Light Nonabelian Monopoles: Constructing Dual Nonabelian Superconductor of More General Types

Kenichi Konishi,[1,3] Stefano Bolognesi[2,3] and Giacomo Marmorini[2,3]

[1] *Physics Department, "E. Fermi", University of Pisa, Italy*
[2] *Scuola Normale Superiore, Pisa, Italy*
[3] *INFN, Sezione di Pisa, Italy*

In this talk we discuss a new class of $\mathcal{N}=1$ supersymmetric $U(N)$ gauge theories, in which the low-energy magnetic effective gauge group contains multiple nonabelian factors, $\prod_i SU(r_i)$, supported by light monopoles carrying associated charges. These nontrivially generalize the physics of r-vacua found in softly broken $\mathcal{N}=2$ supersymmetric QCD and yield dual nonabelian superconductor of a more general type. The matching between classical and quantum $(r_1, r_2, \ldots)$ vacua leads to nontrivial hints about the nonabelian duality.

§1. Why get interested in nonabelian monopoles?

Nonabelian monopoles can be the key for understanding confinement in QCD. In spite of many papers on (semi-classical) monopoles their true properties still elude us. The central fact is that the nonabelian monopoles,[1)] unlike their abelian counterpart, is essentially quantum mechanical. Only a fully quantum mechanical treatment can tell about their physical properties.[2),3)] Thanks to some exact results in $\mathcal{N}=1,2,4$ supersymmetric gauge theories found in the last ten years or so,[4)–12)] however, we now have certain solid knowledge about them. Here we discuss the quantum mechanical behavior of nonabelian monopoles in general, and in the context of a $U(N)$ theory, systems with low-energy $SU(r_1)\times SU(r_2)\times SU(r_3)\ldots$ magnetic gauge symmetry.[13)]

§2. Bosonic $SU(N)$ theory

First consider a bosonic $SU(N)$ theory with

$$\mathcal{L}=\frac{1}{4g^2}(F^A_{\mu\nu})^2+\frac{1}{g^2}|(\mathcal{D}_\mu\phi)^A|^2-V(\phi),$$

$$\langle\phi\rangle=\begin{pmatrix} v_1\cdot\mathbf{1}_{r_1\times r_1} & & \\ & v_2\cdot\mathbf{1}_{r_2\times r_2} & \\ & & \ddots\end{pmatrix},$$

$$SU(N)\to\frac{SU(r_1)\times SU(r_2)\times U(1)^{N-r_1-r_2+1}}{\boldsymbol{Z}_{r_1}\times\boldsymbol{Z}_{r_2}}. \tag{2·1}$$

The monopoles can be (semiclassically) constructed by embedding 't Hooft-Polyakov monopoles in various broken $SU(2)$ subgroups, thus one finds r_1 monopoles living on $(i,N+1)$ subspaces, $i=1,2,\ldots,r_1$; r_2 monopoles in $(j,N+1)$ subspaces, $j=$

$r_1+1, r_1+2, \ldots, r_1+r_2$; $r_1 r_2$ monopoles living on (i,j) subspaces; and so on. In softly broken $\mathcal{N}=2$ SQCD, with a small adjoint scalar mass, we do know that there appear in the infrared only vacua with magnetic gauge group

$$SU(r) \times U(1)^{N_c-r+1}, \qquad r = 0, 1, 2, \ldots, \frac{N_f}{2}, \, \tilde{N}_c = N_f - N_c.$$

The symmetry breaking pattern (2·1) cannot be realized quantum mechanically in this system.[10)] The problem is thus highly a nontrivial one.

§3. Phases of softly broken $\mathcal{N}=2$ gauge theories

Let us now recall what happens in $\mathcal{N}=2$ gauge theories, softly broken by the adjoint scalar mass $\mu\Phi^2$. In the case of $SU(n_c)$ gauge theory, the vacua in confinement phase is completely classified by an integer r, $r = 0, 1, \ldots, \frac{n_f-1}{2}$. The effective gauge group is $SU(r) \times U(1)^{n_c-r}$. The "dual quarks" of the r vacua are identified as the GNO monopoles, which have become massless by the quantum effects. The special case is the $r = \frac{n_f}{2}$ vacua, which are nontrivial, strongly interacting SCFT. The infrared degrees of freedom are relatively nonlocal set of monopoles and dyons, carrying nonabelian charges.

In the case of $USp(2n_c)$ and $SO(n_c)$ gauge groups (with vanishing bare quark masses), all of the confining vacua are of this special type (deformed SCFT).[10)]

Table I. Phases of $SU(n_c)$ gauge theory with n_f flavors. $\tilde{n}_c \equiv n_f - n_c$.

label (r)	Deg. Freed.	Eff. Gauge Group	Phase	Global Symmetry
0	monopoles	$U(1)^{n_c-1}$	Confinement	$U(n_f)$
1	monopoles	$U(1)^{n_c-1}$	Confinement	$U(n_f-1) \times U(1)$
$\leq [\frac{n_f-1}{2}]$	NA monopoles	$SU(r) \times U(1)^{n_c-r}$	Confinement	$U(n_f-r) \times U(r)$
$n_f/2$	rel. nonloc.	-	Confinement	$U(n_f/2) \times U(n_f/2)$
BR	NA monopoles	$SU(\tilde{n}_c) \times U(1)^{n_c-\tilde{n}_c}$	Free Magnetic	$U(n_f)$

Table II. Phases of $USp(2n_c)$ gauge theory with n_f flavors with $m_i \to 0$. $\tilde{n}_c \equiv n_f - n_c - 2$.

	Deg. Freed.	Eff. Gauge Group	Phase	Global Symmetry
1st Group	rel. nonloc.	-	Confinement	$U(n_f)$
2nd Group	dual quarks	$USp(2\tilde{n}_c) \times U(1)^{n_c-\tilde{n}_c}$	Free Magnetic	$SO(2n_f)$

$$\mathcal{W}(\phi, Q, \tilde{Q}) = \mu \, \mathrm{Tr}\Phi^2 + m_i \tilde{Q}_i Q^i, \qquad m_i \to 0.$$

§4. Why nonabelian monopoles are intrinsically quantum mechanical

It is clear, as in the results summarized above, that the presence of semi-classically degenerate set of monopoles is not sufficient for us to conclude that there are going to be nonabelian monopoles in the infrared. They are intrinsically quantum mechanical. In particular, the "unbroken" group H can break itself dynamically at low energies. This happens, for instance, in the pure $\mathcal{N}=2$ $SU(N)$ theories, where all surviving monopoles are abelian. It turns out that the semiclassical classical r vacua survive as such if $r < \frac{n_f}{2}$; they are replaced in the infrared by the $r' = n_f - r$ vacua if $r > \frac{n_f}{2}$. The latter corresponds to the Seiberg's dual theory, $SU(n_f - r)$.

Another important issue is that the nonabelian monopoles are multiplets of the dual gauge group, $\tilde{H}$, and not of H itself. Certain difficulties found in the quantization of semiclassical nonabelian monopoles can be attributed to this.

In order to illustrate these issues, we study now[13)] a wider class of models, $U(N)$ gauge theories with a chiral superfield Φ in the adjoint representation; N_f of quarks superfields, and with a generic superpotential,

$$\mathcal{W} = W(\Phi) + \widetilde{Q}_i^a \, m_i(\Phi)_a^b \, Q_b^i :$$

$i = 1, 2, \ldots N_f$ $a, b = 1, 2, \ldots N$ are the color indices. The model has a global $U(N_f)$ symmetry in the limit, $m_i(\Phi) \to m(\Phi)$. As in Ref. 10), we keep the quark mass functions $m_i(\Phi)$ generic and all different at first, and send them to equal value only at the end. This allows us to keep track of the number of vacua, and as a consequence, to allow us to conclude unambiguously what happens to each of the semiclassical vacua, in the fully quantum mechanical limit.

§5. Classical vacua

Classical vacua are solutions of the equations

$$[\Phi, \Phi^\dagger] = 0\,; \tag{5·1}$$
$$0 = Q_a^i (Q^\dagger)_i^b - (\widetilde{Q}^\dagger)_a^i \widetilde{Q}_i^b\,; \tag{5·2}$$
$$Q_a^i \, \frac{\delta m_i(\Phi)_a^b}{\delta \Phi_c^d} \, \widetilde{Q}_i^b + \frac{\delta W(\Phi)}{\delta \Phi_c^d} = 0\,; \tag{5·3}$$
$$m_i(\Phi)_a^b Q_b^i = 0 \qquad \text{(no sum over } i)\,; \tag{5·4}$$
$$\widetilde{Q}_i^b \, m_i(\Phi)_b^a = 0 \qquad \text{(no sum over } i). \tag{5·5}$$

$$W(\Phi) = \sum_k a_k \, \mathrm{Tr}\,(\Phi^k), \qquad [m_i(\Phi)]_{ab} = \sum_k m_{i,k} \Phi_{ab}^{k-1}.$$

We shall choose $m(\Phi)$ to be quadratic or higher, assume also that the equation

$$m(z) = 0$$

has more than one solutions,

$$z = v^{(1)}, v^{(2)}, v^{(3)}, \ldots.$$

Note that

- (5·1) allows us to take Φ in a diagonal form, with $\mathrm{diag}\,\Phi = (\phi_1, \phi_2, \ldots)$; each diagonal element ϕ_i is either the solution of

- $m(\phi_i) = 0$, satisfying

$$Q_c^i = \widetilde{Q}_i^c = \sqrt{-\frac{W'(\phi_c^*)}{m_i'(\phi_c^*)}} \neq 0;$$

- or of the solution of $W'(a_j) = 0$ implying

$$Q_c^i = \widetilde{Q}_i^c = 0.$$

To be concrete, we consider the case in which r_1 of the ϕ_i are the roots of $m_i(x) = 0$, close to v_1 and r_2 of the ϕ_i are the roots of $m_i(x) = 0$, close to v_2. Other diagonal elements of Φ are taken to be various roots of $W'(\Phi) = 0$. In the flavor-symmetric limit, we are thus considering a vacuum with unbroken $SU(r_1) \times SU(r_2) \times U(1) \times U(1) \times \ldots$ gauge symmetry. The classical VEVS of Φ and $Q, \widetilde{Q}$ in this vacuum are:

$$\langle \phi \rangle = \begin{pmatrix} v_1 \mathbf{1}_{r_1} & & & & \\ & v_2 \mathbf{1}_{r_2} & & & \\ & & a_1 \mathbf{1}_{N_1} & & \\ & & & \ddots & \\ & & & & a_n \mathbf{1}_{N_n} \end{pmatrix},$$

where

$$\sum_{j=1}^{n} N_j + r_1 + r_2 = N,$$

and

$$Q = \begin{pmatrix} d_1 & & & & & & \\ & \ddots & & & & & \\ & & d_{r_1} & & & & \\ & e_1 & & & & & \\ & & \ddots & & & & \\ & & & e_{r_2} & & & \\ & & & & & & \end{pmatrix}, \quad \widetilde{Q} = \begin{pmatrix} \widetilde{d}_1 & & & & & & \\ & \ddots & & & & & \\ & & \widetilde{d}_{r_1} & & & & \\ & \widetilde{e}_1 & & & & & \\ & & \ddots & & & & \\ & & & \widetilde{e}_{r_2} & & & \\ & & & & & & \end{pmatrix},$$

where

$$d_c = \widetilde{d}_c = \sqrt{-\frac{W'(v^{(1)})}{m'(v^{(1)})}}, \qquad e_c = \widetilde{e}_c = \sqrt{-\frac{W'(v^{(2)})}{m'(v^{(2)})}}.$$

The multiplicity of the (r_1, r_2) vacua and symmetry breaking pattern are given by

$$\mathcal{N} = \binom{N_f}{r_1} \times \binom{N_f}{r_2} \times \prod_{i=1}^{n} N_i, \tag{5·6}$$

$$U(N_f) \to U(r_1 - s) \times U(s) \times U(r_2 - s) \times U(N_f - r_1 - r_2 + s).$$

Note that Eq. (8) does not imply a color-flavor locked form, in contrast to the cases with $W(\Phi) = \mu\,\Phi^2$, $m_i(\Phi) = m_i$ (constant masses), studied earlier.[10] The meson VEVs are given by

$$\widetilde{Q}Q = \begin{pmatrix} -\frac{W'(v^{(1)})}{m'(v^{(1)})}\mathbf{1}_{r_1-s} & & & & & \\ & -\left(\frac{W'(v^{(1)})}{m'(v^{(1)})}+\frac{W'(v^{(2)})}{m'(v^{(2)})}\right)\mathbf{1}_s & & & & \\ & & -\frac{W'(v^{(2)})}{m'(v^{(2)})}\mathbf{1}_{r_2-s} & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}. \tag{5·7}$$

§6. Quantum vacua

The above semiclassical analysis is adequate if all the VEVs are large as compared to the scale Λ of the theory. Otherwise, a quantum mechanical treatment is required. To understand the fully quantum situation, it is necessary to appeal to the recent work by Cachazo, Douglas, Seiberg and Witten,[11] inspired by those by Dijkgraaf and Vafa.[12] They found a complete solution for the chiral composite operator VEVs,

$$M = \widetilde{Q}\,\frac{1}{z-\Phi}\,Q; \qquad R(z) = -\frac{1}{32\pi^2}\,\mathrm{Tr}\,\frac{W_\alpha\,W^\alpha}{z-\Phi}, \tag{6·1}$$

where z is a complex variable. Equation (6·1) can be used as generating functions of various gauge-invariant chiral condensates, or seen as the resolvent operators in the chiral ring. The main result is the generalized (Konishi) anomaly equations:

$$\begin{aligned} &\left[W'(z)\,R(z)\right]_- = R(z)^2, \\ &\left[(M(z)\,m(z))_i^j\right]_- = R(z)\,\delta_i^j\,; \qquad \left[(m(z)\,M(z))_i^j\right]_- = R(z)\,\delta_i^j. \end{aligned}$$

The solution for $R(z)$ is (f related to $\langle WW\rangle_i$) :

$$2\,R(z) = W'(z) - \sqrt{W'(z)^2 + f(z)}$$

$\mathcal{N}=1$ (matrix model) curve (a doubly sheeted complex plane):

$$y^2 = W'(z)^2 + f(z), \qquad y = W'(z) - 2\,R(z).$$

The classical poles in z of R become cuts by quantum effects! Various chiral operator VEVs can be expressed as integrals over cycles in this double sheeted Riemann surface. Let us take the quark mass function as

$$m(z) = \mathrm{diag}\,[\,C\,(z - v_i^{(1)})\,(z - v_i^{(2)})\,],$$

$$\frac{1}{m(z)} = \begin{pmatrix} \frac{1}{C(z-v_1^{(1)})(z-v_1^{(2)})} & & \\ & \ddots & \\ & & \frac{1}{C(z-v_{N_f}^{(1)})(z-v_{N_f}^{(2)})} \end{pmatrix},$$

where $v_i^{(1)} \to v^{(1)}$, $v_i^{(2)} \to v^{(2)}$ in the flavor symmetric limit. In the vacuum with $r_1 + r_2$ poles in the physical sheet with r_1 poles near $v^{(1)}$ and r_2 poles near $v^{(2)}$, the exact quantum for the meson resultant is given by

$$M(z) = R(z)\frac{1}{m(z)} - \sum_{i=1}^{r_1+r_2} \frac{R(\widetilde{q}_i)}{z-z_i}\frac{1}{2\pi i}\oint_{z_i}\frac{1}{m(x)}\,dx - \sum_{j=1}^{2N_f-r_1-r_2}\frac{R(q_j)}{z-z_j}\frac{1}{2\pi i}\oint_{z_j}\frac{1}{m(x)}\,dx.$$

Symmetry breaking and the number of vacua are shown precisely to agree with the semiclassical approximation. Exact quantum formula for the meson VEV $\langle Q\tilde{Q}\rangle$ can be easily read off and shown to reduce, in the classical limit, to (5·7).

There is a beautiful classical-quantum (r_1, r_2) vacuum correspondence: namely, all of the classical vacua (r_1, r_2), $(r_1, N_f - r_2)$, $(N_f - r_1, r_2)$, and $(N_f - r_1, N_f - r_2)$ become in the infrared the (r_1, r_2) vacua! In particular, note that

- while $r_{cl} < \min[N_f, N_c]$, $r_{qu} < \frac{N_f}{2}$;
- Total vacuum counting of classical versus quantum vacua gives the right answer;
- If the $SU(r)$, N_f theory is infrared free in the ultraviolet, then it survives as $SU(r)$ theory in the infrared;
- If $SU(r)$, N_f theory is asymptotic free in the ultraviolet, it gets replaced by the Seiberg dual, $SU(N_f - r)$, in the infrared.

§7. Dual group from vortex-monopole systems

The question of quantum nonabelian monopoles and of the dual group has been recently considered from another viewpoint.[14)] Namely, we consider the systems with a hierarchical gauge symmetry breaking in which monopoles and vortices appear together.

- The system is characterized by

$$G \xrightarrow{\langle\phi_1\rangle\neq 0} H \xrightarrow{\langle\phi_2\rangle\neq 0} \emptyset.$$

- Assume exact H_{C+F} broken neither by interactions nor by the VEVs;
- Vortices carry nonabelian flux. For instance, CP^{N-1} for $G = SU(N+1)$, $H = SU(N) \times U(1)/Z_N$; it corresponds to the unbroken $SU(N)_{C+F}$, broken only by a single vortex (as the translational invariance broken by a kink) !
- Monopoles live in $\pi_2(G/H)$; vortices in $\pi_1(H)$;
- If $\pi_1(G) = \emptyset$ then clearly neither monopoles nor vortices are topologically stable;
- In fact, their fluxes match $F_m(H) = F_v(H)$: in other words, nonabelian monopoles are confined by the nonabelian vortices;
- This suggests that the dual group $\widetilde{H}$ (seen in the magnetic variables) is to be identified with H_{C+F} (in the original theory).

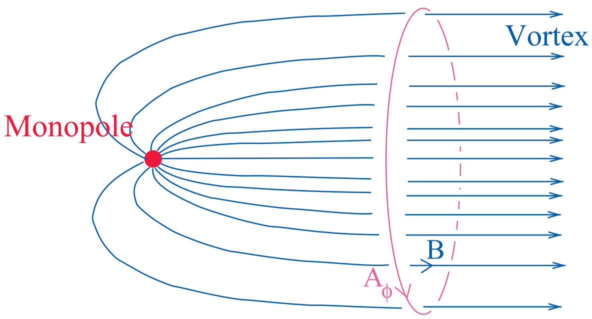

Fig. 1. A monopole-vortex system.

§8. Conclusion

This last observation is of fundamental importance: it implies that dual groups (and related nonabelian monopoles) survive the quantum effects only if the theory under consideration possesses an appropriate set of massless flavors. The latter plays a dual role in the whole discussion: it is needed to render the dual group infrared-free (or conformal), and to define the dual group itself!

The appearance of degenerate monopoles in a semiclassical treatment, therefore, does not mean in itself that nonabelian monopoles appear in the system; even a careful study of "semiclassical quantization" around such backgrounds, might lead us astray. The point is that a semiclassical treatment makes sense where the original "electric" theory is weakly-coupled; but there the dual magnetic theory is strongly coupled, and concepts such as the number of components of a dual gauge multiplet may not be well defined. Vice versa, the regime of a weakly coupled dual theory where the concept of magnetic gauge groups is a well-defined one, can be reached only when the original electric theory is strongly coupled, beyond the reach of a traditional semiclassical approach.

This is why the exact results in the $\mathcal{N}=2$ (and more recent ones in the $\mathcal{N}=1$) theories are of vital importance. As an illustration we have studied here a class of $\mathcal{N}=1$ theories with the following characteristics:

- $U(N)$ theory in the ultraviolet can be realized as $SU(r_1)\times SU(r_2)\times\ldots\times\prod U_i(1)$ in IR;
- Appropriate massless flavors needed;
- Appropriate superpotential $(\tilde{Q}\, m(\Phi)\, Q)$ required also;
- Light monopoles in $(\underline{r}_1, \underline{\mathbf{1}}, \ldots)$, $(\underline{\mathbf{1}}, \underline{r}_1, \ldots)$, etc., can appear as infrared degrees of freedom;
- Another superpotential $W(\Phi)\neq 0$ leads to condensation of these monopoles;
- These systems represent dual nonabelian superconductor of new types;
- As a by-product we find a clear indication that Seiberg's dual quarks are related to the GNO monopoles.

We believe that understanding of confinement in QCD will involve closely related issues.

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