

## Muonium Has Not Yet Decayed!

A brief review of the history and present status of research on muonium and muoniumlike atoms is given. Several new problems, including the Lamb shift in muonium and a sensitive search for muonium to antimuonium conversion, which are of current interest and appear susceptible to experimental advances, are discussed.

### INTRODUCTION

In the 23 years since its discovery, interest in the muonium atom ( $\mu^+e^-$ ) has not decayed. Perhaps this is not at all surprising in view of the current interest in the hydrogen atom, which was discovered about 200 years ago by Cavendish and was studied systematically by Balmer, Lyman and Paschen in the late 19th and early 20th centuries.<sup>1</sup> It seems that a two-body system with simple constituents is revisited by physicists whenever new concepts and theories come up for sensitive testing or whenever new powerful experimental techniques are developed.

The principal reason that muonium (M) continues to be important to fundamental physics is that it is the simplest atom composed of two different leptons.<sup>2</sup> The muon retains a central role as one of the elementary particles in the modern standard theory, but we still have no understanding as to "why the muon weighs" and in all respects behaves simply as a heavy electron. Muonium is an ideal system for determining the properties of the muon, for testing modern quantum electrodynamics, and for searching for effects of weak, strong, or unknown interactions in the electron-muon bound state. Basically,

muonium is a much simpler atom than hydrogen because the proton is a hadron and, unlike a lepton, has a structure that is determined by the strong interactions.

The key to studying muonium was the discovery in 1957 that the  $\mu^+$  originating from  $\pi^+$  decays are polarized, and, furthermore, that the  $e^+$  from  $\mu^+$  decays have an asymmetry in their angular distribution with respect to the  $\mu^+$  spin direction.<sup>3,4</sup> These phenomena are a consequence of parity nonconservation in the  $\pi \rightarrow \mu \rightarrow e$  decay chain and provide the tool for studying muonium. With polarized  $\mu^+$ , polarized M atoms are formed in an electron capture reaction (i.e., an unequal distribution of hyperfine structure states of the ground  $n = 1$  state is formed), and changes in these M state populations induced in a spectroscopy experiment are detectable through the change in angular distribution of the decay positrons.

An important question in 1957 about the  $\pi^+ \rightarrow \mu^+$  decay was the helicity of the  $\mu^+$ . Although it seemed possible initially that studies of induced transitions between M hfs substates might determine the  $\mu^+$  helicity, it turned out that this was not true. It is only the product of the helicity of the muon by the coefficient characterizing the angular distribution of the decay  $e^+$  that can be determined with muonium. The  $\mu^+$  helicity was first measured by Mott scattering to be negative.

The most obvious way to observe M formation is to search for its characteristic Larmor precession frequency. The Larmor precession of  $\mu^+$  with its characteristic frequency

$$f_{\mu} = \frac{\mu_{\mu} H}{\frac{1}{2} h} = 13.5 H \text{ kHz} \quad (1)$$

was observed when  $\mu^+$  were stopped in matter in the original experiment that demonstrated parity nonconservation in the  $\pi \rightarrow \mu \rightarrow e$  decay chain.<sup>3</sup> In Eq. (1)  $\mu_{\mu}$  is the muon magnetic moment and  $H$  is the applied magnetic field. For muonium the Larmor precession frequency for the ground  $n = 1$  state with total angular momentum quantum number  $F = 1$  and with the associated magnetic quantum number  $M = \pm 1$  is given approximately by

$$f_M \simeq \frac{\mu_e H}{h} = 1.40 H \text{ MHz}, \quad (2)$$

where  $\mu_e$  is the electron spin magnetic moment. We note that  $f_M$  is about 100 times larger than  $f_\mu$  because the electron magnetic moment rather than the muon magnetic moment determines  $f_M$ . These two frequencies  $f_\mu$  and  $f_M$  are easily distinguishable.

Several groups attempted to observe the Larmor precession of M when  $\mu^+$  were stopped in various liquids and gases, but initially no evidence for M was found. It was not until about three years later that M was discovered<sup>5</sup> when it was realized that after formation M could be easily depolarized in collisions with paramagnetic molecules present as impurities in the target and hence become undetectable. The effect of collisions on muonium polarization, which proved an obstacle to the discovery of M itself, has become the basis for a rich scientific field of muonium chemistry in gases and solids<sup>6,7</sup> in which the processes of Larmor precession, depolarization by collisions, and microwave-induced transitions studied for the muonium atom are applied.

## GROUND-STATE HYPERFINE STRUCTURE AND ZEEMAN EFFECT

The principal importance of muonium for particle physics and atomic physics relates to the very precise determination of the hfs interval  $\Delta\nu$  in the ground state of muonium and of the muon magnetic moment  $\mu_\mu$ . The experimental method used has been the muonium magnetic resonance method that depends on the formation of polarized M and on the detection of changes in hfs substate populations induced by a resonant microwave magnetic field through the change in the angular distribution of decay positrons. The energy level diagram is shown in Figure 1, together with the transitions observed. The most precise values of  $\Delta\nu$  and of  $\mu_\mu$  come from the most recent measurements of transitions  $\nu_{12}$  and  $\nu_{34}$  at a strong magnetic field of 13.6 kG. Figure 2 indicates the history of measurements of  $\Delta\nu$ . The latest published values are<sup>8</sup>

$$\Delta\nu = 4,463,302.88 (16) \text{ kHz } (0.036 \text{ ppm}) \quad (3)$$

$$\mu_\mu/\mu_p = 3.183,346,1 (11) (0.36 \text{ ppm})$$

in which  $\mu_p$  is the proton magnetic moment. The value of  $\mu_\mu/\mu_p$  is determined essentially from the Zeeman effect in muonium.

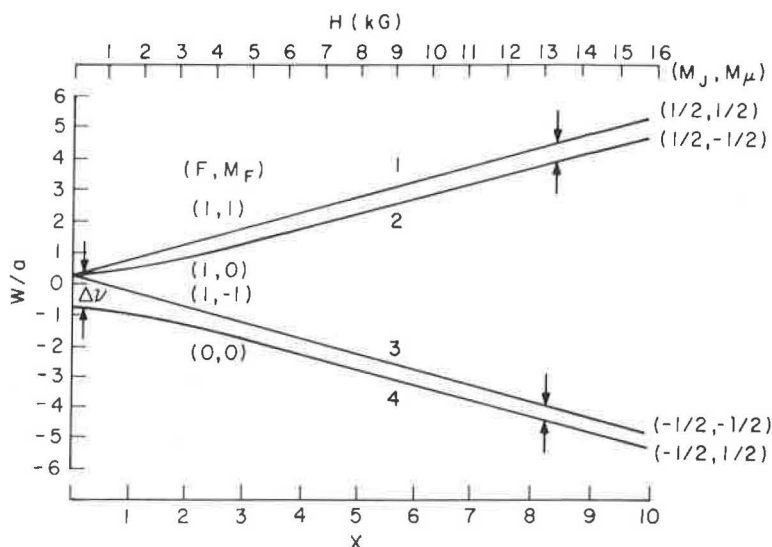


FIGURE 1 Energy levels in ground  $n = 1$  state of muonium based on the Hamiltonian

$$H = a \mathbf{I}_\mu \times \mathbf{J} + \mu_B^s g_J \mathbf{J} \times \mathbf{H} - \mu_B^s g_\mu \mathbf{I}_\mu \times \mathbf{H}$$

$$x = (g_J \mu_B^s + g_\mu \mu_B^s) H / (h \Delta \nu)$$

$$a = h \Delta \nu \simeq 4463 \text{ MHz.}$$

The muon magnetic moment, or its ratio relative to the proton or electron magnetic moment, is determined most accurately at present from the above value of  $\mu_\mu/\mu_p$  obtained from muonium. There is another measurement of  $\mu_\mu/\mu_p$  of comparable accuracy obtained in a muon spin resonance ( $\mu$ SR) experiment with  $\mu^+$  in liquid bromine in good agreement with the value from muonium. The combined best value is

$$\mu_\mu/\mu_p = 3.183,345,47 (95) (0.30 \text{ ppm}). \quad (4)$$

The value of the muon mass  $m_\mu$ , or of the ratio of muon mass to electron mass  $m_\mu/m_e$ , is obtained from  $\mu_\mu/\mu_p$  and values of  $\mu_e/\mu_p$ ,  $g_e$  and  $g_\mu$  through the equation

$$\frac{m_\mu}{m_e} = \frac{\mu_e \mu_p g_\mu}{\mu_p \mu_\mu g_e}. \quad (5)$$

The current value is

$$\frac{m_\mu}{m_e} = 206.768,259\ (62)\ (0.30\ \text{ppm}) \tag{6}$$

in which the error is dominated by that of  $\mu_\mu/\mu_p$  in Eq (4).

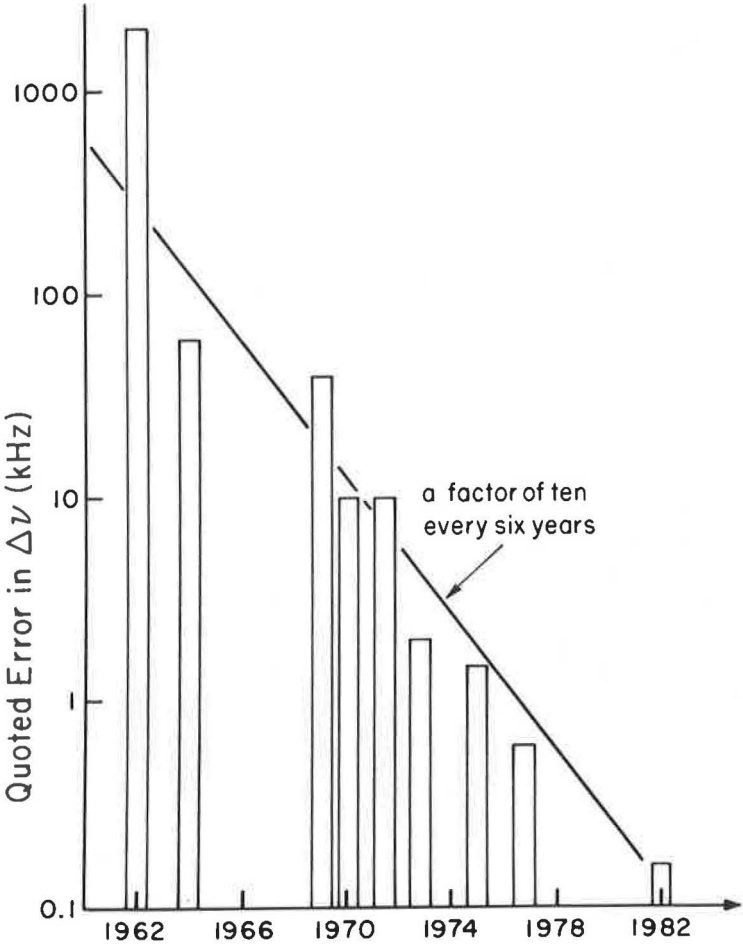


FIGURE 2 Quoted errors in the determination of the hyperfine structure interval  $\Delta\nu$  for muonium in experiments over the time period from 1962 to 1982. The experiments have been done by the Yale (Yale–Heidelberg) group at Nevis Laboratory of Columbia University and at Los Alamos National Laboratory, and by the Chicago group at the University of Chicago and at the Space Radiations Effects Laboratory.

The fundamental constants of the muon  $m_\mu$  and  $\mu_\mu$  are of course of vital importance. Thus a precise value of  $\mu_\mu/\mu_p$  is required for comparison of theory and experiment for the muon  $g_\mu - 2$  value, and a precise value of  $m_\mu$  is required for the interpretation of muonic atom spectra and for the determination of the mass of the muon neutrino.

Within the framework of quantum electrodynamics, the energy levels of muonium as a two lepton bound state can, in principle, be evaluated exactly. In practice, calculations are made by perturbation theory as expansions in powers of the fine structure constant  $\alpha$  and the mass ratio  $m_e/m_\mu$ . Successive terms become more complicated to calculate. We note that calculation of the energy levels of  $\mu^+e^-$  is considerably simpler than calculations for  $e^+e^-$  because for muonium the ratio of the masses of the two particles,  $m_e/m_\mu$ , is small whereas for positronium this ratio is 1.

As the result of immense theoretical effort the value of  $\Delta\nu$  has been calculated to a precision of several parts in  $10^7$ . All terms including those of relative order  $\alpha^3$  and  $\alpha^2 m_e/m_\mu$  have been evaluated. The theoretical value can be written<sup>9,10</sup>

$$\Delta\nu = \left\{ \frac{16}{3} \alpha^2 c R_\infty \left( \frac{\mu_\mu}{\mu_p} \right) \left( \frac{\mu_p}{\mu_B} \right) \left( 1 + \frac{m_e}{m_\mu} \right)^{-3} \right\} (1 + \epsilon_{\text{QED}}), \quad (7)$$

in which  $R_\infty$  is the Rydberg constant,  $\mu_B$  is the electron Bohr magneton, and  $\epsilon_{\text{QED}}$  represents the virtual radiative and relativistic recoil terms. The term in the first bracket is the leading Fermi term  $\Delta\nu_F$ . The values of the fundamental constants used to evaluate  $\Delta\nu$  are the following:

$$\begin{aligned} \alpha^{-1} &= 137.035,963 (15) (0.11 \text{ ppm}); \\ c &= 2.997,924,580 (12) \times 10^{10} \text{ cm/s} (0.004 \text{ ppm}); \\ R_\infty &= 1.097,373,152, 1 (11) \times 10^5 \text{ cm}^{-1} (0.001 \text{ ppm}); \quad (8) \\ \mu_\mu/\mu_p &= 3.183,345,47 (95) (0.30 \text{ ppm}); \\ m_\mu/m_e &= 206.768,259 (62) (0.30 \text{ ppm}). \end{aligned}$$

Hence the value for  $\Delta\nu_{\text{th}}$  is

$$\Delta\nu_{\text{th}} = 4,463,304.5 (1.7) (1.0) \text{ kHz } (0.4 \text{ ppm}), \quad (9)$$

in which the 1.7 kHz uncertainty comes from combining a 1.3 kHz uncertainty from  $\mu_\mu/\mu_p$  with a 1.0 kHz uncertainty from  $\alpha$ . The 1.0 kHz theoretical uncertainty is the estimated contribution from uncalculated terms contributing to  $\epsilon_{\text{QED}}$ . A hadronic vacuum polarization term of 0.22 (4) kHz is included in Eq. (9).

The good agreement between the experimental and theoretical values for  $\Delta\nu$

$$\Delta\nu_{\text{th}} - \Delta\nu_{\text{exp}} = (1.6 \pm 1.9) \text{ kHz} \quad (10)$$

provides one of the important tests of the validity of quantum electrodynamics and of the assumption that the muon behaves like a heavy electron.

An alternative approach is to equate  $\Delta\nu_{\text{exp}}$  of Eq. (3) to  $\Delta\nu_{\text{th}}$  of Eq. (7) and hence determine  $\alpha$ . The result

$$\alpha^{-1} = 137.035,988 (20) (0.15 \text{ ppm}) \quad (11)$$

is in good agreement with the value of  $\alpha$  obtained from the ac Josephson effect as listed above. From this viewpoint we confirm the condensed matter theory of the ac Josephson effect. The value of  $\alpha$  from M is in excellent agreement with that from the electron  $g-2$  value<sup>11</sup>:  $\alpha^{-1} = 137.035,999 (10)$ .

The standard electroweak theory predicts<sup>12</sup> an axial vector-axial vector coupling contribution to  $\Delta\nu$  of 0.07 kHz or a fractional contribution of  $1.6 \times 10^{-8}$ . This is about  $\frac{1}{2}$  the experimental error in  $\Delta\nu$ , but about  $\frac{1}{25}$  the present theoretical error in  $\Delta\nu$ . Recent high-energy colliding beam experiments have measured the charge asymmetry in  $e^+e^- \rightarrow \mu^+\mu^-$ , which is believed due to this weak neutral current coupling between leptons. As yet there has been no measurement of a weak interaction energy contribution in an atom, and it would be of interest to measure this term in muonium where low momentum transfer is involved.

Further improvement in the precision of measurement of  $\Delta\nu$  and of  $\mu_\mu/\mu_p$  will probably require the use of line-narrowing techniques,<sup>13</sup> which have indeed already been applied in earlier experiments on muonium. The latest and most precise values for  $\Delta\nu$  and for  $\mu_\mu/\mu_p$ , given in Eq. (3), involved the determination of the center of a resonance line to about  $1/3000$  of its linewidth—a not unprecedented but small number. Since systematic errors often occur as a fraction of a linewidth, it is desirable to achieve as narrow a resonance line as possible. At LAMPF there is the possibility of having a high intensity pulsed  $\mu^+$  beam associated with the proton storage ring now under construction at the Los Alamos National Laboratory. With such a pulsed  $\mu^+$  source, line-narrowing techniques could be employed and an improvement in the precision of measurement of  $\Delta\nu$  and of  $\mu_\mu/\mu_p$  by about a factor of 5 and 10, respectively, should be possible.

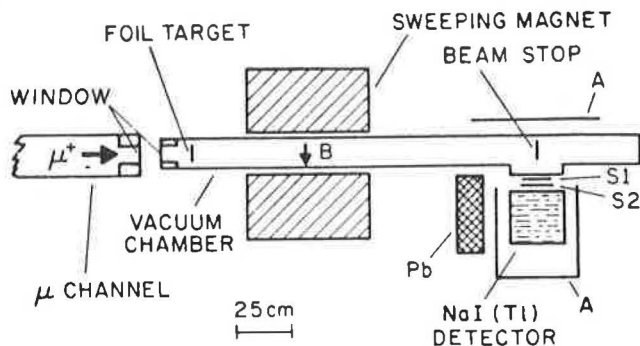
Active theoretical work is in progress to evaluate additional higher order terms in  $\epsilon_{\text{QED}}$  so that the theoretical accuracy will be several parts in  $10^8$  or about 0.1 kHz in  $\Delta\nu$ .

## NEW PROBLEMS

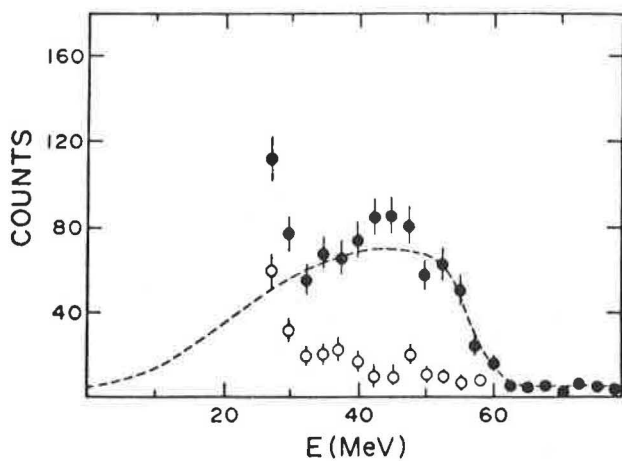
Perhaps the outstanding new problems in muonium research are the measurement of the Lamb shift and a sensitive search for the conversion of muonium to antimuonium, i.e.,  $\mu^+e^- \rightarrow \mu^-e^+$ . A precise measurement of the Lamb shift in the  $n = 2$  state would provide an ideal test of quantum electrodynamics free of the effect of proton structure which complicates the interpretation of the Lamb shift in hydrogen.<sup>9,14</sup> The  $M \rightarrow \bar{M}$  conversion would violate muon number conservation, but is predicted by some of the gauge theories of the electroweak interaction and is an interesting case of a flavor-changing transition between degenerate states.<sup>15</sup>

Both of these measurements require that muonium be in vacuum because of the important and deleterious effect of collisions of  $M$  with atoms. Obtaining  $M$  in vacuum has been regarded as a worthwhile objective for many years. Early attempts sought to obtain thermalized  $M$  from surfaces but proved unsuccessful. Muonium has recently been obtained in vacuum by passing  $\mu^+$  in the kinetic energy range below 1 MeV through thin foils.<sup>16</sup> Muonium in its ground  $n = 1$  state with kinetic energy up to about 20 keV emerges from the foil through the process of electron capture by  $\mu^+$ . The experimental setup and results are shown in Figure 3. Referring to Figure 3a, the





(a)



(b)

FIGURE 3 (a) Schematic diagram of the experimental apparatus for the observation of muonium in vacuum.  $S_1$ ,  $S_2$ , and  $A$  are plastic scintillation counters. The beam passes through a 125- $\mu\text{m}$  Mylar window at the end of the  $\mu$  channel and a 50- $\mu\text{m}$  Ti window entering the vacuum chamber, and is collimated to 7.5-cm diameter by polyethylene collimators. (b) Measured NaI spectrum for 25- $\mu\text{m}$  Be foil target. Solid circles are data obtained with the apparatus evacuated; open circles are with 6-Torr helium. The dashed curve is a fit to the data based on the Michel spectrum. Below the incident beam momentum of 28 MeV/c,  $\gamma$ -ray background dominates. Muonium is ionized by the helium gas and hence does not reach the beam stop shown in Figure 3a.

neutral component of the beam emerging from the foil target strikes the beam stop, and the  $e^+$  from  $\mu^+$  decay with their characteristic Michel spectrum (Figure 3b) are observed with a NaI (Tl) detector.

Having now a technique to obtain a substantial intensity of  $M$  atoms in vacuum, it is realistic to attempt a measurement of the  $2\ ^2S_{1/2}$  to  $2\ ^2P_{1/2}$  Lamb shift in  $M$  and also a sensitive search for  $M \rightarrow \bar{M}$  conversion. With regard to the Lamb shift measurement, although our observations on muonium in vacuum<sup>16</sup> only prove that the  $n = 1$  ground state is formed, data on the passage of protons through thin foils do establish that the  $n = 2\ ^2S_{1/2}$  state of  $H$  as well as the  $n = 1$  state is formed, and indeed we expect the number of  $M(2S)$  atoms emerging from the foil to be about 0.1 of the number of  $M(1S)$  atoms. The scheme of experiments now in progress at LAMPF and TRIUMF involves the application of microwave radiation at the predicted  $2S_{1/2} \rightarrow 2P_{1/2}$  resonance frequency of 1140 MHz and observation of the resulting  $2P_{1/2} \rightarrow 1S_{1/2}$  UV line. The experimental setup for the LAMPF experiment is shown in Figure 4. Both groups are now reporting the observation of the Lamb shift transition.

An early search was made for the  $M \rightarrow \bar{M}$  conversion when  $\mu^+$  were stopped in Ar gas by looking for a muonic Ar x-ray which would be expected if the conversion occurred.<sup>15</sup> The  $M$ - $\bar{M}$  coupling can be ascribed to an effective four-Fermion coupling with constant  $G$ :

$$H_{MM} = G \bar{\psi}_\mu \gamma_\lambda (1 + \gamma_5) \psi_e \bar{\psi}_\mu \gamma^\lambda (1 + \gamma_5) \psi_e + \text{h.c.} \quad (12)$$

This early experiment was rather insensitive due to the effect of  $M$ -Ar collisions in inhibiting the  $M \rightarrow \bar{M}$  conversion and it established only that  $G \leq 5800\ G_F$ , in which  $G_F$  is the Fermi weak interaction coupling constant. Subsequently an  $e^- - e^-$  colliding beam experiment<sup>15</sup> that looked for  $\mu^-\mu^-$  production established that  $G \leq 610\ G_F$ . More recently, an experiment at TRIUMF<sup>17</sup> using  $M$  formed in powders, where the interpretation is more complicated and depends on the interactions of  $M$  in the powder, has set a limit of  $G \leq 42\ G_F$ .

A sensitive search for this interesting  $M \rightarrow \bar{M}$  conversion now seems possible using  $M$  in vacuum and using a detector to observe

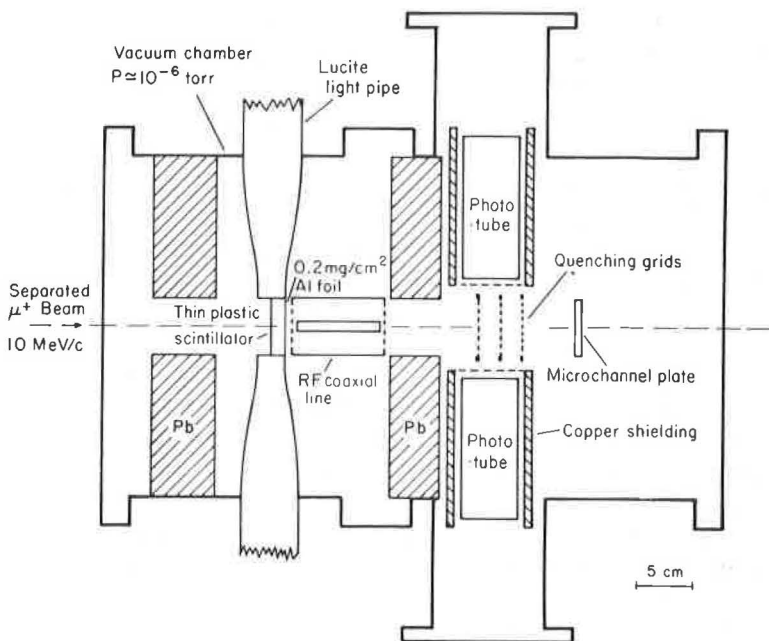


FIGURE 4 Schematic diagram of the apparatus to observe the Lamb shift transition in muonium. The signal will be a delayed triple coincidence count between an incoming  $\mu^+$ , a Lyman  $\alpha$  photon in a photomultiplier, and a  $M(1S)$  atom detected in the microchannel plate.

a fast  $e^-$  from  $\mu^-$  decay. The question of flavor-changing transitions is of central importance now in particle physics, and a sensitive search for  $M \rightarrow \bar{M}$  conversion at the level of  $G \lesssim 0.1 G_F$  would be very significant.

Measurements of the optical spectra of muonium by techniques of laser spectroscopy are conceivable now using pulsed sources of muons and having muonium in vacuum. In particular, the  $1S \rightarrow 2S$  two-photon transition might be studied as has been done for hydrogen<sup>18</sup> and for positronium.<sup>19</sup> A precise measurement of this transition would constitute a measurement of the Lamb shift in the  $1S$  state of muonium or, alternatively, a determination of  $m_\mu/m_e$ .

## THE MUONIC HELIUM ATOM

As a final topic in the field of muonium physics, we remark on the muonic helium atom ( $^4\text{He } \mu^- e^-$ ) which consists of an  $\alpha$  particle, a

negative muon, and an electron. This atom is of interest to atomic physics as an unusual three-body problem in which the three particles all have different masses and no Pauli principle applies. Studies of the hfs interval and Zeeman effect have provided the most precise direct measurement of the  $\mu^-$  magnetic moment (its equality to the  $\mu^+$  magnetic moment then constitutes a test of CPT invariance) and a sensitive test of the  $\mu^-e^-$  interaction.

The muonic helium atom is similar to  $\mu^+e^-$  in the sense that the  $({}^4\text{He}\mu^-)^+$  can be considered as a small heavy pseudonucleus with charge  $+e$  and with the magnetic moment of  $\mu^-$ . Hence the electron in this atom will have an orbit similar to that in  $\mu^+e^-$ , and the system may be considered to consist of an atom within an atom. The hfs interval will be approximately the same as for  $\mu^+e^-$  but the hfs levels will be inverted because of the negative sign of the  $\mu^-$  magnetic moment.

The muonic helium atom was discovered in an experiment at SREL<sup>20</sup> by observation of its characteristic Larmor precession frequency when  $\mu^-$  were stopped in He gas with a small admixture of Xe. The only trick in the experiment is associated with the fact that  $({}^4\text{He}\mu^-)^+ (1S)$ , which is the state the  $\mu^-$  quickly enters in the He gas, is forbidden energetically from capturing an electron from a He atom, so that an electron donor gas (e.g., Xe) must be present. The hfs transition at weak magnetic field was first observed at SIN<sup>21</sup> where a search problem was encountered due to the state of the theoretical predictions for  $\Delta\nu$  at the time. Subsequently a measurement at strong magnetic field was performed at LAMPF,<sup>22</sup> which determined  $\Delta\nu$  with somewhat improved accuracy and also determined  $\mu_{\mu^-}/\mu_p$ .

The theoretical value for  $\Delta\nu$  can be written as in Eq. (7)

$$\Delta\nu = \Delta\nu_F (1 + \epsilon'_{\text{QED}}). \quad (13)$$

The value of  $\Delta\nu_F$  is proportional to the expectation value  $\langle \delta(r_{e\mu}) \rangle$  and thus involves the electron-muon correlation. The term  $\epsilon'_{\text{QED}}$  represents the higher order contributions from QED. The present theoretical value<sup>23,24</sup> is limited by the estimated error in  $\Delta\nu_F$  arising from inaccuracy in the Schrödinger wavefunction. The experimental and theoretical values for  $\Delta\nu$  are in good agreement within the uncertainty of  $\Delta\nu_{\text{th}}$  as indicated in Eq. (14).

$$\begin{aligned}\Delta\nu_{\text{exp}} &= 4,465,004 \text{ (29) MHz (6.5 ppm)} \\ \delta\nu_{\text{th}} &= 4,465.0 \text{ (0.3) MHz.}\end{aligned}\tag{14}$$

Very recently the hfs interval has been measured also in the ( $^3\text{He}\mu^-e^-$ ) atom, an interesting system in which two distinct magnetic moments  $\mu_{^3\text{He}}$  and  $\mu_{\mu^-}$  contribute to  $\Delta\nu$ . The pseudonucleus ( $^3\text{He}\mu^-$ ) $^+$  actually can be in two different spin states  $I = 1$  and  $I = 0$ ; only the  $I = 1$  configuration has a magnetic moment and hfs interaction with the electron.

## CONCLUSION

Theoretical developments and questions have adequately motivated precise and sensitive experimental studies of muonium and muoniumlike atoms over the past 20 years. As long as new improved experimental possibilities at meson factories can be realized, the stimulus for ever more precise and sensitive studies of muonium and other simple muonic atoms will continue, and better measurements will motivate improved calculations. Hence still more sensitive tests will be made of the quantum field theory best understood to date, quantum electrodynamics, and indeed to the level at which weak interaction effects predicted by the unified electroweak theory and at which virtual effects due to strong interactions are observed. We predict that active interest in muonium will continue.

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VERNON W. HUGHES

*Yale University,  
New Haven, Connecticut 06520*

GISBERT ZU PUTLITZ

*Physikalisches Institut der Universität Heidelberg,  
D-6900 Heidelberg,  
Federal Republic of Germany*

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