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Coulomb's law corrections and fermion field localization in a tachyonic de Sitter thick braneworld

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Abstract. Following recent studies which show that it is possible to localize gravity as well as scalar and gauge vector fields in a tachyonic de Sitter thick braneworld, we investigate the solution of the gauge hierarchy problem, the localization of fermion fields in this model, the recovering of the Coulomb law on the non-relativistic limit of the Yukawa interaction between bulk fermions and gauge bosons localized in the brane, and confront the predicted 5D corrections to the photon mass with its upper experimental/observational bounds, finding the model physically viable since it passes these tests. In order to achieve the latter aims we first consider the Yukawa interaction term between the fermionic and the tachyonic scalar fields $MF(T)\Psi\bar{\Psi}$ in the action and analyze four distinct tachyonic functions $F(T)$ that lead to four different structures of the respective fermionic mass spectra with different physics. In particular, localization of the massless left-chiral fermion zero mode is possible for three of these cases. We further analyze the phenomenology of these Yukawa interactions among fermion fields and gauge bosons localized on the brane and obtain the crucial and necessary information to compute the corrections to Coulomb's law coming from massive KK vector modes in the non-relativistic limit. These corrections are exponentially suppressed due to the presence of the mass gap in the mass spectrum of the bulk gauge vector field. From our results we conclude that corrections to Coulomb's law in the thin brane limit have the same form (up to a numerical factor) as far as the left-chiral massless fermion field is localized on the brane. Finally we compute the *corrections to the Coulomb's law for an arbitrarily thick brane scenario* which can be interpreted as 5D corrections to the photon mass. By performing consistent estimations with brane phenomenology, we found that the predicted corrections to the photon mass, which are well bounded by the experimentally observed or astrophysically inferred photon mass, are far beyond its upper bound, positively testing the viability of our tachyonic braneworld. Moreover, the 5D parameters that define these corrections possess the same order, providing naturalness to our model, however, a fine-tuning between them is needed in order to fit the corresponding upper bound on the photon mass.

Keywords: Cosmic strings, domain walls, monopoles, extra dimensions, cosmological applications of theories with extra dimensions

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1 Introduction

In recent decades, the emergence of phenomenological braneworld models [1]–[15] suggested the existence of extra compact and extended dimensions. Within the framework of these models it was possible to geometrically reformulate the gauge hierarchy problem [4]–[10], to address the cosmological constant problem (see, [1, 16], for instance) and to localize the known matter fields of the standard model in braneworld scenarios with a simple dimensional reduction [17]–[40], among other issues.

With regard to the hierarchy problem, the models proposed in [4]–[6] make use of a spacetime that constitutes the direct product of a (3+1)-dimensional manifold and a d-dimensional torus. In this setup, the Standard Model particles are trapped in a 3-brane while gravity can propagate in the whole bulk spacetime and the hierarchy between the Planck scale and the fundamental scale of physics, M_*, can be very large if $RM_* \gg 1$, where R labels the compactification radius of the extra dimensional torus. Thus, the hierarchy is explained in terms of the *size* of the higher dimensional world. However, astrophysics and cosmology place significant bounds on this models. A new family of models which makes use of a compact hyperbolic manifold instead of a torus yields an exponential hierarchy between the Planck and electroweak scales with coefficients of the same order as a consequence of the *topology* of the extra space [7]–[9]. Within the framework of these models both particle physics and cosmology find a plausible description when confronted with experimental/observational data.

Another kind of improvement within the braneworld paradigm was proposed by introducing warped extra dimensions with Z_2-symmetry. The orbifold nature of the underlying higher dimensional manifold with consistent boundary conditions involves the introduction of delta function thin branes at its singular points, reducing the number of needed extra dimensions to a single one [10].

Afterwards, due to the intrinsic singularities that braneworld models possess at the position of the branes, there were proposed several scenarios in which the fifth dimension was modeled by bulk scalar fields, extending the idea of thin branes to thick brane configurations [41]–[46]. Recently, thick brane models have been proposed in gravity minimally or non-minimally coupled to scalar fields originating from supergravity theories which can be modeled by sigma models, opening thereby the possibility of linking the phenomenology of thick branes to more fundamental theories [47]. This research line pretends to understand the standard model physics from a higher dimensional point of view in order to address, reformulate and/or solve several open problems such as the gauge hierarchy problem. As a primary requirement of consistency, these models need to localize not only gravity, but also the matter field content of the Standard Model, i.e. scalar, vector (gauge), and spinor fields on the brane. Moreover, braneworld models are also required to yield the Newton and Coulomb laws, for instance, in the respective weak field and non-relativistic limits, since we need to recover the physical laws of our 4D world from the higher dimensional perspective as a correspondence principle. Finally, once these braneworld configurations reproduce the experimentally observed laws on the 3-brane we need to compute the corresponding corrections that they get from the extra dimensional world and to confront them with experimental/observational data that will tell us whether a given braneworld model is physically feasible or not. This confrontation of predictions versus observations (or versus experimental data) constitutes an stringent test on the viability of the braneworld paradigm.

In this work we will focus on *i*) the geometrical solution to the gauge hierarchy problem, *ii*) the study of the localization of spin–1/2 fermions and *iii*) the computation of the 5D corrections to the Coulomb's law to confront them with observations/experimental data within the context of a braneworld model generated by a bulk tachyonic condensate scalar field along with 5D gravity (see [23, 24, 46] for braneworld models of this type).

The first objective will be reached by orbifolding the spacetime along the extra dimension and placing two 3-branes at the fixed points of the orbifold. In one brane the 4D gravity will be localized whereas in the second one the electro-weak interacting Standard Model fields will be hosted. By further considering the fundamental Higgs field in the later 3-brane, TeV mass scales can be derived from Planck mass scales by a symmetry breaking mechanism which involves parameters with no large hierarchy among them. With regard to the second goal, it will be afforded by using the conventional mechanism that employs a Yukawa coupling between the fermion and the background scalar field, with the interaction factor $F(T(w))$ restricted to be an odd function, and performing a suitable dimensional reduction that leads to positive results. The third aim will be achieved by considering the Yukawa interaction between 5D fermions and bosons, recovering the 4D Coulomb's law and computing the corrections to it coming from the higher dimensional world.

The localization of spin–1/2 fermion fields has been performed in several thick braneworld configurations that make use of different scalar fields [25]–[36]. Noteworthy recent works reported a new mechanism for localizing fermions with a different Yukawa interaction between the background scalar fields and the bulk fermion, where the scalar function can be an even function [35]–[36]. The action for the tachyonic scalar field which models the

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fifth dimension of our work was originally proposed in [48] within the framework of string theory. The introduction of this tachyonic field in the thin braneworld paradigm was proposed in [23], however, the corresponding 5D spacetime possesses physical singularities at the place where the branes are positioned. A further development of this model was presented in [24], where it was shown that it is not possible to localize both gravity and matter fields on the braneworld due to the shape of the used warp factor. A thick braneworld generalization of this model was presented in [46] and it was shown that 4D gravity can be localized on it. Moreover, it was proved that the relevant field configuration which gives rise to the braneworld model is *stable* under the whole sector of small scalar fluctuations [49]. The scalar curvature which corresponds to this model is positive definite and asymptotically approaches a 5D Minkowski spacetime, in contrast with all of the models, to the best of our knowledge, previously reported in the literature. Thus, this model is completely regular and asymptotically flat, instead of (A)dS$_5$. Quite recently it was also reported that in this braneworld it is possible to localize different matter fields as gauge vector fields [38] and massive (self-interacting) scalar fields [39]. In both of these cases the spectrum for the massive KK modes presents a mass gap which allows us to study in a better way the physics of the massless bound states, specially within the context of computing the higher dimensional corrections to the Coulomb's law that come from the interaction between fermions and gauge bosons localized on the same brane. Thus, the present tachyonic scalar field braneworld turns out to be interesting from the phenomenological viewpoint compared to previous works since it allows us to localize gravity as well as massive scalars, gauge vector fields and fermions, preparing the arena for a consistent treatment of the Standard Model fields within the thick braneworld model generated by the interaction of gravity with a bulk tachyonic scalar field.

Another important issue related to the physics of the aforementioned braneworld consists in confronting its extra dimensional predictions to observations and/or experimental data. As mentioned above, once the matter fields are localized on the 3-brane, we must recover the physical laws of our 4D world in certain limit. Here we show that the Coulomb law is rendered by our model when considering a Yukawa interaction between bulk fermions and gauge bosons localized on the 3-brane. We further compute the corrections to Coulomb law that come from the massive KK gauge bosons and consider that they constitute corrections to the photon mass. We then show that when confronting the predicted corrections to the photon mass within the framework of our tachyonic braneworld model, they are far beyond the upper bound established with the aid of experimental data and astrophysical observations, yielding a viable model from the phenomenological point of view.

The paper is organized as follows: section 2 contains a brief review of the tachyonic scalar field braneworld model [46]; section 3 shows that the gauge hierarchy problem can be geometrically explained within this model; section 4 presents four cases where we discuss the problem of fermion localization on the brane; in section 5 we compute corrections to Coulomb's law coming from the extra dimensional world for two point fermions interacting with a gauge boson in the brane limit. We perform the same computation for a braneworld with arbitrary thickness and confront our result with the upper bound on the photon mass coming from several terrestrial/extraterrestrial experiments, and astrophysical observations in section 6. Finally, we conclude in section 7 with a general discussion of our results.

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2 The thick tachyonic braneworld model

Here we shall give a brief motivation and a review of the derivation of our tachyonic scalar field braneworld model. The effective action from which our model was inspired possesses a string theory origin and was proposed for the first time in [48] as a supersymmetric version of the Dirac-Born-Infeld action which describes the dynamics of tachyonic and massless modes on the world-volume of a non-BPS D-brane within type II string theory in Minkowski spacetime (for a clear construction of this action also see [50–52]). Within the context of string theory, D-branes give rise to *i*) stable BPS states and *ii*) unstable objects such as brane-antibrane configurations and non-BPS D-branes (for a review on this issue see [53, 54]). It turns out that unstable non-BPS brane configurations in Type II string theories can decay to stable D-branes through a condensation mechanism, namely, a non-BPS Dp-brane can condense to a BPS D(p-1)–brane in particular. Moreover, the non-BPS Dp-branes living in type II string theories are related to BPS D(p+1)-brane-antibrane systems by condensation of the tachyon field hosted on this brane-antibrane configuration. Therefore, the tachyonic effective field theory which describes the dynamics of a non-BPS D-brane in string theory possesses a BPS D-brane that is physically viable, i.e. stable. It is remarkable that by studying the world-volume theory of the massless modes on this BPS D-brane, it was established that the world-volume action precisely adopts the Dirac-Born-Infeld form without higher derivative corrections [55]. As it was mentioned above, this effective action was further considered in the braneworld [23] and supergravity [47] realms motivated by the hope of relating the phenomenology of these models to more fundamental theories like string theory, making it physically interesting.

Thus, within the braneworld paradigm, the 5D action for the thick brane model generated by a tachyon condensate scalar field with the complicity of gravity reads

$$S=\int d^5x\sqrt{-g}\left(\frac{1}{2\kappa_5^2}R-\Lambda_5\right)-\int d^5x\sqrt{-g}V(T(w))\sqrt{1+g^{AB}\partial_A T(w)\partial_B T(w)}, \tag{2.1}$$

where R is the 5D scalar curvature, Λ_5 is the bulk cosmological constant, and $\kappa_5^2=8\pi G_5$ with G_5 being the 5D Newton constant; T is a real tachyonic scalar field[1] which depends only on the extra dimension and $V(T)$ denotes its self-interaction potential. Here we use the signature $(-+++\,+)$ and the Ricci tensor is defined upon contraction of the first and third indices of the Riemann tensor $R_{NQ}=R^M_{NMQ}$, where $M,N,P,Q=0,1,2,3,5$. The corresponding Einstein equations with a cosmological constant in five dimensions are

$$G_{AB}=-\kappa_5^2\,\Lambda_5 g_{AB}+\kappa_5^2\,T^{bulk}_{AB}, \tag{2.2}$$

where the bulk energy-momentum tensor is given by

$$T^{bulk}_{AB}=-g_{AB}V(T)\sqrt{1+(\nabla T)^2}+\frac{V(T)}{\sqrt{1+(\nabla T)^2}}\partial_A T\partial_B T. \tag{2.3}$$

The 5D metric ansatz compatible with an induced flat FRW metric on the 3-brane has the form

$$ds^2=e^{2f(w)}\left[-dt^2+a^2(t)\left(dx^2+dy^2+dz^2\right)+dw^2\right], \tag{2.4}$$

[1]The tachyon scalar field considered in the braneworld action (2.1) is real in contrast to the complex tachyon field hosted in the brane-antibrane configuration of the original string theory effective action.

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where $e^{2f(w)}$ and $a(t)$ are the warp factor and the scale factor of the brane, respectively, and w stands for the extended extra dimensional coordinate. Since the G_{05} component of the Einstein tensor vanishes for this metric setup, we are led to the freedom of choosing a tachyonic scalar field depending either on time or on the extra dimension. We have chosen the latter dependence since we are interested in localizing universal fermion fields along the fifth dimension on the considered braneworld model. Time-dependent field configurations are interesting from the viewpoint of cosmology, however, we were unable to find solutions even for this simplified metric ansatz of our model due to the highly non-linear character of the field equations of the scalar tachyon.

On the other hand, here we would like to remark that since we are considering a metric ansatz depending on time, we could, in principle, study more general braneworld models in which the g_{ww} component of the 5D metric as well as the tachyonic scalar field T depend on time. This metric ansatz is in the spirit of the one initially investigated in [56], where an interesting braneworld cosmological solution with no separation of variables, intrinsically different from standard cosmology, was constructed under the assumption that the extra dimensional slice of the metric is not dynamical $\partial_t g_{ww} = 0$.[2] Motivated by this result, we shall further impose this condition, corresponding to the case when the radion is not dynamical in our model, and implying that the warp factor is a function of the extra coordinate alone $f(w)$.

The matter field equation is obtained by variation of the 5D action (2.1) with respect to the condensate tachyonic field. It is expressed as follows:

$$\partial_M \left[\frac{\sqrt{-g} V(T) \partial^M T}{\sqrt{1+(\nabla T)^2}} \right] - \sqrt{-g}\sqrt{1+(\nabla T)^2} \frac{\partial V(T)}{\partial T} = 0. \tag{2.5}$$

The solution for the metric coefficients in (2.4), i.e. for the scale and warp factor, respectively reads [46]

$$a(t) = e^{H\,t}, \qquad\qquad f(w) = \frac{1}{2} \ln \left\{ s \ \mathrm{sech}\left[H\,(2w+c) \right] \right\}, \tag{2.6}$$

indicating a de Sitter symmetry induced on the 3-brane and a warp factor which possesses a decaying behavior and asymptotically vanishes; the tachyon condensate scalar field has the form

$$\begin{aligned} T(w) &= \pm\, b \ \mathrm{arctanh} \left[\frac{\sinh\left[\frac{H\,(2w+c)}{2} \right]}{\sqrt{\cosh\left[H\,(2w+c) \right]}} \right] \\ &= \pm\, b \ \mathrm{arcsinh} \left[\tanh(Hw) \right], \end{aligned} \tag{2.7}$$

and has a kink/antikink profile; while the tachyon condensate potential is given by

$$\begin{aligned} V(T) \,=\, & -\Lambda_5 \ \mathrm{sech}\,(T/b) \sqrt{6 \ \mathrm{sech}^2\,(T/b) - 1} = \\ & -\Lambda_5 \sqrt{\left(1 + \mathrm{sech}\left[H\,(2w+c) \right] \right) \left(1 + \frac{3}{2} \ \mathrm{sech}\left[H\,(2w+c) \right] \right)} \end{aligned} \tag{2.8}$$

[2] In this braneworld cosmological solution the first Friedmann equation is generically related to an expression quadratic in the brane energy density instead of being linear in it as in standard cosmology, favoring the accelerated expansion of the Universe. Moreover, the relevant dynamical equations that describe the cosmological evolution in the brane do not contain the effective 4D Newton's constant, but only on the 5D Newton's constant (i.e. the Planck mass scale of the fundamental theory) and are independent of the metric on the bulk and in particular of the time evolution of g_{ww}, giving a consistent justification for the condition $\partial_t g_{ww} = 0$.

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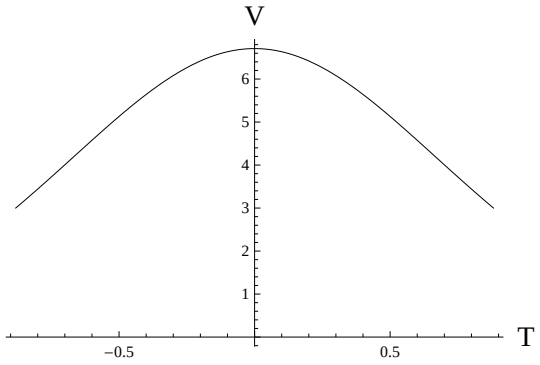

Figure 1. The profile of the tachyonic scalar field self-interaction potential $V(T)$. Here we have set $\Lambda_5 = -3$ and $\kappa_5^2 = 0.5$ for simplicity. Due to the bounded character of the tachyonic field, the potential is bounded from below as well.

and represents a positive definite potential with a maximum at the origin and possesses *finite* minima at the boundaries of the 5D manifold, i.e. at $w \to \pm\infty$ (see figure 1). It should be stressed that in the original tachyonic effective string field theory the self-interaction potential $V(T)$ is symmetric under $T \to -T$ and displays a maximum at $T = 0$ as for our solution, but it possesses its minima at $T = \pm\infty$ where it vanishes. This latter feature is the only difference between the properties of the tachyonic self-interaction potential of our solution and the one of the effective string field theory.

In this solution H and c are integration constants, and we have set

$$s = -\frac{6H^2}{\kappa_5^2\,\Lambda_5} = 4b^2H^2 \qquad \text{and} \qquad b = \sqrt{\frac{-3}{2\,\kappa_5^2\,\Lambda_5}} \tag{2.9}$$

with an arbitrary negative bulk cosmological constant $\Lambda_5 < 0$.

The 5D curvature scalar of this braneworld field configuration is given by the expression

$$R = -\frac{14}{3}\kappa_5^2\,\Lambda_5\,\mathrm{sech}\,[\,H\,(2w + c)]\,, \tag{2.10}$$

which is a positive definite invariant and asymptotically vanishes, yielding an asymptotically 5D Minkowski spacetime. By looking at the action (2.1) it is easy to see that the overall effective cosmological constant of the 5D spacetime possesses two important contributions: the negative bulk cosmological constant Λ_5 and the tachyonic self-interaction potential $V(T)$ which asymptotically adopts the value $-\Lambda_5$, yielding an asymptotically flat 5D spacetime, i.e. a braneworld which interpolates between two 5D Minkowski spacetimes.

Thus, this non-trivial, completely regular braneworld configuration created by a cooperative work between the minimally coupled 5D gravity and the bulk tachyonic scalar field possesses a 3-brane with de Sitter symmetry, i.e. a brane representing an accelerated expanding universe that, in principle, can also model the inflating stages in the evolution of our Universe.

It is worth mentioning that the scale factor that is responsible for the aforementioned expansion or inflation of the brane universe plays a crucial role in the generation of the 5D braneworld model since when it vanishes the whole field configurations blows up, indicating that there is no consistent flat limit for the 3-brane. This is an interesting peculiarity of our expanding braneworld model since being asymptotically flat along the fifth dimension,

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it contains a 3-brane with positive spatial curvature, a positive definite tachyon potential, a negative bulk cosmological constant and constitutes a non-perturbative field configuration with respect to a Minkowski 3-brane.

We would like to finally remark that the tachyonic effective action (2.1) has found several interesting applications within the framework of braneworld cosmology [58] and string cosmology [59–66]. In [62] some aspects of canonical quantization of this field theory coupled to gravity were studied, where the tachyonic scalar field was used as the definition of time within quantum cosmology. In [67] solar system constraints were imposed on its parameters by considering this action as a scalar-tensor model. Moreover, recent studies of tachyon inflation within the N-formalism, which considers a prescription for the small Hubble flow slow-roll parameter ϵ_1 as a function of a large number of e-folds, have lead to a consistent analysis of observables in the light of the Planck 2015 data and show the viability of some models of this class [68].

3 Geometrical reformulation of the gauge hierarchy problem

In this section we shall show that the gauge hierarchy problem can be geometrically solved by orbifolding the extra dimension and introducing a thin 3-brane, where the Standard Model fields live, located some distance away from another 3-brane where the 4D gravity is localized. The known gauge hierarchy is generated by a symmetry breaking mass scale which relates TeV to Planck masses with no large hierarchy among the parameters of the model.

Thus, once it has been proven that our thick tachyonic de Sitter braneworld model supports the localization of 4D gravity, consistently recovering Newton's law in the thin brane limit [46], it is of physical relevance to address the issue of deriving physical TeV mass scales from fundamental masses of the Planck scale order through a consistent mechanism like the Randall-Sundrum one, for instance. In order to achieve this aim we shall impose Z_2-symmetry along the fifth dimension (given by the replacement $w \longrightarrow |w|$) and supplement our braneworld model with a thin 3-brane in order to have two branes supporting (3+1)-dimensional field theories, one located at the origin $w = 0$ and the other one at $w = w_0$. By performing such a procedure we are mathematically orbifolding the extra dimension to a segment, where the 3-branes are located at the fixed points of the orbifold and represent the boundaries of the 5D spacetime, reducing the original range of the locally non-compact extra dimension $-\infty \leq w \leq \infty$ to the effective one $0 \leq w \leq w_0$.

Here we shall further consider that the thick Planck brane is positioned at the origin of the extra dimension and localizes 4D gravity, whereas a thin TeV brane will be located at a certain distance, w_0, from the Planck brane and will host the electro-weak interacting Standard Model fields. It is worth noticing that we could equally well consider that the thick Planck brane is located at w_0 and the Standard Model TeV brane is at the origin of the fifth dimension. We call it TeV brane even when we know now that the Higgs mass is approximately 125 GeV.

By recalling the main result obtained Randall and Sundrum in [10] when considering the 4D effective action for the fundamental Higgs field, it turns out that any fundamental mass parameter m_0 of the higher-dimensional theory gives rise to the following 4D physical mass

$$m = e^{f(w_0)} m_0 = 2Hb \sqrt{\mathrm{sech}\,(2Hw_0)}\; m_0, \tag{3.1}$$

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when measured in the TeV brane, located at w_0, with the effective 4D metric that has been rescaled according to its 4D conformal weight. Therefore, when $e^{f(w_0)} = 2Hb\,\sqrt{\mathrm{sech}\,(2Hw_0)}$ is of order 10^{-15} we obtain TeV physical mass scales from Planck ones within the framework of our model.

Let us compute now the effective 4D Planck mass of our thick de Sitter tachyonic braneworld. By integrating the action (2.1) over the fifth dimension and comparing the curvature term to the canonical 4D Einstein-Hilbert action, it is easy to see that all we need is to perform the following integration

$$M_{\rm Pl}^2 = M_*^3 \int_{-\infty}^{\infty} e^{3f(w)} dw = \frac{M_*^3 H^2\, b^3 \Gamma\left(-\frac{1}{4}\right)^2}{\sqrt{2\pi}} \approx 9.6\;\; M_*^3 H^2\, b^3, \tag{3.2}$$

which is finite along the whole fifth dimension, as expected. Thus, the effective 4D $M_{\rm Pl}$ depends on the expansion parameter of the 3-brane H, besides the fundamental higher dimensional parameters M_* and b. This implies that in order to properly derive the scale of 4D gravitational interactions we can combine the values of these parameters in different ways. In fact, there is more freedom when assigning certain values to these parameters when comparing to the spatially flat braneworld case where the H parameter vanishes.

By further assigning to H the currently observed value of the Hubble parameter $H \approx 10^{-60} M_{\rm Pl}$ and by considering TeV mass scales for the mass parameter $M_* \approx 10^{-15} M_{\rm Pl} \sim 1\,\mathrm{TeV}$ we obtain $b^{-1} \approx 2 \times 10^{-55} M_{\rm Pl}$ from eq. (3.2) in order to recover the correct 4D gravitational couplings on the thick brane.

Therefore, the physically relevant mass ratio which allows us to obtain TeV physical mass scales from Planck ones

$$\frac{m}{m_0} = 10^{-15} \tag{3.3}$$

can be achieved when

$$H\, w_0 = \frac{1}{2}\mathrm{arccosh}\left[\left(2bH\,\frac{m_0}{m}\right)^2\right] \approx \frac{1}{2}\mathrm{arccosh}\left(10^{20}\right) \approx 23, \tag{3.4}$$

since $2bH \approx 10^{-5}$ and therefore we do not need a large hierarchy between the compactification scale $\mu_c \equiv \frac{1}{w_0}$ and the Hubble parameter H in order to generate the desired mass hierarchy within the framework of our model.

We finally would like to remark that this geometric reformulation of the gauge hierarchy problem leads to the question about the stability of the brane separation w_0. It turns out that the introduction of the TeV 3-brane some distance away from the Planck 3-brane in order to achieve the desired hierarchy gives rise to a new fine-tuning on the position of the TeV 3-brane, and therefore, to the need of stabilizing this brane separation. This issue can successfully be addressed with the aid of the generalized Golberger-Wise mechanism proposed by DeWolfe *et al.* in [41] by associating to the brane separation a canonical scalar field with interaction terms on both 3-branes while taking into account the back-reaction of the TeV brane. In this work it was shown that generically the brane separation w_0 remains stable when modeled by this canonical scalar field with a set of quartic self-interacting potentials, obtaining the desired hierarchy from the Planck scale to TeV scale masses and resolving the fine-tuning problem of the Higgs mass in a stable braneworld scenario.

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4 Localization of spin-1/2 fermion fields

In this section we shall investigate the localization of spin-1/2 fermion bulk matter fields on a tachyon condensate de Sitter thick braneworld model by considering a very weak gravitational interaction between gravity and the fermionic fields, so that the brane solution given in the previous section remains valid even in the presence of generalized 5D bulk matter. As a generic feature, the 5D profile of the fermion fields obey a Schrödinger equation when assuming that the corresponding 4D Dirac equations are satisfied. The mass spectra of the fermion fields on the de Sitter thick brane will also be discussed by analyzing the potential of, and by analytically solving, the corresponding Schrödinger equation for their KK massive modes related to the 4D fermionic fields in four different cases. These different situations are constructed in order to analyze four mass fermionic spectra with an intrinsic different structure that leads to a different behaviour of the KK fermionic massive modes that could, in principle, affect the fermion physics of our 3-brane in a different way.

In 5D spacetime fermions are four-component spinors and their Dirac structure can be described by $\Gamma^M = e^M_{\bar{M}}\Gamma^{\bar{M}}$ with $e^M_{\bar{M}}$ being the vielbein and $\{\Gamma^M, \Gamma^N\} = 2g^{MN}$. In this section $\bar{M}, \bar{N}, \cdots = 0, 1, 2, 3, 5$ and $\bar{\mu}, \bar{\nu}, \cdots = 0, 1, 2, 3$ denote the 5D and 4D local Lorentz indices, respectively, and $\Gamma^{\bar{M}}$ are the gamma matrices in 5D flat spacetime. In our set-up, the vielbein is given by

$$e_M^{\ \bar{M}} = \begin{pmatrix} \mathrm{e}^f \hat{e}_\mu^{\ \bar{\nu}} & 0 \\ 0 & \mathrm{e}^f \end{pmatrix}, \tag{4.1}$$

$\Gamma^M = \mathrm{e}^{-f}(\hat{e}^\mu_{\ \bar{\nu}}\gamma^{\bar{\nu}}, \gamma^5) = \mathrm{e}^{-f}(\gamma^\mu, \gamma^5)$, where $\gamma^\mu = \hat{e}^\mu_{\ \bar{\nu}}\gamma^{\bar{\nu}}$, $\gamma^{\bar{\nu}}$ and γ^5 are the usual flat gamma matrices in the 4D Dirac representation. The generalized Dirac action of a spin-1/2 fermion with a mass term can be expressed as [18]

$$S_{\frac{1}{2}} = \int d^5x\sqrt{-g}\left[\bar{\Psi}\Gamma^M\left(\partial_M + \omega_M\right)\Psi - M\bar{\Psi}F(T)\Psi\right]. \tag{4.2}$$

Here ω_M is the spin connection defined as $\omega_M = \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}}$ with

$$\begin{aligned}\omega_M^{\bar{M}\bar{N}} = &\frac{1}{2}e^{N\bar{M}}\left(\partial_M e_N^{\bar{N}} - \partial_N e_M^{\bar{N}}\right) - \frac{1}{2}e^{N\bar{N}}\left(\partial_M e_N^{\bar{M}} - \partial_N e_M^{\bar{M}}\right)\\ &-\frac{1}{2}e^{P\bar{M}}e^{Q\bar{N}}\left(\partial_P e_{Q\bar{R}} - \partial_Q e_{P\bar{R}}\right)e_M^{\bar{R}},\end{aligned} \tag{4.3}$$

and $F(T)$ is an arbitrary general scalar function of the tachyon condensate scalar field.[3] We will discuss about the properties of the scalar function $F(T)$ later in the context of the localization of KK fermion modes. The non-vanishing components of the spin connection ω_M for the background metric (2.4) has the form

$$\omega_\mu = \frac{1}{2}(\partial_w f)\gamma_\mu\gamma_5 + \hat{\omega}_\mu, \tag{4.4}$$

here $\hat{\omega}_\mu = \frac{1}{4}\bar{\omega}_\mu^{\bar{\mu}\bar{\nu}}\Gamma_{\bar{\mu}}\Gamma_{\bar{\nu}}$ is the spin connection derived from the metric $\hat{g}_{\mu\nu}(x) = \hat{e}_\mu^{\ \bar{\mu}}(x)\hat{e}_\nu^{\ \bar{\nu}}(x)\eta_{\bar{\mu}\bar{\nu}}$. Thus, the equation of motion corresponding to the variation of the action (4.2) whit respect to $\bar{\Psi}$ can be written as

$$\left[\gamma^\mu(\partial_\mu + \hat{\omega}_\mu) + \gamma^5\left(\partial_w + 2\partial_w f\right) - \mathrm{e}^f MF(T)\right]\Psi = 0, \tag{4.5}$$

where $\gamma^\mu(\partial_\mu + \hat{\omega}_\mu)$ is the 4D Dirac operator on the brane.

[3] The only condition that this function must satisfy in order to yield 4D chiral fermions upon dimensional reduction is to be odd in w.

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Next, we will investigate the KK modes for the 5D Dirac equation (4.5), and write the spinor in terms of 4D effective fields. On account of the fifth gamma matrix γ^5, we anticipate the left– and right-handed projections of the 4D part to behave differently. We shall consider the following ansatz for the general chiral decomposition in (4.5):

$$\Psi = \mathrm{e}^{-2f}\left(\sum_n \Psi_{Ln}(x)L_n(w) + \sum_n \Psi_{Rn}(x)R_n(w)\right), \tag{4.6}$$

where $\Psi_{Ln}(x) = -\gamma^5\Psi_{Ln}(x)$ and $\Psi_{Rn}(x) = \gamma^5\Psi_{Rn}(x)$ are the left-handed and right-handed components of a 4D Dirac field, respectively. Further, we shall assume that $\Psi_{Ln}(x)$ and $\Psi_{Rn}(x)$ satisfy the 4D Dirac equations. Therefore the KK modes $L_n(w)$ and $R_n(w)$ should satisfy the following coupled equations:

$$\Big(\partial_w + \mathrm{e}^f MF(T)\Big)L_n(w) = \quad m_n R_n(w), \tag{4.7a}$$

$$\Big(\partial_w - \mathrm{e}^f MF(T)\Big)R_n(w) = -m_n L_n(w). \tag{4.7b}$$

where m_n is the fermionic 4D mass arising from the separation of variables (4.6). From the above coupled equations, we can obtain the Schrödinger-like equations for the left– and right-chiral KK modes of fermions:

$$\Big(-\partial_w^2 + V_L(w)\Big)L_n = m_n^2 L_n, \tag{4.8}$$

$$\Big(-\partial_w^2 + V_R(w)\Big)R_n = m_n^2 R_n, \tag{4.9}$$

where the corresponding left and right potentials read

$$V_L(w) = \mathrm{e}^{2f}M^2F^2(T) - \mathrm{e}^f f'MF(T) - \mathrm{e}^f M\partial_w F(T), \tag{4.10a}$$

$$V_R(w) = \mathrm{e}^{2f}M^2F^2(T) + \mathrm{e}^f f'MF(T) + \mathrm{e}^f M\partial_w F(T). \tag{4.10b}$$

We can perform a dimensional reduction on (4.2) in order to obtain the standard model 4D action for a massless fermion and a series of massive chiral fermions

$$\begin{aligned} S_{\frac{1}{2}} &= \int d^5x\sqrt{-g}\ \bar{\Psi}\left[\Gamma^M(\partial_M + \omega_M) - MF(T)\right]\Psi \\ &= \sum_n \int d^4x\sqrt{-\hat{g}}\ \bar{\Psi}_n\left[\gamma^\mu(\partial_\mu + \hat{\omega}_\mu) - m_n\right]\Psi_n, \end{aligned} \tag{4.11}$$

where the following orthonormalization conditions for L_n and R_n need to be satisfied in order to perform the dimensional reduction:

$$\int_{-\infty}^{+\infty} L_m L_n dw = \delta_{mn}, \tag{4.12}$$

$$\int_{-\infty}^{+\infty} R_m R_n dw = \delta_{mn}, \tag{4.13}$$

$$\int_{-\infty}^{+\infty} L_m R_n dw = 0. \tag{4.14}$$

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It is easy to see that if one sets $m_n = 0$ in the expressions (4.7a) and (4.7b), then one gets an easy way to calculate the zero modes for the left– an right-chiral fermions

$$L_0 \propto e^{-M\int e^f F(T(w))dw}, \tag{4.15a}$$

$$R_0 \propto e^{M\int e^f F(T(w))dw}. \tag{4.15b}$$

In the next subsections we will investigate four different profiles of the function $F(T)$ in order to localize the 4D fermions on the thick 3-brane. To achieve this goal we require the effective potentials V_L and V_R to possess a minimum and to be symmetric with respect to their position on the thick brane along the extra dimension. Therefore we will demand the function $F(T(w))$ to be an odd function in w. Here we should point out that different tachyonic coupling functions $F(T)$ lead to different KK massive spectra for the fermionic fields, yielding a different physical behaviour of the fermions in the extra dimensional world. These different mass spectra in the bulk will, in principle, affect in a different way the physics of the 4D fermions localized on the 3-brane which represents our world. However, their coupling to other bulk fields will render a different effective 4D physics on the 3-brane depending on the physical properties of the latter fields.

Therefore, in what follows we shall consider a quantum field theory motivated Yukawa interaction term between the bulk fermion fields and gauge bosons localized on the brane and then we shall study the physics of four tachyonic coupling functions $F(T)$ that correspond to four different mass spectra of the KK fermion fields when they are localized on the 3-brane.

4.1 Case I: $F(T) = T/b$

In this case we shall investigate a simple interaction in the action (4.2) between the 5D fermionic fields and the tachyon condensate scalar field by taking $F(T) = T/b$, where we divide the T field by $b = \sqrt{\frac{-3}{2\,\kappa_5^2\,\Lambda_5}}$ in order to make the function $F(T)$ adimensional and make the parameter M to encode all the relevant units for the interaction term of the 5D action. For this field configuration we have the following potentials for L_n and R_n 5D Dirac fermions

$$V_L = MsH\,\mathrm{sech}(Hw)\left[\frac{M}{H}\frac{\mathrm{sech}(Hw)\,\mathrm{arcsinh}^2[\tanh(Hw)]}{1+\tanh^2(Hw)} + \frac{1}{\sqrt{s}}\left(\mathrm{arcsinh}[\tanh(Hw)]\frac{\tanh(2Hw)}{\sqrt{1+\tanh^2(Hw)}} - 1\right)\right], \tag{4.16a}$$

$$V_R = MsH\,\mathrm{sech}(Hw)\left[\frac{M}{H}\frac{\mathrm{sech}(Hw)\,\mathrm{arcsinh}^2[\tanh(Hw)]}{1+\tanh^2(Hw)} - \frac{1}{\sqrt{s}}\left(\mathrm{arcsinh}[\tanh(Hw)]\frac{\tanh(2Hw)}{\sqrt{1+\tanh^2(Hw)}} - 1\right)\right]. \tag{4.16b}$$

Both potentials have the same asymptotic behavior $V_{R,L}(w \to \pm\infty) \to 0$, the critical value (maximum and minimum) of the right and left potentials when $w = 0$ are, respectively, $V_R(w=0) = \sqrt{s}\,HM^2$ and $V_L(w=0) = -\sqrt{s}\,HM^2$. Both of the potentials have a very complicated form and it is impossible to find an explicit solution for the Dirac fermion fields when trying to analytically solve the Schrödinger equations (4.8)–(4.9). However, these

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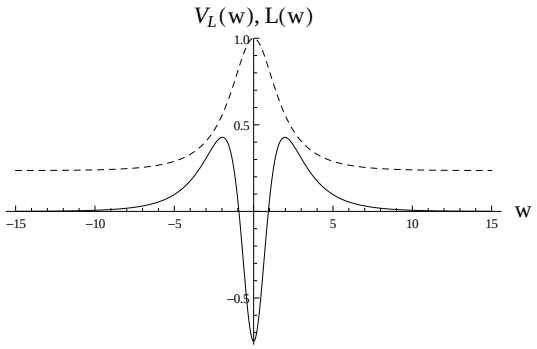

Figure 2. The profile of the V_L potential (solid black line) and the non-localized left-chiral zero mode L_0 (dashed black line) along the fifth dimension for the case I. Here we have set $H = 1/2$, $M = 1$ and $s = 1$.

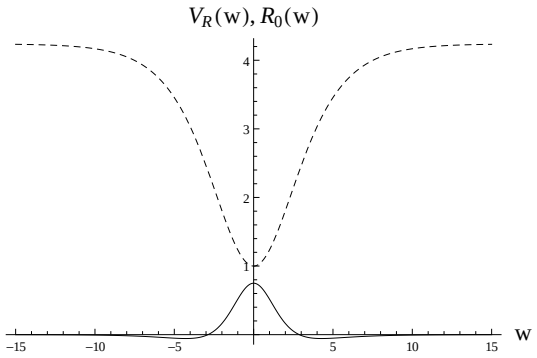

Figure 3. The profile of the V_R potential (solid black line) and the non-localized right-chiral massless fermion zero mode R_O (dashed black line) along the extra dimension for the case I. Here $H = 1/2$, $M = 1$ and $s = 1$ as well.

potentials do not allow us to localize the fermion zero modes, the form of the L_0 can easily be found numerically as show in figure 2. While the potential is of volcano type, which allows, in principle, the existence of bound states, the zero mode is not localized on the 3-brane because it asymptotically tends to a positive definite constant, indicating that the bottom of the volcano potential is not deep enough to localize fermion fields.

The shape of the potential V_R predicts the lack of localized right bound states since it constitutes a barrier potential. Figure 3 shows the shape of this potential and the massless KK zero mode of the spectrum.

Thus, for the above analyzed case I there are no, neither left nor right, fermionic bound states localized on the considered 5D braneworld model generated by gravity in complicity with the bulk tachyonic scalar field. Therefore, we need to explore more complicated functions $F(T)$ in order to achieve the desired fermion field localization on the aforementioned braneworld model.

4.2 Case II: $F(T) = \frac{\mathbf{sinh}(2T/b)}{2\left[1-\mathbf{sinh}^2(T/b)\right]}$

We shall now propose one case in which the left KK ground state is localized in our braneworld model. Therefore, we shall consider a new $F(T)$ for which we obtain the following expressions

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for the left and right potentials

$$V_L = MH^2G[MG\sinh^2(Hw) - \cosh(Hw)], \tag{4.17a}$$
$$V_R = MH^2G[MG\sinh^2(Hw) + \cosh(Hw)], \tag{4.17b}$$

where $G = \sqrt{\frac{-6}{\kappa_5^2\Lambda_5}} = \frac{\sqrt{s}}{H} = 2b$.

Both of these potentials have the same asymptotic behaviour $V_{R,L}(w \longrightarrow \pm\infty) \longrightarrow \infty$, giving rise to infinitely high well potentials, which means in turn that the mass spectra of both left– and right-chiral fermions consists of an infinite set of discrete massive bound states localized on the thick 3-brane. The critical values of the right and left potentials take place when $w = 0$ and are given by $V_R(w = 0) = H^2MG$ and $V_L(w = 0) = -H^2MG$, respectively. Thus, both of the potentials possess a tower of discrete KK bound states, the only essential difference is that the left-chiral KK fermionic ground state is massless (see figure 4), while the right-chiral KK fermionic ground state is a massive one.

The general solution for both the left and right KK bound states can be expressed in terms of confluent Heun functions as follows

$$L_n = e^{MG\cosh(Hw)}\left[K_1\,\mathrm{HeunC}\left(4MG, -\frac{1}{2}, -\frac{1}{2}, 2MG, \Omega_{n_-}, \frac{1}{2}+\frac{1}{2}\cosh(Hw)\right) + K_2\sqrt{2+2\cosh(Hw)}\,\mathrm{HeunC}\left(4MG, \frac{1}{2}, -\frac{1}{2}, 2MG, \Omega_{n_-}, \frac{1}{2}+\frac{1}{2}\cosh(Hw)\right)\right], \tag{4.18}$$

$$R_n = e^{MG\cosh(Hw)}\left[k_1\,\mathrm{HeunC}\left(4MG, -\frac{1}{2}, -\frac{1}{2}, -2MG, \Omega_{n_+}, \frac{1}{2}+\frac{1}{2}\cosh(Hw)\right) + k_2\sqrt{2+2\cosh(Hw)}\,\mathrm{HeunC}\left(4MG, \frac{1}{2}, -\frac{1}{2}, -2MG, \Omega_{n_+}, \frac{1}{2}+\frac{1}{2}\cosh(Hw)\right)\right], \tag{4.19}$$

where K_1, K_2, k_1, k_2 are arbitrary constants and $\Omega_{n_\pm} = \frac{(3\pm 8MG)H^2+8m_n^2}{8H^2}$.

Returning to our goal, we need to know, in particular, the explicit expression for the left and right KK ground states. By proceeding to calculate these expressions as usual we get

$$L_0 \propto e^{-MG\cosh(Hw)}, \tag{4.20a}$$
$$R_0 \propto e^{MG\cosh(Hw)}, \tag{4.20b}$$

implying that just the massless left-chiral fermionic zero mode is localized on the 3-brane.

We must emphasize that the shape of the potential V_R predicts the existence of an infinite tower of discrete massive bound states localized on the brane along with the presence of a non-localized massless zero mode. Figure 5 shows the shape of the right potential and the delocalized massless zero mode from the brane.

4.3 Case III: $F(T) = \frac{\mathbf{arctanh}[\mathbf{sinh}(T/b)]}{\sqrt{2\,\mathbf{sech}^2(T/b)-1}\,\left(1+\mathbf{arctanh}^2[\mathbf{sinh}(T/b)]\right)}$

In general, the localization of spin–$\frac{1}{2}$ fermions is obtained in a more artisanal way when compared to the localization of gravity, scalar and/or gauge vector fields. This is why we shall undertake the task of finding field configurations with a little whimsical $F(T)$, like the one considered here in case III, that allows us to localize fermion fields on the 3-brane. For the configuration corresponding to case III we get again a left potential V_L of volcano type

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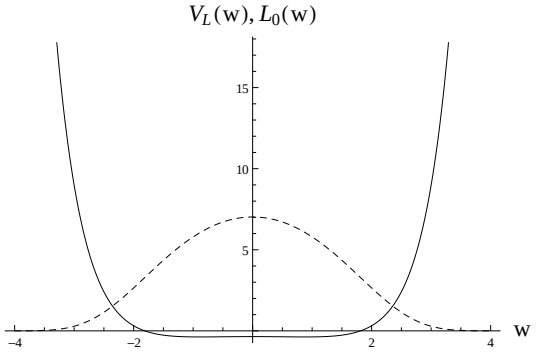

Figure 4. The profile of the V_L potential (solid black line) and the localized left-chiral zero mode L_0 scaled by a factor of 10 (dashed black line) along the fifth dimension for case II. Here we have set $H = 1$, $M = 1/2$ and $s = 1/2$.

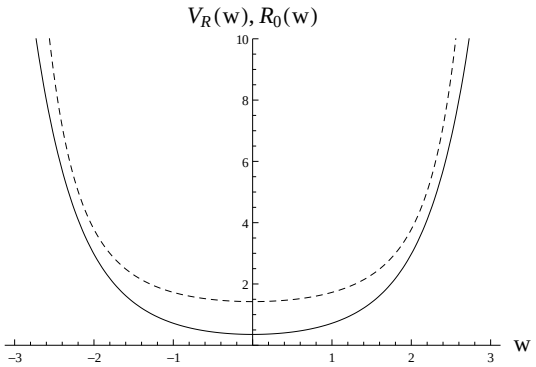

Figure 5. The profile of the V_R potential (solid black line) and the non-localized right-chiral zero mode R_O (dashed black line) along the extra dimension in case II. Here we have set $H = 1.0$, $M = 1/2$ and $s = 1/2$.

(see figure 6), while the shape for the right potential V_R is conceived as a barrier potential as shown in figure 7. The expression for both the left and right potentials reads

$$V_L(w) = \frac{2H^2Mb\left[(2Mb+1)H^2w^2-1\right]}{\left(1+H^2w^2\right)^2}, \tag{4.21a}$$

$$V_R(w) = \frac{2H^2Mb\left[(2Mb-1)H^2w^2+1\right]}{\left(1+H^2w^2\right)^2}. \tag{4.21b}$$

These two potentials have the same vanishing asymptotic behavior $V_{R,L}(w \longrightarrow \pm\infty) = 0$, indicating the lack of a mass gap in their corresponding mass spectra; the critical values (maximum and minimum) of the right and left potentials is achieved when $w = 0$ and are respectively given by $V_R(w = 0) = 2H^2Mb$ and $V_L(w = 0) = -2H^2Mb$. Only the left potential supports a left-chiral zero mode L_0 localized on the brane. The volcano potential V_L supports a tower of continuous KK massive modes non-localized on the 3-brane. On the other hand, the right potential V_R represents a barrier potential, a fact which indicates that right-chiral fermions cannot be localized on the 3-brane.

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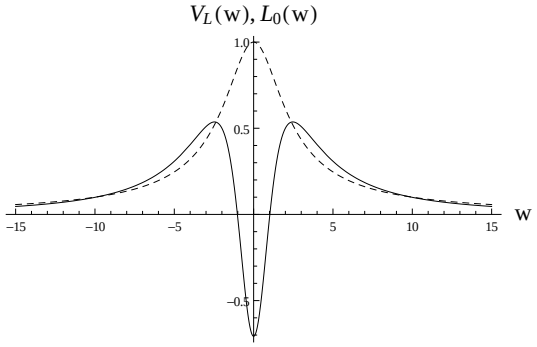

Figure 6. The profile of the V_L potential (solid black line) and the localized left-chiral ground state L_0 (dashed black line) along the fifth dimension for case III. Here $H = 1/2$, $M = 1$ and $s = 2$.

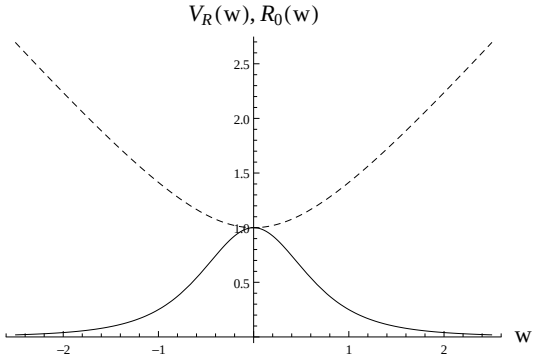

Figure 7. The profile of the V_R potential (solid black line) and the non-localized right-chiral zero mode R_0 (dashed black line) along the extra dimension for case III. Here we have set $H = 1$, $M = 1/2$ and $s = 1$.

By making use of the relations given by (4.15) we can easily find expressions for the massless zero modes R_0 and L_0 supported by the potentials (4.21)

$$L_0 \propto e^{-M\int \frac{Hw\sqrt{s}}{(1+H^2w^2)}dw} = \left(1+H^2w^2\right)^{-Mb}, \tag{4.22a}$$

$$R_0 \propto e^{M\int \frac{Hw\sqrt{s}}{(1+H^2w^2)}dw} = \left(1+H^2w^2\right)^{Mb}. \tag{4.22b}$$

Moreover, by solving the Schrödinger equation corresponding to the left potential V_L (4.21a) we can also obtain the general solution for the KK excitations with arbitrary mass and see that the continuous spectrum of KK massive modes can be expressed in terms of confluent Heun functions in the following form

$$\begin{aligned} L_n = {} & C_1\left(1+H^2w^2\right)^{1+Mb}\,\mathrm{HeunC}\left(0,-\frac{1}{2},1+2Mb,-\frac{1}{4}\frac{m_n^2}{H^2},\eta_n,-H^2w^2\right)+ \\ & C_2\left(1+H^2w^2\right)^{1+Mb}\,w\,\mathrm{HeunC}\left(0,\frac{1}{2},1+2Mb,-\frac{1}{4}\frac{m_n^2}{H^2},\eta_n,-H^2w^2\right), \end{aligned} \tag{4.23}$$

where C_1 and C_2 are arbitrary constants, and the parameters η_n is given by $\eta_n = \frac{2(1+Mb)H^2+m_n^2}{4H^2}$.

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We can conclude that case III yields a left-chiral massless fermion zero mode localized on the 3-brane of our model, along with a continuum of massive KK fermionic excitations delocalized from the brane, whereas all right-chiral KK fermionic modes are non-localized on the brane.

4.4 Case IV: $F(T) = \frac{2\tanh(T/b)}{\sqrt{1-\sinh^2(T/b)}}$

In this case we have for both the left and right potentials a modified Pöschl-Teller configuration which has been carefully studied in several modern physics scenarios. This function $F(T)$ allows us to have KK discrete and continuous mass spectra separated by a mass gap from the massless zero mode. The size of these mass gaps largely depend on the value of 4D and 5D parameters as shown by the following expressions:

$$V_L(w) = M\left[Ms\tanh^2(2Hw) - 2H\sqrt{s}\,\mathrm{sech}^2(2Hw)\right], \tag{4.24a}$$
$$V_R(w) = M\left[Ms\tanh^2(2Hw) + 2H\sqrt{s}\,\mathrm{sech}^2(2Hw)\right]. \tag{4.24b}$$

By substituting the value of s in (4.24) and recalling that $b = \sqrt{\frac{-3}{2\kappa_5^2\Lambda_5}}$ according to (2.9), we can recast the potentials $V_{R,L}$ as

$$V_L(w) = 4MH^2b\left[Mb\tanh^2(2Hw) - \mathrm{sech}^2(2Hw)\right], \tag{4.25a}$$
$$V_R(w) = 4MH^2b\left[Mb\tanh^2(2Hw) + \mathrm{sech}^2(2Hw)\right]. \tag{4.25b}$$

The asymptotic behaviour for the potentials has the form $V_{R,L}(w \longrightarrow \pm\infty) = M^2\,s = 4M^2H^2b^2$ and is positive definite, a fact which in general ensures the existence of a mass gap between the bound states of the corresponding mass spectra. The critical values (maximum and minimum) of the right and left potentials when $w = 0$ are respectively $V_R(w=0) = 4MH^2b$ and $V_L(w=0) = -4MH^2b$. The massless zero modes for both potentials can be written as follows

$$L_0 \propto \mathrm{sech}^{Mb}(2Hw), \tag{4.26a}$$
$$R_0 \propto \cosh^{Mb}(2Hw). \tag{4.26b}$$

From these expressions it is clear that just the left-chiral fermion field possesses a localized zero mode on the 3-brane. The general solution for the L_n's is given in terms of hypergeometric functions ${}_2\mathrm{F}_1$ and reads

$$L_n \propto \cosh^{1+Mb}(2Hw)\ {}_2\mathrm{F}_1\left(s_n, r_n; \frac{1}{2}; -\sinh^2(2Hw)\right), \tag{4.27}$$

for even n and

$$L_n \propto \cosh^{1+Mb}(2Hw)\sinh(2Hw)\ {}_2\mathrm{F}_1\left(s_n+\frac{1}{2}, r_n+\frac{1}{2}; \frac{3}{2}; -\sinh^2(2Hw)\right), \tag{4.28}$$

for odd n, where the parameters s_n and r_n are given by

$$s_n = \frac{1}{2}(n+1), \qquad r_n = Mb - \frac{1}{2}(n-1). \tag{4.29}$$

The number of bound states for the left-chiral fermion L_n is finite, they are labeled by $n = 0, 1, 2, \ldots, < Mb$ and the corresponding KK mass spectrum is described by

$$m^2_{L_n} = 4H^2\,(Mb - n)\,n. \tag{4.30}$$

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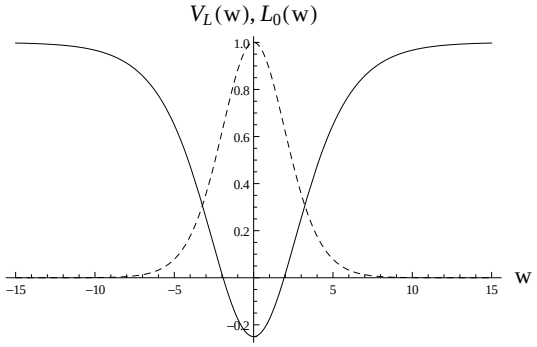

Figure 8. The profile of the V_L potential (solid black line) and the localized left-chiral ground state L_0 (dashed black line) along the fifth dimension for case IV. Here we have set $H = 1/4$, $M = 1$ and $s = 1$.

If we take into account that, by definition, $b > 0$, we can infer that when $Mb < 1$ there is a single left-chiral bound state, the massless zero L_0, as depicted in figure 8, and therefore it is only possible to localize left-chiral fermions on the 3-brane. On the contrary, in order to obtain a finite number of massive KK excited modes we must impose the condition $Mb > 1$.

For the potential of right-chiral fermions, as shown in (4.25b), it is not possible to localize the massless zero mode. Thus, the only way to ensure the existence of a finite number of localized bound states for the right-chiral massive fermions consists in imposing the condition $Mb > 1$.

The general expression for the KK right-chiral bound states in this case is given by

$$R_n \propto \cosh^{Mb}(2Hw)\ {}_2\mathrm{F}_1\left(\frac{1+n}{2}, Mb - \frac{1+n}{2}; \frac{1}{2}; -\sinh^2(2Hw)\right), \tag{4.31}$$

for even n and

$$R_n \propto \cosh^{Mb}(2Hw)\sinh(Hz)\ {}_2\mathrm{F}_1\left(1+\frac{n}{2}, Mb - \frac{n}{2}; \frac{3}{2}; -\sinh^2(2Hw)\right), \tag{4.32}$$

for odd n.

It is worth emphasizing that the massless zero mode R_0 given in (4.26b) is not a localized fermionic bound state, therefore the ground state for right-chiral fermions corresponds to the first massive bound state (with $n = 0$), as illustrated in figure 9, and is denoted by

$$R_{m_0} \propto \mathrm{sech}^{Mb-1}(2Hw), \tag{4.33}$$

where the mass of the first right-chiral KK bound state obeys the following inequality $m^2_{R_0} = 4H^2\,(2Mb - 1) > 0$.

The number of bound states for the right-chiral fermion fields inferred from the canonical form of the V_R potential is $n = 0, 1, 2, \ldots, < Mb - 1$. For this set of eigenvalues we have the following mass spectrum for the right-chiral fermions $m^2_{R_n} = 4H^2\,[2Mb - (n+1)]\,(n+1)$. We should finally mention that both of the potentials V_R and V_L have a continuous mass spectrum that is achieved when $m_{Ln,Rn} > 4M^2H^2b^2 = M^2\,s$, as it is evident from the asymptotic behaviour of these potentials.

In figure 10 we present the profile of left and right-chiral KK massive modes respectively for $n = 1, 2$ in the above studied case IV.

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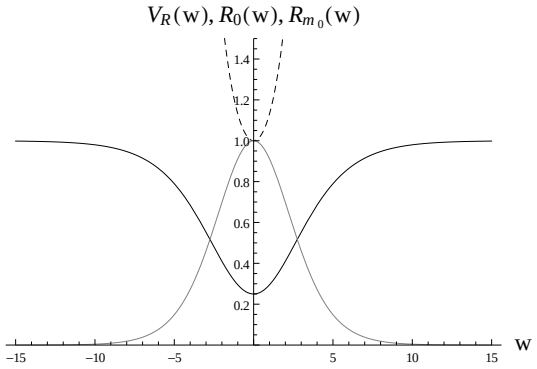
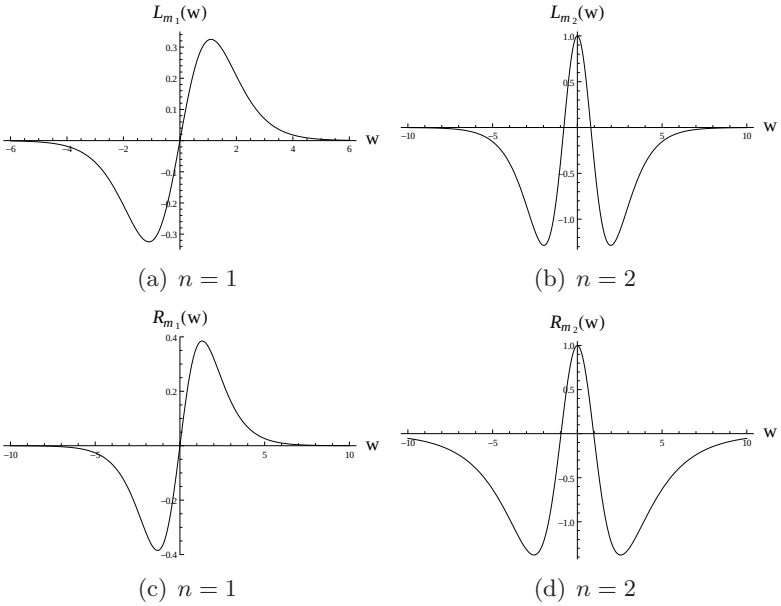


Figure 9. The profile of the V_R potential (solid black line), the localized right-chiral massive ground state R_{m_0} (gray line), and the non-localized massless zero mode R_0 (dashed black line) along the extra dimension for case IV. Here we have also set $H = 1/4$, $M = 1$ and $s = 1$.



(a) $n = 1$ (b) $n = 2$

(c) $n = 1$ (d) $n = 2$

Figure 10. The shape of the left– and right-chiral KK massive modes respectively for $n = 1, 2$ in case IV. The parameters are set to $M = 2$, $b = 4$ and $H = 1/4$.

5 Corrections to Coulomb's law in the thin brane limit

As we mentioned before, a natural and primary condition for a braneworld model to be physically consistent is to render in certain limit the physical laws that we observe in our 4D Universe. Moreover, once these laws are recovered, we must study the corrections they receive from the higher dimensional world. Since we have studied the localization of fermionic fields on the considered tachyonic thick braneworld in the previous section and the localization of gauge bosons fields in this model was accomplished in [38], in this section we are in position to recover the 4D Coulomb law on the 3-brane where we are supposed to live. Moreover, we shall also be able to compute the Coulomb's law modifications that come from the contribution of the KK massive modes of the bulk gauge vector field and to see whether these corrections are phenomenologically viable.

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Thus, in order to achieve this aim, we shall consider a Yukawa interaction between 5D fermions and gauge bosons which constitutes a generalization of the 4D quantum interacting potential given by $L_I = -e\bar{\psi}(x)\gamma^\mu A_\mu(x)\psi(x)$ with the vertex factor $-ie\gamma^\mu$ ([69], see sections 4.7 and 4.8, pages 121–126).

Hence, the generalized 5D interaction between fermions and gauge boson reads [70]

$$S_I = -e_5\int d^4xdw\ \sqrt{-g}\ \bar{\Psi}(x,w)\Gamma^M A_M(x,w)\Psi(x,w), \tag{5.1}$$

where e_5 is a 5D coupling constant and $A_M(x,w)$ represents the generalized 5D gauge vector field that mediates the interaction between the fermion fields under the gauge condition $A_5 = 0$ and the KK vector modes decomposition

$$A_\mu(x,w) = \sum_n a_\mu^{(n)}(x)\rho_n(w)e^{-f/2}, \tag{5.2}$$

where $\rho_n(w)$ is the profile of the massive gauge boson along the fifth dimension. We shall suppose as well that the 4D fermions are associated to the left-chiral KK massless zero mode L_0 of the last three cases considered in the previous section. The zero mode of this gauge field has recently been shown to be localized on our braneworld model given by (2.1)–(2.9) in [38]. Then, by performing the dimensional reduction we can confirm the similarity between the Newton potential for two point particles interacting with massive KK tensor modes and the Coulomb potential for two point charges interacting with massive KK gauge vector field modes. Let us tart by considering the following action:

$$\begin{aligned} S_I &\supset \sum\!\!\!\!\!\!\int_n d^4xdw\ \sqrt{-\hat{g}}\ \mathrm{e}^{5f}(-e_5)\mathrm{e}^{-2f}\bar{\psi}_0(x)L_0(w)\mathrm{e}^{-f}\gamma^\mu a_\mu^{(n)}(x)\mathrm{e}^{-f/2}\rho_n(w)\mathrm{e}^{-2f}\psi_0(x)L_0(w) \\ &= -e_5\sum\!\!\!\!\!\!\int_n dw\ \mathrm{e}^{-f/2}\ \rho_n(w)L_0^2(w)\int d^4x\sqrt{-\hat{g}}\ \bar{\psi}_0(x)\gamma^\mu a_\mu^{(n)}(x)\psi_0(x) \\ &= \int d^4x\ \sqrt{-\hat{g}}\ \Big\{ - e\bar{\psi}_0(x)\gamma^\mu a_\mu^{(0)}(x)\psi_0(x) - \sum\!\!\!\!\!\!\int_{n\neq 0}\ e_n\bar{\psi}_0(x)\gamma^\mu a_\mu^{(n)}(x)\psi_0(x)\Big\}, \end{aligned} \tag{5.3}$$

where the $\sum\!\!\!\!\!\!\int_n$ stands for summation or integration (or both) with respect to n, depending on the respective discrete or continuous (or mixed) character of the $a_\mu^{(n)}(x)$ and $e_n(w)$. By taking into account the form of the gauge vector modes $\rho_0(w)$ and $\rho_n(w)$ from [38]

$$\rho_0(w) = \frac{\sqrt{H}(\pi/2)^{1/4}}{2\Gamma(5/4)}\mathrm{sech}^{1/4}(2Hw), \tag{5.4}$$

$$\rho_n(w) = \left[\sum_{\pm} C_\pm(\sigma)P_{1/4}^{\pm i\sigma}\left(\tanh(2Hw)\right)\right], \tag{5.5}$$

where $P_{1/4}^{\pm i\sigma}$ are associated Legendre functions of first kind of degree $1/4$ and order $\pm i\,\sigma$ with $\sigma = \sqrt{\frac{m^2}{4H^2} - \frac{1}{16}}$, which imposes the condition $m \geq H/2$, we find the next relations between the couplings e, e_5 and $e_n(w)$:

$$e = e_5\int dw\ \mathrm{e}^{-f/2}\ \rho_0(w)L_0^2(w) = e_5\frac{(2\pi)^{1/4}}{\Gamma\left(\frac{1}{4}\right)b^{1/2}}\int dw\ L_0^2(w) = e_5\frac{(2\pi)^{1/4}}{\Gamma\left(\frac{1}{4}\right)b^{1/2}}, \tag{5.6}$$

where e is the usual 4D charge of the fermion localized on the brane and e_n's are 4D effective couplings defined as follows

$$e_n \equiv e_5 \int dw\ \mathrm{e}^{-f/2}\ \rho_n(w) L_0^2(w) = e \frac{\Gamma\left(\frac{1}{4}\right) b^{1/2}}{(2\pi)^{1/4}} \int dw\ \mathrm{e}^{-f/2}\ \rho_n(w) L_0^2(w). \tag{5.7}$$

In the non-relativistic limit the Coulomb potential (and its corrections) between two charged fermions is determined by the KK photon exchange process and turns out to be

$$\begin{aligned} V(r) &= \frac{e^2}{4\pi r} + \int_{m_0}^{\infty} dm \frac{e_n^2}{4\pi r} \mathrm{e}^{-mr} \\ &= \frac{1}{4\pi r} \left[e^2 + e_5^2 \int_{m_0}^{\infty} dm\ \mathrm{e}^{-mr} \left(\int dw\ \mathrm{e}^{-f(w)/2}\ \rho_n(w) L_0^2(w) \right)^2 \right], \end{aligned} \tag{5.8}$$

where $m_0 = H/2$ is the first excited KK massive mode of the gauge vector field. In this way it is easy to see that the Coulomb potential arises from the vector zero mode, while its corrections come from the massive KK vector excitations.

If we pay attention to the formula (5.6) we realize that the existing relationship between the 4D charge e and the coupling constant e_5 does not depend on the form of the left-chiral KK ground state L_0 since it is normalized to unity. On the other hand, the integral in the r.h.s. of the expression (5.8) for the extra dimensional corrections to the Coulomb potential will always render a constant (which depends on the mass m of the KK gauge field) as far as we suitably define a Dirac delta function with the aid of the left-chiral zero mode L_0 in the thin brane limit (see further subsections for concrete examples). Thus, under this definition of the delta function, the squared integral with respect to w in (5.8), with the prefactor $\mathrm{e}^{-f(w)/2}\ \rho_n(w)$ multiplying a Dirac delta function, will lead to the value $\rho_n(0)^2$ since the Dirac delta function is located at the origin of the fifth dimension $w = 0$. This circumstance makes us conclude, despite the heuristic proposals employed for the function $F(T)$, that the corrections to Coulomb's law associated with the massive KK gauge vector modes in the thin brane limit do not depend on the explicit form of the function $F(T)$ and are, in this sense, model independent as it will be shown in the following examples.

In the following subsections we will analytically compute the Coulomb potential $V(r)$ in the thin brane limit, which is not an easy analytical task, but is still affordable for the three previously studied cases in which the left-chiral massless fermion localization on the 3-brane is feasible.

5.1 Corrections to Coulomb's law in case II

In order to compute the Coulomb's law corrections for the case II, let us begin by calculating the 4D effective coupling constants e_n. To do that we shall make use of the fermionic localization mechanism described above with the odd function $F(T) = \frac{\sinh(2T/b)}{1-\sinh^2(T/b)}$. In this case the normalized fermion zero mode reads

$$L_0(w) = \left(\frac{H}{\mathrm{K}_0\left(2\mathrm{MG}\right)} \right)^{\frac{1}{2}} e^{-MG\cosh(Hw)}, \tag{5.9}$$

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where K_0 is the modified Bessel function of second kind. By substituting the warp factor (2.6) and the expression for $\rho_n(w)$ in (5.7) we obtain

$$e_n = e\frac{\sqrt{H}\,\Gamma\left(\frac{1}{4}\right)}{\sqrt{2}\,(2\pi)^{1/4}\,\mathrm{K}_0\,(2\mathrm{MG})}\int dw\,\cosh^{\frac{1}{4}}(2Hw)\,e^{-2MG\cosh(Hw)}\times \left[\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}\left(\tanh(2Hw)\right)\right]. \tag{5.10}$$

By making use of the following definition of the Dirac delta function[4] which corresponds to the thin brane limit when $H\to\infty$:

$$\delta(w) = \lim_{H\to\infty}\frac{He^{-2MG\cosh(Hw)}}{\mathrm{K}_0\,(2\mathrm{MG})}, \tag{5.11}$$

we finally get the following expression for the e_n's

$$e_n = e\frac{\Gamma\left(\frac{1}{4}\right)}{\sqrt{2H}\,(2\pi)^{1/4}}\left[\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}\,(0)\right]. \tag{5.12}$$

Once we have these 4D effective couplings at hand we can write the Coulomb potential as follows

$$\begin{aligned} V(r) &= \frac{e^2}{4\pi r}\left[1+\frac{\left[\Gamma\left(\frac{1}{4}\right)\right]^2}{2H\sqrt{2\pi}}\int_{m_0}^{\infty} dm\;\mathrm{e}^{-mr}\left|\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}\,(0)\right|^2\right] \\ &= \frac{e^2}{4\pi r}\left[1+\frac{\left[\Gamma\left(\frac{1}{4}\right)\right]^2}{\sqrt{2\pi}\,H}\int_{m_0}^{\infty} dm\;\mathrm{e}^{-mr}\left|\frac{\Gamma\left(1+i\sigma\right)}{\Gamma\left(\frac{3}{8}-\frac{i\sigma}{2}\right)\Gamma\left(\frac{9}{8}-\frac{i\sigma}{2}\right)}\right|^2\right], \end{aligned} \tag{5.13}$$

where we have taken into account the fact that the normalization constants for the associated Legendre functions are given by $C_{\pm}(\sigma) = \frac{|\Gamma(1\mp i\sigma)|}{\sqrt{2\pi}}$, as well as the following relation

$$P_{\nu}^{\mu}(0) = \frac{2^{\mu}\sqrt{\pi}}{\Gamma\left(\frac{1-\nu-\mu}{2}\right)\Gamma\left(1+\frac{\nu-\mu}{2}\right)}. \tag{5.14}$$

Thus, the Coulomb potential can be written in the form

$$V(r) = \frac{e^2}{4\pi r}\left[1+\Delta V\right], \tag{5.15}$$

where the correction ΔV reads

$$\Delta V = \frac{\left[\Gamma\left(\frac{1}{4}\right)\right]^2}{\sqrt{2\pi}\,\Gamma\left(\frac{3}{8}\right)^2\Gamma\left(\frac{9}{8}\right)^2}\frac{e^{-Hr/2}}{Hr}\left(1+\mathcal{O}\left(\frac{1}{Hr}\right)\right). \tag{5.16}$$

When performing this computation, in (5.13) we have expanded the prefactor that multiplies the exponential function in the integrand with respect to $m_0 = H/2$ (which corresponds to $\sigma = 0$) since the corrections to the Coulomb potential are dominated by the sector of small massive KK vector modes [71].

[4] It is straightforward to check that this definition possesses all the properties of the normalized to unity delta distribution function.

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5.2 Corrections to Coulomb's law in case III

We now will calculate the explicit form of the Coulomb potential $V(r)$ following the same procedure as in case II. Here $F(T) = \frac{\operatorname{arctanh}[\sinh(T/b)]}{\sqrt{2\operatorname{sech}^2(T/b)-1}\left[1+\operatorname{arctanh}^2(\sinh(T/b))\right]}$ and for this function the normalizable left-chiral KK massless zero mode is

$$L_0(w) = \left[\frac{H\ \Gamma\left(2Mb\right)}{\sqrt{\pi}\,\Gamma\left(2Mb-\frac{1}{2}\right)}\right]^{\frac{1}{2}}\left(1+w^2H^2\right)^{-Mb}, \qquad Mb > \frac{1}{4}, \tag{5.17}$$

where the inequality condition follows in order to render a convergent integral.

By taking into account the expressions for the warp factor (2.6) and the gauge function $\rho_n(w)$ (5.5) we can compute the expression for the coupling constants e_n's (5.7) and get the same result as in the previous case:

$$e_n = e\frac{\Gamma\left(\frac{1}{4}\right)}{\sqrt{2H}\,(2\pi)^{1/4}}\left[\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}\left(0\right)\right], \tag{5.18}$$

when defining the Dirac delta function as shown below, in the thin brane limit, when $H \to \infty$:

$$\delta(w) = \lim_{H\to\infty}\ \frac{H\ \Gamma\left(2Mb\right)}{\sqrt{\pi}\,\Gamma\left(2Mb-\frac{1}{2}\right)}\left(1+w^2H^2\right)^{-2Mb}. \tag{5.19}$$

By substituting the expression (5.18) into equation (5.8) we get the same form for the Coulomb potential (5.15), where its correction is again defined as in (5.16), obtaining the same result as in the above studied case II.

5.3 Corrections to Coulomb's law for case IV

We shall further proceed to perform the analytical calculation of $V(r)$ for case IV. Let us compute first the 4D effective couplings e_n. In order to achieve this goal we shall make use of the function $F(T) = \frac{2\tanh(T/b)}{\sqrt{1-\sinh^2(T/b)}}$ in the fermionic localization mechanism for which the normalizable left-chiral zero mode reads

$$L_0(w) = \left[\frac{2H\Gamma\left(\frac{1}{2}+Mb\right)}{\pi^{1/2}\Gamma\left(Mb\right)}\right]^{\frac{1}{2}}\operatorname{sech}(2Hw)^{Mb}. \tag{5.20}$$

By substituting the expression for the warp factor (2.6) and the expression for $\rho_n(w)$ in (5.7) we obtain

$$\begin{aligned} e_n &= e\frac{2^{1/4}\sqrt{H}\,\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{\pi^{3/4}\Gamma\left(Mb\right)}\int dw\,\operatorname{sech}(2Hw)^{2Mb-\frac{1}{4}}\left[\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}\left(\tanh(2Hw)\right)\right] \\ &= e\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(Mb-\frac{1}{8}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{2^{3/4}\pi^{1/4}\sqrt{H}\,\Gamma\left(Mb\right)\Gamma\left(\frac{3}{8}+Mb\right)}\left[\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}\left(0\right)\right], \end{aligned} \tag{5.21}$$

where now we have applied the following definition for the normalized Dirac delta function

$$\delta(w) = \lim_{H\to\infty}\ \frac{2H\Gamma\left(\frac{3}{8}+Mb\right)}{\pi^{1/2}\Gamma\left(Mb-\frac{1}{8}\right)}\operatorname{sech}(2Hw)^{2Mb-\frac{1}{4}}, \qquad Mb > \frac{1}{8} \tag{5.22}$$

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in the thin brane limit when $H \to \infty$. The above result leads us to the following form of the Coulomb potential

$$V(r) = \frac{e^2}{4\pi r}\left[1+\frac{1}{2^{\frac{3}{2}}\sqrt{\pi}H}\left(\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(Mb-\frac{1}{8}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{\Gamma(Mb)\Gamma\left(Mb+\frac{3}{8}\right)}\right)^2\int_{m_0}^{\infty} dm\, \mathrm{e}^{-mr}\left|\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}(0)\right|^2\right]$$
$$= \frac{e^2}{4\pi r}\left[1+\frac{1}{\sqrt{2\pi}\,H}\left(\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(Mb-\frac{1}{8}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{\Gamma(Mb)\Gamma\left(Mb+\frac{3}{8}\right)}\right)^2\int_{m_0}^{\infty} dm\, \mathrm{e}^{-mr}\left|\frac{\Gamma(1+i\sigma)}{\Gamma\left(\frac{3}{8}-\frac{i\sigma}{2}\right)\Gamma\left(\frac{9}{8}-\frac{i\sigma}{2}\right)}\right|^2\right]. \quad (5.23)$$

After replacing the integration constants $|C_{\pm}(\sigma)|$ and using the formula (5.14) in the expression for the Coulomb potential (5.23), it can be written in the form of (5.15), where the correction ΔV now reads

$$\Delta V = \frac{1}{\sqrt{2\pi}}\left(\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(Mb-\frac{1}{8}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{\Gamma(Mb)\Gamma\left(Mb+\frac{3}{8}\right)\Gamma\left(\frac{3}{8}\right)\Gamma\left(\frac{9}{8}\right)}\right)^2 \frac{e^{-Hr/2}}{Hr}\left(1+\mathcal{O}\left(\frac{1}{Hr}\right)\right). \quad (5.24)$$

Thus, for all the above considered cases, the corrections to Coulomb's law are exponentially suppressed in the thin brane limit $H \to \infty$, making the tachyonic braneworld model physically viable. This result is due to the existence of a mass gap in the spectrum of KK gauge field excitations reported in [38]. If there was no such a mass gap, the corrections were not exponentially suppressed, but polynomially suppressed. Notwithstanding they will be still be very small according to the results obtained in the well-known Randall-Sundrum model [11].

6 Corrections to Coulomb's law in a thick brane scenario, case IV

At this point the corrections made for the different cases discussed above are valid only in the limit of thin branes, in which we assumed that the first massive mode of the gauge bosons $m = \frac{H}{2}$ predicted in [38] is very large. However, in some cases we can also analyze the corrections to Coulomb's law from another more realistic point of view, i.e. within another valid approximation for thick brane scenarios.

Let us start by performing the integral (5.21) for e_n without the thin brane limit assumption

$$e_n = e\frac{2^{1/4}\sqrt{H}\,\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{\pi^{3/4}\,\Gamma(Mb)}\int dw\,\mathrm{sech}(2Hw)^{2Mb-\frac{1}{4}}$$
$$\times\left[\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}(\tanh(2Hw))\right]. \quad (6.1)$$

In order to facilitate this integration it is convenient to introduce the following variable $w = \frac{\mathrm{arctanh}(x)}{2H}$, which leads to

$$e_n = e\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{\sqrt{H}\,(2\pi)^{3/4}\,\Gamma(Mb)}\int dx\,\left(1-x^2\right)^{Mb-\frac{9}{8}}\left[\sum_{\pm} C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}(x)\right]. \quad (6.2)$$

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When we integrate over the entire fifth dimension using formula ET II 316(6) of the Gradshteyn and Ryzhik handbook [72], the expression for (6.2) results in

$$e_n = e\,\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{\sqrt{H}\,(2\pi)^{3/4}\,\Gamma\left(Mb\right)}\int_{-1}^{1}dx\,\left(1-x^2\right)^{Mb-\frac{9}{8}}\left[\sum_{\pm}C_{\pm}(\sigma)P_{1/4}^{\pm i\sigma}\left(x\right)\right]=$$
$$e\,\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{1}{2}+Mb\right)}{\sqrt{H}\,(2\pi)^{3/4}\,\Gamma\left(Mb\right)}\left(\sum_{\pm}C_{\pm}(\sigma)\frac{\pi\,2^{\pm i\sigma}\Gamma(2Mb-\frac{1}{4}\pm i\frac{\sigma}{2})\Gamma(2Mb-\frac{1}{4}\mp i\frac{\sigma}{2})}{\Gamma(2Mb-\frac{3}{8})\Gamma(2Mb+\frac{3}{8})\Gamma(\frac{3}{8}\mp i\frac{\sigma}{2})\Gamma(\frac{9}{8}\mp i\frac{\sigma}{2})}\right), \tag{6.3}$$

where $Mb>\frac{1}{8}$. We then need to square the couplings e_n and integrate this expression over all continuous KK massive modes along the lines of (5.8)

$$\int_{m_0}^{\infty}dm|e_n|^2e^{-mr}. \tag{6.4}$$

Before making this calculation, we will perform the following change of variable $m=\frac{H}{2}\sqrt{1+16\sigma^2}$, then the above integral reads

$$e^2\frac{2^{3/2}\,\Gamma\left(\frac{1}{4}\right)^2\Gamma\left(\frac{1}{2}+Mb\right)^2}{\pi^{3/2}\,\Gamma\left(Mb\right)^2}\int_0^{\infty}d\sigma\,\frac{e^{-\frac{1}{2}Hr\sqrt{1+16\sigma^2}}}{\sqrt{\frac{1}{\sigma^2}+16}}\times$$
$$\left|\sum_{\pm}C_{\pm}(\sigma)\frac{\pi\,2^{\pm i\sigma}\Gamma(2Mb-\frac{1}{4}\pm i\frac{\sigma}{2})\Gamma(2Mb-\frac{1}{4}\mp i\frac{\sigma}{2})}{\Gamma(2Mb-\frac{3}{8})\Gamma(2Mb+\frac{3}{8})\Gamma(\frac{3}{8}\mp i\frac{\sigma}{2})\Gamma(\frac{9}{8}\mp i\frac{\sigma}{2})}\right|^2. \tag{6.5}$$

It seems impossible to do this integral analytically, however, as we have previously assumed when computing (5.16), we shall consider that the contribution to the mass integral (6.5) is dominated by the first KK continuous excitation modes. Therefore we can expand the prefactor of the exponential around $\sigma=0$, using $C_{\pm}(\sigma)=\frac{|\Gamma(1\mp i\sigma)|}{\sqrt{2\pi}}$, and we obtain the following approximate result for the above integral

$$e^2\frac{2^{3/2}\,\Gamma\left(\frac{1}{4}\right)^2\Gamma\left(\frac{1}{2}+Mb\right)^2}{\pi^{3/2}\,\Gamma\left(Mb\right)^2}\int_0^{\infty}d\sigma\,\frac{e^{-\frac{1}{2}Hr\sqrt{1+16\sigma^2}}}{\sqrt{\frac{1}{\sigma^2}+16}}\times$$
$$\left|\sum_{\pm}C_{\pm}(\sigma)\frac{\pi\,2^{\pm i\sigma}\,\Gamma(2Mb-\frac{1}{4}\pm i\frac{\sigma}{2})\Gamma(2Mb-\frac{1}{4}\mp i\frac{\sigma}{2})}{\Gamma(2Mb-\frac{3}{8})\Gamma(2Mb+\frac{3}{8})\Gamma(\frac{3}{8}\mp i\frac{\sigma}{2})\Gamma(\frac{9}{8}\mp i\frac{\sigma}{2})}\right|^2\sim$$
$$e^2\left(\frac{2^6\,\Gamma\left(\frac{1}{4}\right)^2\Gamma\left(\frac{1}{2}+Mb\right)^2\Gamma\left(2Mb-\frac{1}{4}\right)^4}{(2\pi)^{\frac{1}{2}}\Gamma\left(\frac{3}{8}\right)^2\Gamma\left(\frac{1}{8}\right)^2\Gamma(Mb)^2\Gamma\left(2Mb-\frac{3}{8}\right)^2\Gamma\left(2Mb+\frac{3}{8}\right)^2}\right)\times$$
$$\frac{\mathrm{e}^{-\frac{1}{2}Hr}}{Hr}\left(1+\mathcal{O}\left(\frac{1}{Hr}\right)\right), \tag{6.6}$$

finally the expression for the corrected Coulomb's potential reads

$$V=\frac{e^2}{4\pi r}\left[1+\Delta V\right], \tag{6.7}$$

where the correction is given by

$$\Delta V=\gamma\left(Mb\right)\frac{e^{-\frac{1}{2}Hr}}{Hr}\left(1+\mathcal{O}\left(\frac{1}{Hr}\right)\right), \tag{6.8}$$

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and the constant function $\gamma(Mb)$ explicitly depends on the 5D parameters M and b and possesses the form

$$\gamma\left(Mb\right)=\left(\frac{8\,\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{1}{2}+Mb\right)\Gamma\left(2Mb-\frac{1}{4}\right)^{2}}{(2\pi)^{\frac{1}{4}}\Gamma\left(\frac{3}{8}\right)\Gamma\left(\frac{1}{8}\right)\Gamma\left(Mb\right)\Gamma\left(2Mb-\frac{3}{8}\right)\Gamma\left(2Mb+\frac{3}{8}\right)}\right)^{2}.$$

This result is particularly relevant when directly compared to the one obtained in the thin brane limit (5.24), because the corrections to Coulomb's law hold their shape for large and small mass scales dictated by the Hubble parameter H since the first excited state possesses $m=H/2$.

It is worth noticing that even when the 5D parameters are of the same order, providing naturalness for our braneworld, i.e. a model without large hierarchies, we need to perform a fine-tuning on these parameters, $Mb=\frac{3}{16}\pm\epsilon$, where ϵ is an infinitesimal parameter, in order to achieve the value $\gamma(Mb)\approx 1\times 10^{-33}$, considering that in our epoch $H_0\approx 1\times 10^{-33}\,\mathrm{eV}$. Thus, by making this assumption we have established a proportionality relationship between the 5D metric parameter b (which depends on Λ_5 and κ_5) and the fermion coupling constant M, simplifying indeed the potential as follows

$$V=\frac{e^{2}}{4\pi r}\left(1+\frac{e^{-\frac{1}{2}Hr}}{r}\right). \tag{6.9}$$

We should mention as well that the corrections to this potential are still far beyond the upper experimental bound observed on the photon mass, making the present braneworld model phenomenologically viable. In fact, by confronting our result to actual experimental observations, we can consider a realistic scenario by setting the Hubble parameter to its present value $H=H_0\approx 1\times 10^{-33}\,\mathrm{eV}$, leading to an estimation for the photon mass of the order $m_\gamma\approx 5\times 10^{-34}\,\mathrm{eV}$, while several estimates made until recent years [73]–[88] have reported greater bounds on the photon mass. For instance, one of the most recent works [76] reported an estimated upper bound for the photon mass of order $m_\gamma\approx 1.6\times 10^{-4}\,\mathrm{eV}$ based on the anomalous magnetic moment of the electron, whereas the most stringent estimation made so far on the basis of galactic magnetic fields [82] reported an upper bound that goes like $m_\gamma\approx 3\times 10^{-27}\,\mathrm{eV}$.

It must be emphasized as well that the mass corrections predicted by (6.9) resemble the Proca mass term in the potential, however, our corrections go like r^{-2} instead of r^{-1} and hence are not directly comparable to the corrections obtained within the electromagnetic theory of Proca. In this sense we do not expect exactly the same results. In addition, our upper bound for the mass of the photon is far from the one reported until now in the literature. This is because in our braneworld model the predicted mass corrections to the photon mass emerge in a picture where the Universe is in expansion and are determined by the Hubble parameter. This places our prediction far from all bounds reported in the literature so far.

Additionally to the experimental bounds on the photon mass, we can consider more stringent astrophysical/cosmological bounds as well [78]–[88]. In both of these cases the 5D parameters M and b remain the same order, but the aforementioned fine-tuning changes from case to case as shown in table 1. Namely, it turns out that by fixing the upper bound on the photon mass m_γ we determine the value of $\gamma(M,b)$, since $m_\gamma=\gamma(M,b)/2$, which corresponds to a certain relation between the fermion coupling constant M and the metric parameter b that leaves them of the same order, but modifies the order of the needed fine-tuning.

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Type of measurement	Author(s) and year	Upper bound on m_γ (eV)	Mb
Radio-wave interferometer	Froome (1958) [78]	2.4×10^{-7}	$\frac{3}{16}\pm2.72\times10^{-5}$
Crab Nebula Pulsar	Warner et al. (1969) [79]	2.9×10^{-8}	$\frac{3}{16}\pm9.45\times10^{-6}$
Pulsar Emission	Bay et al. (1972) [80]	1.7×10^{-13}	$\frac{3}{16}\pm2.29\times10^{-8}$
Short pulses radiation	Brown et al. (1972) [81]	7.9×10^{-1}	$\frac{3}{16}-2.30\times10^{-2}$
Galactic magnetic field	Chibisov (1976) [82]	3×10^{-27}	$\frac{3}{16}\pm3.04\times10^{-15}$
Torque on toroid balance	Lakes (1998) [83]	1×10^{-17}	$\frac{3}{16}\pm1.76\times10^{-10}$
Gamma ray bursts (GRB980703)	Schaefer (1999) [84]	2.4×10^{-11}	$\frac{3}{16}\pm2.72\times10^{-7}$
Gamma ray bursts (GRB980703)	Schaefer (1999) [84]	3.4×10^{-6}	$\frac{3}{16}\pm1.03\times10^{-4}$
Torque on rotating magnetized toroid	Tu et al. (2006) [85]	1×10^{-19}	$\frac{3}{16}\pm1.76\times10^{-11}$
Magnetohydrodynamics of solar wind	Ryutov (2007) [86]	1×10^{-18}	$\frac{3}{16}\pm5.57\times10^{-11}$
Proca galactic field	Aldelberger et al. (2007) [87]	1×10^{-26}	$\frac{3}{16}\pm5.55\times10^{-15}$
Electron anomalous magnetic moment	Accioly et al. (2010) [76]	1.6×10^{-4}	$\frac{3}{16}\pm7.02\times10^{-4}$,

Table 1. In this table we show different upper bounds on the photon mass obtained through distinct experimental data and astrophysical/cosmological observations. These bounds modify the order of the needed fine-tuning between the 5D parameters b and M, but leave them of the same order of magnitude, providing naturalness for our model.

7 Conclusions and discussion

With this work we contribute to the program of constructing a consistent braneworld scenario from both the high energy physics and cosmological points of view using the tachyonic de Sitter thick braneworld constructed in [46]. We first started by addressing the gauge hierarchy problem with a geometrical approach as in the Randall-Sundrum model and showed that TeV mass scales can be produced from Planck mass scales through a symmetry breaking mechanism that requires a hierarchy of order 20, namely, $Hw_0 \approx 23$ between these parameters of the braneworld model. The second aim was to localize fermion fields in the 3-brane, as part of the whole set of Standard Model matter fields, since gravity, scalar and gauge vector fields were already localized in this expanding braneworld scenario. A third goal consisted in computing the corrections to Coulomb's law coming from the extra dimensional nature of the KK massive gauge vector excitations. This task was accomplished for three different cases in the thin brane limit as well as for a particular thick brane scenario. In the latter case we were able to confront the corrections to the photon mass predicted by the model with experimental and astrophysical/cosmological upper bounds previously established on the photon mass. All of these objectives were successfully achieved.

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This work was carried out by considering four different functions $F(T)$, which establish the concrete 5D Yukawa interaction between fermions and the tachyon condensate scalar field. This coupling functions correspond to different Schrödinger potentials with different asymptotic behaviour along the extra dimension. Moreover, they lead to different mass spectra for the massive KK fermionic modes that influence in a different way the effective physics of the fermionic zero modes localized on the 3-brane which represents our world.

In the first case we set $F(T)$ proportional to the field T. However, for this configuration neither left– nor right-chiral fermions are localized in the 3-brane (although the left Schrödinger potential is of volcano type) since the left-chiral zero mode asymptotically tends to a positive constant and is therefore delocalized from it.

For case II we have field configurations endowed with left and right Schrödinger potentials with infinitely high walls which have discrete mass spectra for the KK modes. The left-chiral fermionic massless zero mode, as well as an infinite number of discrete massive bound states are localized on the brane, whereas the right-chiral zero mode is delocalized from it.

For the case III we found a Schrödinger potential of volcano type for left-chiral fermions, where only the ground state corresponding to the KK massless zero mode is localized and glued to the continuous KK massive spectrum. On the other hand, the corresponding right Schrödinger potential does not localize any right-chiral fermions on the 3-brane.

In the case IV we got modified Pöschl-Teller potentials with mass gaps which allows us to localize both left– and right-chiral fermions on the brane and to get discrete KK mass spectra where the left-chiral massless zero mode is separated from the continuous massive spectrum of KK excitations; for this scenario the right-chiral KK massless zero mode is non-localized on the 3-brane.

As mentioned above, after localizing the fermion fields, our third objective was to make use of the results presented here and in [38], which show that it is possible to localize gauge fields in our braneworld model, in order to study the interaction between photons and fermions localized on the brane. We further performed the computation of the corrections to the Coulomb's law coming from the massive gauge vector modes by considering the aforementioned cases II, III and IV. The computed corrections to the Coulomb's potential exponentially decay due to the presence of a mass gap in the spectrum of the gauge vector fields. Thus, these corrections decay much faster than $1/r$ due to the exponential function that quickly removes all small contributions from the KK massive gauge vector modes in the limit of thin branes.

Moreover, for case IV, it was possible to obtain a novel result that displays the corrections to Coulomb's law in a tachyonic de Sitter braneworld scenario of *arbitrary thickness*, allowing us to get an idea of what would be the effects of the electromagnetic interaction between localized fermions, due to non-localized massive gauge bosons. When confronting the estimated correction to the Coulomb law in terms of the photon mass predicted by our braneworld model with experimental and astrophysical/cosmological upper bounds on the photon mass we find that our prediction is far away from being detected in the near future, leaving the 5D parameters of the braneworld of the same order and providing naturalness to the model itself. However, in order to recover the present value of the Hubble constant (making the braneworld model realistic) we need to perform a fine-tuning on these parameters in the above considered case.

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