Chapter 30

Right Unitarity Triangles and Tri-Bimaximal Mixing from Discrete Symmetries and Unification

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Abstract

We discuss a recently proposed new class of flavour models which predicts both close to tri-bimaximal lepton mixing (TBM) and a right-angled Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle, $\alpha \approx 90^\circ$. The ingredients of the models include a supersymmetric (SUSY) unified gauge group such as $SU(5)$, a discrete family symmetry such as A_4 or S_4, a shaping symmetry including products of Z_2 and Z_4 groups as well as spontaneous CP violation. The vacuum alignment in such models allows a simple explanation of $\alpha \approx 90^\circ$ by a combination of purely real or purely imaginary vacuum expectation values (vevs) of the flavon fields responsible for family symmetry breaking.

30.1. Motivation

Albeit the great success of the Standard Model (SM) of particle physics, its flavour sector is still puzzling. The SM flavour puzzle can be roughly divided into three aspects, which are first the hierarchies of the observed fermion masses, second the pattern of the observed mixing angles, and third the origin of CP violation.

Here we are concerned mainly with two of those aspects. The first one concerns the mixing angles. The fact that the leptonic mixing angles turned out to be close to TBM [1] has led to increasing interest in non-Abelian discrete family symmetries for flavour model building. Nevertheless, in many realistic models another shaping symmetry has to be invoked to forbid unwanted operators in the (super-)potential. These shaping symmetries can shed some light on the second aspect of the flavour puzzle we are concerned with, the origin of CP violation, as was recently shown in [2].

Experimental results point towards a right-angled CKM unitarity triangle with $\alpha = (89.0^{+4.4}_{-4.2})^\circ$ [3]. This can be understood in terms of a simple phase sum rule [4]. As we will revise later it becomes clear from this sum rule, that mass matrices with purely real and purely imaginary elements can lead to a right-angled CKM unitarity triangle, see also [5]. These special phases in turn can be the result of a spontaneously broken discrete symmetry [2].

In combination with a unified gauge group this proliferates an attractive framework to describe mixing angles and CP violation in the quark and the lepton sector as a result of spontaneously broken discrete family and shaping symmetries.

30.2. The Quark Mixing Phase Sum Rule

First we revise the phase sum rule from [4]. For the mass matrices M_u and M_d in the Lagrangian we use the convention

$$\mathcal{L}_Y = -\overline{u^i_L}(M_u)_{ij}u^j_R - \overline{d^i_L}(M_d)_{ij}d^j_R + H.c.\ . \tag{30.1}$$

They are diagonalised by bi-unitary transformations

$$V_{u_L} M_u V^\dagger_{u_R} = \text{diag}(m_u, m_c, m_t) \quad \text{and} \quad V_{d_L} M_d V^\dagger_{d_R} = \text{diag}(m_d, m_s, m_b)\ , \tag{30.2}$$

where V_{u_L}, V_{u_R}, V_{d_L} and V_{d_R} are unitary 3×3 matrices. The CKM matrix V_{CKM} is given by

$$V_{\text{CKM}} = V_{u_L} V^\dagger_{d_L} = U^{u_L}_{12}{}^\dagger U^{u_L}_{13}{}^\dagger U^{u_L}_{23}{}^\dagger U^{d_L}_{23} U^{d_L}_{13} U^{d_L}_{12}\ , \tag{30.3}$$

where the U_{ij} matrices are unitary rotation matrices in the i-j plane, for instance,

$$U_{12} = \begin{pmatrix} c_{12} & s_{12}e^{-\mathrm{i}\,\delta_{12}} & 0 \\ -s_{12}e^{\mathrm{i}\,\delta_{12}} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\ . \tag{30.4}$$

For hierarchical quark mass matrices with a texture zero in the 1-3 element it is straightforward to derive the following approximate expressions for the quark mixing angles (for more details see [4])

$$\theta_{23}e^{-\mathrm{i}\,\delta_{23}} = \theta^d_{23}e^{-\mathrm{i}\,\delta^d_{23}} - \theta^u_{23}e^{-\mathrm{i}\,\delta^u_{23}}\ , \tag{30.5}$$

$$\theta_{13}e^{-\mathrm{i}\,\delta_{13}} = -\theta^u_{12}e^{-\mathrm{i}\,\delta^u_{12}}(\theta^d_{23}e^{-\mathrm{i}\,\delta^d_{23}} - \theta^u_{23}e^{-\mathrm{i}\,\delta^u_{23}})\ , \tag{30.6}$$

$$\theta_{12}e^{-\mathrm{i}\,\delta_{12}} = \theta^d_{12}e^{-\mathrm{i}\,\delta^d_{12}} - \theta^u_{12}e^{-\mathrm{i}\,\delta^u_{12}}\ . \tag{30.7}$$

From these formulas we obtain for α

$$90^\circ \approx \alpha = \arg\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right) \approx \delta^d_{12} - \delta^u_{12} \quad \text{with} \quad \delta^{d/u}_{12} = \arg\left(\frac{M^{d/u}_{12}}{M^{d/u}_{22}}\right)\ . \tag{30.8}$$

As a direct consequence it becomes obvious, that a relative phase difference of 90° in the 1-2 mixing is enough to describe the CP violation in the quark sector, see also [5]. The simplest realisation of this would be mass matrices with purely real and purely imaginary elements.

In the following we discuss a recent idea, how this can be accomodated in the context of flavour models with discrete family and shaping symmetries.

30.3. The Method: Discrete Vacuum Alignment

The class of models, we discuss here, is based on the method of discrete vacuum alignment [2], which has as its ingredients a discrete family (like A_4 or S_4) and shaping symmetry (like a product of Z_n's), spontaneous

CP violation and a SUSY unified gauge group. The unified gauge group is not strictly necessary, but it is very powerful, because it relates the mixing and the CP violation in the quark and the lepton sector to each other.

The method can be described in a simple algorithm. First, use the family symmetry to align the flavon vevs, so that only one complex parameter x is left undetermined, e.g. $\langle\phi\rangle \propto (0,0,x)^T$ or $\langle\phi\rangle \propto (x,x,x)^T$. Then add for each flavon ϕ the following type of terms to the superpotential

$$P\left(\frac{\phi^n}{\Lambda^{n-2}} \mp M^2\right) \ , \tag{30.9}$$

which are allowed by the discrete Z_n shaping symmetries, and where M and Λ are real mass parameters. By solving the F-term condition, $F_P = 0$, the phase of the flavon vev is fixed to be

$$\arg(\langle\phi\rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n} q \ , & q = 1, \ldots, n \quad \text{for "}-\text{" in Eq. (30.9)} \ , \\ \frac{2\pi}{n} q + \frac{\pi}{n} \ , & q = 1, \ldots, n \quad \text{for "}+\text{" in Eq. (30.9)} \ . \end{cases} \tag{30.10}$$

If the shaping symmetries are only Z_2 or Z_4 symmetries the phases can easily be arranged to fulfill the phase sum rule in Eq. (30.8).

30.4. One Example Model: $SU(5) \times A_4$

As an example we sketch now the A_4 model from [2], where an S_4 model is given as well. The A_4 model has the symmetry $SU(5) \times A_4 \times Z_4^4 \times Z_2^2 \times U(1)_R$ and five flavons with the alignments

$$\langle\phi_1\rangle \propto \begin{pmatrix}1\\0\\0\end{pmatrix} , \ \langle\phi_2\rangle \propto \begin{pmatrix}0\\-\mathrm{i}\\0\end{pmatrix} , \ \langle\phi_3\rangle \propto \begin{pmatrix}0\\0\\1\end{pmatrix} , \ \langle\phi_{23}\rangle \propto \begin{pmatrix}0\\1\\-1\end{pmatrix} , \ \langle\phi_{123}\rangle \propto \begin{pmatrix}1\\1\\1\end{pmatrix} \ . \tag{30.11}$$

Note that only $\langle\phi_2\rangle$ has a purely imaginary vev, while all other vevs are real. To demonstrate the method of discrete vacuum alignment we discuss the simple alignment superpotential for $\phi_{1,2,3}$ (for the others see [2]):

$$W = \ P_1\left(\frac{(\phi_1\cdot\phi_1)^2}{M^2_{\Upsilon_{1;1}}} - M_1^2\right) + P_2\left(\frac{(\phi_2\cdot\phi_2)^2}{M^2_{\Upsilon_{2;2}}} - M_2^2\right) + P_3(\phi_3\cdot\phi_3 - M_3^2) + A_i(\phi_i\star\phi_i) + O_{ij}(\phi_i\cdot\phi_j) \ , \tag{30.12}$$

where M_Υ labels messenger masses. We use the standard "$SO(3)$ basis" for which "$\cdot$" is the usual $SO(3)$ inner product and the symmetric "$\star$" product is defined analogous to the cross product but with a relative plus sign instead of a minus sign.

The F-term conditions $F_{A_i} = F_{O_{ij}} = 0$ give the directions of the flavon vevs and their mutual orthogonality. The vev of ϕ_3 (charged only under a Z_2) is fixed to be real while the vevs of ϕ_2 and ϕ_3, which are charged under Z_4's can be chosen to be either real or imaginary and we pick the phases from Eq. (30.11).

The five-dimensional matter fields are organised in triplets under A_4 and the tenplets are A_4 singlets. Therefore in our conventions the flavon vevs form rows of the down-type quark Yukawa matrix. The up-type quark Yukawa matrix is given by the inner product of two flavon vevs apart from the 3-3 element, which is generated on the renormalisable level to account for the large top mass. With the symmetries and the field content (for details see [2]) we obtain in the quark sector

$$Y_d = \begin{pmatrix} 0 & \mathrm{i}\,\epsilon_2 & 0 \\ \epsilon_{123} & \epsilon_{23}+\epsilon_{123} & -\epsilon_{23}+\epsilon_{123} \\ 0 & 0 & \epsilon_3 \end{pmatrix} \quad \text{and} \quad Y_u = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & a_{23} \\ 0 & a_{23} & a_{33} \end{pmatrix} \ , \tag{30.13}$$

where the ϵ_i and a_{ij} are real coefficients. First note that $\delta_{12}^d = \arg((Y_d)_{12}/(Y_d)_{22}) = 90°$, due to the purely imaginary 1-2 element of Y_d, and $\delta_{12}^u = 0°$, because Y_u is real. The 1-3 elements in Y_d and Y_u vanish and the sum rule from Eq. (30.8) can be applied successfully.

In the lepton sector we obtain for the Yukawa matrices and the right-handed neutrino mass matrix

$$Y_e^T = -\frac{3}{2}\begin{pmatrix} 0 & \mathrm{i}\,\epsilon_2 & 0 \\ \epsilon_{123} & -3\epsilon_{23}+\epsilon_{123} & 3\epsilon_{23}+\epsilon_{123} \\ 0 & 0 & \epsilon_3 \end{pmatrix}\,,\; Y_\nu = \begin{pmatrix} 0 & a_{\nu_2} \\ a_{\nu_1} & a_{\nu_2} \\ -a_{\nu_1} & a_{\nu_2} \end{pmatrix}\,,\; M_R = \begin{pmatrix} M_{R_1} & 0 \\ 0 & M_{R_2} \end{pmatrix}\,. \tag{30.14}$$

The first thing to note here, is that we do not use standard GUT relations, but instead use $y_\tau/y_\mu = -3/2$ and $y_\mu/y_s \approx 9/2$, which fit much better to current data for the quark and lepton masses and a CMSSM like scenario with $\mu > 0$ [6].

In the neutrino sector only two of the three neutrinos are massive by construction since we have introduced only two right-handed neutrinos and the mass pattern is normal hierarchical. For the mixing we obtain exact tri-bimaximal mixing in the neutrino sector, which is disturbed by corrections coming from the charged lepton sector inducing, for instance, a non-vanishing $\theta_{13}^{\mathsf{PMNS}} \approx 3°$. It is also interesting to note, that we predict all CP phases in the lepton sector, which turn out to be very close to $0°$ or $180°$.

30.5. Another Example

The $SU(5)\times A_4$ model in [7] can also be read as another example of this class of models, if the flavon $\tilde{\phi}_{23}$ is split into two flavons

$$\langle\tilde{\phi}_{23}\rangle \to \langle\tilde{\phi}_2\rangle + \langle\tilde{\phi}_3\rangle \quad \text{where} \quad \langle\tilde{\phi}_2\rangle = \begin{pmatrix} 0 \\ -\mathrm{i} \\ 0 \end{pmatrix} \tilde{\epsilon}_{23} \quad \text{and} \quad \langle\tilde{\phi}_3\rangle = \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix} \tilde{\epsilon}_{23}\,. \tag{30.15}$$

In this model the sum rule, Eq. (30.8), is not applicable, because there are no texture zeros in the 1-3 elements, but the agreement with the experimentally determined CKM phase is still very good, which is closely related to the use of the GUT relation $y_\mu/y_s \approx 9/2$. In fact, the CKM phase can be predicted in this model from the precisely known values for the electron mass, the muon mass and the Cabibbo angle and we obtain

$$\delta_{\mathsf{CKM}}^{\mathsf{pred}} = 69.9° \quad \text{while} \quad \delta_{\mathsf{CKM}}^{\mathsf{exp}} = (68.8^{+4.0}_{-2.3})°\,. \tag{30.16}$$

The fit to the quark masses and mixing parameters and the charged lepton masses in this model is quite good with a χ^2 per degree of freedom of about 1.6.

In the neutrino sector we have added a fifteen dimensional representation of $SU(5)$ giving a universal contribution to the neutrino masses, which can result in quasi-degenerate neutrino masses. All the mixing parameters are close to tri-bimaximal and the phases are fixed with $\delta_{\mathsf{PMNS}} \approx 90°$, $\alpha_1 \approx 9°$ and $\alpha_2 \approx 0°$. This has interesting phenomenological consequences. For example in Fig. 30.1 we have shown the prediction for neutrinoless double beta decay, which depends in this setup only on the neutrino mass scale and the sign of Δm^2_{31}.

30.6. Summary and Conclusions

Discrete symmetries are not only powerful in describing leptonic mixing angles, but they can also be used to predict the right-angled CKM unitarity triangle by means of spontaneous CP violation. In combination with a unified gauge group this gives close relations between the CP violation in the quark and the lepton sector. In fact, in this new class of models all physical phases can be predicted up to a discrete choice. For example in the A_4 and S_4 model from [2] apart from $\alpha \approx 90°$ in the quark sector, the leptonic Dirac and Majorana CP phases are all close to $0°$, $90°$, $180°$ or $270°$. These predictions, especially for the leptonic Dirac CP phase, can be tested at ongoing and forthcoming neutrino experiments

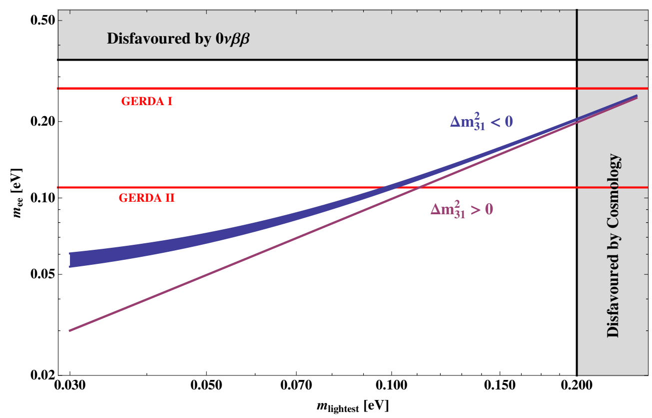

Figure 30.1. The effective mass m_{ee} in the setup from [7] relevant for neutrinoless double beta decay as a function of the mass m_{lightest} of the lightest neutrino, for an inverted neutrino mass ordering ($\Delta m^2_{31} < 0$, upper line) and for a normal mass ordering ($\Delta m^2_{31} > 0$, lower line). The bands represent the experimental uncertainties of the mass squared differences. The mass bounds from cosmology [8] and from the Heidelberg-Moscow experiment [9] are displayed as grey shaded regions. The red lines show the expected sensitivities of the GERDA experiment in phase I and II [10].

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