REMARKS ON THE NEUTRINO GAUGE GROUP

B r u n o   T o u s h e k

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It is generally recognised that if a theory is invariant under the transformation

$$\psi' = e^{i\gamma_5 \alpha}\psi \qquad (1)$$

in which ψ is the operator of the neutrino field the mass of the neutrino must be zero. On the other hand Heisenberg has proposed a unitary field theory in which the properties of all particles should be deduced from a single four state spinor. The theory is invariant under the full Pauli group containing (1) and this immediately rises the question how arises the result from such a theory.

To study this apparent contradiction a simplified version of Heisenberg's theory, which is characterised by the following postulates was investigated.

(I) Schurr's lemma holds for a Majorana spinor.

(II) The theory is invariant under (1).

(III) The theory is invariant under the proper inhomogeneous Lorentz group so well as under the reflection.

$$R\,\psi(x)\,R^{+} = i\gamma_4\,\psi(\tilde{x}) \qquad (2)$$

(IV) There exists a unique vacuum state χ_0.

It can then be proven that this theory cannot give Majorana particles of mass $m \neq 0$ from which it follows, that massive particles of spin 1/2 which result from such a theory must necessarily be "degenerate" i.e. for a given mass there must be at least 4 states available for any given value of the momentum.

The case of a simple degeneracy can now be studied in more detail. For the description of a massive particle of spin I/2 it is necessary that from the "interpolating" field ψ one can deduce two asymptotic Majorana fields φ_1 and φ_2 If , for example one has $\psi \rightarrow \varphi_1$, in the sense of a weak convergence it is clear that φ_1 must not transform like ψ under (1). It then follows from a consideration of the Pauli group that under (1) one must have

$$\varphi_2' = e^{-i\gamma_5 \alpha} \varphi_2 \tag{3}$$

It follows from Schurr's lemma that for the description of massive particles it should be possible to form a functional $f[\psi]$, which asymptotically must satisfy $f[\psi] \rightarrow \varphi_2$. The theory has such a functional namely

$$f[\psi] = \hat{\psi} = \lambda^2 \gamma_5 \gamma_\nu \psi(x) \bar{\psi}(x) \gamma_5 \gamma_\nu \psi(x) \tag{4}$$

(λ is a constant of the dimensions of a length). Generalising these results we can show the following conclusion regarding theories of the Heisenberg type.

1. In the subspace $\mathfrak{H}_1$ of the total Hilbert space $\mathfrak{H}$ the reflection R can be split in a form which exactly corresponds to $R = CP$.

2. A massive spin I/2 particle never corresponds to a given single power of ψ but always to inhomogeneous non-linear constructions.

3. There is no element in the actual Heisenberg theory which would safeguard a degeneracy capable of producing masses. Such an element could, however, be introduced by symmetrizing the "kinetic energy" by adding a term $\bar{\hat{\psi}} \gamma \hat{\psi}$. This would automatically introduce $R = CP$ for the whole Hilbert space.

4. A Heisenberg type theory in which Schurr's lemma is valid for a single Majorana field has the attractive feature that in agreement with our experience all massive spin 1/2 particles are degenerate, but need not have more than four states. In the Heisenberg's actual theory they must have at least eight states.

5. This leads to the proposal of investigating a Heisenberg type theory based on a single Majorana field as a possible model for the "small world" composed of neutrinos, electrons, μ-mesons and γ-rays.
