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NNLO Mixed QCD–EW Corrections to Drell–Yan Processes in the Resonance Region

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Abstract. We review the application of the pole approximation to the QCD–electroweak corrections of $O(\alpha_s\alpha)$ to W- and Z-boson production at hadron colliders and present results for the numerically dominant corrections, which arise from the combination of the QCD corrections to the production with electroweak corrections to the decay of the W/Z boson. We compare our results to simpler approximations based on naive products of NLO QCD and electroweak correction factors or leading-logarithmic approximations for QED final-state radiation. Finally, we estimate the effect of the $O(\alpha_s\alpha)$ corrections on the measurement of the W-boson mass.

INTRODUCTION

The Drell–Yan-like production of W and Z bosons, $\mathrm{pp}/\mathrm{p\bar{p}} \to V \to l_1\bar{l}_2 + X$, is one of the most prominent classes of particle reactions at hadron colliders. The large production rate and the clean experimental signature of the leptonic vector-boson decay allow these processes to be measured with great precision and render them the most important "standard-candle" processes at the LHC. Of particular relevance for precision tests of the Standard Model is the potential of the Drell–Yan processes at the LHC for high-precision measurements in the resonance regions, where the effective weak mixing angle might be extracted from data with LEP precision. The W-boson mass M_W can be determined from a fit to the distributions of the lepton transverse momentum ($p_{\mathrm{T},l}$) and the transverse mass ($M_{\mathrm{T},\nu l}$) of the lepton pair, allowing for a sensitivity below 10 MeV (see Ref. [1] and references therein).

The Drell–Yan-like production of W or Z bosons is one of the theoretically best understood processes. The current state of the art includes QCD corrections at next-to-next-to-leading-order (NNLO) accuracy, supplemented by leading higher-order soft-gluon effects or matched to QCD parton showers up to NNLO, and electroweak (EW) corrections at next-to-leading order (NLO) and leading universal corrections beyond (see, e.g., references in Ref. [2]). Thus, in addition to the N^{3}LO QCD corrections, the next frontier in fixed-order computations is given by the mixed QCD–EW corrections of $O(\alpha_s\alpha)$, which can affect observables relevant for the M_W determination at the percent level. Currently these effects are approximated, e.g., in a parton-shower framework where the virtual NLO corrections and the first emitted photon or gluon are treated exactly, while further emissions are generated in the collinear approximation (see Refs. [3, 4] and references therein). However, a full NNLO calculation at $O(\alpha_s\alpha)$ is necessary for an ambiguity-free combination of NLO QCD and NLO EW corrections. Here some partial results for two-loop amplitudes [5, 6, 7] as well as the full $O(\alpha_s\alpha)$ corrections to the W/Z decay widths [8, 9] are known. A complete calculation of the $O(\alpha_s\alpha)$ corrections requires to combine the double-virtual corrections with the $O(\alpha)$ EW corrections to W/Z + jet production, the $O(\alpha_s)$ QCD corrections to W/Z + γ production, and the double-real corrections (see references in Ref. [2]).

In a series of two recent papers [2, 10], we have initiated the calculation of the $O(\alpha_s\alpha)$ corrections to Drell–Yan processes in the resonance region via the so-called *pole approximation* (PA). In this contribution we outline the salient features of the PA at $O(\alpha_s\alpha)$ and discuss our numerical results on the dominant corrections in this order, which are the "initial–final" factorizable corrections. We compare them to different versions of a naive product ansatz obtained by multiplying NLO QCD and EW correction factors, and to a leading-logarithmic treatment of photon radiation as provided by the structure-function approach or QED parton showers such as PHOTOS [11]. We further estimate the

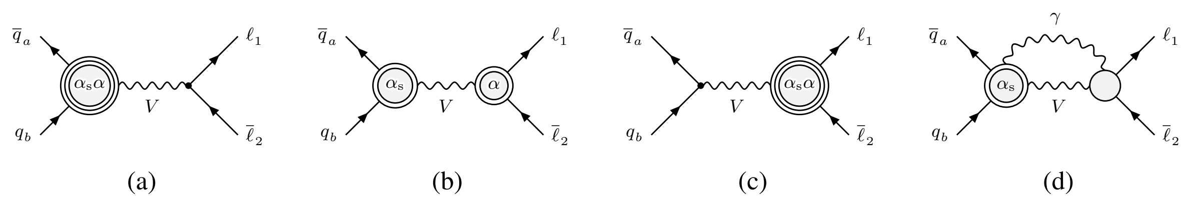

FIGURE 1. The four types of corrections that contribute to the mixed QCD–EW corrections in the PA illustrated in terms of generic two-loop amplitudes: factorizable corrections of initial–initial (a), initial–final (b), and final–final type (c), as well as non-factorizable corrections (d). Simple circles symbolize tree structures, double (triple) circles one-loop (two-loop) corrections.

effect of the NNLO $O(\alpha_s\alpha)$ corrections on the measurement of the W-boson mass.

POLE APPROXIMATION FOR THE MIXED QCD–EW CORRECTIONS

The PA is based on a systematic expansion of the cross section about the pole of the gauge-boson resonance and splits the corrections into factorizable and non-factorizable contributions. The former can be separately attributed to the production and the subsequent decay of the gauge boson, while the latter link the production and decay subprocesses by the exchange of soft photons. The PA has been applied to $O(\alpha)$ corrections in several variants [10, 12, 13] and shows agreement with the known NLO EW corrections up to fractions of 1% near the resonance, i.e. at a phenomenologically satisfactory level. In particular, the bulk of the NLO EW corrections near the resonance is due to the factorizable corrections to the W/Z decay subprocesses, while the factorizable corrections to the production process are mostly suppressed below the percent level, with the non-factorizable contributions being even smaller.

The quality of the PA at NLO motivates its application to the calculation of the NNLO mixed QCD–EW corrections. The structure of the PA for this case has been worked out in detail in Ref. [10] and provides a classification of the $O(\alpha_s\alpha)$ corrections into the four types of contributions shown in Fig. 1 for the case of the double-virtual corrections:[1]

(a) The initial–initial factorizable corrections are given by two-loop $O(\alpha_s\alpha)$ corrections to on-shell W/Z production and the corresponding one-loop real–virtual and tree-level double-real contributions, i.e. W/Z + jet production at $O(\alpha)$, W/Z + γ production at $O(\alpha_s)$, and the processes W/Z + γ + jet at tree level. Results for individual ingredients are known, however, a consistent combination of these building blocks using a subtraction scheme for infrared singularities at $O(\alpha_s\alpha)$ has not been performed yet.

(b) The factorizable initial–final corrections consist of the $O(\alpha_s)$ corrections to W/Z production combined with the $O(\alpha)$ corrections to the leptonic W/Z decay. Their computation is described in detail in Ref. [2]. The main results are presented below.

(c) Factorizable final–final corrections arise from the $O(\alpha_s\alpha)$ counterterms of the lepton–W/Z-vertices, which involve only QCD corrections to the W/Z self-energies [14]. They yield a relative correction below 0.1% [2] and have no impact on the shape of distributions, so that they are phenomenologically negligible.

(d) The non-factorizable $O(\alpha_s\alpha)$ corrections are given by soft-photon corrections connecting the initial state, the intermediate vector boson, and the final-state leptons, combined with QCD corrections to W/Z-boson production. They can be calculated in terms of soft-photon correction factors to squared tree-level or one-loop QCD matrix elements [10] and are numerically below 0.1%. Thus, for phenomenological purposes the $O(\alpha_s\alpha)$ corrections can be factorized into terms associated with initial-state and/or final-state corrections and their combination.

The factorizable initial–initial corrections (a) are the only currently missing $O(\alpha_s\alpha)$ corrections within the PA. Results of the PA at $O(\alpha)$ show that observables such as the $M_{T,\nu l}$ distribution for W production or the M_{ll} distributions for Z production are extremely insensitive to photonic initial-state radiation (ISR) [10]. Since these distributions also do not receive overwhelmingly large QCD corrections, we do not expect significant initial–initial NNLO $O(\alpha_s\alpha)$ corrections to such distributions. Furthermore, they would require $O(\alpha_s\alpha)$-corrected PDFs for a consistent evaluation, which are however not available. On the other hand, the factorizable corrections of the type"initial–final" (b) combine two types of corrections that are sizeable at NLO and deform the shape of differential distributions. Therefore we expect this class of the factorizable corrections to capture the dominant $O(\alpha_s\alpha)$ effects.

[1]For each class of contributions with the exception of the final–final corrections (c), also the associated real–virtual and double-real corrections have to be computed, obtained by replacing one or both of the labels α and α_s in the blobs in Fig. 1 by a real photon or gluon, respectively, and taking corresponding crossed partonic channels, e.g. with quark–gluon initial states, into account.

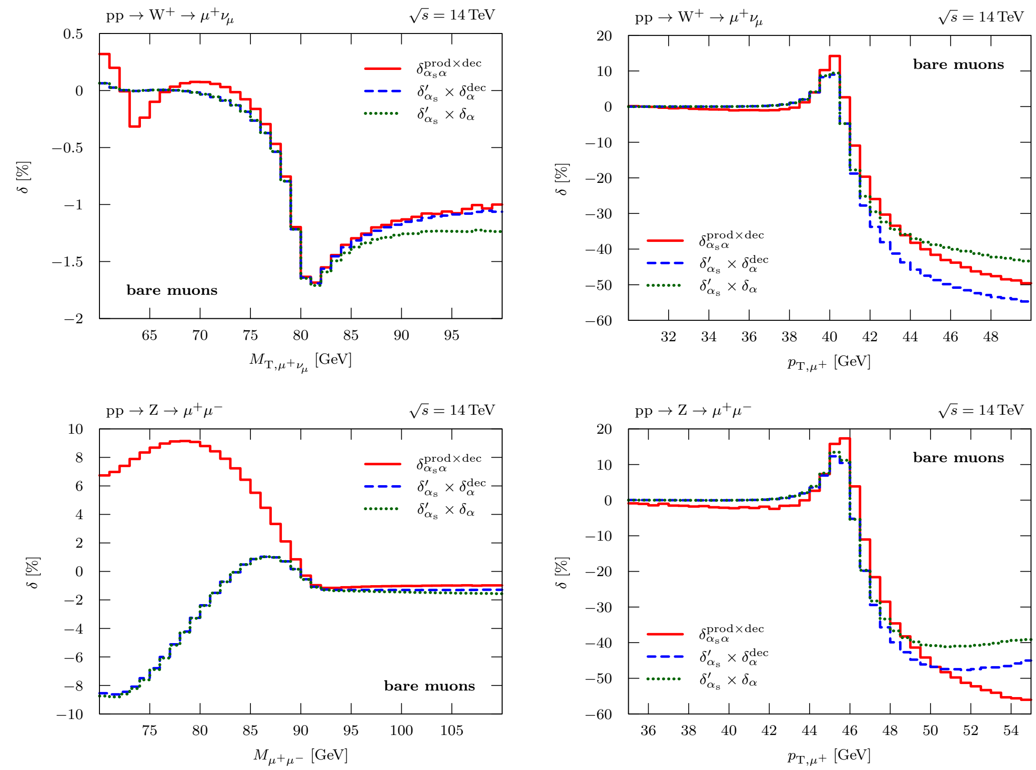

FIGURE 2. Relative factorizable corrections of $O(\alpha_s\alpha)$ induced by initial-state QCD and final-state EW contributions. Above: transverse-mass (left) and transverse-lepton-momentum (right) distributions for W^+ production at the LHC. Below: lepton-invariant-mass distribution (left) and a transverse-lepton-momentum distribution (right) for Z production at the LHC. The naive products of the NLO correction factors δ'_{α_s} and δ_α are shown for comparison. (Taken from Ref. [2].)

NUMERICAL RESULTS FOR THE DOMINANT $O(\alpha_s\alpha)$ CORRECTIONS

In the following we present our results for the dominant $O(\alpha_s\alpha)$ NNLO corrections to the Drell–Yan cross section in the resonance region, which are given by the initial–final factorizable corrections (Figure 1 (b)). We consider isolated ("bare") muons using the setup and input parameters of Ref. [2]. The corresponding corrections for "dressed leptons" using a recombination with collinear photons show the same features, but are typically smaller by a factor of two [2].

Our default prediction $\sigma^{\mathrm{NNLO}_{s\otimes ew}}$ is obtained by adding the factorizable initial–final NNLO corrections $\Delta\sigma^{\mathrm{NNLO}_{s\otimes ew}}_{\mathrm{prod}\times\mathrm{dec}}$ to the sum $\Delta\sigma^{\mathrm{NLO}_s} + \Delta\sigma^{\mathrm{NLO}_{ew}}$ of the full NLO QCD and EW corrections, where all contributions are consistently evaluated with NLO PDFs. The numerically negligible non-factorizable and factorizable final–final corrections are not included. Figure 2 shows the numerical results for the relative $O(\alpha_s\alpha)$ initial–final factorizable corrections

$$\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_s\alpha} \equiv \Delta\sigma^{\mathrm{NNLO}_{s\otimes ew}}_{\mathrm{prod}\times\mathrm{dec}} / \sigma^{\mathrm{LO}} \tag{1}$$

for the $M_{\mathrm{T},\nu l}$ and the $p_{\mathrm{T},l}$ distributions for W^+ production at the LHC. For Z production, the results for the M_{ll} distribution and a transverse-lepton-momentum (p_{T,l^+}) distribution are displayed. In order to check the validity of simpler estimates of the NNLO QCD–EW corrections, the plots also show the product $\delta'_{\alpha_s}\delta_\alpha$ of the QCD and EW correction factors

$$\delta'_{\alpha_s} \equiv \Delta\sigma^{\mathrm{NLO}_s}/\sigma^{\mathrm{LO}}, \qquad \delta_\alpha \equiv \Delta\sigma^{\mathrm{NLO}_{ew}}/\sigma^0, \tag{2}$$

which arises in the relative difference of our default NNLO prediction $\sigma^{\mathrm{NNLO}_{s\otimes ew}}$ and a naive product ansatz $\sigma^{\mathrm{NNLO}_{s\otimes ew}}_{\mathrm{naive\ fact}} = \sigma^{\mathrm{NLO}_s}(1+\delta_\alpha)$. Note that the LO prediction σ^{LO} is evaluated with LO PDFs, whereas σ^0 is evaluated using NLO PDFs. The relative NLO EW corrections are defined in two different versions: First, based on the full $O(\alpha)$ correction (δ_α), and second, based on the dominant EW final-state correction of the PA ($\delta_{\alpha}^{\mathrm{dec}}$). Any large deviations between $\delta_{\alpha_s\alpha}^{\mathrm{prod}\times\mathrm{dec}}$ and $\delta'_{\alpha_s}\delta_{\alpha}^{(\mathrm{dec})}$ can be attributed to the double-real emission corrections, which do not take the reducible form of a product of two NLO corrections, in contrast to the other initial–final factorizable contributions [2]. The difference of the naive products defined in terms of $\delta_{\alpha}^{\mathrm{dec}}$ and δ_{α} indicates the impact of the missing $O(\alpha_s\alpha)$ corrections beyond the initial–final corrections considered in our calculation and therefore also provides an error estimate of the PA, and in particular of the omission of the corrections of initial–initial type.

For the $M_{\mathrm{T},\nu l}$ distribution for W$^+$ production (upper left plot in Figure 2), the mixed NNLO QCD–EW corrections amount to approximately -1.7 % around the resonance, which is about an order of magnitude smaller than the NLO EW corrections. Both variants of the naive product provide a good approximation to the full result in the region around and below the Jacobian peak, which is dominated by resonant W production. This can be attributed to well-known insensitivity of the observable $M_{\mathrm{T},\nu l}$ to ISR effects already seen for the NLO corrections [10]. For larger $M_{\mathrm{T},\nu l}$, the product $\delta'_{\alpha_s}\delta_{\alpha}$ based on the full NLO EW correction factor deviates from the other curves, which signals the growing importance of effects beyond the PA. However, the deviations amount to only few per-mille for $M_{\mathrm{T},\nu l} \lesssim 90$ GeV.

The corrections to the $p_{\mathrm{T},l}$ distributions (right plots in Figure 2) are small far below the Jacobian peak, but rise to about 15% (20%) on the Jacobian peak at $p_{\mathrm{T},l} \approx M_V/2$ for the case of the W$^+$ boson (Z boson) and then drop to almost -50% at $p_{\mathrm{T},l} = 50$ GeV. This enhancement of corrections above the Jacobian peak arises already in the NLO QCD results (see e.g. Fig. 8 in Ref. [10]) where the recoil due to real QCD radiation shifts events with resonant W/Z bosons above the Jacobian peak. The naive product ansatz deviates from the full result $\delta_{\alpha_s\alpha}^{\mathrm{prod}\times\mathrm{dec}}$ by 5–10% at the Jacobian peak, where the PA is expected to be the most accurate. This can be attributed to the strong influence of the recoil induced by ISR on $p_{\mathrm{T},l}$, which implies a larger effect of the double-real emission corrections on this distribution, which are not captured correctly by the naive products. The two versions of the naive products display larger deviations than in the $M_{\mathrm{T},\nu l}$ distribution, which signals a larger impact of the missing $O(\alpha_s\alpha)$ initial–initial corrections.[2]

In the M_{ll} distribution for Z production (lower left plot in Figure 2), corrections up to 10% are observed below the resonance. This is consistent with the large NLO EW corrections from photonic final-state radiation (FSR) that shifts the reconstructed value of M_{ll} away from the resonance $M_{ll} = M_{\mathrm{Z}}$ to lower values. The naive products $\delta'_{\alpha_s}\delta_{\alpha}^{(\mathrm{dec})}$ approximate the full initial–final corrections $\delta_{\alpha_s\alpha}^{\mathrm{prod}\times\mathrm{dec}}$ reasonably well for $M_{ll} \geq M_{\mathrm{Z}}$ but completely fail already a little below the resonance where they do not even reproduce the sign of the full correction $\delta_{\alpha_s\alpha}^{\mathrm{prod}\times\mathrm{dec}}$. This failure can be understood from the fact that the appropriate QCD correction factor for the events that are shifted below the resonance by photonic FSR is given by its value at the resonance $\delta'_{\alpha_s}(M_{ll} = M_{\mathrm{Z}}) \approx 6.5$% [2], whereas the naive product ansatz simply multiplies the corrections locally on a bin-by-bin basis.

Approximating $O(\alpha_s\alpha)$ corrections by leading logarithmic final-state radiation

As is evident from Fig. 2, a naive product of the QCD and EW correction factors (2) is not adequate to approximate the NNLO QCD–EW corrections for all observables. A promising factorized approximation for the dominant initial–final corrections can be obtained by combining the full NLO QCD corrections to W/Z production with a leading-logarithmic (LL) approximation for FSR. For this purpose we have employed a structure-function approach [15] and a simulation of FSR using PHOTOS [11]. Both approaches take the interplay of the recoil effects from jet and photon emission properly taken into account, but neglect certain subdominant finite contributions. In order to compare to our result for the $O(\alpha_s\alpha)$ corrections, we only generate a single photon emission in both implementations of the LL approximation and use the same input-parameter scheme for α as in $\delta_{\alpha_s\alpha}^{\mathrm{prod}\times\mathrm{dec}}$ (see Ref. [2] for details).

In Fig. 3 we compare our best prediction (1) for the factorizable initial–final $O(\alpha_s\alpha)$ corrections for W$^+$ and Z production to the combination of NLO QCD corrections with the two FSR approximations. For the structure-function approach (denoted by LL^{1}FSR), the intrinsic uncertainty of the LL approximation is illustrated by the band width resulting from varying the QED scale Q within the range $M_V/2 < Q < 2M_V$ for $V = \mathrm{W,Z}$. We observe a clear improvement compared to the naive product approximations investigated above, in particular for the M_{ll} distribution in Z production, which is correctly modelled by both FSR approximations, whereas the naive products completely failed to describe this distribution. In the $M_{\mathrm{T},\nu l}$ spectrum of the charged-current process one also finds good agreement of the different results below the Jacobian peak and an improvement over the naive product approximations in Fig. 2. The description of the $p_{\mathrm{T},l}$ distributions is also improved compared to the naive product approximations, but some differences remain in the charged-current process.

[2] These deviations should be interpreted with care, since the peak region $p_{\mathrm{T},l} \approx M_V/2$ corresponds to the kinematic onset for V + jet production where fixed-order predictions break down and QCD resummation is required for a proper description.

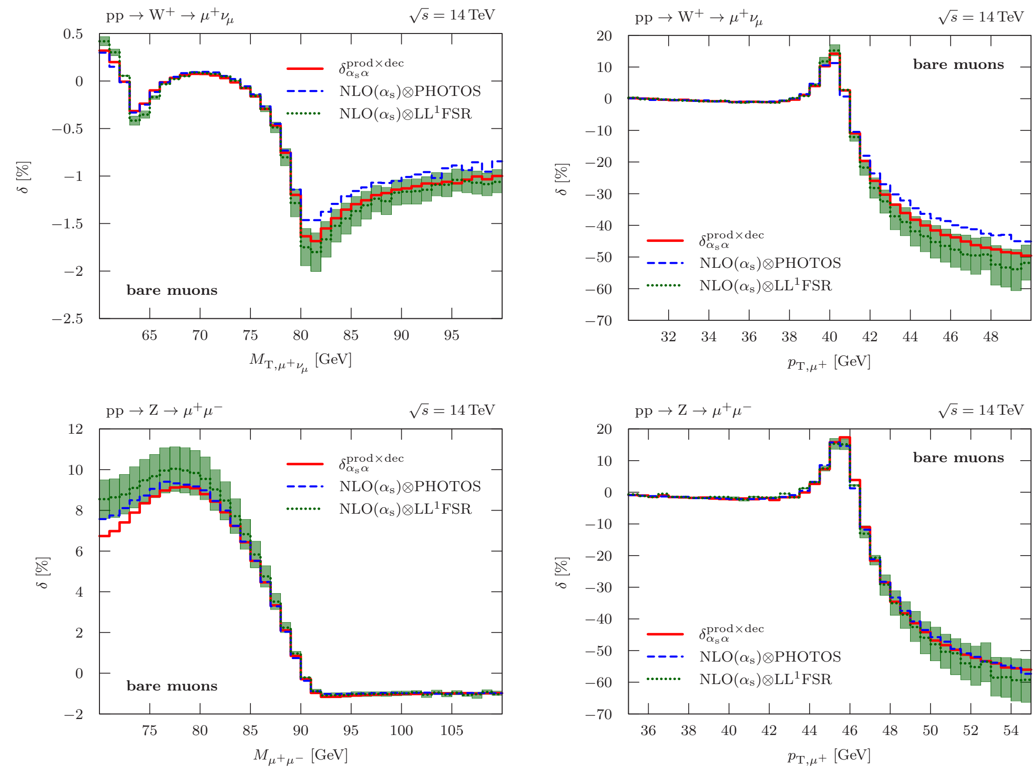

FIGURE 3. Comparison of the approximation obtained from PHOTOS and from the structure-function (LL[1]FSR) approach for the relative $\mathcal{O}(\alpha_s\alpha)$ initial-state QCD and final-state EW corrections to our best prediction $\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_s\alpha}$. Above: transverse-mass (left) and transverse-lepton-momentum (right) distributions for W^+ production at the LHC. Below: lepton-invariant-mass distribution (left) and a transverse-lepton-momentum distribution (right) for Z production at the LHC. (Taken from Ref. [2].)

Impact on the W-boson mass extraction

In order to estimate the effect of the $\mathcal{O}(\alpha_s\alpha)$ corrections on the M_W measurement at the LHC we have performed a χ^2 fit of the $M_{\mathrm{T},\nu\ell}$ distribution in the interval $M_{\mathrm{T},\nu\ell} = [64,\ 91]$ GeV . We treat the $M_{\mathrm{T},\nu\ell}$ spectra calculated in various theoretical approximations for a reference mass $M_\mathrm{W}^\mathrm{OS} = 80.385$ GeV as "pseudo-data" that we fit with "templates" calculated using the LO predictions σ^0 for different values of M_W^OS. The best-fit value $M_\mathrm{W}^\mathrm{fit,th}$ quantifying the impact of a higher-order correction in the theoretical cross section σ^th is then obtained from the minimum of the function

$$\chi^2(M_\mathrm{W}^\mathrm{fit,th}) = \sum_i \left[\sigma_i^\mathrm{th}(M_\mathrm{W}^\mathrm{OS}) - \sigma_i^0(M_\mathrm{W}^\mathrm{fit,th})\right]^2 / (2\Delta\sigma_i^2)\,, \tag{3}$$

where the sum over i runs over $M_{\mathrm{T},\nu l}$ bins in steps of 1 GeV. Here σ_i^th and σ_i^0 are the integrated cross sections in the i-th bin, uniformly rescaled so that the sum over all bins is identical for all cross sections. We assume a statistical error of the pseudo-data, taking $\Delta\sigma_i^2 \propto \sigma_i^\mathrm{th}$. We do not attempt to model detector effects that are expected to affect the different theory predictions in a similar way and to cancel to a large extent in our estimated mass shift.

Using the prediction $\sigma^{\mathrm{NLO}_\mathrm{ew}}$ as the pseudo-data σ^th in (3) we estimate the mass shift due to the NLO EW corrections as $\Delta M_\mathrm{W}^{\mathrm{NLO}_\mathrm{ew}} \approx -90$ MeV (-40 MeV) for bare muons (dressed leptons) [2]. We have also estimated the effect of multi-photon radiation and obtained a mass shift $\Delta M_\mathrm{W}^\mathrm{FSR} \approx 9$ MeV relative to the result of the fit to the NLO EW prediction for bare muons. These values are comparable to previous results reported in Ref. [16].[3] To estimate the impact of the initial–final $\mathcal{O}(\alpha_s\alpha)$ corrections we consider the mass shift obtained by using our best prediction (1) relative to

[3]The results of Ref. [16] cannot be compared directly to our results, since different event-selection criteria are used. Note that the role of pseudo-data and templates is reversed in Ref. [16] so that the mass shift has the opposite sign.

that obtained for the sum of the NLO QCD and EW corrections. We obtain $\Delta M_{\mathrm{W}}^{\mathrm{NNLO}} \approx -14$ MeV (-4 MeV) for bare muons (dressed leptons) [2], which provides a simple estimate of the impact of the full $O(\alpha_s\alpha)$ corrections on the M_{W} measurement.

CONCLUSIONS

The precision-physics program in Drell–Yan-like W- and Z-boson production at the LHC requires a further increase in the accuracy of the theoretical predictions, where the mixed QCD–electroweak corrections of $O(\alpha_s\alpha)$ represent the largest component of fixed-order radiative corrections after the well established NNLO QCD and NLO electroweak corrections. In this contribution, we have reviewed the major results of our two recent papers [2, 10], where we have established a framework for evaluating the $O(\alpha_s\alpha)$ corrections to Drell–Yan processes in the resonance region using the pole approximation and presented the calculation of the non-factorizable and most important factorizable corrections. The non-factorizable corrections [10] and the factorizable corrections corresponding solely to the W/Z decay subprocesses [2] turned out to be phenomenologically negligible. Moreover, an analysis of the NLO corrections in pole approximation suggests that the factorizable corrections corresponding to the production subprocess, which are yet unknown, will have a minor impact on the observables relevant for the W-boson mass measurement.

We have summarized our numerical results [2] of the dominant factorizable corrections of $O(\alpha_s\alpha)$, which arise from the combination of sizeable QCD corrections to the production with large EW corrections to the decay subprocesses. Naive product approximations fail to capture these corrections in distributions that are sensitive to QCD initial-state radiation and therefore require a correct treatment of the double-real-emission part of the NNLO corrections. Naive products also fail to capture observables that are strongly affected by a redistribution of events due to final-state real-emission corrections, such as the invariant-mass distribution of the neutral-current process. A combination of the NLO QCD corrections and a collinear approximation of real-photon emission through a QED structure-function approach or a QED parton shower such as PHOTOS provides a significantly better agreement with our results. In particular, for the invariant-mass distribution in Z-boson production both collinear approximations model the redistribution of events due to final-state radiation, which is responsible for the bulk of the corrections in this observable.

We have estimated the effect of the $O(\alpha_s\alpha)$ corrections on the M_{W} measurement to ≈ -14 MeV for the case of bare muons and ≈ -4 MeV for dressed leptons. These corrections therefore have to be properly taken into account in the W-boson mass measurements at the LHC, which aim at a precision of about 10 MeV.

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