

ELECTRON COOLING WITH MAGNETIC FIELD

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An elementary discussion is given of the basic equations for slowing of a proton in an electron gas with magnetic field. For large impact parameter the result is sensitive to the way in which the divergent integral is cut off at the lower limit. The most straightforward procedure gives the same force along the direction of the proton velocity, but only one half of that at right angles to it, as compared with previous results. However, a compensating modification of the small impact parameter contribution is found. The combined result is as before.

1. INTRODUCTION

The theory of electron cooling has been developed especially by Derbenev and Skrinsky.¹ See also Rosenbluth.² Helpful simplified versions have been given by Mills³ and Sørensen.⁴ Calculation of a first approximation to the momentum transfer from a passing proton to a given electron allows, on squaring, an estimate of the corresponding energy transfer, and summing over all electrons gives the mean rate of loss of energy by the proton. This gives immediately the mean force exerted by the medium in the direction opposite to the proton velocity. In the non-magnetic case the medium does not, by symmetry, exert any mean force at right angles to the proton velocity and so (ignoring fluctuations) the cooling force is completely known. But when a magnetic field is present, and the proton does not move exactly along it, there is no such symmetry. The force on the proton has other components and the calculation of energy loss is not enough. Derbenev and Skrinsky give a fairly sophisticated argument to complete the analysis, and Sørensen⁴ just quotes their result at this point.

Here a simple and elementary discussion is made. We estimate the disturbance in the space distribution of the electrons caused by the passage of the proton. Those behind it have been displaced a little towards it, causing a negative charge concentration, which pulls the proton back and also a little sideways in the magnetic case.

For large impact parameter, it is found that for the force perpendicular to the proton velocity the result depends on just how the divergent integral is cut off at the lower limit. While one procedure gives the expected result, a more careful treatment gives a different result. However, it is seen that a closer examination of the small impact-parameter region

produces a compensating term, so that the combined result is as before.

2. NO MAGNETIC FIELD

Consider first the case of no magnetic field. Suppose for simplicity that the electrons have zero temperature and let us work in the frame of reference in which they are at rest. Let \mathbf{V} be the velocity of the proton (assumed nonrelativistic)—which is constant in the first approximation. We wish to calculate the force exerted by the proton on the medium (and then by change of sign that exerted by the medium on the proton) at some moment—which is taken as the time origin $t = 0$ —when it arrived at some point which we take as origin of coordinates $\mathbf{r} = 0$.

The force on an electron at position \mathbf{r} at time $t = 0$ is then

$$\mathbf{F}(\mathbf{r}) = -e^2 \mathbf{r} / |\mathbf{r}|^3. \quad (1)$$

In a first approximation the electrons have not been disturbed by the proton and integrating Eq. (1) over a fully symmetrical electron distribution gives zero. A second approximation is required.

In the second approximation we still ignore any perturbation of the motion of the relatively massive proton, but estimate the displacement of electrons and its effect on the force.

The force on an electron at position \mathbf{r} at time t , due to the given proton, is

$$-e^2(\mathbf{r} - \mathbf{V}t) / |\mathbf{r} - \mathbf{V}t|^3 = \mathbf{F}(\mathbf{r} - \mathbf{V}t).$$

In first approximation the electron maintains its unperturbed position \mathbf{r} , but in second approximation suffers a displacement

$$\Delta(\mathbf{r}) = (1/m_e) \int_{-\infty}^0 dt' \int_{-\infty}^{t'} dt'' \mathbf{F}(\mathbf{r} - \mathbf{V}t'') \quad (2)$$

or

$$\Delta(x, y, z) = (1/m_e V^2) \int_z^{\infty} dz' \int_{z'}^{\infty} dz'' \mathbf{F}(x, y, z''), \quad (3)$$

where m_e is the electron mass and the z -axis has been taken in the direction of \mathbf{V} . This causes an increment of the force on the electron at $t = 0$

$$\delta \mathbf{F}(\mathbf{r}) = [\Delta(\mathbf{r}) \cdot \partial / \partial \mathbf{r}] \mathbf{F}(\mathbf{r}). \quad (4)$$

So finally the mean force on the proton is

$$\mathbf{F} = -n_e \int d^3 \mathbf{r} \delta \mathbf{F}(\mathbf{r}), \quad (5)$$

where n_e is the number of electrons per unit volume in the undisturbed medium.

In the non-magnetic case only the z -component of Eq. (5) (i.e., along \mathbf{V}) is non-zero (by symmetry). For this,

$$\begin{aligned} \delta F_z &= \Delta_x \partial F_z / \partial x + \Delta_y \partial F_z / \partial y + \Delta_z \partial F_z / \partial z \\ &= \Delta_x \partial F_x / \partial z + \Delta_y \partial F_y / \partial z + \Delta_z \partial F_z / \partial z, \end{aligned} \quad (6)$$

where we have used

$$\partial F_z / \partial x = \partial F_x / \partial z, \quad \partial F_z / \partial y = \partial F_y / \partial z \quad (7)$$

(which follow from \mathbf{F} being proportional to an electrostatic field, of zero curl). Then

$$\begin{aligned} \bar{F}_z &= -n_e \iint dx dy \int dz (\Delta_x \partial F_x / \partial z + \Delta_y \partial F_y / \partial z + \Delta_z \partial F_z / \partial z) \\ &= n_e \iint dx dy \int dz (F_x \partial \Delta_x / \partial z + F_y \partial \Delta_y / \partial z + F_z \partial \Delta_z / \partial z) \end{aligned} \quad (8)$$

(by partial integration over z). Evaluating the derivatives of Δ by differentiating Eq. (3) with respect to the integration limit z ,

$$\begin{aligned} \bar{F}_z &= -[n_e / (m_e V^2)] \iint dx dy \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz' \times \\ &\quad \{F_x(x, y, z) F_x(x, y, z') + F_y(x, y, z) F_y(x, y, z') + \\ &\quad F_z(x, y, z) F_z(x, y, z')\} \end{aligned} \quad (9)$$

$$\begin{aligned} &= -[n_e / (2m_e V^2)] \int \int dx \\ &\quad dy \left\{ \left[\int_{-\infty}^{\infty} dz F_x(x, y, z) \right]^2 + \right. \\ &\quad \left[\int_{-\infty}^{\infty} dz F_y(x, y, z) \right]^2 + \\ &\quad \left. \left[\int_{-\infty}^{\infty} dz F_z(x, y, z) \right]^2 \right\}. \end{aligned} \quad (10)$$

The last of the three integrals vanishes, because $F_z(x, y, z) = -F_z(x, y, -z)$. Changing to cylindrical polar coordinates (θ, ρ, z) the others give, using Eq. (1)

$$\begin{aligned} \bar{F}_z &= -[n_e / (2m_e V^2)] \int_0^{2\pi} d\theta \int_{\rho_{\min}}^{\rho_{\max}} d\rho \rho \times \\ &\quad \left\{ \left[\int_{-\infty}^{\infty} dz e^2 \rho \cos \theta / (z^2 + \rho^2)^{3/2} \right]^2 + \right. \\ &\quad \left. \left[\int_{-\infty}^{\infty} dz e^2 \rho \sin \theta / (z^2 + \rho^2)^{3/2} \right]^2 \right\} \end{aligned} \quad (11)$$

$$\begin{aligned} &= -[n_e / (2m_e V^2)] 2\pi \int_{\rho_{\min}}^{\rho_{\max}} d\rho \\ &\quad \times \rho \{ (2e^2 / \rho)^2 / 2 + (2e^2 / \rho)^2 / 2 \} \quad (12) \\ &= [4\pi n_e e^4 / (m_e V^2)] \log(\rho_{\max} / \rho_{\min}), \quad (13) \end{aligned}$$

where ρ_{\max} and ρ_{\min} are maximum and minimum impact parameters beyond which the approximations made are inadequate. Interaction between electrons has been neglected. When it is allowed for, remote electrons are shielded from the proton by the tendency of nearer electrons to cluster round it. This leads to formulae for ρ_{\max} . On the other hand, treating Δ as a perturbation is not sensible for small ρ , when Δ becomes comparable with ρ . This determines a ρ_{\min} below which it is not sensible to extend the integral. Of course it remains then to estimate the real effects of small impact parameters which are thereby neglected. Discussion by two-body collision theory shows them to be relatively unimportant when the log in Eq. (13) is large. Formulae for ρ_{\min} and ρ_{\max} are given in many places, including Ref. 1.

So much for zero temperature. But the force on the proton can depend only on the relative motion of proton and electrons. So we can write immediately

$$\mathbf{F} = -[4\pi n_e e^4 / m_e] \int d^3 \mathbf{V}_e (\mathbf{V} - \mathbf{V}_e) / (\mathbf{V} - \mathbf{V}_e)^3 f(\mathbf{V}_e) \log(\rho_{\max} / \rho_{\min}), \quad (14)$$

where $f(\mathbf{V}_e)$ is a more general distribution in velocity space normalized so that

$$\int d^3 \mathbf{V}_e f(\mathbf{V}_e) = 1.$$

This is the standard result. We proceed to modify it for the magnetic case.

3. MAGNETIC CASE

Following Derbenev and Skrinsky, the collisions are divided into two types—or the $dx dy$ integration into two parts—with a gray area in between that it is hoped is not too important. It is assumed that $V_p \lesssim V_e$,^{1,4} and we consider a “flattened” distribution,¹ with V_e negligible in the direction of the magnetic field. We suppose that $\rho_{\min} \ll r_L \ll \rho_{\max}$, where r_L is the Larmor radius of the electrons.

3.1 $\rho \ll r_L$

The electrons perform small Larmor circles around the (strong) magnetic field. But when the proton passes close enough to the electron, the latter may be regarded as moving freely in a straight line during the small time interval of most importance. So such collisions are perhaps allowed for just by replacing ρ_{\max} by r_L in Eq. (14). Extending the integration right up to r_L , rather than say $r_L/10$, is of no importance when the log is sufficiently large. It will turn out, however, that in the magnetic case there is an important supplementary contribution from this small ρ region.

3.2 $\rho \gg r_L$

In these conditions the force exerted by the proton on the electron changes little while the electron performs a revolution. We have the conditions for an adiabatic approximation. As regards electron motion transverse to the magnetic field \mathbf{B} , only a slow drift is induced of the center of the Larmor circle. But as regards motion along \mathbf{B} , the electron is accelerated freely towards the proton. Only the component of \mathbf{F} along \mathbf{B} displaces the electron significantly, but this component has just the same effect as before. So we replace Eq. (4) by

$$\delta \mathbf{F}(\mathbf{r}) = (\boldsymbol{\Delta} \cdot \hat{\mathbf{B}}) (\hat{\mathbf{B}} \cdot \partial / \partial \mathbf{r}) F(\mathbf{r}), \quad (15)$$

with $\boldsymbol{\Delta}$ again given by Eq. (3), \mathbf{F} by Eq. (1), and \mathbf{B} by Eq. (5). \mathbf{B} is a unit vector in the direction of \mathbf{B} . Strictly speaking this procedure is not legitimate right down to $\rho = r_L$, but again we cut off the integral at this point, because as will be seen, the lower limit appears only in a large logarithm.

In more detail, Eq. (15) reads

$$(\delta F_x, \delta F_y, \delta F_z) = (\Delta_x \hat{B}_x + \Delta_y \hat{B}_y + \Delta_z \hat{B}_z) (\hat{B}_x \partial / \partial x + \hat{B}_y \partial / \partial y + \hat{B}_z \partial / \partial z) (F_x, F_y, F_z). \quad (16)$$

Without loss of generality, we choose \mathbf{B} to lie in the y - z plane. Fortunately many of the 27 terms disappear and many others do not contribute finally by symmetry considerations. One finds

$$\bar{F}_x = 0 + \dots \quad (17)$$

$$\begin{aligned} \bar{F}_y = & -n_e \iint dx \\ & \times dy \left\{ \hat{B}_y \hat{B}_z \int dz \Delta_y \partial / \partial z F_y + \right. \\ & \left. \hat{B}_z \hat{B}_y \int dz \Delta_z \partial / \partial y F_y \right\} + \dots \end{aligned} \quad (18)$$

$$\begin{aligned} \bar{F}_z = & -n_e \iint dx \\ & \times dy \left\{ \hat{B}_y^2 \int dz \Delta_y \partial / \partial y F_z + \right. \\ & \left. \hat{B}_z^2 \int dz \Delta_z \partial / \partial z F_z \right\} + \dots \end{aligned} \quad (19)$$

where the terms $+\dots$ do not contribute when x and y integrations are performed, because of integrands antisymmetric with respect to reflections in zx or zy planes.

The second term in Eq. (18) can be rewritten, by partial integration over y , as

$$+ n_e \hat{B}_y \hat{B}_z \iint dx dy \int dz F_y \partial / \partial y \Delta_z + \text{surface terms}, \quad (20)$$

where the surface terms arise from integration limits at $(x^2 + y^2)^{1/2} = r_L$ or ρ_{\max} . Ignoring surface terms for the moment, this can be further rewritten

$$+ n_e \hat{B}_y \hat{B}_z \iint dx dy \int dz F_y \partial / \partial z \Delta_y \quad (21)$$

[using $\partial \Delta_z / \partial y = \partial \Delta_y / \partial z$, which follows from Eqs. (2) and (7)]. On partial integration with respect to z , this is seen to equal the first term of Eq. (18). Then (apart from the surface term)

$$\mathcal{F}_y = -2n_e \hat{B}_y \hat{B}_z \iint dx dy \int dz \Delta_y \partial / \partial z F_y. \quad (22)$$

For \mathcal{F}_z , using $\partial F_z / \partial y = \partial F_y / \partial z$, we have

$$\begin{aligned} \mathcal{F}_z &= -n_e \iint dx \\ &\times dy \left\{ \hat{B}_y^2 \int dz \Delta_y \partial / \partial z F_y \right. \\ &\left. + \hat{B}_z^2 \int dz \Delta_z \partial / \partial z F_z \right\}. \quad (23) \end{aligned}$$

The integrals in Eqs. (22) and (23) occurred already in Eq. (8) and were evaluated following that equation. Substituting the results

$$\begin{aligned} \mathcal{F}_x &= 0 \\ \mathcal{F}_y &= 2\hat{B}_y \hat{B}_z (-2\pi n_e e^4 / m_e V^2) \log(\rho_{\max} / r_L) \quad (24) \\ \mathcal{F}_z &= \hat{B}_y^2 (-2\pi n_e e^4 / m_e V^2) \log(\rho_{\max} / r_L). \end{aligned}$$

On resolving into components parallel and perpendicular to \mathbf{B} ,

$$\begin{aligned} \mathcal{F}_{\parallel} &= \mathcal{F}_y \hat{B}_y + \mathcal{F}_z \hat{B}_z \\ &= (-2e^4 n_e \pi / m_e V^2) \log(\rho_{\max} / r_L) 3\hat{B}_y \hat{B}_z \quad (25) \end{aligned}$$

and

$$\begin{aligned} \mathcal{F}_{\perp} &= \mathcal{F}_z \hat{B}_y - \mathcal{F}_y \hat{B}_z \\ &= (-2e^4 n_e \pi / m_e V^2) \log(\rho_{\max} / r_L) \\ &\times (\hat{B}_y^3 - 2\hat{B}_y \hat{B}_z^2). \quad (26) \end{aligned}$$

We may write, in terms of components of \mathbf{V} parallel and perpendicular to \mathbf{B} ,

$$\hat{B}_y = V_{\perp} / V, \quad \hat{B}_z = V_{\parallel} / V,$$

whence

$$\begin{aligned} \mathcal{F}_{\parallel} &= (-2e^4 n_e \pi / m_e V^5) \log(\rho_{\max} / r_L) 3V_{\perp}^2 V_{\parallel} \\ \mathcal{F}_{\perp} &= (-2e^4 n_e \pi / m_e V^5) \log \\ &(\rho_{\max} / r_L) (V_{\perp}^2 - 2V_{\parallel}^2) V_{\perp} \quad (27) \end{aligned}$$

as quoted by Derbenev and Skrinsky¹ and Sørensen.⁴

It remains, however, to discuss the surface terms referred to in expression (20). Their explicit form is

$$\begin{aligned} -n_e \hat{B}_y \hat{B}_z \int dx dz \left\{ \left[\Delta_z F_y \right]_{y=(\rho_{\max}^2 - x^2)^{1/2}} \right. \\ \left. - \left[\Delta_z F_y \right]_{y=-(\rho_{\max}^2 - x^2)^{1/2}} \right. \\ \left. - \left[\Delta_z F_y \right]_{y=(r_L^2 - x^2)^{1/2}} \right. \\ \left. - \left[\Delta_z F_y \right]_{y=-(r_L^2 - x^2)^{1/2}} \right\}. \quad (28) \end{aligned}$$

When Δ_z is calculated from Eqs. (3) and (1) it is found to diverge logarithmically, because of the long range of the Coulomb force. So screening has to be taken into account here. We do so crudely by cutting off the integrals at $z = \rho_{\max}$. Then

$$\begin{aligned} \Delta_z &= -(1/m_e V^2) \int_z^{\rho_{\max}} dz' \int_{z'}^{\rho_{\max}} dz'' \\ &\times e^2 z'' / (x^2 + y^2 + z''^2)^{3/2} \\ &= (e^2 / m_e V^2) \int_z^{\rho_{\max}} dz' [(x^2 + y^2 + z''^2)^{-1/2}]_{z'}^{\rho_{\max}} \\ &= (e^2 / m_e V^2) \int_z^{\rho_{\max}} dz' \{ (\rho^2 + \rho_{\max}^2)^{-1/2} \\ &\quad - (\rho^2 + z'^2)^{-1/2} \} \\ &= (e^2 / m_e V^2) \{ (\rho_{\max} - z) / (\rho^2 + \rho_{\max}^2)^{-1/2} \\ &\quad - \log[(\rho_{\max} + (\rho_{\max}^2 + \rho^2)^{1/2}) / \\ &\quad (z + (z^2 + \rho^2)^{1/2})] \}. \quad (29) \end{aligned}$$

Substitution into expression (28) and examination of the various terms shows that the only one that can contain the large log, and so compete with the term already included in Eqs. (24), comes from the log term in Eq. (29) and the second term in expression (28):

$$\begin{aligned}
& -n_e \hat{B}_y \hat{B}_z (e^2/m_e V^2) \int dx dz [F \log\{(\rho_{\max} + (\rho_{\max}^2 + r_L^2)^{1/2})/(z + (z^2 + r_L^2)^{1/2})\}] \Big|_{y=-(r_L^2-x^2)^{1/2}}^{y=(r_L^2-x^2)^{1/2}} \\
& = (-e^2/m_e V^2) n_e \hat{B}_y \hat{B}_z \int dx dz [-e^2 y (z^2 + r_L^2)^{-3/2} \log\{(\rho_{\max} + (\rho_{\max}^2 + r_L^2)^{1/2})/ \\
& \quad (z + (r_L^2 + z^2)^{1/2})\}] \Big|_{y=-(r_L^2-x^2)^{1/2}}^{y=(r_L^2-x^2)^{1/2}} \\
& = (e^4/m_e V^2) 2n_e \hat{B}_y \hat{B}_z \int_{-r_L}^{r_L} dx \int_{-\infty}^{\infty} dz (r_L^2 - x^2)^{1/2} (z^2 + r_L^2)^{-3/2} \log\{(\rho_{\max} + (\rho_{\max}^2 + r_L^2)^{1/2})/(z + (r_L^2 + z^2)^{1/2})\} \\
& = (e^4/m_e V^2) \pi n_e \hat{B}_y \hat{B}_z \int dz r_L^2 (z^2 + r_L^2)^{-3/2} \log\{(\rho_{\max} + (\rho_{\max}^2 + r_L^2)^{1/2})/ \\
& \quad (z + (r_L^2 + z^2)^{1/2})\}. \tag{30}
\end{aligned}$$

This integral can be done exactly. It turns out that the logarithm, because it varies rather slowly, can be replaced for large (ρ_{\max}/r_L) by its approximate value at $z = 0$

$$\log(\rho_{\max}/r_L).$$

We do not retain the factor 2 in the argument of the log since we assume $\rho_{\max} \gg r_L$. With

$$\int_{-\infty}^{\infty} dz r_L^2 / (z^2 + r_L^2)^{3/2} = 2,$$

Eq. (30) becomes

$$\hat{B}_y \hat{B}_z (2\pi n_e e^4/m_e V^2) \log(\rho_{\max}/r_L). \tag{31}$$

The surface term (31) is half the magnitude and opposite in sign to that already given for \mathcal{F}_y in Eqs. (24). The final result for \mathcal{F}_y is then

$$\mathcal{F}_y = \hat{B}_y \hat{B}_z (-2\pi n_e e^4/m_e V^2) \log(\rho_{\max}/r_L). \tag{32}$$

Instead of Eqs. (27) we then find

$$\left. \begin{aligned}
\mathcal{F}_{\parallel} &= (-2e^4 n_e \pi / m_e V^5) \{ \log(\rho_{\max}/r_L) \} 2V_{\perp}^2 V_{\parallel} \\
\mathcal{F}_{\perp} &= (-2e^4 n_e \pi / m_e V^5) \{ \log(\rho_{\max}/r_L) \} \\
&\quad \times (V_{\perp}^2 - V_{\parallel}^2) V_{\perp}
\end{aligned} \right\}, \tag{33}$$

Could there be some reason for ignoring the surface term? One could note that the main term in Δ_z , the log term in Eq. (29), is essentially constant near $z \approx 0$, $\rho \approx r_L$. A uniform displacement of the whole medium should have no effect, even when deflected by the magnetic field. So one could anticipate a complementary term from a full treatment of the region $\rho \leq r_L$. And indeed when one calculates the force on the proton due to the cylinder of electrons originally within a distance r_L from its trajectory, now displaced in the y -direction by an amount

$$(-e^2/m_e V^2) \hat{B}_z \hat{B}_y \log(\rho_{\max}/r_L), \tag{34}$$

we obtain just the opposite of expression (31). This large constant term in the displacement is built up as the proton approaches from far away to a distance of order r_L . It is common to all electrons near the proton. It has not already been allowed for when the non-magnetic formula (13) is used for close collisions, since in the non-magnetic case the corresponding effect is just an unimportant translation of the cylinder $\rho < r_L$ in its longitudinal direction. It is only the magnetic field that deflects it into the transverse direction.

Note that the use of r_L as upper and lower limit, for close and distant collisions, respectively, is not really justified by the accuracy of the formulae. It affects the answer little because these limits occur only in large logarithms. But it remains that the gray area in the neighborhood of $\rho \sim r_L$ has not been properly treated. But any additional contribution from the region will not involve ρ_{\min} or ρ_{\max} , and not, therefore, the large logarithms that we have supposed to dominate.

The results given in Eq. (27) were for $V_{e\parallel} = 0$. But the magnetic field is unaltered by Lorentz transformations with velocity in the \hat{B} direction, and the components of force are unchanged by such transformation with non-relativistic velocity. So for electrons with velocity $V_{e\parallel}$ along \hat{B} we have only to replace V_{\parallel} by $V_{\parallel} - V_{e\parallel}$ everywhere in Eq. (27). Then for a distribution $f(\mathbf{V}_e)$ the formulae are

$$\begin{aligned} \mathcal{F}_{\parallel} &= (-2e^4 n_e \pi / m_e) V_{\perp}^2 \int 3 f(\mathbf{V}_e) (V_{\parallel} - V_{e\parallel}) |\mathbf{V} - \mathbf{V}_{e\parallel}|^{-5} \log(\rho_{\max} / r_L) d^3 \mathbf{V}_e \\ \mathcal{F}_{\perp} &= (-2e^4 n_e \pi / m_e) V_{\perp} \int f(\mathbf{V}_e) \{V_{\perp}^2 - 2(V_{\parallel} - V_{e\parallel})^2\} |\mathbf{V} - \mathbf{V}_{e\parallel}|^{-5} \log(\rho_{\max} / r_L) d^3 \mathbf{V}_e \end{aligned} \quad (35)$$

We keep $\log(\rho_{\max} / r_L)$ under the integral sign since r_L / ρ_{\max} depends on $V_{e\perp}^2$ —although this is not very important.

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