

A DEMOCRITIAN APPROACH TO
ELEMENTARY PARTICLE PHYSICS

H. Pierre Noyes

Stanford Linear Accelerator Center
Stanford University, Stanford, California

I. INTRODUCTION.

Theoretical physicists are caught in a maze of exciting experimental results. Many of us are charmed by the situation, and not a few believe that "charm" is Ariadne's thread, which will guide us out of this labyrinth to a decisive confrontation. Like the elusive quarks to which it is tied, and the concept of "strangeness" which led us to them, this thread can be counted on to lead us around many new corners into new puzzles. But this discovery also gives us an opportunity to ask whether some of the puzzles are of our own making, whether we might not climb up over the walls rather than being confined to using familiar search strategies.

The approach I take in this seminar is first to listen to some of our critics. We claim to be studying the basic phenomena on which all natural science rests, yet¹

"...good mathematical physicists who are not in the field make no attempt to master it - or else have made an attempt and given it up. ... It seems that to think particle physics you have to give assent to a private language which is only compatible with the language of the rest of physics at the expense of following a whole series of very complicated rules of procedure."

I trust that most of you are sensitive enough to our delicate relationships to sources of both public and private funds to realize that this type of evaluation could easily lead to our supply train being cut off at the pass.

*Work supported by the Energy Research and Development Administration.

Our problem goes deeper than the use of technical jargon. We often fail to formulate our hypotheses in such a way that they can be disproved. Indeed, some of us may even do this deliberately.^{2,3} When we ask our experimental colleagues to test one of our hypotheses, we should first ask ourselves whether we have specified a possible outcome which would convince us that our hypothesis was wrong. We should also be prepared to take the responsibility for our mistakes when experiment goes against us, and to try to pinpoint past mistakes in framing and presenting new hypotheses. Of course I am stating the obvious; I wish this obvious statement had more connection with practice.

Many of us are unaware of the non-trivial connections between the acausal aspects of quantum mechanics and the disillusionment with deterministic science and its accompanying technology following World War I.⁴ I know from sad experience that few physicists now assent to the proposition that quantum mechanics is not the product of experimental experience and hence innocent of philosophy, let alone Forman's contention⁴ that it is at least partly the result of an active search within physics for a rupture with the past capable of breaking the chains of classical determinism. Even fewer take the trouble to study available historical records before making dogmatic statements.

I realize that mentioning criticism from outside our own "charmed" circle carries little weight with many of you. But Einstein is still a name that I can conjure with. He never accepted the radical rupture with classical causality incorporated in quantum mechanics.⁵ Yet his criticisms did not bear experimental fruit until the 1960's. As I will discuss below, his chosen experiment crisis went against his philosophical

conclusions. Paradoxically, many physicists have chosen to ignore his criticism of quantum mechanics, while swallowing at one gulp his acceptance of the space-time continuum. In contrast, Bridgman⁶ abstracted from Einstein the methodology which he judged to have led to success - the operational approach - and then used this weapon to attack the foundations of both general relativity and quantum mechanics. Most of Bridgman's criticisms are still unanswered, but as I indicate below some of them can be approximately met. Yet I find Bridgman's belief that by adopting operational definitions physics can disentangle itself from the quagmire of philosophy at best a naive faith.

Of more immediate technical interest to the problems of elementary particle physics is the discussion of quantum electrodynamics by Bohr and Rosenfeld.⁷ This theory (QED) forms a bridge between classical and quantum physics thanks to the fact that, assuming sources with continuously variable mass and charge, the only dimensional constants which enter the theory are c and \hbar . Thus, the wavelength of the radiation can be arbitrarily large compared to the detection apparatus, and the structure of the theory can be explored using classically describable equipment. Quantum mechanics enters the analysis only through the assumption that the positions and momenta of the parts used to detect the electromagnetic radiation obey the Heisenberg uncertainty relations. Two years of work enabled Bohr and Rosenfeld to construct the limits of measurability of the electromagnetic field in finite volumes (not at points) by designing sufficiently complicated (conceptual) apparatus, and proved that these limits coincide with the commutation relations derived more simply via "second quantization." But if mass (as they point out) or

charge are quantized, the scale invariance of the theory disappears, and with it the correspondence limit on which their derivation is based. Consequently, from an operational point of view, there is no available justification for the second quantization of the "matter field." This suggests that we should be cautious in accepting any aspect of current "quantum field theories" that cannot be related to observable "massless" radiation. In particular, the properties of Feynman diagrams that have served as a heuristic guide - even in S-matrix theory - are suspect.

II. METHODOLOGY

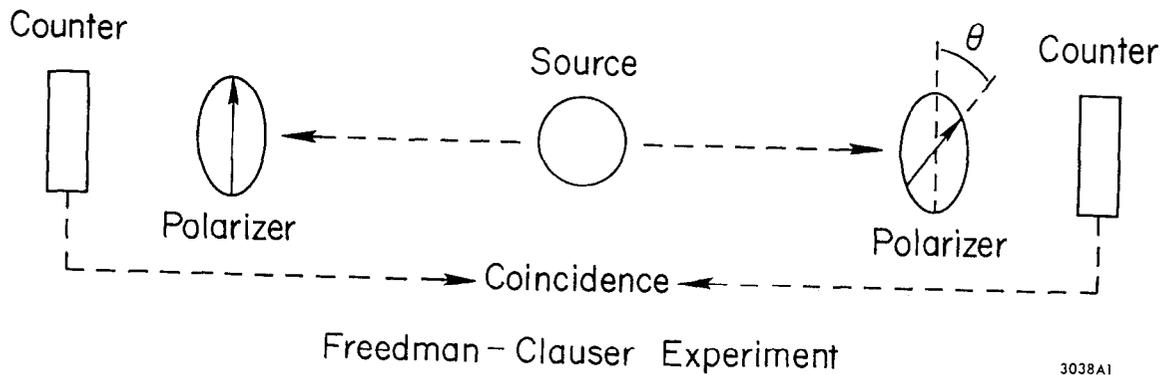
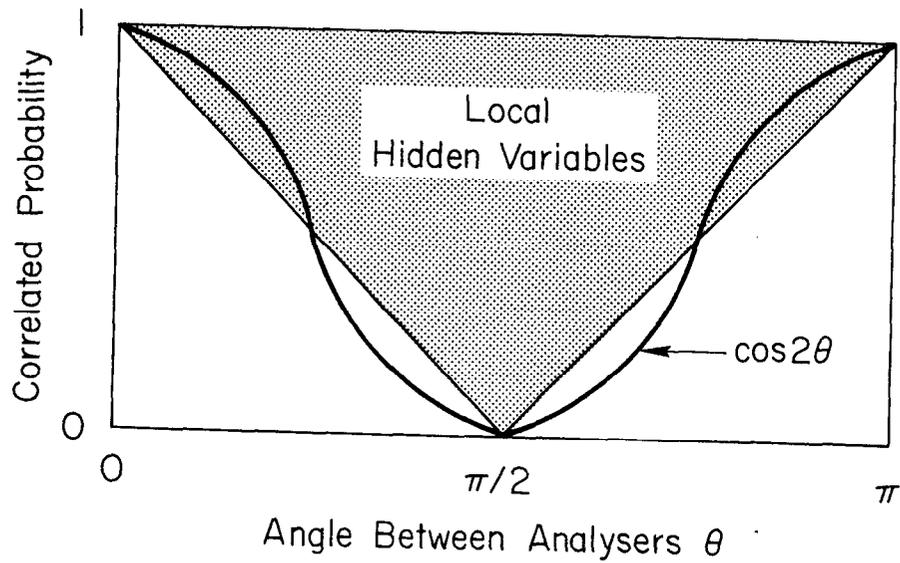
Although my introductory remarks are critical of current theory, my intent is not to be destructive. Quantum field theory has provided a guide to the experimental discovery of many new and exciting phenomena. Our problem is more an embarrassment of riches than a poverty of possibilities. Under like circumstances, it has often proved useful to pare down the number of hypotheses needed to correlate the data to a minimum (Occam's Razor) and to relate these hypotheses as closely as possible to laboratory operations actually carried out (Bridgman's operational analysis). Once this analysis has been performed, it might prove possible to synthesize new structures which have previously been obscured by the bric-a-brac. A complementary approach is to try to isolate a few philosophical principles which are compatible with the analysis, and to explore what minimal set of hypotheses might be added in order to synthesize from them an explanatory structure capable of meeting the problems posed.

Fortunately, several decades of work by a few philosophers, theorists, and experimental physicists have considerably narrowed the field of

theories we need to consider if we wish to make use of experimental results usually interpreted by conventional quantum mechanics as a first approximation. Many of these results can be explained by introducing "local hidden variables" which are classically describable, but subject to unknown randomizing forces analogous to the collisions between point molecules in classical statistical mechanics. However, Bell has shown that if these randomizing forces are not themselves subject to macroscopic correlations (e.g., changing their effect at one slit when the other slit is opened or closed in a double slit experiment), the assumption of statistical independence limits the type of statistical correlation possible in the measurement of two systems. For example, if a spin-0 system decays to two spin- $\frac{1}{2}$ systems whose correlated polarizations are subsequently measured by two polarimeters, the correlated prediction of any local hidden variable theory must lie within the shaded triangle indicated in Figure 1. In contrast, quantum mechanics predicts the unique correlation curve $\cos 2\theta$, which lies outside this region for half the observable range. Hence, an experimental measurement of this curve in the critical interval can either disprove the quantum mechanical prediction or disprove the whole class of local hidden variable theories considered by Bell.

This is a specific example of the "Einstein-Podolsky-Rosen" paradox which Einstein raised as a criticism of quantum mechanics. In a classical deterministic theory, if one spin is measured at one detector and the original system determines the sum of the two spins, we can predict from one measurement what the result will be in the second detector, even though the two detection events cannot be connected by a light signal. In contrast,

BELL'S THEOREM



3038A1

Fig. 1 Local hidden variable theories allow correlations only inside the triangle, while quantum mechanics predicts the unique curve, $\cos 2\theta$. The Freedman-Clauser result gives the quantum mechanical curve and excludes local hidden variable theories.

quantum mechanics predicts that if one particle is detected, there is a statistical correlation between that measurement and what happens at the other detector, even though no causal chain (with $v < c$) can connect the two detections. Thus, the quantum mechanical prediction is non-local in a way that is irreconcilable with relativistic causality, no matter how we introduce local randomization.

A closely related experiment (in a higher spin system) has been performed by Freedman and Clauser.⁹ Their result is consistent with the quantum mechanical prediction, and excludes local hidden variable theories with high probability. Of course, it is always possible to introduce non-local hidden variables which can describe any macroscopic correlation one wishes - but this really does seem to be multiplying hypotheses unnecessarily; if we adopt Occam's Razor, we should accept the quantum mechanical explanation, or something equivalent to it.

If we ignore for the moment the preparation of the correlated system and the structure of the two polarimeters and detectors, the only technical apparatus we need to describe the experimental result is the wave function for two free particles and the Born interpretation of the wave function giving the statistical probability (including correlations) of detecting the two particles. So we concentrate now on the interpretation of this wave function, and in particular its time-development. We have discussed this in detail elsewhere¹⁰, initially following Phipps's critique^{11,12} of the passage from classical to quantum mechanics. In classical Hamilton-Jacobi theory, we have not only the dynamical variables $\underline{p}_k, \underline{q}_k$ but also the initial constants of the motion $\underline{P}_k, \underline{Q}_k$. In the usual "derivations" of quantum mechanics, these degrees of freedom are apparently lost.

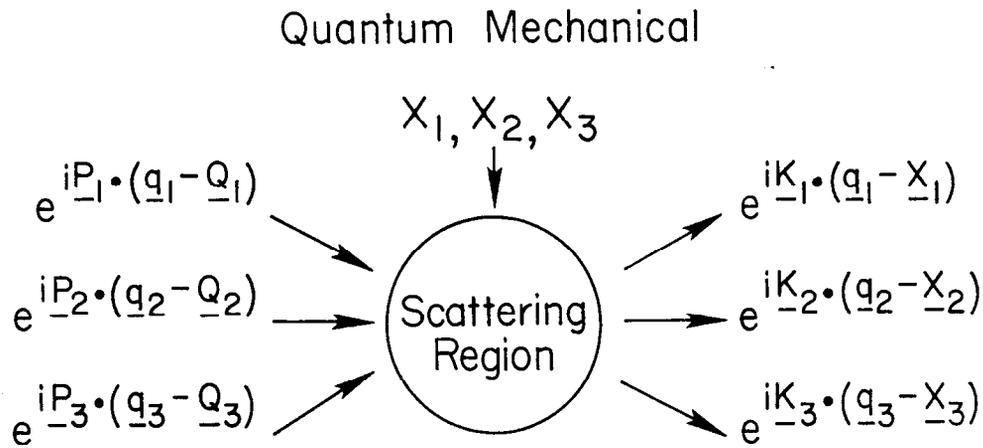
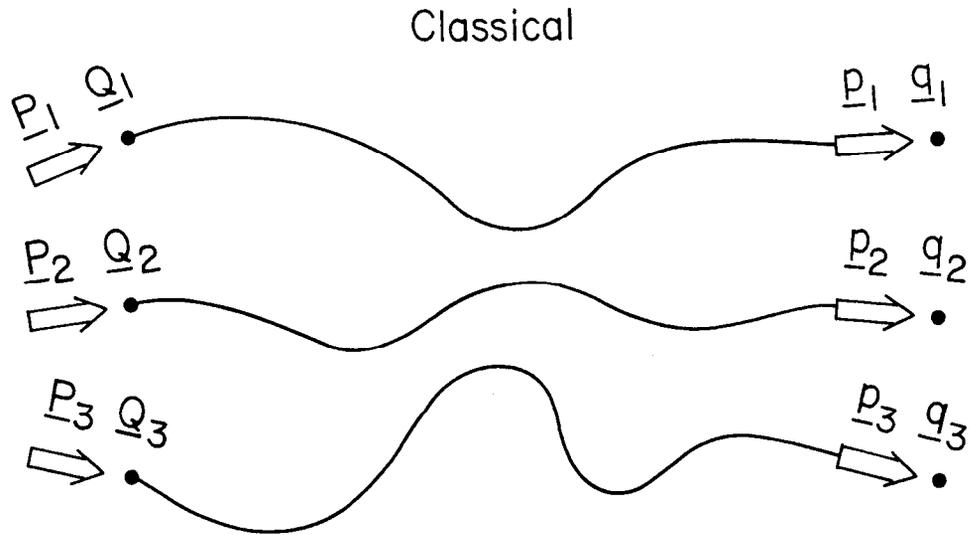
As he justly remarks¹³:

"There is nothing to be happy about in a theory that claims to embody a formal "Correspondence," yet absent-mindedly mislays half the classical canonical variables in the process, then covers its nakedness with a fog of blather about "mind," which could just as well be the "God" whose sensorium provided Newton with such convenient cover in circumstances of like embarrassment. I'm pretty absent-minded myself, but when it comes to counting parameters I'll take on any performing horse (or non-performing physicist)."

Phipps has shown that, in the quantum limit he defines, these degrees of freedom can be retained, and provide the conventional Schroedinger wave function with a phase factor $\exp -i\sum_{\mathbf{k}} \mathbf{P}_{\mathbf{k}} \cdot \mathbf{Q}_{\mathbf{k}}$. Thus, in this limit, the predictive consequences of the usual theory are retained; by assuming that the randomness of quantum processes arises from the abrupt change in these parameters when "virtual" processes are completed and join the fixed past Phipps supplies not only a natural explanation of the severance of phase chains but also a quantal basis for the second law of thermodynamics. This was the original starting point for my interest in the "Fixed Past-Uncertain Future" interpretation of quantum mechanics.¹⁰ (See Figure 2.)

I have noted¹⁴ that when one starts from scattering boundary conditions these parameters occur naturally, if one requires translational invariance. The scattering process is then attributed to an abrupt change in these parameters. If one sums all such changes consistent with (macroscopic) energy and momentum conservation in such a way as to preclude their occurrence as hidden variables, one recovers the usual scattering formalism, with the significant difference that the T-matrix is now an arbitrary function which one can use to describe any quantum process with N_A free particles in and N_B free particles out. The description

FIXED PAST \Rightarrow UNCERTAIN FUTURE



$$\Psi_{\text{PHIPPS}} = e^{-i\sum_k P_k \cdot Q_k} \Phi_{\text{schroed}}(q_k, t)$$

$$\rightarrow e^{-i\sum_k K_k \cdot X_k} \Phi_{\text{schroed}}(q_k, t)$$

3038A3

Fig. 2 Classical Hamilton-Jacobi theory contains both the constants of the motion P_k, Q_k and the dynamical variables p_k, q_k . Ordinary quantum mechanical presentations discuss only the latter variables. Phipps retains the constants of the motion as a phase factor of the Schrodinger wave function, and interprets their sudden change (in the quantum limit) as the source of quantum mechanical irreversibility.

is covariant if T (after removing the usual phase space factors) is a Lorentz scalar, but is not necessarily the matrix element of some "interaction." In fact it need not even be unitary. Thus, we have explicitly exhibited a quantum mechanical scattering formalism in which the kinematic (descriptive) element is cleanly separated from the dynamics (calculation of T). In the paper¹⁴ I go part way toward Bridgman's operational requirements in that my description is carried through using only counts in detectors, plus devices to change energy and momentum; my description is "Democritean" in that, with those exceptions, it contains only particles and the void. By introducing external e.m. fields in the usual way one can then, to first order in $e^2/\hbar c$, describe the operative devices, and by extrapolation to atomic dimensions, their structure. As Finkelstein has pointed out¹⁵, one can define momentum via De Broglie wavelength, using a grating; since a regular array of detectors is a grating, we could further simplify our operational procedure.

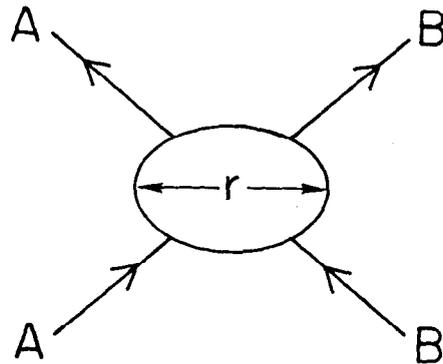
III. AN INTERMEDIATE DYNAMICAL THEORY

Our immediate task is to construct a dynamical theory using only the covariant multiparticle wave function for free particles of finite mass containing the associated, and empirically tested, connections between mass, energy, and momentum. For this purpose we can adopt the simple, but profound, analysis used by Wick¹⁶ to derive the range of nuclear forces in Yukawa's meson theory. If the trajectories of two systems are defined precisely enough so that they coincide within a region of linear dimension r during a time δt , the limiting velocity of special relativity tells us that they can be coherent in this region only if $r < c\delta t$. Applying the Heisenberg uncertainty relation, we find

that $c\delta t \leq \hbar c/\delta E$. If these two systems communicate with some particle of finite mass m , Heisenberg's principle and the mass-energy relation allow this particle to be present if $\hbar c/\delta E \leq \hbar/mc$. If, following Newton, we assume that the overall process conserves momentum, when the two systems subsequently separate so far that the uncertainty relation will no longer support the presence of the particle of mass m , they do not need to share their momenta in the same way as they did initially, and hence can scatter (cf. Figure 3). We further predict that if the energy of the initial system is high enough, the particle of mass m can emerge in the final state along with the two initial systems. The importance of this derivation for our theory is that this Wick-Yukawa mechanism can generate scattering between free particle systems without the necessity of introducing the concept of "interaction." Thus, we can hope to generate at least a partial theory of quantum scattering by making empirical observations of two-particle scattering, and use these observations as input for a dynamical scheme to calculate the behavior of systems containing three or more particles.

This picture of scattering processes already has profound qualitative implications, even in the "non-relativistic" regime where there is no particle creation. In a classical scattering theory based on finite-range "forces", we can study the effects of pairwise scatterings in dilute systems, and then predict uniquely and deterministically the effect of a double scattering in which one of the particles in the first scattering subsequently scatters from a third particle (cf. Figure 4). However, in a quantum mechanical system, study of two-particle scatterings does not allow us to predict uniquely and unambiguously the wave function within

WICK-YUKAWA MECHANISM



$$r < c \Delta t \leq c \frac{\hbar}{\Delta E} \leq \frac{c\hbar}{mc^2} = \frac{\hbar}{mc}$$

limiting uncertainty mass
velocity principle -energy

Finite Mass

Momentum Conservation

3038A2

Fig. 3 Wick's derivation of the range of forces in Yukawa's meson theory using only the uncertainty relations and special relativity.

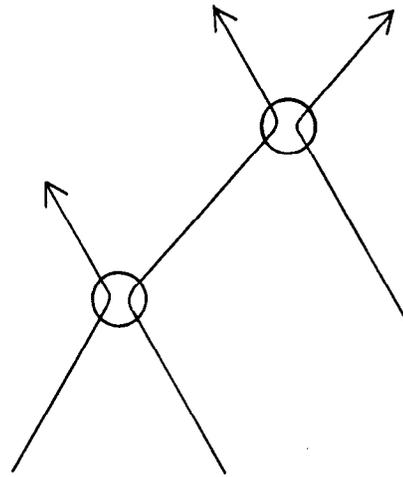
the range of forces. Further, since the scattering process is statistical, the wave function emerging from the first scattering can interfere with this unknown wave function in the second scattering, making the result both unpredictable and irreversible. I have called this the "eternal triangle effect" for reasons discussed elsewhere^{17,3}. Perhaps even more surprising is the fact that the non-local effect caused by the presence of the third particle (even with structureless particles, zero angular momentum, and strictly finite range "potentials") does not fall off with the range of forces, but exists throughout a region of dimension $|a|/R$, where a is the two-body s-wave scattering length and R the range of forces. In fact Efimov¹⁸ has proved in this case that even when the two-body systems contain no bound state, this effect generates a number of three-body bound states which goes to infinity like $(1/\pi) \ln |a|/R$ as $|a|$ goes to infinity. In this limit the bound state spectrum corresponds precisely to the two-body bound state spectrum generated by a "potential" which falls off like $1/r^2$. Note that this effect depends only on the on-shell scattering parameter a , and is independent of the details of the potential inside the range R ; it therefore should persist in a "zero-range" theory, providing that limit can be well defined in the three-body problem.

As an intermediate step toward a zero-range theory we first consider the covariant boundary condition model for three-particle systems developed by Brayshaw^{19,20,21,22}. Two-body scattering data are fitted using a boundary condition at finite radius, whose analytic structure is prescribed by the Faddeev-type three-body equations in which these amplitudes will be used. Outside this radius the hadrons are described,

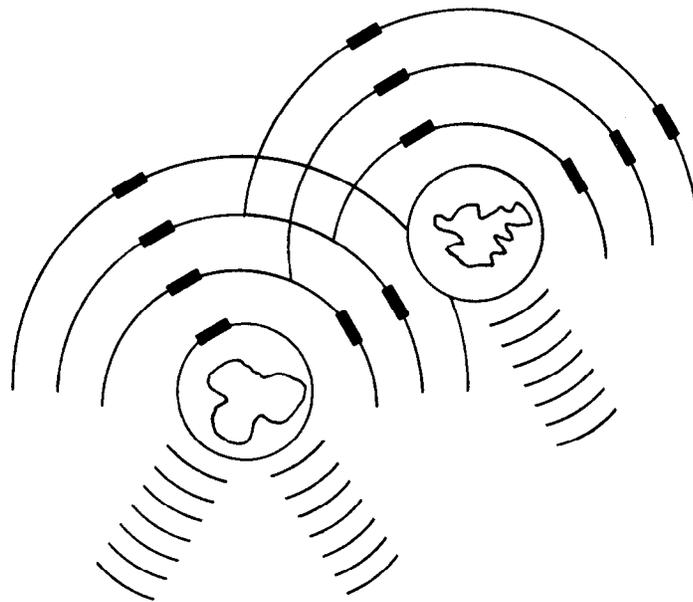
as in a zero-range theory, by free particle wave functions; the inside region may be thought of as the regime that can only be described by quarks, whose function in this model is simply to provide the boundary condition. Brayshaw's results for the non-strange three-hadron systems are summarized in Figure 5. Starting from the $\pi\text{-}\pi$ $I=0$ s-waves at a few hundred Mev (the ϵ_0 region) the boundary condition radius is found to be $\hbar/4m_\pi c$. Using this to calculate the $I=1, 0^- 3\pi$ state, he finds¹⁹ that there is a three-body bound state close in mass to m_π ; if this is identified with the π , the model then predicts the $I=1$ s-wave $\pi\text{-}\pi$ scattering length to be $a_{0,0}^0 \sim 0.23 \hbar/m_\pi c$, in rather good agreement with other estimates of this poorly known parameter. We know from other work that of the three parameters a_{0,m_ρ}^0 , and Γ_ρ only one is independent, so this calculation also determines the $\pi\text{-}\pi$ $I=1$ p-wave, or "rho-meson." Given the rho, it is then possible to calculate the $I=0, 1^- 3\pi$ system and find¹⁹ a single resonance at the position of the ω ; adding the $K\bar{K}$ channel sharpens this resonance to about the observed width. Using the ρ it is possible to get the A_2 , while using both the ρ and the $\pi\text{-}\pi$ s-wave it is also possible to get the A_1 and explain why the A_1 is so difficult to detect experimentally.²² Thus, all the low-lying states of the 3π system can be obtained using only the one parameter $r_{\pi\pi}$. Equally, or perhaps even more, significant is the fact that Brayshaw can prove that the model yields no other low-lying bound states or resonances in this system.

We know from other work that, given the $\pi\text{-}\pi$ and $\pi\text{-}N$ phase-shifts, the main features of the $\pi\pi N$ system emerge, so Brayshaw has not yet examined this system using his model. Further, if one accepts the Chew-Low

ETERNAL TRIANGLE EFFECT



Classical Determinism



Quantum Mechanical Irreversibility
+
Statistical Prediction

3038 A 4

Fig. 4

For finite range systems, study of the scattering of pairs allows a unique prediction of double scattering in a three particle system. For the quantum mechanical three-body problem, the interference between the scattered wave from the first scattering interferes with the second scattering, making the result not only statistically unpredictable but also novel. Thus, the future cannot be unambiguously predicted from the past, and systems evolve. The effect does not fall off with the range of forces R , but instead depends on the dynamical scattering length a and its ratio to R .

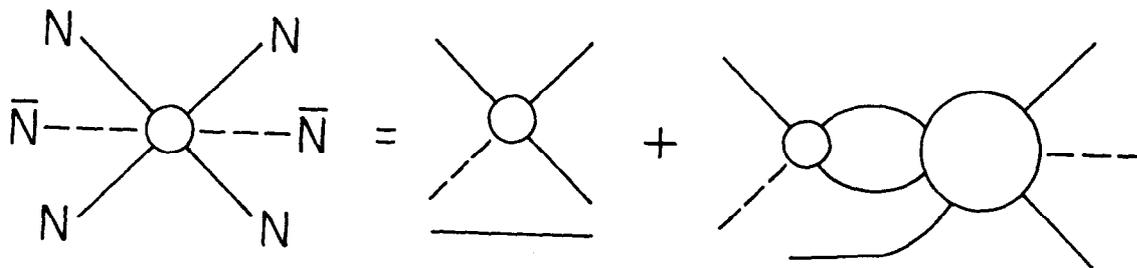
bootstrap²³ connecting the P_{11} and P_{33} states, the pion-nucleon mass ratio and G^2 are at least approximately co-determined by self-consistency. In a boundary condition model, the associated πN radius is approximately $\hbar/M_N c$. For the $NN\pi$ system we need, in addition to the P_{11} state already discussed, the NN radius of $\hbar/2m_\pi c$ and the logarithmic derivative for the singlet and triplet NN S-waves above pion production threshold. As we have shown²⁰, these suffice to determine the most important features of the nuclear force - the deuteron and "singlet deuteron." Unpublished work by Brayshaw shows that many other features of the NN phase shifts also emerge in a satisfactory way²¹, as well as interesting properties of πd scattering and related phenomena. Finally, since we now have calculated the singlet and triplet nucleon scattering lengths and effective ranges, Brayshaw¹⁹ can calculate the NNN system, and finds a triton at about -7.2 Mev, which is as good or better than the binding energy obtained with "realistic nucleon-nucleon potential models." We conclude that the covariant boundary condition model goes a long way toward explaining all of the most significant features of the low-lying states of non-strange three-hadron systems.

Although the basic physics underlying Brayshaw's thinking is that inside the boundary radii lie super-strong quark systems which communicate with the external hadrons in first approximation only through the boundary condition parameters, the actual values of the radii required by experiment - $\hbar/4m_\pi c$, $\hbar/M_N c$, $\hbar/2m_\pi c$ - correspond precisely to the a priori estimate of where the $\pi\pi$, πN , and NN systems become so strongly "interacting" that the two-particle incident channels get lost in a hadronic soup. Thus, it may be possible to think of the boundary condition radii

as a convenient way of parameterizing this estimate, or equivalently as the radii of the "diffractive discs" generated by unitarity due to the opening of particle production channels at high energy. Thus, lacking any evidence for free quarks, Occam's Razor would advise us to try to construct a model which has only free particles, plus the unitarity requirements of particle production. I am actively engaged in trying to construct such a model as a zero-range limit of the successful boundary condition model, but so far all my efforts contain flaws. I therefore must confine myself here to indicating how such a theory might go, with no guarantee that it will be successful.

The conceptual outline of my zero-range theory is indicated in Figure 6. The starting point for hadrons would be to postulate that the π is a bound state of the nucleon-antinucleon system, and then try to calculate the nucleon as a bound state of a nucleon plus a nucleon-antinucleon pair, using the pion bound state in the nucleon-antinucleon channel as the only input. The kinematics would be the same as for Brayshaw's successful $\pi - 3\pi$ bound state calculation (three equal masses), but with different quantum numbers and different dynamical input. If this works, the self-consistency condition would determine a relation between the residue at the bound state pole (G^2) and the m_π/M_N mass ratio. The second relation would come from the Chew-Low bootstrap, leaving no free parameters in the theory. By introducing particle production processes, I should then be able to predict the boundary condition radii, and start to reproduce Brayshaw's successes from a theory including anti-nucleons, and hence some approximation to crossing, without relying on phenomenology.

ZERO RANGE THEORY
(conceptual)



$$N\bar{N} \rightarrow \pi \quad (\text{postulate})$$

$$N\bar{N}N \rightarrow N \quad (\text{Chew-Low Bootstrap})$$

Extension (QED)

$$e\bar{e} \rightarrow \gamma$$

$$e\bar{e}e \rightarrow e$$

Extension (E.M.-weak)

$$e + \bar{e} \rightarrow \pi + \pi$$

$$\rightarrow \nu_e + \bar{\nu}_e$$

$$\rightarrow \psi$$

3038A5

Fig. 6 Summary of a conceptual zero range theory (see text).

If the hadronic calculations work, the next step will be to attempt to construct a theory for charged particles, starting from the assumption that the photon is a 1^- zero mass bound state of a spin $\frac{1}{2}$ particle and its antiparticle. The consistency condition would then be that this bound state generates the particle as a bound state of a particle and a particle-antiparticle pair. In this instance, I would have to be able to reconstruct the renormalized perturbation theory of QED up to the point where hadronic processes enter. Since even Compton scattering, from this point of view, is a five-body problem, you can see why I intend to start with hadrons rather than QED. To bring hadrons into the QED calculation, I would initially limit the high energy behavior by adding to the assumption that γ is a bound $e\bar{e}$ state; the inelastic threshold $e\bar{e} \rightarrow \pi\pi$ will then serve to remove ultraviolet divergences. Weak interactions enter through the channel $e\bar{e} \rightarrow \nu_e, \bar{\nu}_e$, which can be fixed phenomenologically using the Gipsy. Note that by always formulating the theory in terms of two-body on-shell unitary amplitudes, and equations which generate three (or more) particle unitary on-shell states, I need never encounter infinite quantities. This, basically, is what I mean by a Democritean theory - one which at each stage contains only a finite number of free particles of finite energy.

Although the program just outlined may, in time, generate the equivalent of "coupling constants" self-consistently, it is by no means guaranteed of success. I therefore find it useful to think of coupling constants in another way, which I arrived at by a generalization of an argument given by Dyson.²⁴ The renormalized perturbation series for QED can be viewed as an expansion in the number of particle-antiparticle pairs

which can be present in the system. If this number of pairs N_{qq} is confined to a volume whose linear dimension is their own Compton wavelength, the corresponding electrostatic energy is $\approx N_{qq} e^2 / (\hbar/mc) = N_{qq} (e^2/\hbar c) mc^2$. If we are using a theory in which like charges attract, which corresponds to replacing e^2 by $-e^2$ in the QED series, once we go beyond 137 terms this electrostatic energy can suck additional pairs out of the vacuum with a net gain in energy, and the energy of the system collapses to minus infinity. Dyson argued that this proved that the QED series is not absolutely convergent. I would use the same observation to conclude that we cannot, by electromagnetic measurement, define what we mean by more than 137 charged particle pairs within their own Compton wavelength. So stated, $\hbar c/e^2$ becomes a restriction on operationally definable particle number, and hence a "Democritean" definition of what I mean by a "coupling constant."

The generalization of this particulate definition to other "interactions" is immediate (cf. Figure 7). $N_{Gp} \sim 10^{38}$ is the maximum number of gravitating protons which can be defined within their own Compton wavelength using gravitational measurements. In this case the Dyson singularity is not mathematical but physical - they disappear down a Laplacian²⁵ black hole. Similarly, the maximum number of pions which can be defined hadronically within their own Compton wavelength is 10 (using $f^2 \sim 0.1$, rather than 0.08, to compensate for the exponential in the Yukawa potential). For superstrong interactions we can only define isolated systems with quark number 0 or 3. Thus, we have a Democritean description of the familiar sequence 3, 10, 137, $\sim 10^{38}$. If we wish to go behind the apparatus of free particle scattering theory at zero range

COUPLING CONSTANTS

Dyson Argument

ELECTROMAGNETISM

$$\begin{aligned} \Delta E &\approx \frac{N_{qq} \hbar e^2}{\hbar/mc} = N_{qq} \frac{e^2}{\hbar c} mc^2 \\ &= mc^2 \implies N_{qq} \approx 137 \\ &\text{extensions} \end{aligned}$$

GRAVITATION

$$\begin{aligned} \Delta E &\approx N_G \frac{Gm_p^2}{\hbar/m_p c} = N_G \left(\frac{Gm_p}{\hbar c}\right) m_p c^2 \\ &= m_p c^2 \implies N_G \approx 10^{38} \end{aligned}$$

HADRONS

$$\begin{aligned} \Delta E &\approx N_\pi \frac{f^2}{\hbar/m_\pi c} = N_\pi \frac{f^2}{\hbar c} m_\pi c^2 \\ &= m_\pi c^2 \implies N_\pi \approx 10 \end{aligned}$$

QUARKS?

$$N_Q - N_{\bar{Q}} = 0,3$$

$$3, 10, 137, \sim 10^{38}$$

MASSES?

$$m_\pi = \frac{\hbar}{c \times 1.4 \times 10^{-13} \text{cm}} = \frac{2\hbar}{c(e^2/m_e c^2)} = 2 \times 137 m_e$$

m_π / M from Chew-Low-Brayshaw bootstrap

Fig. 7. Basic numbers to be explained by a Democritean theory derived by a generalization of Dyson's argument (see text).

sketched above to a still more fundamental theory, we might start by trying to see how these particular particle numbers arise.

With regard to mass values, we fasten on the unexplained numerical coincidence that $m_{\pi^0} = 2(137)m_e$. A glimmer of explanation might come from the fact, invoked above, that the first anelastic threshold which cuts off (and hence "renormalizes") the QED series for electrons is $e\bar{e} \rightarrow 2\pi^0$. But these π 's are also, by the Dyson argument, the maximum number of electron-positron pairs we can meaningfully discuss in terms of QED. It is therefore tempting to assume that the π^0 is a "bound state" of 137 electron-positron pairs. Since the number is odd, the ground state should be 0^- . If we add $e\bar{\nu}_e$ or $\bar{e}\nu_e$ we might get the π^+ as well as the π^0 . Charge 2 systems would be unstable, thus "explaining" the iso-triplet character of the pion. Since we hope to get m_{π}/M_n from the Chew-Low bootstrap, this leaves only one mass to be chosen (e.g., m_e or M_p). But we need one mass to be chosen arbitrarily in order to relate our elementary picture to macroscopic measurements. The units of length and time are then \hbar/mc and \hbar/mc^2 . These are again allowed since a Democritean theory requires³ a limiting velocity, and the fixed past-uncertain future aspect of quantum mechanics (and experience) require \hbar . Thus, if we can generate the sequence 3, 10, 137, $\sim 10^{38}$ from fundamental philosophical principles, we have a chance of building up constructively to the skeletal theory we have arrived at by applying Occam's Razor to existing quantum scattering theory. A possible route to accomplish this is outlined in the next section.

IV. A CONSTRUCTIVE APPROACH

So far our discussion has concentrated on the dynamical aspects of quantum mechanics, and more specifically on the operational analysis of what

we mean by a particle into whether or not a particle detector fired (i.e., passed through the detector during a spacial and temporal interval defined by the characteristics of the detector) or did not. More careful analysis of quantal phenomena, which cannot be developed here, has led me to the conclusion that all quantum effects which do not support a "classical" description based on the concept of a space-time continuum can be resolved into such dichotomic (yes-no) events. I take the possibility of analyzing any complicated description into a system of discrete choices as a necessity of rational thought. An old characterization of this materialist philosophy is that "atoms and the void suffice." To make this fundamental Democritean postulate somewhat less polemical, I phrase it as:

"SOMETHING DIFFERS FROM NOTHING" (I)

Starting from this necessity, we recognize, experientially, that not all discriminations are made (can be made?) "simultaneously"; and hence that we must meet a sequence of choices, which (again experientially) can be modified by previous discriminations. Causal theories assert that this experience is unreal in the sense that these sequences, being determined by the initial discrimination, are already contained in the initial discrimination. Thus, there is only one choice - whether the world exists or not. Thus, causal theories end up by being anti-rational in our sense, since nothing determines the difference between something and nothing until an additional postulate is added. This may be a good starting point for a mystical philosophy - or madness - but quantum mechanics has provided us with a rational alternative. If we accept the existence of sequential discriminations, and hence in a primitive sense time, we still do not have to believe that the past determines the future. In fact, if we accept the

analysis of the Freedman-Clauser experiment given above, we can rationally make the postulate that:

"PAST SEQUENCES DETERMINE NOW ONLY FUTURE PROBABILITIES" (II)

If we take (I) and (II) seriously, this commits us to what I have described as the fixed past-uncertain future interpretation of quantum mechanics, or some philosophical equivalent. This should not sound too heretical to most quantum physicists. Indeed, when I discussed this with Rudolph Peierls a few years ago, he stated his objection to my calling this an "interpretation" of quantum mechanics by saying "But that is quantum mechanics."

These two philosophical postulates still do not suffice, for me, to indicate the direction in which to develop a theory. We still need some way to discriminate sequences, or order within a sequence, or both. For my third basic postulate, I rely on historical experience within natural philosophy to recognize that the start of new and exciting developments has often been connected with isolating aspects of sequences which, more or less, recur and which can be used to give further structure to the description of the past. The most general way I have thought of to isolate this aspect of successful theories is to postulate that:

"SEQUENCES CONSERVE SOMETHING" (III)

Historical examples of this principle are easy to cite. Quantitative predictive astronomy in Chaldea grew out of the recognition, in temple records, that the helical risings of planets and other astronomical objects recurred at regular periods which could be recovered by numerical analysis. Leucippus and Democritus postulated that the invariant objects were atoms. Ptolemy based his astronomical system on the principle of

circular motion. For Copernicus, Ptolemy had not been precise enough in applying this principle (because of the use of equants), and insisted on couching his description in terms of uniform circular motion, thus finding some simplicity in centering the motion of the solar system on the center of the earth's orbit, rather than on the earth. Kepler made his system fully heliocentric, and arrived at the conservation of elliptical paths, equal areas in equal times, and the ratio of the cubes of the radial distances to the squares of periodic times. For Descartes, who broke the circle, the basic principle was the conservation of uniform linear motion. For Newton it was conservation of linear and angular momentum. For Lavoisier, mass; for Dalton, atomic mass; for Helmholtz, energy; for Einstein, mass-energy; for Bohr and Sommerfeld, adiabatic invariants. Any of you can supply equally cogent examples. These principles and examples are summarized in Figure 8.

To proceed from these principles to a specific physical theory we therefore seek first for conserved quantities which have so far survived the buffeting of intense experimental investigation, and try to state the goal of the theory which could connect back to the regularities left behind after our surgical application of Occam's Razor.

My choice of candidates for conserved quantum numbers is given in Figure 9. These are quantized charge, baryon number, muon number, electron number, and (for "spin- $\frac{1}{2}$ particles") the z-component of spin at a single locus. Many direct and indirect tests of each of these conservation laws have been performed. I trust all of you will agree that if we found an exception we would be on the track of new and exciting physics. So this postulate meets my own criterion of being disprovable by

CONSTRUCTIVE APPROACH

Democritean Postulate

"Something differs from nothing"

(atoms and the void suffice)

Evolutionary Postulate

"Past sequences determine now only future possibilities"

(fixed past → uncertain future)

Conservation Postulate

"Sequences conserve something"

Historical Examples

Chaldean astronomers: periods of Helical risings

Leucippus, Democritus: atoms

Ptolemy: circular motion

Copernicus: uniform circular motion

Kepler: ellipses, equal areas in equal times, R^3/T^2

Descartes: uniform linear motion

Newton: linear and angular momentum

Lavoisier: mass Dalton: atomic mass

Helmholtz: energy Einstein: mass-energy

Bohr-Sommerfeld: adiabatic invariants

Fig. 8 Basic postulates of the constructive approach.

CANDIDATES FOR
CONSERVED QUANTUM NUMBERS

- Charge
- Baryon number
- Muon number
- Lepton number
- z component of spin
at a single locus

BASIC SYSTEMS

$p, n, \mu, e, \nu_\mu, \nu_e$ + antiparticles + spin \rightarrow 256 possibilities
at one locus

COMPOSITE SYSTEMS

$\pi \supset N \uparrow \bar{N} \downarrow$ etc.

$\gamma \uparrow - e \uparrow \bar{e} \uparrow, \nu_e \uparrow \bar{\nu}_e \uparrow$

$g \uparrow \supset \nu_e \uparrow \bar{\nu}_e \uparrow \nu_\mu \uparrow \bar{\nu}_\mu \uparrow$

Goal is to account for

3, 10, 137, $\sim 10^{38}$

$\sim 10^5$ (weak decays): ~~CP~~ cosmological?

256 saturation

Fig. 9 Absolutely conserved particulate quantum numbers, and how they might be combined to describe more complicated systems.

experiment, yet possible in terms of the framework for discussion I have set forth. The minimal content of this seminar, for those who do not take any of the rest of it seriously, is, I hope, the suggestion that theories should make this assumption fundamental, and experiments should strive vigorously to disprove it.

The next step is to exhibit physical systems that are well defined operationally and which isolate the dichotomic (yes-no) choices implied by these conservation laws. For these I select the proton, neutron, muon, electron, their respective antiparticles, and (since they are all spin- $\frac{1}{2}$) their spin states (explored via the exclusion principle). The baryons (p,n) illustrate the charged-neutral dichotomy. The leptons (μ, e) illustrate the muon-electron dichotomy. The fact that one pair is charged-neutral and the other charged-charged illustrates the (secondary) baryon-lepton dichotomy, and provides a first clue for dynamics. A second dynamical clue is that the leptons (but, so far as we know, not the baryons) also have neutral counterparts (neutrinos) that are not "particles", if we use mass as a defining characteristic of a particle, but still carry the muon-electron quantum number dichotomy. Using also the particle -antiparticle and spin-up spin-down dichotomies, the exclusion principle then tells us that there can be 256 distinct systems at a single locus. Everything else is to be built up by sequences of comparisons ("time") and additional loci ("space").

Although we now have a descriptive quantal system to identify the "illustrations" and discriminate between these 256 possibilities operationally, we must obviously carry out sequences of comparisons, and in the course of these operations define what we mean by different "loci." In this process we can expect to encounter complex systems which will not have

the same fundamental descriptive significance but which can persist long enough to cause "counts in detectors", and hence be described as "particles" in a more directly experimental sense. For the hadrons, the lowest mass system will be the pion - a 0^- , $I=1$ system which in terms of quantum numbers is a bound state of a nucleon-antinucleon pair, but ultimately unstable. For the leptons, if the success of QED is any guide, we know that there is a massless 1^- "bound state" of $e\bar{e}$, which can enter quantum particulate phenomena due to "pair creation", and is further restricted by having a "correspondence limit" in "classical" electromagnetic fields. From the point of view of quantum numbers, it could just as well be a $\nu_e\bar{\nu}_e$ state, and thus provide a link to the "weak interactions." With this as a guide, we can speculate that the other "classical field" has a quantum "graviton" describable as a $\nu_e\bar{\nu}_e\nu_\mu\bar{\nu}_\mu$ system in a 2^- state. The only phenomenon then left over that does not have a particulate description would be the CP-violating "super-weak" decay. One possibility is that this phenomenon arises cosmologically due to the accumulation of time-ordered events during the evolution of the universe, and hence is not directly related to "interactions" described in terms of particulate systems. Linking cosmology to the elementary particle parameters is an old game. A recent example is the fact noted by Hoyle and Narlikar²⁶ that the Hubble radius in units of the pion Compton wavelength ($\sim 10^{40}$) is associated with the number which, in our language, we would call the maximum number of pions we could define by gravitational means within their own Compton wavelength.

We have now abstracted to a dimensionless level a few specific numbers which we view as the task of a fundamental theory to derive

and which, by reversing the abstractions used above, we might hope to use to "derive" all of physics as we now understand it. These are (1) the sequence 3, 10, 137, $\sim 10^{38}$ as the maximum number of particles we can operationally define within their own Compton wavelength using quark, hadronic, electromagnetic or gravitational means of identification; (2) saturation of exact quantal description without requiring spatial extension at 256 possibilities; (3) explanation of the instability of systems with respect to "weak decays" in terms of some number of order 10^5 ; (4) cosmological explanation due to time development of the Hubble constant and the K_L - K_S "super-weak" transitions.

Although for this audience I have presented these requirements inductively starting from familiar facts in elementary particle physics and cosmology, they were in part arrived at by another route, to which I now turn. Some time ago Amson, Bastin, Kilmister, and Parker-Rhodes presented a deductive scheme²⁷ which yields the sequence 3, 10, 137, $\sim 10^{38}$ and terminates. I initially arrived at the generalization of the Dyson argument given above in trying to understand how a sequence of pure numbers could have anything to do with "coupling constants"³, and this year noted that the number 256, which also occurs in their theory, has a very natural connection with the absolute particulate conservation laws listed above. I therefore conclude by discussing a few pieces of their scheme in the hope that it will stimulate further thinking along these lines for some of you.

From the point of view I have been developing, I like to state the basic postulate of Amson, Bastin, Kilmister, and Parker-Rhodes as:

"THINGS ARE THE SAME OR DIFFERENT" (III')

I call it III' rather than IV because it is not obviously equivalent to my conservation postulate, but also is not obviously independent of it. In formal logical terms, it would be interesting to derive one from the other or to find what additional postulates are needed to incorporate both in the same system. In a primitive sense, it appears to be different because it includes the concept of identity, while my conservation postulate does not explicitly require such a concept. I have made a brief case above for the importance of conservation laws in physics, but the concept of identity has also proved powerful, particularly in quantum mechanics - as in the phenomena we associate with Bose-Einstein and Fermi-Dirac statistics, and the Gibbs paradox. I do not have time here to explore the deep philosophical issues involved in the choice between III or III', or their ordering in a deductive scheme if they both turn out to be logically compatible with II.

If we start introducing mathematical concepts, postulate I (SOMETHING DIFFERS FROM NOTHING) we can start off with the symbols 0,1. One aim of Amson, Bastin, Kilmister, and Parker-Rhodes is to construct hierarchies based on this symbolization. Their basic postulate (III') then defines the logical operation of symmetric difference, or the arithmetic operation of addition, modulo 2 (cf. Figure 10). To extend this operation to more complicated systems they introduce columns containing only zeros and ones, and combine these, elementwise, to form a third column (cf. Figure 10). This defines a symmetric operation on columns, with interesting group properties which are proved in their paper.²⁷ Although their initial paper can be read without introducing any fundamental distinction between 0 and 1, it is convenient to distinguish the null column (all zeros); from

COMBINATORIAL BASE HIERARCHY

OF

AMSON, BASTIN, KILMISTER, and PARKER-RHODES

DISCRIMINATORY POSTULATE

"Things are the same or different"

(Discriminating between the same things produces a different result than discriminating between different things.)

2 elements (0,1)

$$0 + 0 \rightarrow 0, 0 + 1 \rightarrow 1, 1 + 0 \rightarrow 1, 1 + 1 \rightarrow 0$$

Columns

$$B_n(x,y) = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \rightarrow \begin{pmatrix} x_1+y_1 \\ \vdots \\ x_n+y_n \end{pmatrix}$$

$$B_n(x,x) = (0)_n ; B(x,y) = B(y,x)$$

Exclude null column

Lowest level (4 elements)

$$\begin{array}{l} \text{Non-trivial results} \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{array}$$

Fig. 10 Starting point of the combinatorial base hierarchy.

our Democritean point of view this distinction is not just convenient but necessary for the interpretations we intend to make.

Although various hierarchies can be constructed in this way, the one which concerns us - the combinatorial base hierarchy - starts from columns of height 2. Excluding the null column, the trivial result $B_n(x,0) = (x)_n = B_n(0,x)$, and taking account of the symmetry we have three possibilities, given in Figure 10. These form a "discriminately closed set" in the sense that any two members of the set combine under the binary operation to give a third member of the set. In the paper²⁷ they show that it is also possible to define three discriminately closed subsets, which are given in Figure 11. This is most easily done in terms of a mapping isomorphism onto non-singular 2×2 matrices of which these are eigenvectors; but there is also a logical definition which does not require matrix multiplication.

The middle level of the hierarchy can be obtained by rearranging the mapping matrices as columns with four elements and finding the discriminately closed set. This contains $2^3 - 1 = 7$ columns, out of the $4^2 = 16$ possibilities, which are exhibited in Figure 11. Again there are the same number of discriminately closed subsets, just as at the lowest level there are $2^2 - 1 = 3$ columns (and DCsS) out of $2^2 = 4$ possibilities. Repeating the construction gives the last level of the hierarchy with $2^7 - 1 = 127$ columns out of $16^2 = 256$ possibilities. Although the construction is non-unique in terms of a specific labeling of slots in the columns, the cardinal numbers of DCsS are provably unique. The last level of the hierarchy is the last because attempted repetition of the construction, provably, should yield $2^{127} - 1 \sim 10^{38}$ columns which obviously cannot be

Lowest Level (cont'd)

$2^2 - 1 = 3$ discriminately closed subsets (DCsS)

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Mapping Isomorphism

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Middle Level

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$4^2 = 16 \text{ elements} \quad 2^3 - 1 = 7 \text{ DCsS}$$

Last Level

$$16^2 = 256 \text{ elements} \quad 2^7 - 1 = 127 \text{ DCsS}$$

Why Last?

$$2^{127} - 1 > (256)^2$$

Fig. 11 The three levels of the hierarchy.

exhibited in terms of the 256^2 possibilities.

These results are summarized in Figure 12. The connection with physics is attempted by noting that the construction "preserves information" (explicitly discriminate closure) in passing from one level to another, and that if the successively articulated systems are considered together, including lower levels, the construction generates the cardinal sequence 3, 10, 137, $\sim 10^{38}$ for which we are looking. Further, since the levels intercommunicate, this need only be a first approximation to the "coupling constants" when we go to more complicated sequences. Interestingly, the failure to close at the $(256)^2$ level, and hence the generation of unstable sequences at that level, gives us an estimate of $\sim 10^5$ for the inverse "coupling constant" that could correspond to "weak decays." Finally, the last level at which indefinite repetition of the comparison between columns can occur contains only 256 possibilities, and hence might be connected with the number we arrived at above from the absolute particulate conservation laws.

It is clearly a long way from this kind of abstract mathematical system to an elementary particle theory capable of being disproved by experiment, but to me it looks like a promising beginning which could, in principle, provide an explanatory framework for all of physics. The scheme is much richer in structure than I have been able to explain in this very brief account. As an example, let me show, at the middle level, how we might start to use it to explain the "stability" of certain structures. Assume we start with some random assemblage of the 16 possible columns. Discriminating between two identical columns leads to a null result, but the other columns fall into either an open or a closed

SUMMARY OF
COMBINATORIAL BASE HIERARCHY

		<u>Lowest Level</u>	
$2^2 = 4$	columns	$2^2 - 1 = 3$	DCsS
		<u>Middle Level</u>	
$4^2 = 16$	columns	$2^3 - 1 = 7$	DCsS
		<u>Last Level</u>	
$16^2 = 256$	columns	$2^7 - 1 = 127$	DCsS
		<u>Termination</u>	
		$(256)^2 < 2^{127} - 1$	

TENTATIVE CONNECTIONS WITH PHYSICS

1. Information preservation

$$3; 3 + 7 \rightarrow 10; 10 + 127 \rightarrow 137, \text{Max } 2^{127} - 1 + 127 \approx 10^{38}$$

$$\Rightarrow 3, 10, 137, \sim 10^{38}$$

2. Comparisons at $(256)^2$ level fail to close indicates instability

$$\Rightarrow (256)^2 \sim 10^5 \text{ for instability}$$

3. Termination at 256

$$\Rightarrow \text{absolute particulate conservation laws}$$

Fig. 12 How the combinatorial base hierarchy might connect to physics.

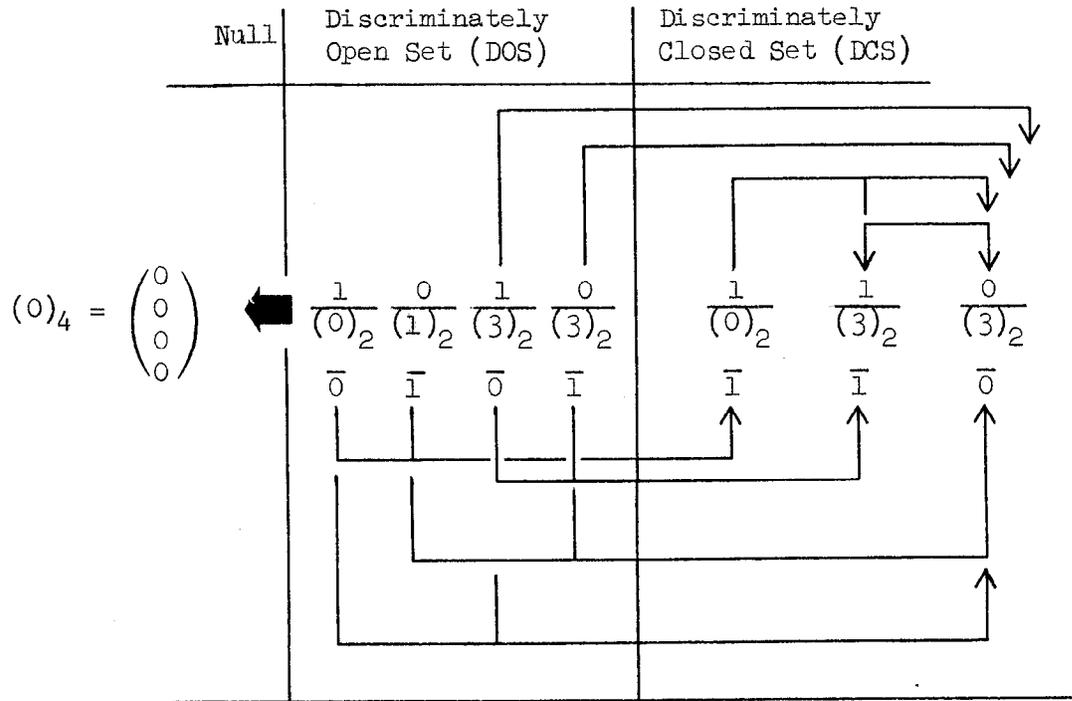
set (cf. Figure 13). Discriminating any two of the members of the open set leads to a member of the closed set, as can be readily checked. But, as we have already seen, once in the closed set one never gets out of it. So, statistically, we will settle down from 15 to 7 possible columns. But if this goes on within the closed set, as can also be seen from Figure 13, we will settle down from 7 to 3 columns. These, except for zeros which can no longer change, are just the three members of the lowest level! Thus, there is a natural ordering of processes once one adds some rule (in this case randomness) to the scheme. Whether this will provide a useful clue to the dynamics only the uncertain future can decide.

Conclusion

I hope to have shown you that by taking some thought it is possible to at least approach very fundamental questions in physics, and in that way open for consideration new schemes that, though firmly grounded in experimental fact, need not lead to conventional conclusions. Whether this is physics - which I prefer to call natural philosophy - or poetry I leave for you to decide. If you reach the latter conclusion, I rest my defense on the words of John Keats:

"... and if it is so that it [poetry] is not so fine a thing as philosophy - For the same reason that an eagle is not so fine a thing as a truth - Give me this credit - Do you not think that I strive to know myself." ²⁰

STABILIZATION
AT THE MIDDLE LEVEL



$$(0)_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(1)_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$(3)_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Fig. 13 Stabilization at the middle level (see text).

REFERENCES

1. E. W. Bastin, report to the Science Research Council (of the U.K.) April, 1976.
2. J. M. Ziman, Public Knowledge (Cambridge University Press, London, 1968).
3. H. P. Noyes, "Non-Locality in Particle Physics," SLAC-PUB-1405 (Rev. November 1975).
4. Paul Forman, "Weimar Culture, Causality, and Quantum Theory, 1918-1927: Adaptation by German Physicists and Mathematicians to a Hostile Intellectual Environment." Historical Studies in the Physical Sciences 3,1 (1971).
5. P. Schlipp, ed. Albert Einstein, Philosopher-Scientist (Library of Living Philosophers, Vol. 7, Open Court, LaSalle, Illinois, 1969)
6. P. W. Bridgman, The Logic of Modern Physics, Macmillan, New York, 1928; The Nature of Physical Theory, Princeton University Press, Princeton, 1936.
7. N. Bohr and L. Rosenfeld, Kgl. Danske Vidensk. Selskab, Math-Fys. 12, No. 8 (1933).
8. J. S. Bell, Physics 1, 195 (1965).
9. S. J. Freedman and J. F. Clauser, Phys. Rev. Letters 28, 938 (1972).
10. H. P. Noyes, Foundations of Physics 5, 37 (1975), Erratum 6, 125(1976).
11. T. E. Phipps, Jr., Dialectica 23, 189 (1969).
12. _____, Foundations of Physics 3, 435 (1973).
13. T. E. Phipps, private communication, quoted in SLAC-PUB-1351, "Fixed Past and Uncertain Future: An Exchange of Correspondence Between Pierre Noyes, John Bell, and Thomas E. Phipps, Jr." (Stanford Linear Accelerator Center, Stanford, California, 1973.)
14. H. P. Noyes, Foundations of Physics 6, 83 (1976).
15. D. Finkelstein, private communication.
16. G. C. Wick, Nature 142, 993 (1938).
17. H. P. Noyes, Bull. Am. Phys. Soc. 16, 20 (1971).
18. V. Efimov, Phys. Letters 33B, 563 (1970).
19. D. D. Brayshaw, Phys. Rev. D11, 2583 (1975).

20. D. D. Brayshaw and H. P. Noyes, Phys. Rev. Letters 34, 1582 (1975).
21. D. D. Brayshaw, "A New Dynamics of NN Scattering," SLAC-PUB-1686 (December 1975).
22. D. D. Brayshaw, Phys. Rev. Letters 36, 73 (1976).
23. G. F. Chew, Phys. Rev. Letters 9, 233 (1962).
24. F. J. Dyson, Phys. Rev. 85, 631 (1952).
25. P. S. Laplace, Exposition du Systeme du Monde, Vol. II (Paris, 1795), p. 305 (English translation: W. Flint, London, 1809). I am indebted to E. Guth for finding this reference for me. It is quoted in W. C. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation (E. H. Freeman, San Francisco, 1970), as follows: "A luminous star, of the same density as the earth, and whose diameter should be 250 times larger than that of the Sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible."
26. F. Hoyle and J. V. Narlikar, Action at a Distance in Physics and Cosmology, Freeman, San Francisco (1974).
27. Ted Bastin, Studia Philosophica Gandensia 4, 77 (1966).
28. John Keats - Letter 159 (Rollins' numbering) - journal - letter of February 14 - May 3, 1819, to George and Georgiana Keats in America.