γ, f, g, h, v_1, v_2 are defined in Eqs. (IV.18,20) but now $\mu_1 = \mu_2 = \mu$. Also, in this limit

$$R^2 = |\vec{Q}|^2[E_2^2|\vec{Q}|^2 + 4\mu^2E_2^2 + \mu^4 + 2k.p_1\mu^2] - \mu^2(2k.p_1)^2 \qquad \text{(IV.24)}$$

which can be shown to be positive definite.

The cross section through second Born approximation for spin zero pairs produced in a nucleus with a Yukawa form factor $F(Q^2) = \mu^2/(|\vec{Q}|^2 + \mu^2)$ is[30]

$$\frac{d\sigma}{dE_1 dE_2 d\Omega_1 d\Omega_2} = \frac{Z^2\alpha^3}{2\pi^2}\left|\vec{p}_1\right|\left|\vec{p}_2\right| \frac{S(k)}{k}\frac{(-A)}{k.p_1 k.p_2} \qquad \text{(IV.25a)}$$

where

$$A = \frac{F^2(Q^2)}{|\vec{Q}|^4}[(k.p_1)(k.p_2) + Q^2E_1E_2] + 2Z\alpha\frac{F(Q^2)}{|\vec{Q}|^2}A' + O(Z\alpha)^2 \qquad \text{(IV.25b)}$$

and

$$A' = [Q^2(E_2-E_1) + 2E_2k\cdot p_2 - 2E_1k\cdot p_1] \cdot \left[\frac{\pi}{2} - 2\tan^{-1}\frac{|\vec{Q}|}{\mu} + \tan^{-1}\frac{|\vec{Q}|}{2\mu}\right]\frac{1}{4|\vec{Q}|}$$

$$+ (C + G)[(k\cdot p_1)(k\cdot p_2) + Q^2E_1E_2]$$

$$+ \tfrac{1}{2} B_1 \left[\left\{(E_1 + E_2)Q^2 + 2E_1k\cdot p_1 - 2E_2k\cdot p_2\right\}E_1 + 2k\cdot p_1k\cdot p_2\right] \qquad \text{(IV.25c)}$$

$$- \tfrac{1}{2} B_2 \left[\left\{(E_1 + E_2)Q^2 + 2E_2k\cdot p_2 - 2E_1k\cdot p_1\right\}E_2 + 2k\cdot p_1k\cdot p_2\right]$$

The corresponding cross section for spin $\frac{1}{2}$ pairs is twice this result if conditions (II.5) hold. The electron positron asymmetry for a Yukawa nuclear charge distribution for spin zero pairs is then

$$\mathcal{R} = \frac{d\sigma_{int}}{d\sigma} = \frac{2Z\alpha|\vec{Q}|^2}{F(Q^2)} \frac{A'}{(k\cdot p_1)(k\cdot p_2) + Q^2E_1E_2} + O(Z\alpha)^3 \qquad \text{(IV.26)}$$

which reduces to (IV.9) when $\mu \to \infty$. As we have discussed in Section II, the result (IV.26) may also be used for electron (spin $\frac{1}{2}$) pair production.

V. SECOND BORN CORRECTIONS TO WIDE-ANGLE ELECTRON BREMSSTRAHLUNG

In Born approximation the bremsstrahlung cross section for an electron scattering on a static spherical nucleus (see Fig. 3) is

$$\frac{d\sigma}{dk d\Omega' d\Omega_k} = \frac{Z^2\alpha^3}{2\pi^2} \frac{F^2(Q^2)}{Q^4} \frac{|\vec{k}||\vec{p}'|}{E} (W_{\frac{1}{2}}^{BR}) \tag{V.1}$$

where $W_{\frac{1}{2}}^{BR}$ can be obtained from the pair production result of Eq. (II.1) by a crossing relation[3]

$$W_{\frac{1}{2}}^{BR} = - W_{\frac{1}{2}}^{PP} \; (p_1 \to p', \; p_2 \to -p, \; k \to -k). \tag{V.2}$$

If the electron had spin zero, the cross section would be given by Eq. (V.1) with $W_{\frac{1}{2}}^{BR}$ replaced by W_0^{BR}:

$$W_0^{BR} = + 2W_0^{PP} \; (p_1 \to p', \; p_2 \to -p, \; k \to -k) \tag{V.3}$$

where W_0^{PP} is given by Eq. (II.3). We now notice that

$$W_{\frac{1}{2}}^{BR} - W_0^{BR} = - \frac{[k\cdot(p-p')]^2 + Q^2[(E-E')^2 - \frac{1}{2}(p-p')^2]}{k.p \; k.p'} + O(m^2) \tag{V.4}$$

Thus if $(E-E')^2 \ll E^2$, $[k\cdot(p-p')]^2 \ll (k\cdot p)^2 + (k\cdot p')^2$ and $p\cdot p' \ll E^2$, then the spin zero cross section is a very good approximation to the actual electron spin $\frac{1}{2}$ expression.[31] The spin zero cross section through order $Z^3\alpha^4$ can be obtained immediately from the results of Section IV:

$$\frac{d\sigma}{dk d\Omega_k d\Omega'} = \frac{Z^2\alpha^3}{2\pi^2} \frac{|\vec{k}||\vec{p}'|}{E} \frac{A^{BR}}{k\cdot pk\cdot p'} \tag{V.5a}$$

where

$$A^{BR} = 2A^{PP}(p_1 \to p', \; p_2 \to -p', \; k \to -k) \tag{V.5b}$$

and A^{PP} is defined in Eq. (IV.25c).

The ratio of the second and first Born contributions can again be taken to be independent of spin to lowest order in θ' if $(E-E')^2 \ll E^2$, using arguments similar to those given for pair production in Section II. On the other hand, if the emitted photon has large energy, the amplitude corresponding to Fig. 3(a) will usually dominate and the higher Born corrections are then the same as those for a (nearly on-shell) electron scattering from momentum $\vec{p}$ to $\vec{p} - \vec{Q}$.

VI. NUMERICAL RESULTS FOR WIDE-ANGLE PAIR PRODUCTION

The results of Section IV for the second Born corrections to large angle asymmetric electron pair production are applicable to the coincidence measurements performed by Asbury, et al., at DESY[7]. In this experiment, counts were binned according to differences in electron and positron angle, energy, or transverse momentum for pairs with invariant pair mass $m(e^+e^-) = 770 \pm 50$ MeV (the region of the ρ^0) produced on a carbon target, and for $300 < m(e^+e^-) < 550$ MeV on lead. For the latter region, previous measurements[4] have shown that the production rate for symmetric pairs agrees with the QED predictions.

In Fig. (4) we have shown a comparison of our result (IV.25) for $\epsilon(\delta)$, defined in Eq. (I.1), and the carbon experimental results, binned according to the electron and positron energy difference. The dotted line shows (for reference only) the ratio of differential cross sections for coplanar, mirror symmetric pairs, with $\delta = E_1 - E_2$ and $\frac{1}{2}(E_1 + E_2) = E_0$. A Yukawa charge distribution is used, with μ adjusted to the nuclear electromagnetic radius. The solid curve is obtained by averaging the cross sections over an angular acceptance of $\theta_{1,2} = \theta_0 \pm .07\ \theta_0$.[32] This is only a first approximation to the experimental acceptance ("window"). A detailed comparison with experiment would require integration over the coplanarity acceptance, the bremsstrahlung distribution, as well as the actual angular acceptance dependence on δ, and finally an averaging over the bin size. Despite these qualifications, Fig. (3) shows that the second Born corrections are small for the carbon data. On the other hand, the contribution of the Compton diagram corresponding to virtual ρ^0 diffraction production does fit the data and within a phase angle of $\varphi = 15^\circ \pm 25^\circ$ is consistent with a purely absorptive ($\varphi = 0^\circ$) Compton amplitude[7]. The inclusion of second Born corrections should make only a minor correction to this analysis.

The effect of the ρ^0 Compton amplitude for the lead data is very small (see Fig. 3a of Ref. 7) since the invariant pair mass is well below the ρ^0 region. In Fig. (5) we have shown the contribution to $\epsilon(\delta)$ from the inferference of second Born approximation, again assuming a Yukawa nuclear charge distribution, coplanarity, and spin zero electrodynamics. The solid curve is obtained from averaging over the lepton angular region $\theta_0 \pm .07\ \theta_0$.[32] Even with this rough approximation to the acceptance window, and the expected importance of higher Born corrections, the results are in qualitative agreement with the experimental results. Agreement with the $|\delta| \leq 300$ MeV data would be improved further if the finite bin size and exact experimental acceptance were taken into account.

We have also evaluated the asymmetry ϵ for lead as a function of $\theta_1 - \theta_2$. Appreciable asymmetries for the differential cross section are obtained, but they are reduced typically by a factor of 5 when the large energy acceptance ($E_0 \pm 500$ MeV) is averaged over. Our results thus account for the negligible asymmetry for this case noted by the experimentalists[7]. We have not compared our results for asymmetries binned to transverse momentum difference since the analysis depends very critically on the detailed experimental acceptance.

Thus despite the many approximations made in our analysis, the sign and magnitude of the asymmetry in the lead results has been explained. We summarize here the limits and approximations used in our calculation (in addition to the estimates of the experimental acceptance): Spin zero electrodynamics, zero electron mass, and a one parameter Yukawa nuclear charge distribution were employed. Compton contributions, terms higher than second order in the potential, and radiative corrections were not included. Finally, it was assumed that the asymmetric corrections to the inelastic contribution to the cross section scaled the same as the elastic contributions. Even with these approximations, our results are reliable enough to satisfactorily explain the large asymmetries in the lead data, and to provide the experimentalist with a reliable estimate of the importance of second Born effects in pair production[33].

ACKNOWLEDGEMENTS

We wish to thank G. Greenhut, D. R. Yennie, Y. S. Tsai, and S. D. Drell for helpful discussions. We are also grateful to J. Primack and J. Royer for checking some of our calculations, and to R. Simonds for discussions of his experiment as well as his assistance in the numerical analysis.

REFERENCES

1. H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) A146 (1934).
2. J. D. Bjorken, S. D. Drell, and S. C. Frautschi, Phys. Rev. 112, 1409 (1958).
3. S. D. Drell and J. D. Walecka, Ann. Phys. (N.Y.) 28, 19 (1964).
4. J. G. Asbury, W. K. Bertram, U. Becker, P. Joos, M. Rohde, A.J.S. Smith, S. Friedlander, C. L. Jordan, and S.C.C. Ting, Phys. Rev. 161, 1344 (1967), and references therein.
5. A review of recent pair production experiments has been given by Roy Weinstein, Report to the International Symposium on Electron and Photon Interactions at High Energies, Stanford (1967), p. 409. Preliminary results have also been reported by K. J. Cohen, S. Homma, P. D. Luckey, and L. S. Osborne, Bull. Am. Phys. Soc. 13, 97 (1968), for symmetric and asymmetric electron pair production on carbon.
6. The ρ^0-meson Compton contribution is discussed by S. Drell, International Conference on Electron and Photon Interactions, Hamburg (1965), and K. A. Ter-Martirosyan and K. G. Boreskov, Phys. Letters 25B, 223 (1967). For a complete discussion of the Compton contribution to pair production, including the effect of nucleon resonances, see S. J. Brodsky, A. C. Hearn and R. Parsons (in preparation).
7. J. G. Asbury, U. Becker, W. K. Bertram, P. Joos, M. Rohde, A.J.S. Smith, C. L. Jordan, and S.C.C. Ting, Phys. Letters 25B, 565 (1967), and Phys. Rev. Letters 18, 65 (1967). Also see S.C.C. Ting, Report to the International Symposium on Electron and Photon Interactions at High Energy, Stanford (1967), p. 452, and lecture given at the

International School of Physics 'Ettore Majorana,' Erice (Trapani), Sicily, July 1967.

8. H. A. Bethe and L. C. Maximon, Phys. Rev. 93, 768 (1954) and H. Davies, H. A. Bethe, and L. C. Maximon, Phys. Rev. 93, 788 (1954).

9. The recoil energy of the nucleus is assumed to be negligible. Our calculation of the interference of second and first Born amplitudes is only appropriate to the elastic pair-production events, where the nucleus returns to its ground state. The excited state contributions in the intermediate state are also not taken into account by the static approximation. However, we can argue that the real (inferfering) part of a resonance contribution will roughly vanish when integrated over the loop momentum (see Fig. 1c), since it changes sign as the excitation energy ranges through the resonance. See R. R. Lewis, Jr., Phys. Rev. 102, 544 (1956), and S. D. Drell and S. Fubini, Phys. Rev. 113, 741 (1959).

10. The momentum transfer to the nucleus is assumed to be small compared to the inverse of any nuclear moment.

11. R. R. Lewis, Jr., Phys. Rev. 102, 537 (1956). See also S. D. Drell and R. M. Pratt, Phys. Rev. 125, 1394 (1962).

12. S. D. Drell, Ann. of Phys. 4, 75 (1958).

13. S. D. Drell and J. S. Trefil, Phys. Rev. Letters 16, 552 (1966); S. M. Berman and S. D. Drell, Phys. Rev. 133, B791 (1964), and M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966).

14. R. Simonds and B. Richter (to be published).

15. D. R. Yennie, D. G. Ravenhall, and R. N. Wilson, Phys. Rev. 95, 500 (1954); R. Herman, B. C. Clark, and D. G. Ravenhall, Phys. Rev. 132, 414 (1963), and references therein.

16. A distorted wave method is being used by G. Greenhut and D. R. Yennie to obtain an estimate of the higher Born corrections to wide angle pair production (private communication).

17. Our notation is that of J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill Book, Co., Inc., New York, 1964). In particular $p_1 \cdot p_2 = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2$, and for a static nucleus $Q^2 = -|\vec{Q}|^2$.

18. The electron mass m has been neglected in (II.2) and (II.3) except in terms which can give a contribution independent of m when integrated over an undetected lepton's angle. The pair production differential cross section including nuclear recoil, inelastic form factors, and all terms in the lepton mass is given in Ref. 3. The virtual and "soft" photon radiative corrections to the lepton lines for (II.1) are given in Ref. 2. The "hard" photon corrections have been calculated for the symmetric pair experiments by B. Huld (to be published in Phys. Rev.) and S. J. Brodsky (unpublished).

19. The exception is when the nucleus recoils along the incident direction; the spin 0 Born corrections dips to zero, whereas the spin $\frac{1}{2}$ cross section is very small but finite due to spin contributions. See G. R. Henry, Phys. Rev. 153, 1649 (1967). In practice, the coincidence experiments always involve appreciable rms transverse momentum transfer to the nucleus because of the experimental acceptance, and the value of the cross section at the dip is unimportant.

20. The factor of two can be understood from an examination of the Dirac equation for a zero mass electron in the two component theory:

$$[\vec{\sigma}\cdot(\vec{p} - e\vec{A}) + (V - E)]\varphi = 0$$

$$[-\vec{\sigma}\cdot(\vec{p} - e\vec{A}) + (V - E)]\chi = 0$$

The free, positive energy solutions for the decoupled wave functions φ and χ have positive and negative helicity respectively. Propagator perturbation theory can be developed (to first order in $\vec{A} = \vec{\epsilon}\, e^{i\vec{k}.\vec{x}}$, all orders in the static potential V) from the equation of motion

$$[(\vec{p} - e\vec{A})^2 - (E-V)^2 - e\,\vec{\sigma}\cdot\vec{\nabla} \times \vec{A} - i\,\vec{\sigma}\cdot\vec{\nabla}\,V]\varphi = 0.$$

The effects of the spin terms can thus be isolated and shown to give a negligible contribution to the Born cross section when conditions (II.5) hold; in particular, the last term gives contributions suppressed by a factor of θ. The spin $\frac{1}{2}$ cross section is double the spin 0 result because φ and χ give equal helicity-conserving contributions. Complete treatments of the two component theory in quantum electrodynamics have been given by L. M. Brown, Phys. Rev. 111, 957 (1958), references therein, and in Ref. 15.

21. We have also found a similar result for the trident process (electroproduction of electron pairs); twice the spin 0 result is a good approximation for near-symmetric (tripod) triple-coincident kinematics. Also, the differential cross section for pair production plus the emission of one photon can also be obtained from twice the corresponding spin 0 result in certain kinematical regions. The application to the bremsstrahlung process is discussed in Section V. As a general rule, it is beneficial to first study the spin 0 result before undertaking the more complicated spin $\frac{1}{2}$ calculation: the

cross sections are often similar, and in any case can be used to determine the important kinematical regions and efficient grids for numerical integrations.

22. In practice, the cross section is dominated by configurations in which the angle of the undetected lepton is also small.

23. For convenience we use the Lorentz gauge, but this result is independent of gauge because the magnetic current interaction with the photon dominates the cross section here. For spin 0 pairs, one should use the Coulomb gauge.

24. We have ignored the lepton mass here. $\tilde{\Gamma}$ is obtained by reordering the γ matrices in Γ.

25. The corresponding correction to electron scattering was first given by W. A. McKinley and H. Feshbach, Phys. Rev. 74, 1759 (1949), and R. H. Dalitz, Proc. Roy. Soc. (London) A206, 509 (1951).

26. We would like to thank Dr. D. R. Yennie for discussions on this point.

27. See Ref. 11, Appendix I. As usual we must discard the imaginary infrared divergent contributions which correspond to the first order term in the expansion of the infinite Coulomb phase.

28. Note that (IV.12) is an even function of E and can be analytically continued for positive E into the upper half plane.

29. This is immediately apparent from Eq. (IV.15); the denominator $aE_2 + bE_1$, which corresponds to $D(v)$, cancels for the real part.

30. The reader is reminded that $Q^2 = -|\vec{Q}|^2$ and the lepton mass has been neglected here.

31. We are specifically interested in coincidence measurements of wide angle bremsstrahlung which might be used to check quantum electrodynamics in a region where the electron propagator is timelike.

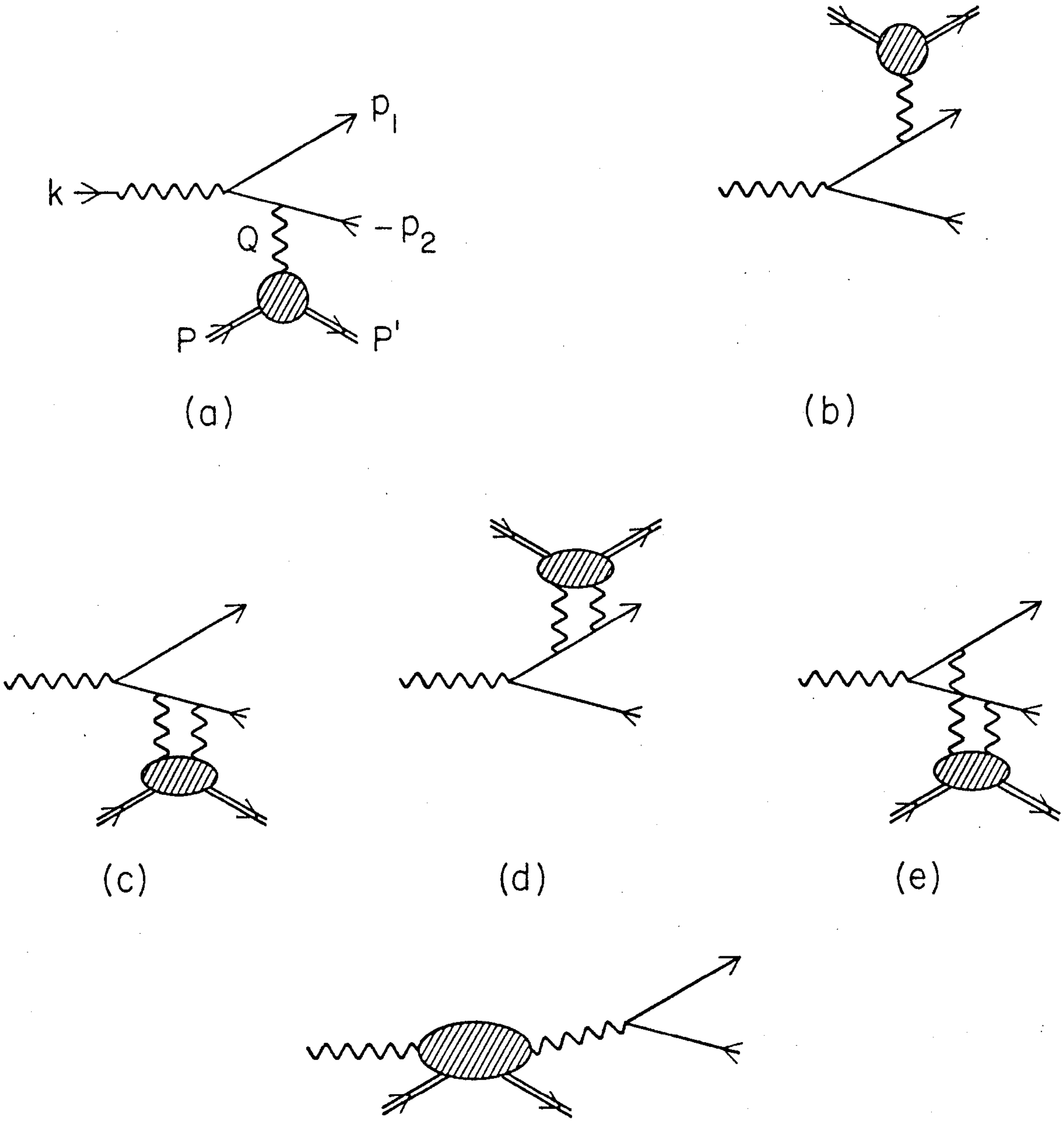
32. We have also reduced ϵ by a factor $1 + \epsilon^2$ in the integrated curves to account for the second Born contribution to the denominator of (I.1).

33. The second Born corrections turn out to be small for the carbon target experiment of Ref. 14. For example, R is + 0.8 percent for an electron detected at 30° with energy of 170 MeV, and - 0.7 percent at 15°, 625 MeV. (R. Simonds, private communication.)

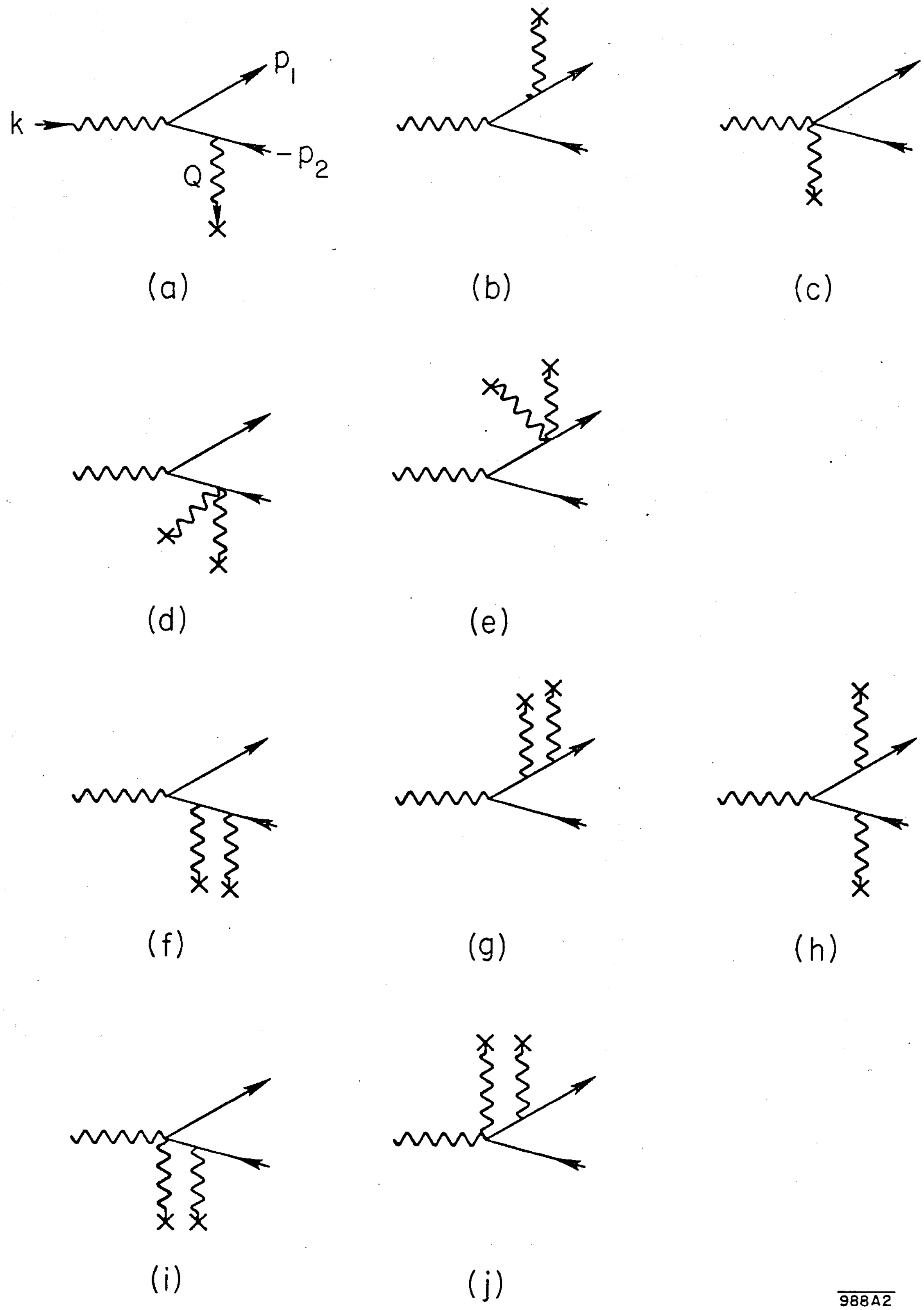
FIGURE CAPTIONS

1. Feynman diagrams for electron pair production. Figures (a) through (e) give the Bethe-Heitler amplitude through second order in the electromagnetic interaction with the nucleus. Diagram (f) represents the virtual Compton contribution to pair production and includes contributions from the nuclear pole terms, nucleon and nuclear excitations, and neutral vector meson production.
2. Feynman diagrams for pair production of spin zero particles in a static electromagnetic field. Figure (a),(b),(c) correspond to first Born approximation. Figures (d) and (e) are "seagull" graphs and lead to the matrix element M_s in Eq. (IV.3) of the text. Figures (f) through (j) give the remainder of the second Born amplitude M_R for spin zero pair production. The amplitudes corresponding to Figures (i) and (j) vanish in the radiation gauge.
3. Born approximation diagrams for electron bremsstrahlung.
4. Electron-positron asymmetry in pair production on carbon for invariant pair mass $m(e^+e^-) = 770 \pm 50$ MeV (the ρ^0-dominated region). The definition of $\epsilon(\delta)$ is given in Section I. The experimental points are the binned results of Ref. 7. The dotted curve is the second Born spin zero result given in Section IV for mirror symmetric (except for energy, $\delta = E_1 - E_2$) pairs produced in a Yukawa charge distribution with $\mu = 204$ MeV chosen to fit the carbon rms radius. The solid curve is obtained by averaging the cross sections over lepton angles in order to approximate the experimental acceptance, as discussed in the text.

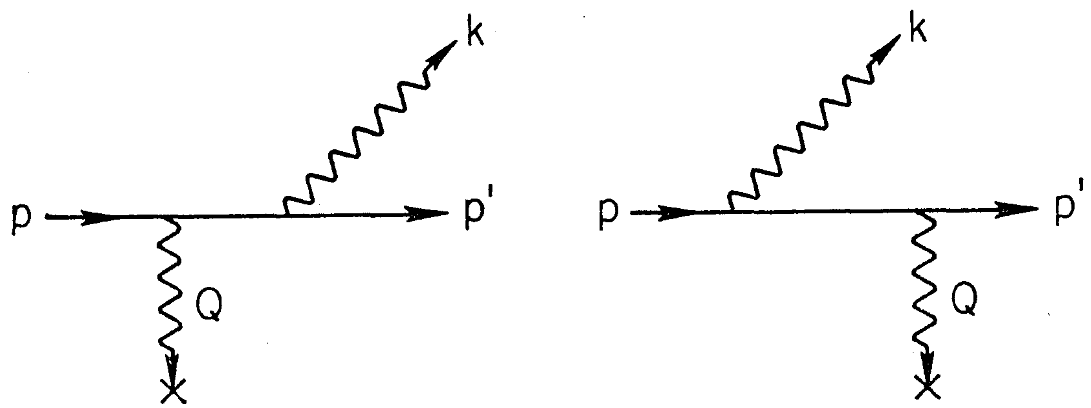
5. Electron-positron asymmetry in pair production on lead for invariant pair mass in the interval $300 \leq m(e^+e^-) \leq 550$ MeV. The experimental points are from Ref. 7. The dotted curve is the predicted differential cross section for mirror symmetric (except for $E_- - E_+ = \delta$) pairs produced in a Yukawa charge distribution with $\mu = 89.5$ MeV chosen to fit the electromagnetic rms radius of lead. The solid curve is obtained from an average over lepton angles to provide a first approximation to the experimental acceptance (see text). The asymmetry for $|\delta| \leq 300$ MeV will be reduced when the theoretical prediction is averaged over the finite binning size and the actual experimental acceptance.



988A1

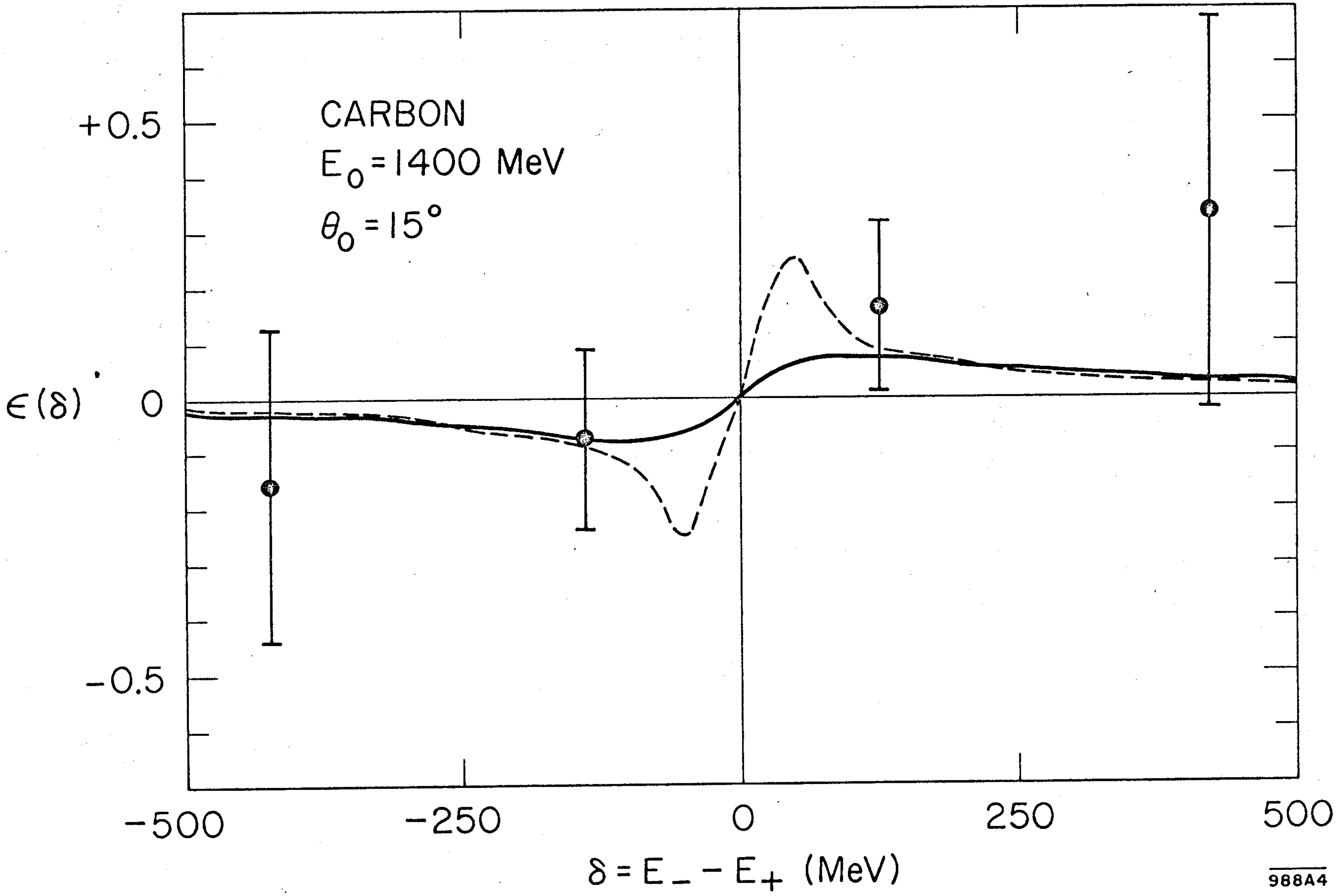


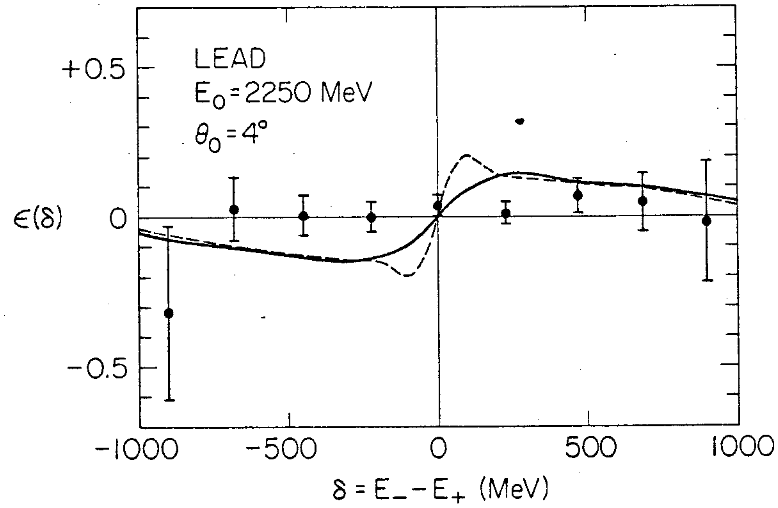
988A2



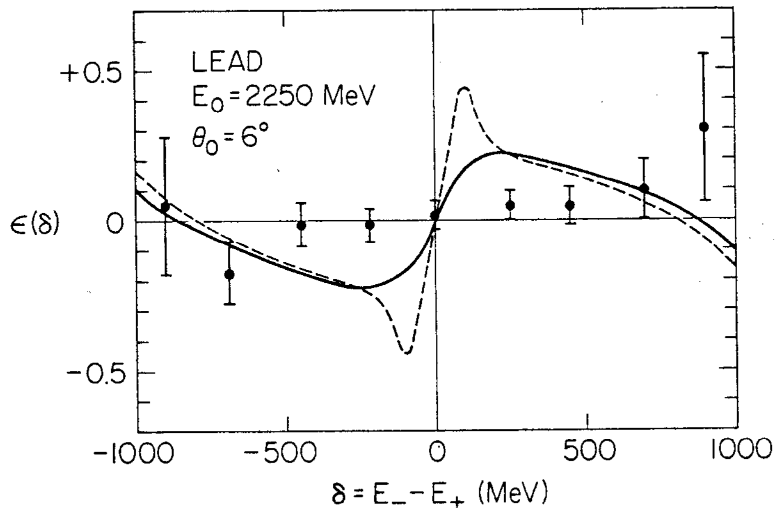
(a) (b)

988A3

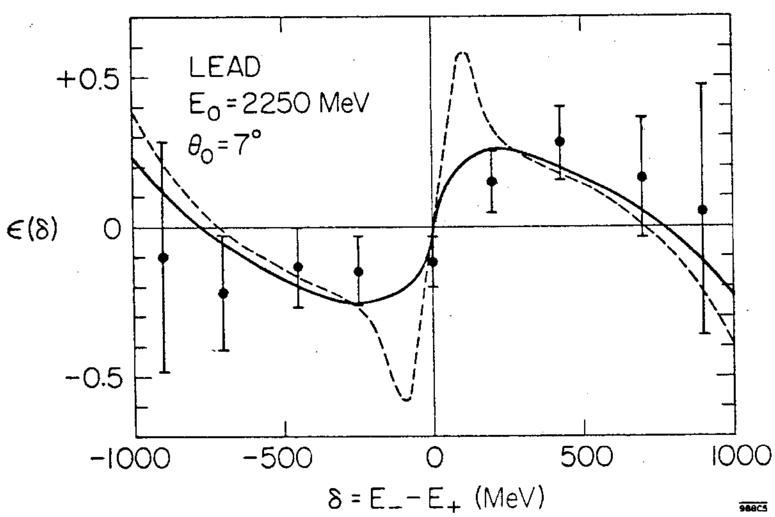

CARBON
$E_0 = 1400$ MeV
$\theta_0 = 15°$
+0.5
$\epsilon(\delta)$
0
−0.5
−500
−250
0
250
500
$\delta = E_- - E_+$ (MeV)
988A4


LEAD
E0=2250 MeV
θ0=4°
+0.5
ϵ(δ) 0
−0.5
−1000 −500 0 500 1000
δ = E− − E+ (MeV)

a.


LEAD
E0=2250 MeV
θ0=6°
+0.5
ϵ(δ) 0
−0.5
−1000 −500 0 500 1000
δ = E− − E+ (MeV)

b.


LEAD
E0=2250 MeV
θ0=7°
+0.5
ϵ(δ) 0
−0.5
−1000 −500 0 500 1000
δ = E− − E+ (MeV)

c.