## Off line subtraction of seismic Newtonian Noise

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A fundamental limit for the sensitivity of an Interferometric Gravitational Wave detector in the low frequency region is originated by the effect of environmental mass density fluctuations. These fluctuations generate stochastic gravitational fields which couple directly to the apparatus test masses, bypassing seismic isolation systems. In this talk the results of a preliminary investigation on the possibility of reducing this kind of noise are reported. We focus on the feasibility of an off-line noise subtraction. In this approach the mass density fluctuations are monitored with an appropriate set of measurement devices. The resulting signals are linearly combined with the output of the interferometer to obtain a partial cancellation of the noise.

### 1 Introduction

Density mass fluctuations are continuously generated in the environment by a variety of mechanisms. Examples are atmospheric pressure fluctuations, infrastructure movements, human activities and seismic fluctuations of the ground.

These mass fluctuations are the sources of a gravitational field which, though very weak, couples directly to the test masses of an interferometric gravitational wave detector. The effect of this coupling on the sensitivity curve of a gravitational wave detector was estimated in a series of works[1,2,3,4,5]. The main result is that this source of noise could be relevant, with the planned sensitivity of the current generation interferometers, in the frequency band between 1 and 10 Hz. Below this range it is rapidly overwhelmed by the seismic noise, above by the thermal noise.

All these estimates are normalized to the power spectrum amplitude of seismic motion, which in some cases is not known very well and probably overestimated [6]. An attempt to explore the feasibility of the reduction of this "Newtonian Noise" is in our opinion worth to be done, especially in the perspective of second generation cryogenic detectors. For these the Newtonian noise could become the fundamental limitation in the low frequency range.

In the following we will focus on seismic generated Newtonian noise. Preliminary, unpublished results shows that atmospheric generated Newtonian noise could be also relevant, in fact the more relevant one. We expect that it will be possible to apply a similar analysis also in this case.

Different strategies can be elaborated in order to reduce the effect of seismic originated Newtonian noise. The more obvious one is the construction of a mold of appropriate depth around test masses, in order to isolate a big enough ground volume from the seismic motion. An accurate modelization is required in order to estimate the effectiveness of this approach. This must take into account, for example, the effect of the diffraction of elastic seismic wave on the mold.

It is also possible to think about the construction of dynamic structures which could provide a screening effect. However this approach appears to be very tricky, and to obtain relevant effects these dynamical structures should probably be very finely tuned.

Another important point is that these solutions require to introduce in the apparatus permanent modifications, which should be avoided as much as possible in order not to add non controlled, systematic effects.

In this preliminary stage of investigation it seems wiser to concentrate our attention on a third approach, which do not require apparatus modifications. This is based on the possibility of monitoring the environmental sources of Newtonian noise and correct "off-line" the output of the experiment using the obtained informations.

## 2 Off line subtraction: generalities.

The Newtonian noise, seen as a random process in the time domain, is a linear function of the mass density fluctuations in the environment. Suppose that we know, at each time, the displacement of each point in the ground from its equilibrium position $\vec{u}(\vec{x}, t)$ The force experimented by an isolated test mass in $x_0$ can be written as

$$\vec{F}(t) = G \int \rho(\vec{x}) \frac{\vec{x} - \vec{x}_0}{|\vec{x} - \vec{x}_0|^3} \vec{\nabla} \cdot \vec{u}(\vec{x}, t) dV \tag{1}$$

In the general case we can write, for the Newtonian part of the output signal of an interferometer, $N(t) = (N|B)$, where $|B)$ is the state of the ground (explicitly, the displacement field $\vec{u}$) and $(N|$ some linear operator which depends on the geometry of the interferometer.

Suppose now that we put a measure instruments, for example an accelerometer, in some point $\vec{x}$ of the ground. The part of the instrument output correlated with seismic motion will be also a linear function of the ground fluctuations, which we can write in the form $(a(x)|B)$. More generally we will write the output of a particular interferometer in the form

$$H(t) = (N|B) + \hat{H}(t), \quad \text{with} \quad \overline{(B|N)\hat{H}} = 0 \tag{2}$$

with the assumption that both $(N|B)$ and $\hat{H}$ are stationary, zero mean stochastic processes. The $\overline{\cdots}$ mean must be conveniently defined. For example it could be the usual $T \to \infty$ limit of the ensemble mean over strips of length $T$. In Eq. (2) $\hat{H}$ represent the fraction of the output uncorrelated with seismic motion, as for example thermal noise, shot noise, gravitational wave signals. On the same footings we will write the output of an accelerometer as

$$A_i(t) = (a_i|B) + \eta_i, \quad \text{with} \quad \overline{(a_i|N)\hat{\eta}_i} = 0 \tag{3}$$

and in this case $\hat{\eta}$ represent the intrinsic instrument noise. We assume that, knowing the output of n different accelerometers, the maximum reduction of Newtonian noise could be obtained by constructing a "subtracted signal" $H_S$ that can be written as

$$H_S(t) = H(t) - \sum_i \int_{-\infty}^{t} w_i(t-t')(N|a_i)A_i(t')dt' \tag{4}$$

or, in the frequency domain,

$$\tilde{H}_S = \tilde{H} - \sum_i w_i(\omega)(N|a_i)\tilde{A}_i = \tilde{H} - \sum_i w_i(\omega)(N|a_i)\left((a_i|B) + \tilde{\eta}_i\right) . \tag{5}$$

Here $w_i(\omega)$ are a set of functions (one for each accelerometer) which must be calculated. The factor $(N|a_i)$ is not a stochastic process, and was introduced for convenience. It represent the coupling of the fluctuations measured by a given accelerometer to the Newtonian noise, and depends only on the interferometer geometry and on the accelerometer position relative to it. With its aid Eq. (5) suggest that what we are doing is to subtract from $H$ the Newtonian noise signal due to the density fluctuation fraction controlled by our acceleration measurements.

It is also evident that in the limit of an infinite number of accelerometers it is always possible to write $(N|B) = \sum_i w_i(N|a_i)(a_i|B)$, so that in this case the subtraction of Newtonian noise should be complete in absence of instrumental noise.

If the noise is stationary, and if the processes $(a_i|B)$ are Gaussian the subtraction procedure (5) is optimal. The $\omega$ dependency of the weights $w_i$ is due to the fact that the optimal signal to subtract must encode the information about seismic dynamics, and this means that it must have memory.

In order to determine the functions $w_i$ we have to fix a well definite quantity that we want to minimize. In the stationary case it is natural to use the frequency integral of the interferometer output power spectrum density, weighted with a function $g(\omega)$ which select the frequency band of interest,

$$\Gamma[w_1, \cdots, w_n] = \int g(\omega)\overline{|H_S|^2}d\omega . \tag{6}$$

As indicated, this is a (quadratic) functional of the unknown functions $w_i(\omega)$. In order to simplify our expression we specialize now to the case $g(\omega) = \delta(\omega - \omega_0)$, so that the expression to be minimized is

$$\Gamma = f_i f_j^\star (C_{ij} + R_{ij}) w_i(\omega_0) w_j^\star(\omega_0) - f_i^\star (N_i + Z_i) w_i^\star(\omega_0) - f_i (N_i + Z_i)^\star w_i(\omega_0) . \tag{7}$$

A summation over repeated index, which label the accelerometer, is understood. Each quantity which appear in this expression admit a simple interpretation. The factor $f_i = (N|a_i)$ has been described yet. The array $C_{ij} = \overline{(a_i|B)(B|a_j)}$ is simply the statistical correlation between the output of $i$-th and $j$-th accelerometer, in absence of instrumental noise. To obtain the correlation between the output of two real instruments we must add the $R_{ij} = \overline{\eta_i \eta_j^\star}$ array, which is the correlation of the intrinsic noises of accelerometer $i$ and $j$.

The vector $N_i = \overline{(N|B)(B|a_i)}$ is the statistical correlation between the fraction of the interferometer output and of the $i$-th accelerometer output of seismic origin. Finally the vector $Z_i = \overline{\hat{H}\eta_i^\star}$ is the correlation between the intrinsic noise of the instrument $i$ and the fraction of the interferometer output uncorrelated with the seism.

Minimizing $\Gamma$ we find easily the optimal weights

$$w_i^{opt}(\omega_0) = f_i^{-1}(C_{ij} + R_{ij})^{-1}(N_i + Z_i)^\star \tag{8}$$

and the reduction of noise power spectrum in $\omega_0$

$$|H|^2 - |H_S|^2 = (C_{ij} + R_{ij})^{-1}(N_i + Z_i)(N_j + Z_j)^\star \tag{9}$$

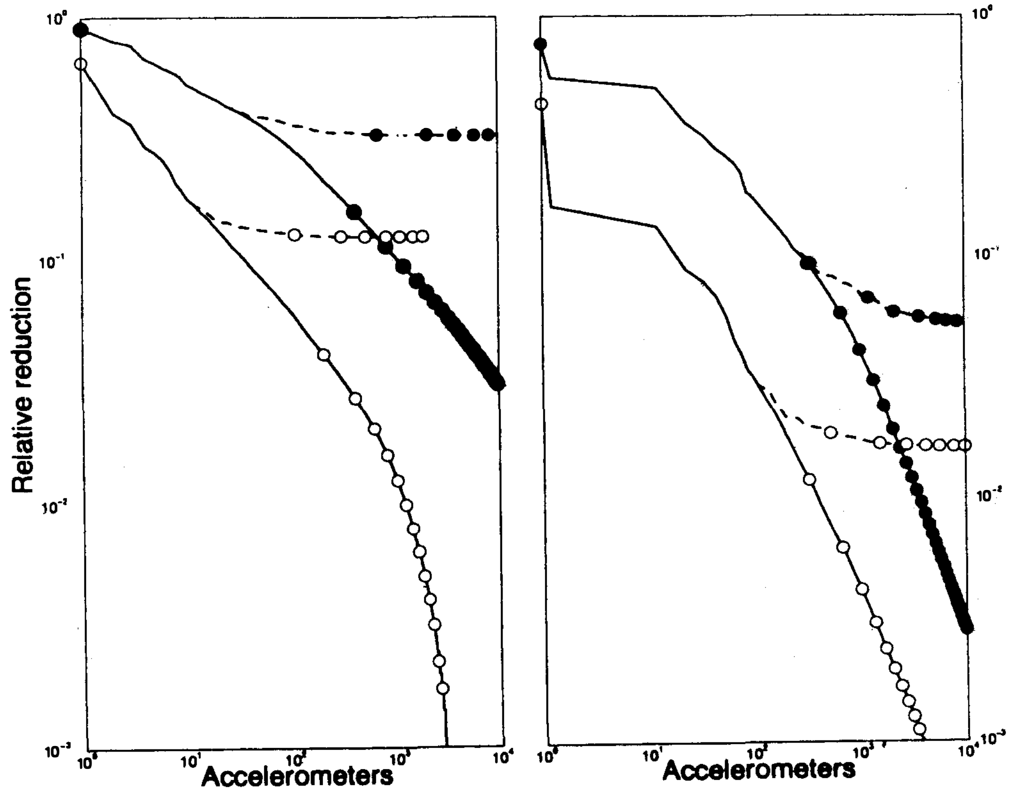


Figure 1: Relative reduction of noise amplitude for Saulson's model (left) and coherent model (right). The empty symbols are for $f_0 = 0.1$ Hz, the filled ones for $f_0 = 1$ Hz. The plots with dotted lines was obtained by constraining the accelerometers to the surface.

This last expression is a nonlinear function of the position and orientation of each accelerometers. Our objective is to optimize these parameters for a given number of instruments, to understand what is the best result we can achieve.

The quantities $C_{ij}$ and $N_i$ depends on the properties of the seismic noise, which are connected in turn to the dynamic of the background motion. While it is relatively easy to measure $C_{ij}$, to obtain experimental informations on $N_i$ we need the sensitivity of the complete interferometer. However both $C_{ij}$ and $N_i$ can be calculated starting from some theoretical model of the ground seismic motion. Some of these calculations can be found in [3], and a detailed account will be published elsewhere [7,8]. For what concern the terms connected with the instrumental noise we note that the non–diagonal part of the $R_{ij}$ matrix and the vector $Z_i$ should be negligible.

An intuitive understanding of the result of the optimization procedure can be obtained by looking at Eq.(9). In order to maximize this expression we can try to move all the accelerometer to the position which are maximally coupled to Newtonian noise. In this way the $N_i$ terms grows. However the accelerometers cannot become too near each other, because in this case the $C_{ij}$ term grows. The optimal configuration is obtained balancing these two factors. Note that if the intrinsic noise $R_{ij}$ is big the $C_{ij}$ term is less important, and all the accelerometers will try to cluster in similar positions and orientations, in such a way to improve statistic.

## 3 Some results.

We studied Eq. (9) numerically, finding the optimal configurations for a fixed number of accelerometers. The starting point is the knowledge of $C_{ij}$ and $N_i$ in function of the positions of accelerometers. In Figure (1) we plot the results obtained for two simple models.

The first is the model used by Saulson to obtain its early estimate of Newtonian noise [1]. Its main assumption is that the seismic motion can be modelized by a partition of the ground in cubic cells which oscillate coherently, but completely uncorrelated each other. The second is a slight modification of the Saulson's model (which we will call coherent model) which takes into account the mass conservation. This from our point of view means that the motions of nearest cells are no more uncorrelated [3].

A first observation is that the subtraction procedure is less effective at higher frequency. This fact has a simple explanation, because the coherence length of the seismic motion (the cell dimension of the Saulson's model) is proportional to the inverse of the frequency. In order to control at some specified level a given volume we need a number of accelerometers which grows as the third power of the frequency.

Another point is that we can use only accelerometers located on the surface without worsening the overall performances of our procedure, if the number of accelerometers is not big (less than 40 for the Saulson's model). Note that the importance of underground measurements is bigger for the Saulson's model. This is a consequence of the fact that in this case surface measurements give us no information at all on the dynamics below the level of the first cell.

The performances of the subtraction procedure are not exceptional: for a reasonable number of accelerometer we can expect a relative reduction of an order of magnitude. We can see that a more coherent seismic dynamic improve the result, and that the two simple models we have used probably underestimate the coherence of the real dynamics. In addition they do not model the contributions to Newtonian noise due to the surface discontinuity, and it turns out that these are the most relevant ones.

We expect to obtain better results with more realistic models. We have evaluated $C_{ij}$ and $N_i$ for the elastic wave model used in [3] to predict the Newtonian noise amplitude and extensive numerical simulations are currently in progress. However we do not expect that a relative reduction of Newtonian noise amplitude greater than two orders of magnitude could be obtained.

## 4 Conclusions and perspectives.

The estimation of the feasibility of an off-line subtraction of Newtonian noise is based on the modelization of the seismic dynamic. From a practical point of view an important issue is to understand what are the effect of a wrong modelization on the performance of the subtraction procedure. The functions $w_i$ optimized for a given model can give poor performances if used in a different situation. Some effects are not easy to model, in particular the dissipative nature of the terrain and the seismic wave scattering generated by the presence of inhomogeneities. Both could be relevant, as they alter the coherence length of seismic motion and can couple otherwise independent normal modes.

A particular model can be a good approximation only in peculiar conditions, for example only on a particular frequency band, or in a particular atmospheric condition, or when the level of human activity is low. The validation of a model is a very important point, that can be achieved with an extensive set of seismic correlation measurement.

Another possible approach is that of a "model-independent" subtraction. The main point is the optimization of the functions $w_i$ by the direct experimental estimation and minimization of the Newtonian noise power spectrum. This can be seen as the training problem for a M–

adaline network with delays, and can be solved with standard adaptive techniques. Numerical simulations are in progress to test the effectiveness of this method. Note that experimentally it is easy to adapt the functions $w_i$, but it is not easy to adapt the positions and the orientations of the accelerometers. This means that a good theoretical model is in any case important in order to guess a good configuration for the instruments.

Our method for the determination of the optimal subtraction procedure is based on the assumption that the stochastic processes we are interested to are stationary. If this is not true the main point is the precise determination of the quantity we are interested to minimize, as the noise power spectrum is no more a useful concept. In some simple cases our formalism requires only minor modifications, for example the redefinition of the $\overline{\cdots}$ average. Further investigations are needed.

As a final comment we stress that our method is independent on the particular instrument used to monitor density fluctuations. We have used accelerometers, but there are a lot of alternative possibilities which can be investigated using the same formalism. A good instrument has a strong overlap $N_i$ with the seismic modes which are maximally coupled to Newtonian noise, and an accurate choice can allow us to obtain major improvements.

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